Huggett (1996) - Wealth Distribution in Life-cycle Economies

Huggett (1996) explores the ability of a consumption-savings life-cycle model to generate a realistic wealth distribution. He shows that uncertain lifespan and incomes shocks are key to getting the right general form of the wealth distribution, but are unable to get the large shares of the very top percentiles of the wealth distribution. He notes that the model seems capable of generating the low wealth levels of the bottom of the distribution (due to pensions, which are not too tightly linked to income). Transfers (inheritance) seem to play a minor role, but that may be as they are not as narrowly focused on just one part of the population (they are spread lump-sum across the whole population).

In terms of replication the only subtlety was that the definition of the exogenous shock transitions was non-standard and so had to be implemented specifically. I had originally misread the exogenous shock as being 19 states, 18 with an 'extra', but in fact the 'extra' is intended to be understood as part of the 18. One thing not clear from paper is what is the 'unnormalized' deterministic earnings profile, since Figure 1 is the normalized earnings, and these need to be multiplied by 0.5289; without this I was unable to replicate Figure 2, with this Figure 2 replicates exactly.[1] Figure 1 of Huggett (1996) is technically computational, but since it just shows some of the calibrated parameter values I omit it as it trivially replicates (it is generated by the replication codes for any interested reader).

The model is a general equilibrium OLG model. The finite-horizon value function problem has one exogenous state (an earnings shock), one endogenous state (assets), and 79 periods. The household value function problem is given by

$$\begin{aligned} V(a,z,j) = &\max_{c,a'} \frac{c^{1-\sigma}}{1-\sigma} + \beta s_{j+1} E_j[V(a',z',j)|z] \\ &\text{subject to } c + a' \leq a(1+r(1-\tau)) + (1-\tau-\theta)e(z,j)w + T + b_j \\ &\qquad a' \geq \underline{\mathrm{a}} \end{aligned}$$

There are $J = 79$ periods and $V(a,z,J+1) = 0$ for all a & z. So household faces income shocks ($e(z,j)$) and solve a consumption-savings problem of choosing consumption c and next period assets a'. There are some basic taxes τ and θ, the later of which is used to fund pensions b_j that are received once retirement age is reached. When people die their assets are redistributed lump-sum accoss the living as T.[2] The lower bound on assets $\underline{\mathrm{a}}$ is either $-w$ or 0 depending on the calibration.

The earnings process $e(z,j)$ consists of a (log) deterministic earnings profile $\bar{y}_j$ which is multiplied by a (log) AR(1) transitory shock, (log of) z_j: so, $e(z,j) = exp(\bar{y}_j + z_j)$, and $z_j = \gamma z_{j-1} + \epsilon$. The ϵ shock has 17 states equally spaced from -4 to +4 standard deviations, with one extra (18th) state at +6 standard deviations; it is normally distributed so transition probabilities can be easily computed by quadrature (same principle as used by Tauchen method).

The initial distribution of the shocks on the z-grid is given by quadrature using σ_{z_1} as the standard deviation of z_1. All households start with zero assets. The simulation of the agents distribution is standard, based on optimal policy function and idiosyncratic shocks.

[1] A friend passed me an old Homework handout by Dean Corbae from 2014 which involved a slightly simplified version of this model, and which describe this 0.5289 and included files containing the deterministic earnings profile. The 0.5289 is related to the share of the population of working age and is used to normalize the model so that average earnings across the whole population are equal to 1. It is not essential in any way to solving the model but means that certain model moments, namely aggregate earnings, are known without computing them.

[2] Paper states the additional constraint that $a' \geq 0$ for $j = J$, but is clear from Figure 2 that this was not actually imposed in code.

The model has three general equilibrium constraints, the first is that the interest rate r equals the marginal product of capital minus the depreciation rate δ (Huggett (1996) considers this in terms of a general equilibrium condition for K/Y, rather than interest rates, but the two are mathematically equivalent). The second is on the benefit rate for pensions b, which must equal the revenue raised by the payroll tax θ. The third is that the the (total across the population of the) lump-sum transfer of accidental bequests T much equal the assets left behind by people on dying.

For more details on the model see Huggett (1996), or the example code at github.com/vfitoolkit/VFItoolkit-matlab-examples (this example is provided in addition to the replication codes, and is intended to show how to use the VFI Toolkit to solve this kind of model).

