

This is a replication of Kulish, Kent, and Smith (2010). Model is a deterministic OLG. Main complexity is the need to deal with changes in the finite-time horizon (number of periods that agents live for). Solves for a few transition paths.

In Figure 10, the middle panel of KKS2010 appears to be incorrect. By definition the 'no health improvement' scenario is that the $v(j, J)$ ($v(s, T)$ in KKS2010 notation) is unchanged, and so the leisure decisions should remain unchanged. Unless I have misunderstood what 'no health improvement' is supposed to mean.

I get quite a different Figure 6 to KKS2010. My understanding is that this should be a plot of Reform 1. In Reform 1 (the 'Baby Bust') according to Table 3 of KKS2010 the growth rate of the age 1 cohort falls instantly to zero. To my mind this fits with what I have in my replication version of Figure 6 as the total population then gradually falls until settling into a new lower level (as seen in the 'growth rate of L'). By contrast there is still positive growth in the total population for the next 20 years according to the 'growth rate of L' in the original Figure 6. I can imagine this happening if, e.g., the growth rate went from its initial 1.2% (n_1 in Table 3) to the value of 0% (n_2 in Table 3) linearly over the first 20 years, but this is not how I understand what the paper says happens in the reform which is instead that we go immediately to n_2 and stay there for 20 years.¹

Figure 7 of KKS2010 is smooth at the 'second' reveal after 20 periods in Reforms 3 and 4 while mine jumps slightly because the change drives a jump in leisure.

For more details on the model see Kulish, Kent, and Smith (2010). For the codes implementing the replication see:

<https://github.com/vfitoolkit/vfitoolkit-matlab-replication>

References

Mariano Kulish, Christopher Kent, and Kathryn Smith. Aging, retirement, and savings: A general equilibrium analysis. *The B.E. Journal of Macroeconomics*, 10(1), 2010. doi: <https://doi.org/10.2202/1935-1690.1808>.

¹Mariano Kulish was kind enough to send me his original codes. Based on lines 116 to 122 of his 'GG.m' my reading is that the interpretation of the reform as a jump (and then possibly second jump) is correct. Note that these lines relate to the initial guess for a path on aggregate capital, not to the actual transition path so it is not entirely clear.

