

## 1. The Paired T-Test (Differences Normal)

The data consists of paired observations. Let:

$X_{1,i}$  = a data point from population 1

$\mu_1$  = true mean of population 1

$X_{2,i}$  = a data point from population 2

$\mu_2$  = true mean of population 2

$D_i = X_{1,i} - X_{2,i}$  = the difference for pair  $i$

$n$  = number of pairs

$\sigma_D^2$  = true variance of the differences

We wish to test:

$$H_0 : \mu_1 - \mu_2 = \delta$$

vs.

$$H_A : \mu_1 - \mu_2 \neq \delta$$

Good graphs for exploring the data include dotplots where the observations for each pair are connected by lines, and QQ plots of the differences.

If based on our prior knowledge and after exploring the data we are willing to assume:

- All of the *pairs* are independent
- The differences follow a normal distribution

Then the test statistic is:

$$t = \frac{\bar{D} - \delta}{\frac{s_D}{\sqrt{n}}}$$

Where:

$\bar{D}$  is the mean of the differences

$s_D$  is the sample standard deviation of the differences

Compare to a  $T$  distribution on  $n - 1$  degrees of freedom.

## 2. Sign Test: see slides.