Chapter 7: One sample tests

(Ott & Longnecker Sections: 10.2)

Part 3

https://dzwang91.github.io/stat324/

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Outline

1. Sign test

2. Tests for population proportion

Sign test

Definition

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- Collect an i.i.d. sample $X_1, \ldots, X_n$ from some population with population median m.
- **Step 1**:
  1. Two-tailed test:
  $$H_0 : m = m_0 \text{ v.s. } m \neq m_0$$
  2. Left-tailed test:
  $$H_0 : m = m_0 \text{ v.s. } m < m_0$$
  3. Right-tailed test:
  $$H_0 : m = m_0 \text{ v.s. } m > m_0$$

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- Under $H_0 : m = m_0$, $B \sim \text{Binomial}(n^*, 0.5)$.
- **Step 4**: calculate p-value. Let b be the observation of B.
  - $H_A : m > m_0$:

    $$\text{p-value} = P(B \geq b \mid B \sim \text{Binomial}(n^*, 0.5))$$

  - $H_A : m < m_0$:

    $$\text{p-value} = P(B \leq b \mid B \sim \text{Binomial}(n^*, 0.5))$$

  - $H_A : m \neq m_0$:

    $$\text{p-value} = 2 \times \min\{P(B \geq b), P(B \leq b)\}.$$

Example

The sanitation department in a large city is considering separating recyclable material out of the trash to save on landfill space and make money selling the recyclables. Based on data from other cities, it is determined that if at least half of the households in the city have more than 4.6 lbs of reclaimable recyclable material, then the separation will be profitable for the city. A random sample of 11 households yields the following data on pounds of recyclable material found in the trash:

14.2, 5.3, 2.9, 4.2, 1.2, 4.3, 1.1, 2.6, 6.7, 7.8, 25.9

Note that the median of the sample is 4.3 lbs.

Example continued

- $H_0 : m = 4.6$ vs. $H_A : m > 4.6$.
- test statistic: B=the number of observations in the sample that are greater than 4.6
- Under H_0, $B \sim Bin(11, 0.5)$.
- Observation of B:

  14.2, 5.3, 2.9, 4.2, 1.2, 4.3, 1.1, 2.6, 6.7, 7.8, 25.9

  so $B_{obs} = 5$.
- p-value$= P(B \geq 5) = 0.726$.
- Set $\alpha = 0.05$, then there is no sufficient evidence to reject the null.
- There is no strong evidence that separation of the recyclables would be profitable in this city.

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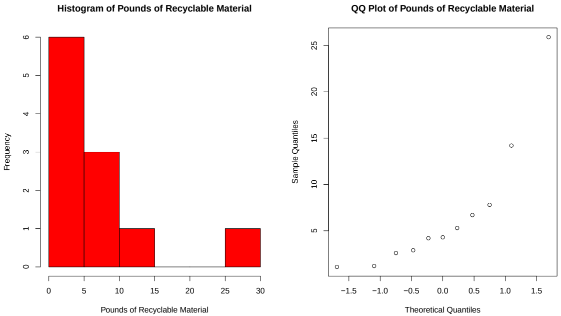
- Comparison between T test and sign test: T test requires the population distribution is normal, but sign test doesn't

Back to example

- Histogram and QQ plot:



The histogram looks neither normal nor symmetric, and the QQ plot also does not support normality. Therefore, we can not use T test.

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Tests for population proportion

- Let π be the population proportion. We want to test

$$\text{(right-tailed test)} \quad H_0 : \pi = \pi_0, \; v.s. \; H_A : \pi > \pi_0;$$

or

$$\text{(left-tailed test)} \quad H_0 : \pi = \pi_0, \; v.s. \; H_A : \pi < \pi_0;$$

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$$\text{(two-tailed test)} \quad H_0 : \pi = \pi_0, \;\; v.s. \;\; H_A : \pi \neq \pi_0.$$

- We have data $X_1, ..., X_n \sim Ber(\pi)$.
- Under $H_0 : \pi = \pi_0$, the sample proportion $P = \frac{\sum_{i=1}^n X_i}{n}$ has

$$\mathbb{E}(P) = \pi_0$$

  and

$$Var(P) = \frac{\pi_0(1 - \pi_0)}{n}.$$

Tests for population proportion

- When $n\pi_0 > 5$ and $n(1-\pi_0) > 5$, the CLT holds:

$$Z = \frac{P - \pi_0}{\sqrt{\frac{\pi_0(1-\pi_0)}{n}}}.$$

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- **Test statistic**:

$$Z = \frac{P - \pi_0}{\sqrt{\frac{\pi_0(1-\pi_0)}{n}}}.$$

  This is Z test!

- **P-value**: Let P_{obs} be the observed sample proportion.
  - $H_A : \pi > \pi_0$: calculate $z_{obs} = \frac{P_{obs} - \pi_0}{\sqrt{\frac{\pi_0(1-\pi_0)}{n}}}$ and calculate $P(Z > z_{obs})$.
  - $H_A : \pi < \pi_0$: calculate $P(Z < z_{obs})$.
  - $H_A : \pi \neq \pi_0$: calculate $2 * P(Z > |z_{obs}|)$.

Example

An accounting firm has a large list of clients, and each client has a file with information about that client. The firm has noticed errors in some of these files, and has decided that it would be worthwhile to know the proportion of files that contain an error. Call the population proportion of files in error π. It was decided to take a simple random sample of size $n = 50$, and use the results of the sample to estimate π. Each selected file was thoroughly reviewed, and classified as either containing an error (call this 1), or not (call this 0). The results are as follows:

Files with an error: 10; Files without any errors: 40.

Example continued

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- Under H_0, $0.1(50) = 5$ and $0.9(50) = 45 > 5$, so we should be able to use the CLT.
- Our observed statistic is $z_{obs} = \frac{0.2-0.1}{\sqrt{\frac{0.1(1-0.1)}{50}}} = 2.357$, so the p-value is $P(Z > 2.357) = 0.009$.
- At the 5% level ($\alpha = 0.05$), we would reject the null, and conclude that too high of a proportion of files are in error. All files should be checked and fixed.

What's the next?

We'll discuss tests for two independent samples in the next lecture.