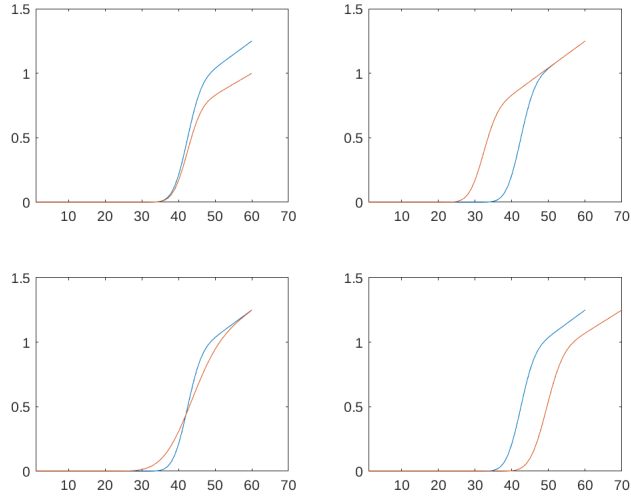


Figure 1: Time Varying Weight on Leisure, $v(s, T)$

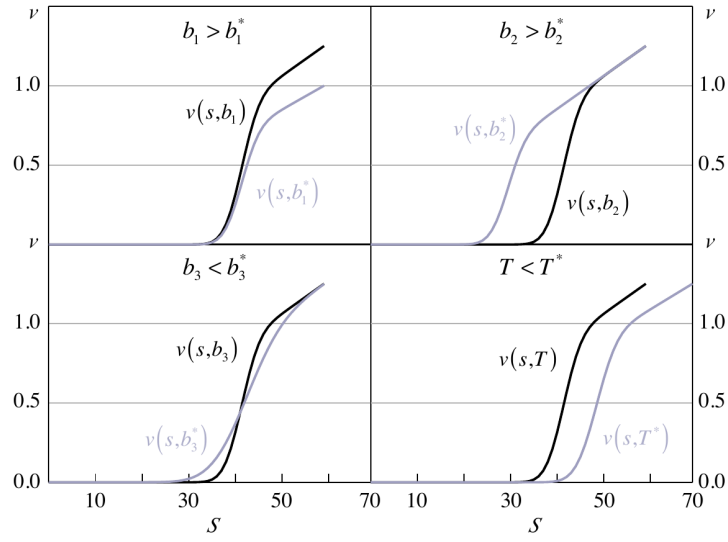


Figure 1: Figure 1 of Kulish, Kent and Smith (2010)

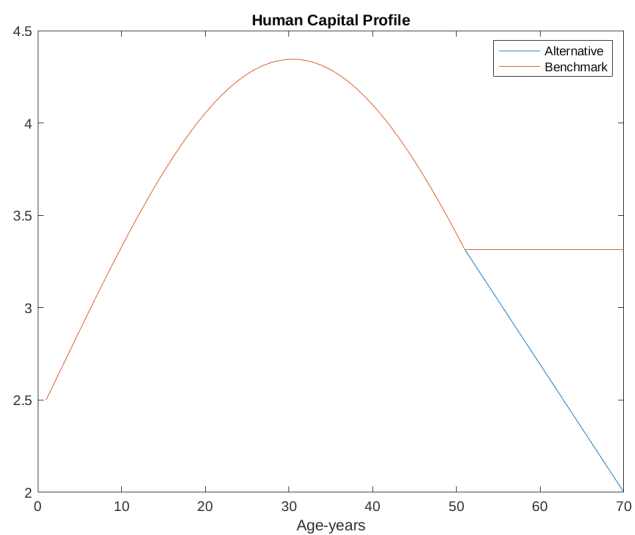
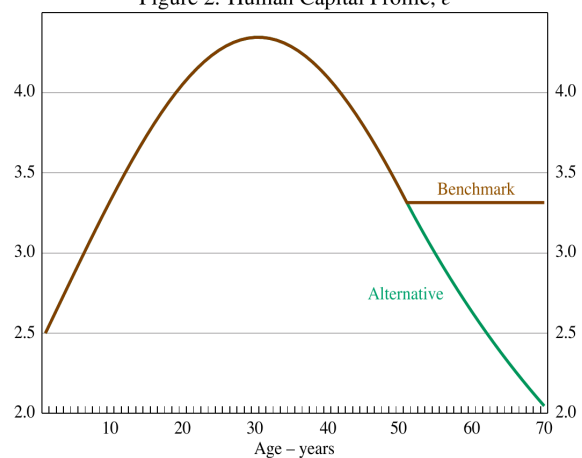


Figure 2: Human Capital Profile, e



Notes: In all figures, unless otherwise specified, the unit of measurement is in terms of the numeraire good (that is, output).

Figure 2: Figure 2 of Kulish, Kent and Smith (2010)