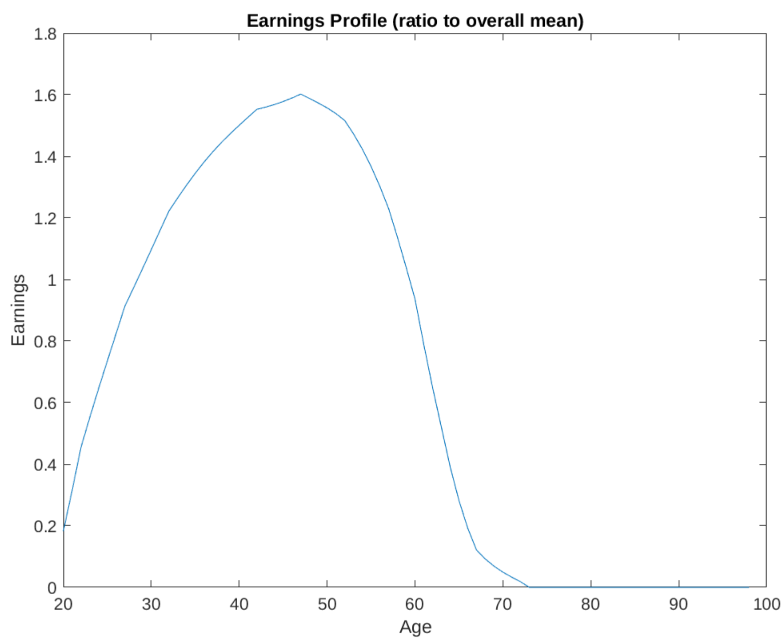
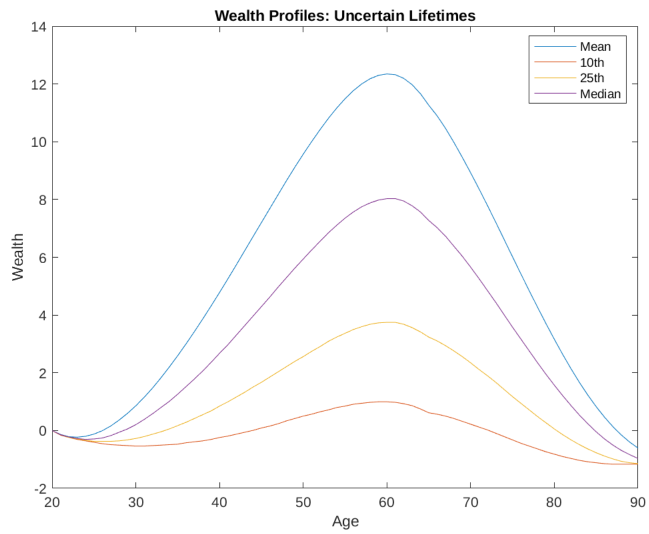
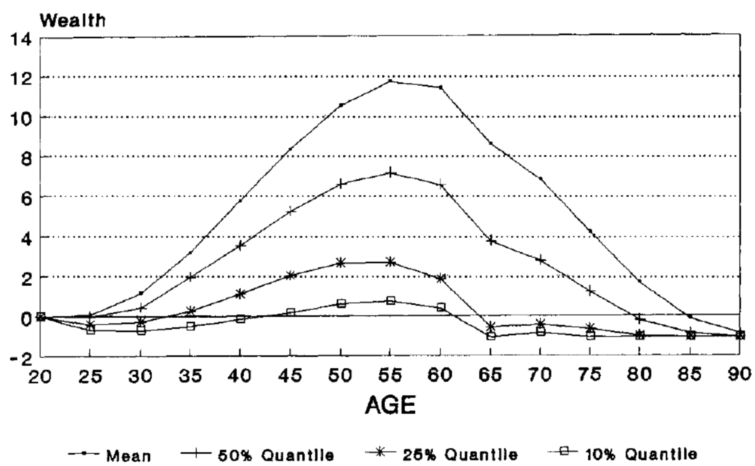


Figure 1: Figure 1 of Huggett (1996)

References

Mark Huggett. Wealth distribution in life-cycle economies. Journal of Monetary Economics, 38: 469–494, 1996.

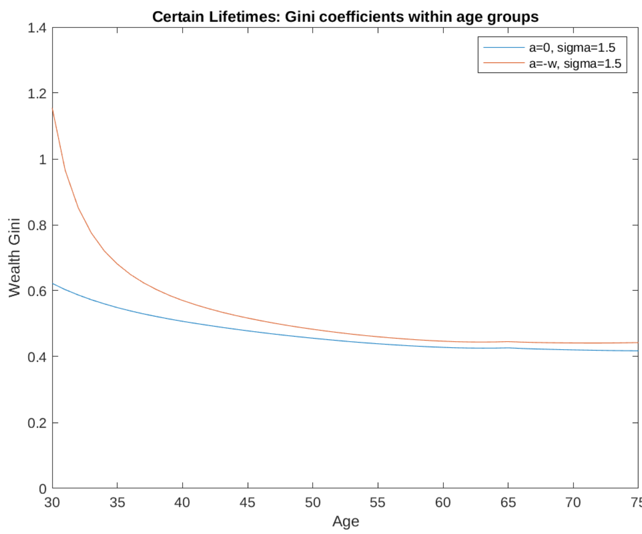




Uncertain Lifetimes

Fig. 2. Wealth profiles.

