Chapter 4: Random Variables and Distributions

(Ott & Longnecker Sections: 4.6-4.10)

https://dzwang91.github.io/stat324/

Part 1

UNIVERSITY OF WISCONSIN–MADISON

What do we study?

Key Concepts: Random variable, discrete random variable, continuous random variable, probability distribution.

Outline

1. Random Variables
2. Probability Distribution
3. Discrete RVs
4. Continuous RVs

Random Variables

A random variable, or RV for short, is the name assigned to the outcome of a random process. RVs are usually denoted by capital letters, like X, Y, etc.
For example, you could define a random variable:

$$X = \text{weight of one ant chosen at random from Billy's ant farm.}$$

All we've really done is given a name "X" to the specific random process of choosing an ant from the farm and weighing it. So, in some ways random variables are simply notation. The reason the variable is called random is because we don't know the weight of the ant exactly until we select and weigh it.

Random Variables

Once the random process that defines a RV is performed, we call the result a realization of the RV. Realizations of RVs are not themselves random, but the process by which a RV is realized is random. Realizations of RVs are usually denoted by lower-case letters, like x, y, etc.

Suppose we realize the RV X defined above, by selecting an ant at random and weighing it. Suppose it weighs 3.1 mg. Then, $x = 3.1$.

Outline

1 Random Variables

2 Probability Distribution

3 Discrete RVs

4 Continuous RVs

Probability Distribution

Every random variable can have different properties depending on the process that it describes, and this is given a special name. The probability distribution of a random variable consists of a description of the possible values that the RV can realize to, along with the probabilities that each realization will occur.

Outline

1 Random Variables

2 Probability Distribution

3 Discrete RVs

4 Continuous RVs

Discrete RV and pmf

Depending on the type of RV, the descriptions of the possible values and probabilities can take different forms.

- Discrete RVs can only realize to certain specific values. Probability distributions for discrete RVs are called probability mass functions, or pmfs, and consist of lists of the values that can be taken by the RV, together with the probabilities of each value. The pmf for an RV X is usually denoted $p(x)$.

Discrete RV example

Let X_{heads} =the number of heads in two tosses of a fair coin. X_{heads} is a discrete RV whose possible values are

Discrete RV example

Let X_{heads} =the number of heads in two tosses of a fair coin. X_{heads} is a discrete RV whose possible values are

0, 1, 2

Discrete RV example

Let X_{heads} =the number of heads in two tosses of a fair coin. X_{heads} is a discrete RV whose possible values are

$$0, 1, 2$$

The pmf of X_{heads} is

x	0	1	2
p(x)	?	?	?

Discrete RV example

Let X_{heads} =the number of heads in two tosses of a fair coin. X_{heads} is a discrete RV whose possible values are

$$0,\ 1,\ 2$$

The pmf of X_{heads} is

x	0	1	2
p(x)	?	?	?

x	0	1	2
p(x)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

Outline

1 Random Variables

2 Probability Distribution

3 Discrete RVs

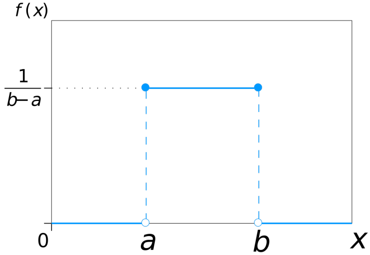
4 Continuous RVs

Continuous RV and pdf

- Continuous RVs take values in specified ranges. Probability distributions for continuous RVs are called probability density functions, or pdfs, and consist of ranges of values that can be taken by the RV, together with a function that lives on those ranges. The area under the function between any two possible realizations of the RV determines the probability that the RV will realize to a value in that range. The pdf for an RV X is usually denoted $f(x)$.

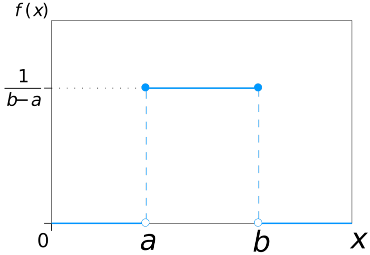
Continuous RV example: the uniform distribution

The Uniform Distribution has equal probability for all values of the Random variable between a and b:



Continuous RV example: the uniform distribution

The Uniform Distribution has equal probability for all values of the Random variable between a and b:



Question: How do we get the number $\frac{1}{b-a}$?

What's the next?

In the next lecture, we will further discuss some important discrete RVs and continuous RVs.