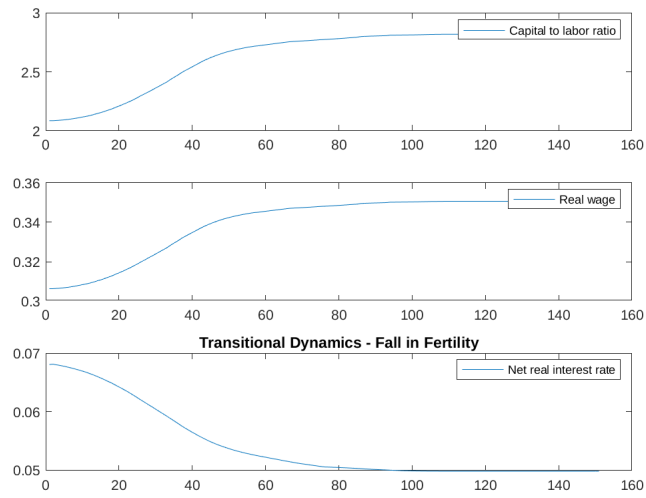


Figure 4: Transitional Dynamics – Fall in Fertility

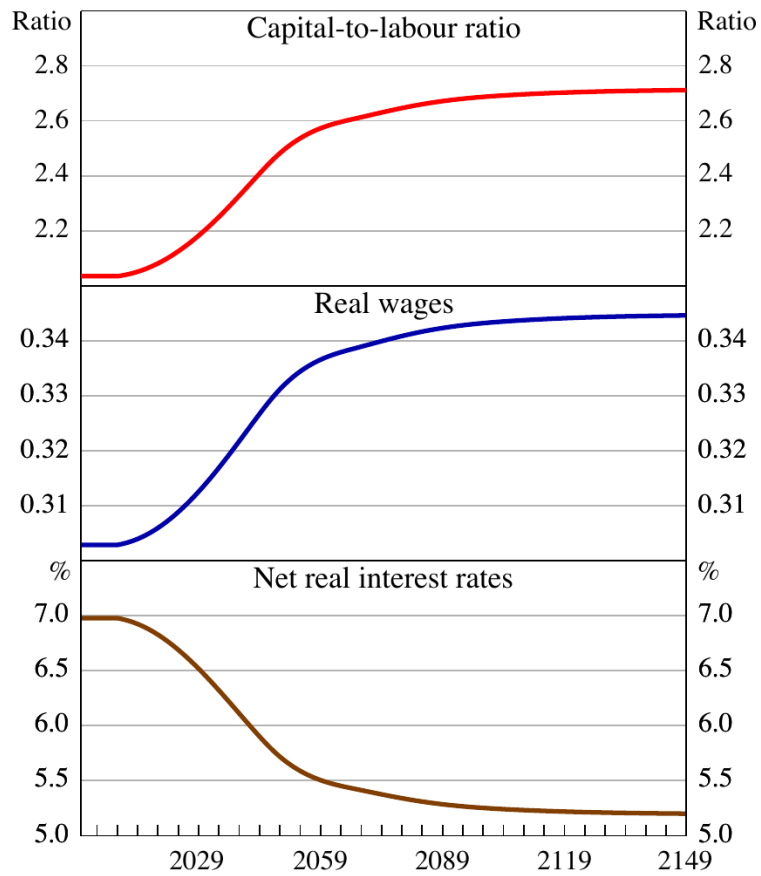


Figure 3: Figure 4 of Kulish, Kent and Smith (2010)