Figure 2: Figure 2 of Huggett (1996)



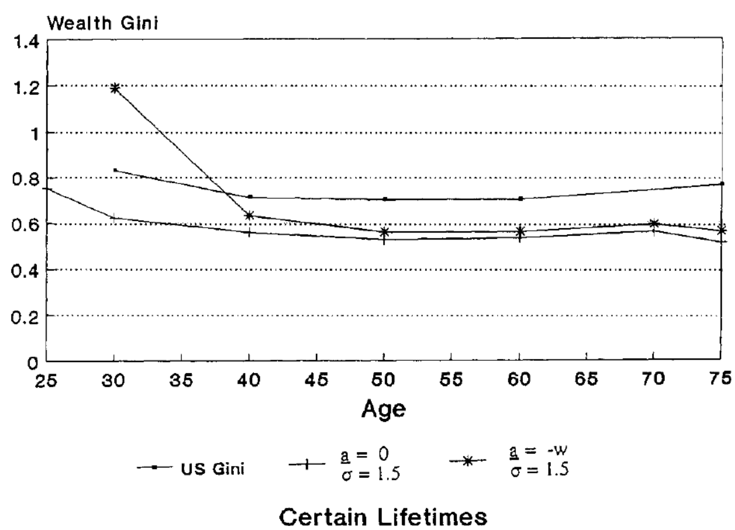
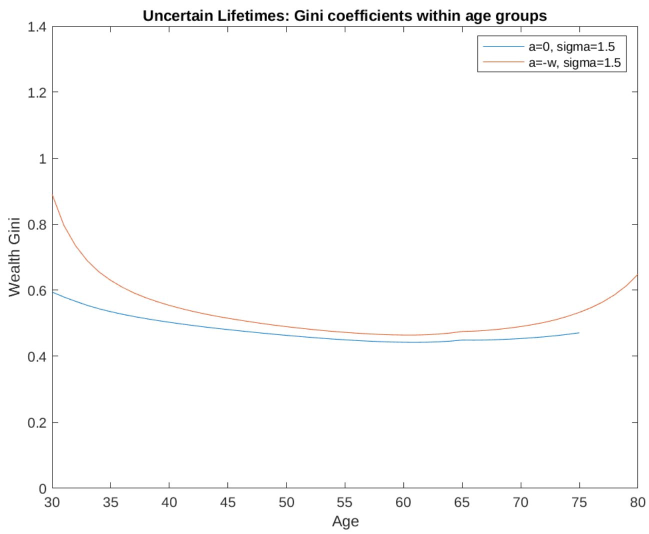


Fig. 4. Gini coefficients within age groups.

Figure 3: Figure 4 of Huggett (1996)



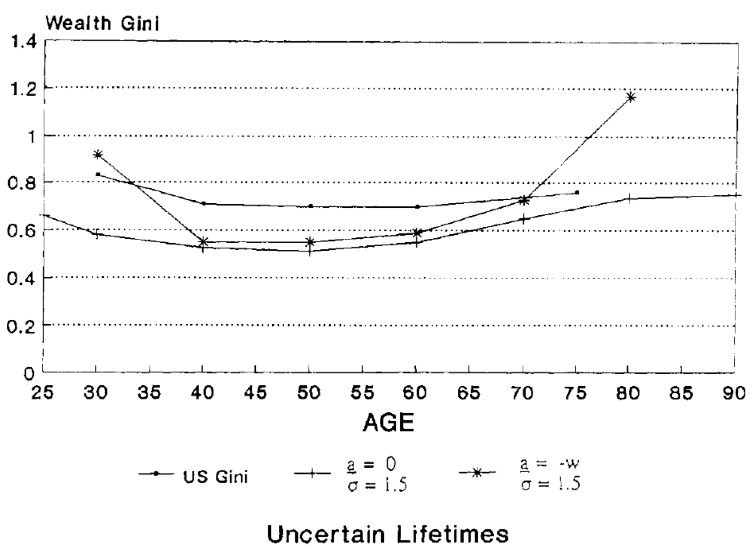


Fig. 5. Gini coefficients within age groups.

