**Assignment #3 — Due Friday 2/22/2019, by 4pm**

1. Let $X \sim N(10, 5^2)$. Answer the following questions about $X$. Calculate these probabilities using Z table.

   (a) Find $P(X \leq 11)$.

   (b) Find $P(X \geq 8.4)$.

   (c) Find $x$ such that $P(X \leq x) = 0.95$.

   (d) Find $P(7.5 \leq X \leq 15)$.

2. To celebrate their 30th birthdays, brothers Mario and Luigi wish to study the distribution of heights of their enemies, the Goombas. Their reasoning is that shorter Goombas are easier to jump on. (Goombas die when Mario and Luigi jump on them.)

   (a) If we assume that the population of Goomba heights is normally distributed with mean 12 inches and standard deviation 6 inches, what is the probability that a randomly chosen Goomba's height is between 13 and 15 inches?

   (b) Koopa Troopas, another enemy of Mario & Luigi, have normally distributed heights, with mean 15 inches and standard deviation 3 inches. How tall must a Koopa Troopa be so that it's taller than the shortest 75% of Goombas? What percentage of Koopa Troopas are this tall, or taller?

   (c) How tall must a Goomba be so that it's taller than the shortest 75% of Koopa Troopas? What percentage of Goombas are this tall, or taller?

3. Let $F$ be an RV that represents the operating temperature in Fahrenheit of one instance of a manufacturing process, and let $F \sim N(90, 25)$. Let $C$ be an RV that represents the same process, but measured in Celsius. Fahrenheit can be converted to Celsius using $\text{Celsius} = \frac{5}{9}(\text{Fahrenheit} - 32)$. (**I would suggest doing these with a hand calculator and normal table as practice for exam conditions, but you may check your answers in R if you wish.**)

   (a) Find the probability that one randomly selected instance of the process will have operating temperature greater than 93.8 Fahrenheit.

   (b) Find the distribution of $C$. The distribution will consist of the 'family' of distribution (for example, Bernoulli, Binomial, Normal, etc.) plus the relevant parameters associated with that family ($\pi$ for a Bernoulli, $n$ and $\pi$ for a Binomial, $\mu$ and $\sigma^2$ for a Normal, etc.).

   (c) Find the probability that one randomly selected instance of the process will have operating temperature below 29 Celsius.

4. Consider a large population which has population mean $\mu$, and population variance $\sigma^2$. We take a sample of size $n = 3$ from this population, thinking of the samples as realizations of the RVs $X_1$, $X_2$, and $X_3$, where the $X_i$ can be considered iid. We are interested in estimating $\mu$.

(a) Consider the estimator $\tilde{\mu} = X_1 + X_2 - X_3$. Is this estimator unbiased for $\mu$? Explain your answer.

(b) Find the variance of $\tilde{\mu}$.

(c) When estimating $\mu$, would you prefer the estimator $\tilde{\mu}$ or the estimator $\bar{X}$? Explain your answer.

(d) Now consider the estimator $\mathring{\mu} = (X_1 + X_2 + X_3)/2$. Is this estimator unbiased for $\mu$? Explain your answer.

(e) Compute the MSE of $\mathring{\mu}$.