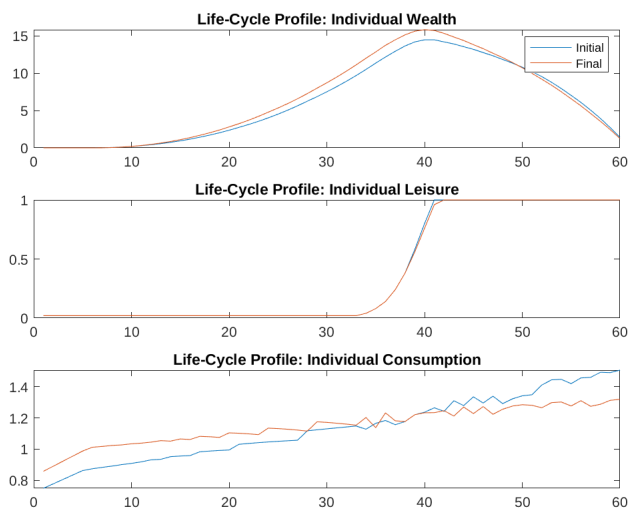


Figure 5: Steady-State Comparisons
Profiles before and after a fall in fertility

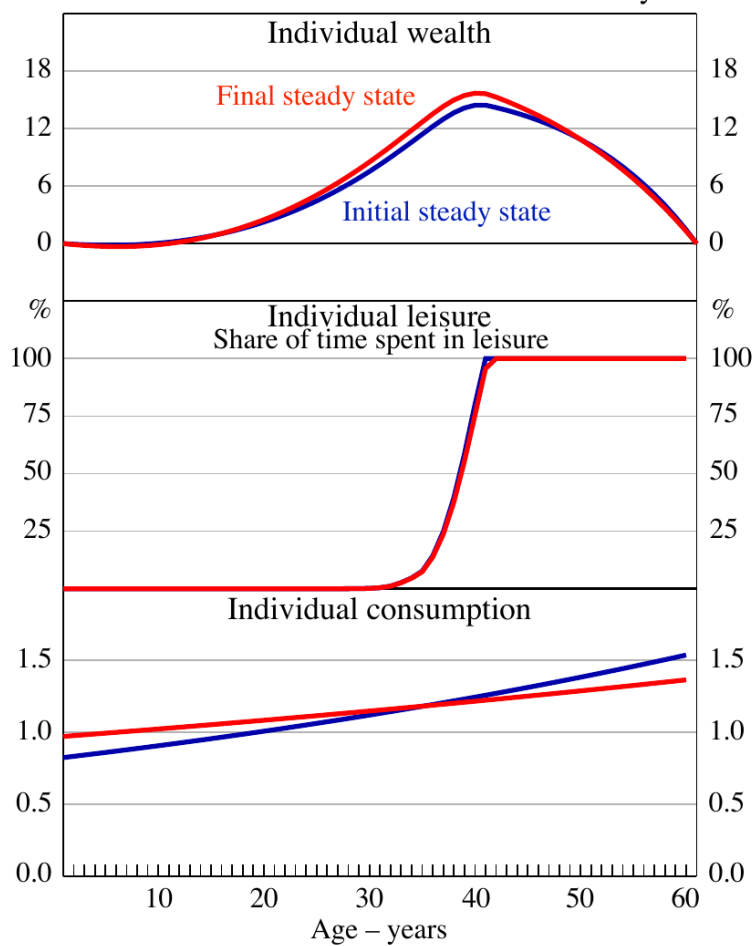


Figure 4: Figure 5 of Kulish, Kent and Smith (2010)