Figure 4: Figure 5 of Huggett (1996)

Table 1: Table 3 of Huggett (1996)

Wealth Distribution (risk aversion coefficient $\sigma = 1.5$)

Credit limit $\underline{a}$	Earnings shock σ_ϵ^2	K/Y	Transfer wealth ration	Wealth Gini	Percentage wealth in the top 1%	5%	20 %	Zero or negative wealth (%)
US Economy		3.0	0.78–1.32	0.72	28	49	75	5.8–15.0
Certain Lifetimes								
0	0.000	4.1	-0.00	0.64	8.3	26.6	63.4	16.1
-w	0.000	4.0	-0.00	0.70	9.3	29.0	68.0	29.7
0	0.045	4.2	-0.00	0.66	9.1	28.8	65.9	15.7
-w	0.045	4.1	0.00	0.70	9.7	30.5	69.2	28.1
Uncertain Lifetimes								
0	0.000	4.0	0.43	0.62	8.1	25.9	61.6	13.9
-w	0.000	3.9	0.39	0.68	8.9	28.2	66.2	26.4
0	0.045	4.1	0.40	0.65	9.2	29.0	65.6	14.6
-w	0.045	4.0	0.36	0.70	9.9	30.9	69.3	26.8

Note: Based on baseline grid size for assets of 2001, and shocks of 18.

Figure 5: Original Table 3 of Huggett (1996)

Table 3
Wealth distribution (risk aversion coefficient $\sigma = 1.5$)

Credit limit $\underline{a}$	Earnings shock σ_ε^2	K/Y	Transfer wealth ratio	Wealth Gini	Percentage wealth in the top 1%	5%	20%	Zero or negative wealth (%)
US economy		3.0	0.78–1.32	0.72	28	49	75	5.8–15.0
Certain lifetimes								
0.0	0.00	2.9	0.0	0.47	2.4	11.6	42.8	14.0
$-w$	0.00	2.8	0.0	0.54	2.7	12.7	46.6	25.0
0.0	0.045	3.2	0.0	0.70	10.8	32.4	68.9	19.0
$-w$	0.045	3.1	0.0	0.74	11.1	33.8	72.3	24.0
Uncertain lifetimes								
0.0	0.00	3.1	1.03	0.46	2.5	11.7	42.8	11.0
$-w$	0.00	3.0	1.07	0.49	2.6	12.1	44.3	12.0
0.0	0.045	3.4	0.84	0.69	10.9	32.9	70.0	17.0
$-w$	0.045	3.2	0.89	0.76	11.8	35.6	75.5	24.0

Table 2: Table 4 of Huggett (1996)

Wealth Distribution (risk aversion coefficient $\sigma = 3.0$)								
Credit limit $\underline{a}$	Earnings shock σ_ϵ^2	K/Y	Transfer wealth ration	Wealth Gini	Percentage wealth in the top 1%	5%	20 %	Zero or negative wealth (%)
US Economy		3.0	0.78–1.32	0.72	28	49	75	5.8–15.0
Certain Lifetimes								
0	0.000	3.3	-0.00	0.66	8.8	27.9	65.9	20.7
-w	0.000	3.2	-0.00	0.79	10.6	32.8	74.9	38.5
0	0.045	3.6	-0.00	0.65	9.1	28.8	65.7	14.2
-w	0.045	3.5	-0.00	0.69	9.6	30.1	68.3	25.1
Uncertain Lifetimes								
0	0.000	3.2	0.72	0.64	8.5	27.1	64.2	18.4
-w	0.000	3.1	0.77	0.77	10.1	31.5	72.3	35.8
0	0.045	3.4	0.54	0.65	9.1	28.8	65.6	14.3
-w	0.045	3.3	0.52	0.69	9.6	30.3	68.3	25.3

Note: Based on baseline grid size for assets of 2001, and shocks of 18.

Figure 6: Original Table 4 of Huggett (1996)

Table 4
Wealth distribution (risk aversion coefficient $\sigma = 3.0$)

Credit limit $\underline{a}$	Earnings shock σ_ε^2	K/Y	Transfer wealth ratio	Wealth Gini	Percentage wealth in the top 1%	5%	20%	Zero or negative wealth (%)
US economy		3.0	0.78–1.32	0.72	28	49	75	5.8–15.0
Certain lifetimes								
0.0	0.00	2.3	0.0	0.51	2.7	13.0	46.1	21.0
$-w$	0.00	2.0	0.0	0.62	3.3	14.7	52.5	29.0
0.0	0.045	2.9	0.0	0.66	10.5	32.0	66.6	3.0
$-w$	0.045	2.8	0.0	0.73	11.4	34.0	73.1	23.0
Uncertain lifetimes								
0.0	0.00	2.5	2.54	0.50	2.6	12.6	45.3	21.0
$-w$	0.00	2.3	4.30	0.61	3.1	14.3	51.1	29.0
0.0	0.045	3.0	1.28	0.72	12.1	35.7	71.7	19.0
$-w$	0.045	2.8	1.75	0.84	13.8	40.4	80.2	40.0