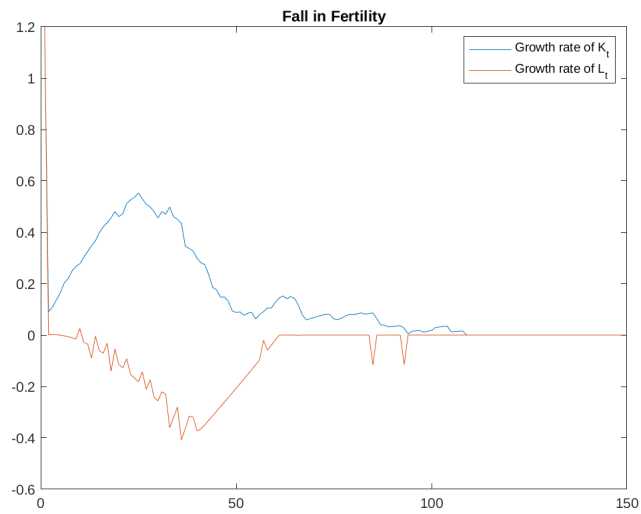


Figure 6: Fall in Fertility

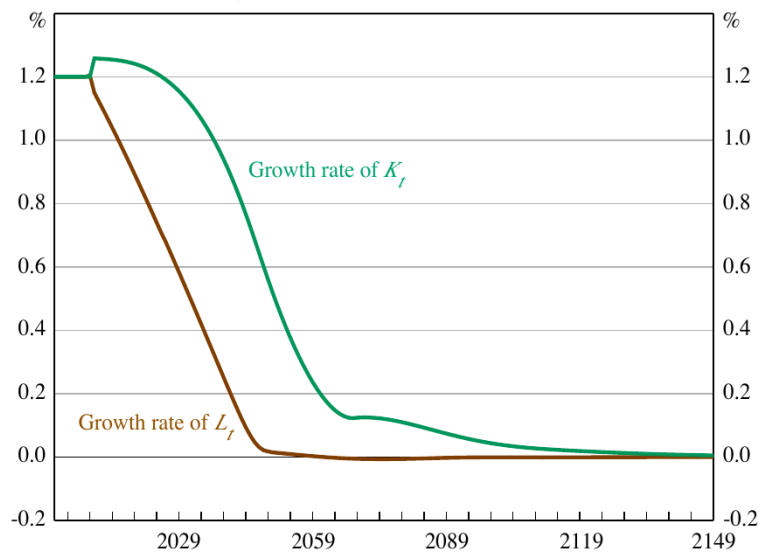


Figure 5: Figure 6 of Kulish, Kent and Smith (2010)

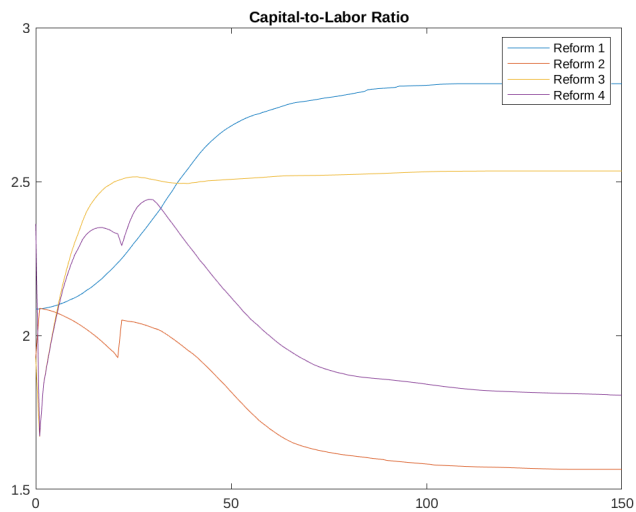


Figure 7: Capital-to-Labor Ratio – Simulations 4.1, 4.2, 4.3 and 4.4

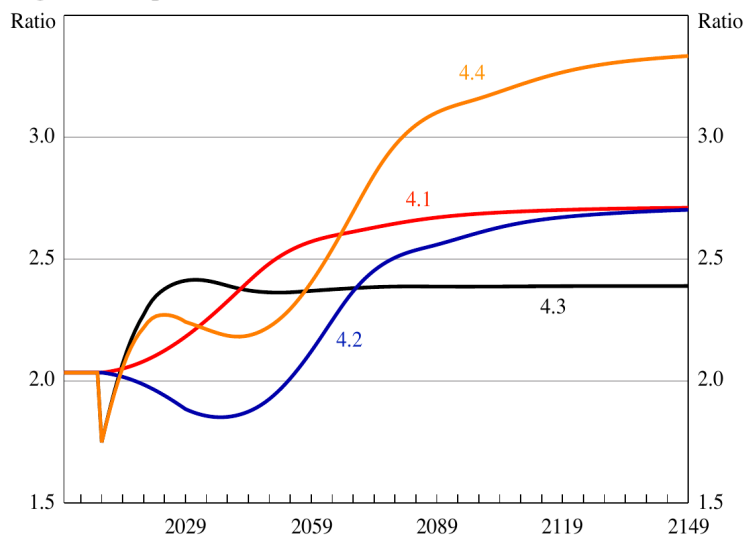


Figure 6: Figure 7 of Kulish, Kent and Smith (2010)

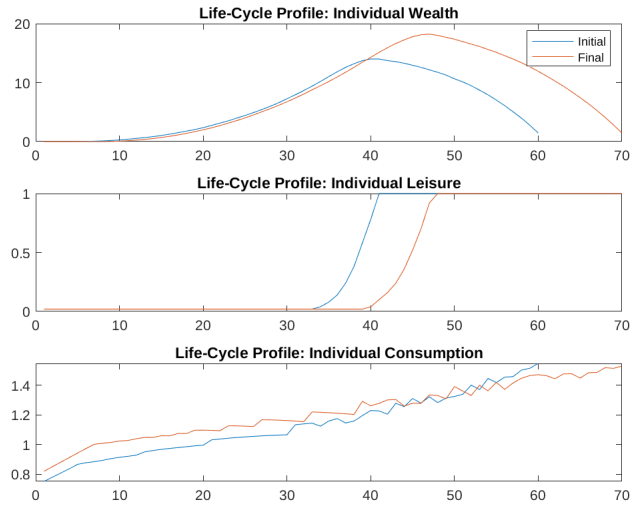
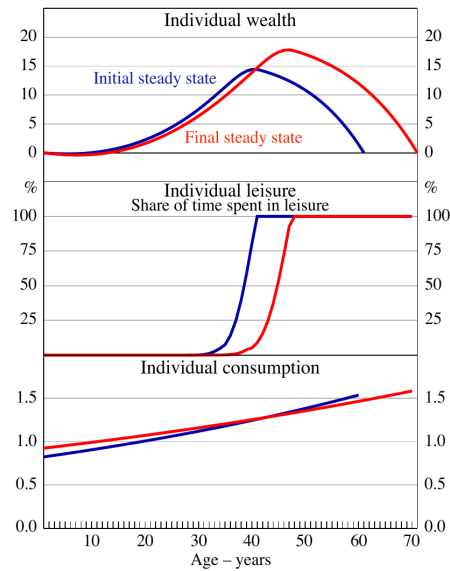


Figure 8: Steady-State Comparisons
Profiles before and after a rise in longevity



Note: The last period shown is 71 years when (under the higher longevity scenario) the person has died and has no wealth left.

Figure 7: Figure 8 of Kulish, Kent and Smith (2010)

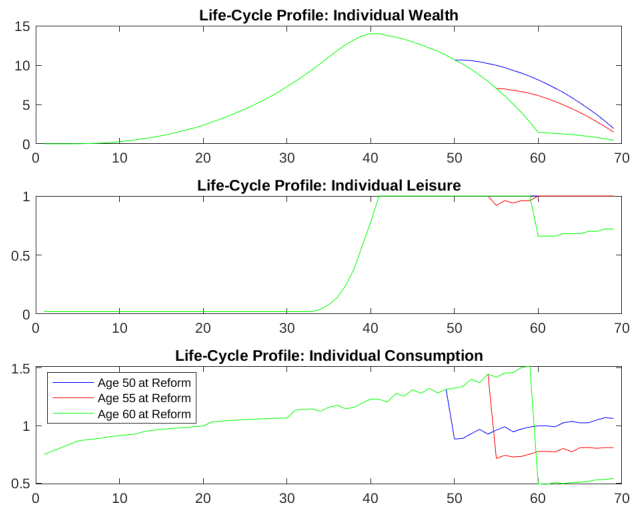


Figure 9: Leisure and Consumption Profiles with Increased Longevity for Agents Aged 50, 55 and 60 at Time of Change

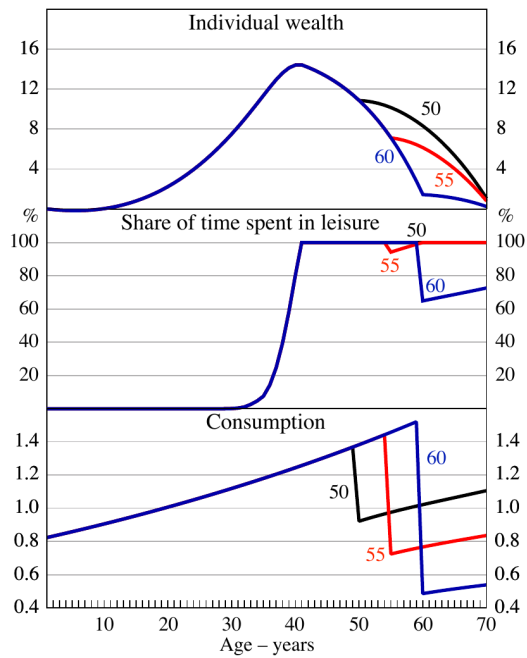


Figure 8: Figure 9 of Kulish, Kent and Smith (2010)

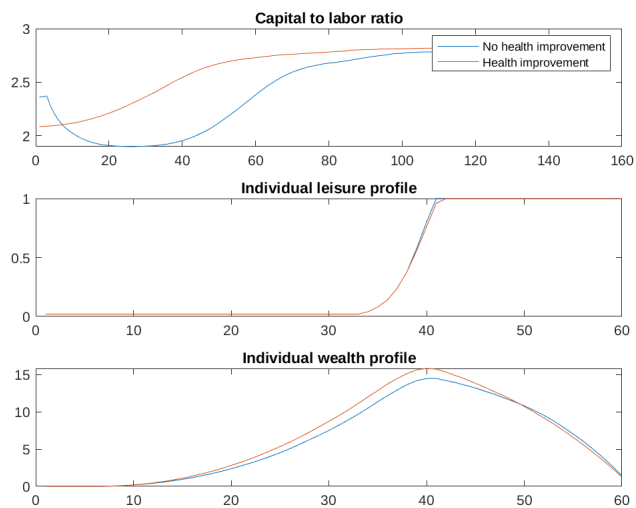


Figure 10: A Rise in Longevity (10 Years) – with and without Better Health

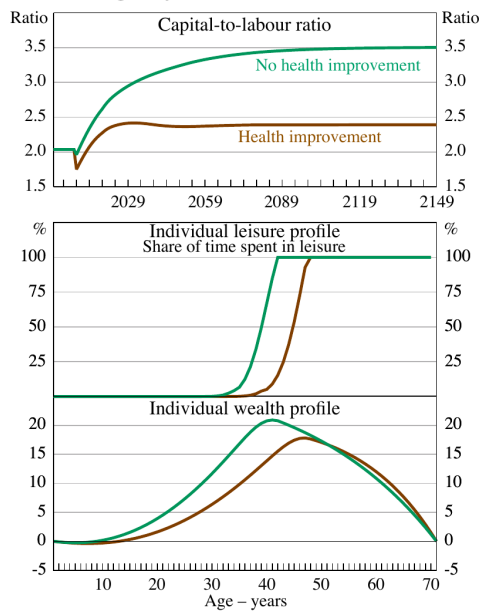


Figure 9: Figure 10 of Kulish, Kent and Smith (2010)

Figure 10: Table 1 of Kulish, Kent and Smith (2010)

Variable	Description	Value
T (years)	Life expectancy	60
n	Cohort growth rate	0.012
e_j	Human capital profile	See Figure 1
b_1	Parameter of $v(j, T)$	3.0
b_2	Parameter of $v(j, T)$	0.7
b_3	Parameter of $v(j, T)$	0.03
β	Human capital profile	0.97
ρ	Human capital profile	3.5
A	Human capital profile	0.4
$1 - \alpha$	Human capital profile	0.55
δ	Human capital profile	0.052

Figure 11: Original Table 1 of Kulish, Kent and Smith (2010)

Table 1: Calibration – Initial Steady State

Variable	Description	Value
T (years)	Life expectancy	60
n	Cohort growth rate	0.012
e_s	Human capital profile	See Figure 2
b_1	Parameter of $v(s, T)$	3.0
b_2	Parameter of $v(s, T)$	0.7
b_3	Parameter of $v(s, T)$	0.03
β	Discount factor	0.97
ρ	Utility function parameter	3.5
A	Total factor productivity	0.4
$1 - \alpha$	Labour's share of income	0.55
δ	Depreciation rate	0.052

Note: See Appendix B for detailed data sources and methods.

Figure 12: Table 4 of Kulish, Kent and Smith (2010)

n (percent)	T (years)				
	60	65	70	75	80
2.4	2.09	1.83	1.56	1.37	1.24
1.2	2.09	1.83	1.56	1.37	1.24
0.0	2.09	1.83	1.56	1.37	1.24
-1.2	2.09	1.83	1.56	1.37	1.24

Figure 13: Original Table 4 of Kulish, Kent and Smith (2010)

Table 4: Capital Intensity (k) in Various Steady States

n (per cent)	T (years)				
	60	65	70	75	80
2.4	1.55	1.65	1.75	1.85	1.94
1.2	2.04	2.21	2.39	2.60	2.77
0.0	2.71	3.03	3.36	3.73	4.12
-1.2	3.67	4.21	4.81	5.48	6.20

Figure 14: Table 5 of Kulish, Kent and Smith (2010)

n (percent)	T (years)				
	60	65	70	75	80
2.4	38.20	30.86	24.71	19.36	14.85
1.2	38.20	30.86	24.71	19.36	14.83
0.0	38.20	30.86	24.71	19.36	14.85
-1.2	38.20	30.86	24.71	19.36	14.85
-2.4	38.20	30.86	24.71	19.36	14.85

Figure 15: Original Table 5 of Kulish, Kent and Smith (2010)

Table 5: Share of Life Spent Working in Various Steady States

Per cent					
n (per cent)	T (years)				
	60	65	70	75	80
2.4	62.5	62.3	62.2	62.0	62.0
1.2	62.8	62.7	62.6	62.6	62.5
0.0	63.0	63.0	63.0	62.9	62.9
-1.2	63.2	63.2	63.2	63.2	63.2
-2.4	63.3	63.3	63.3	63.3	63.4

Figure 16: Table 6 of Kulish, Kent and Smith (2010)

n (percent)	T (years)				
	60	65	70	75	80
2.4	2.30	2.21	2.09	2.01	1.94
1.2	2.30	2.21	2.09	2.01	1.94
0.0	2.30	2.21	2.09	2.01	1.94
-1.2	2.30	2.21	2.09	2.01	1.94

Figure 17: Original Table 6 of Kulish, Kent and Smith (2010)

Table 6: Capital Intensity (k) in Various Steady States: $n_e = 0.02$

n (per cent)	T (years)				
	60	65	70	75	80
2.4	3.30	3.45	3.59	3.72	3.83
1.2	3.62	3.80	3.97	4.12	4.26
0.0	3.98	4.20	4.40	4.59	4.76
-1.2	4.40	4.67	4.91	5.15	5.35

Figure 18: Table 7 of Kulish, Kent and Smith (2010)

n (percent)	T (years)				
	60	65	70	75	80
2.4	34.37	26.98	20.71	15.36	10.62
1.2	34.37	26.98	20.71	15.36	10.62
0.0	34.37	26.98	20.71	15.36	10.62
-1.2	34.37	26.98	20.71	15.36	10.62

Figure 19: Original Table 7 of Kulish, Kent and Smith (2010)

Table 7: Share of Life Spent Working in Various Steady States: $n_e = 0.02$

n (per cent)	Per cent				
	T (years)				
	60	65	70	75	80
2.4	65.9	65.8	65.7	65.5	65.5
1.2	66.1	66.1	65.9	65.8	65.8
0.0	66.3	66.2	66.2	66.1	66.1
-1.2	66.5	66.4	66.5	66.4	66.4