Lab 3

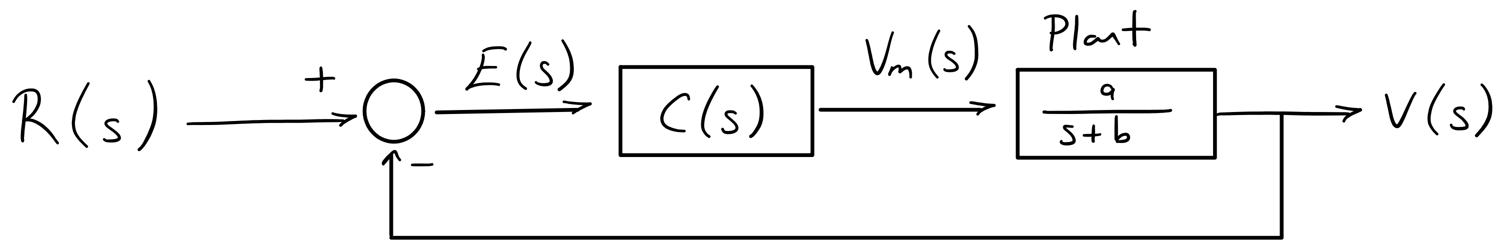
Tuesday, March 29, 2022   9:15 AM

1. Referring to Figure 2, verify that the plant has type 0 (i.e the plant has no pole in the origin). Hence in order to meet SPEC1, you need a controller with a pole at zero. Let $E(s) = R(s) - V(s)$

Define the PI controller $\dfrac{V_m(s)}{E(s)} = K\left(1 + \dfrac{1}{T_I \cdot s}\right)$, $K, T_I > 0$

Write the transfer function from $R(s)$ to $E(s)$ (assuming $D(s) = 0$), and from $D(s)$ to $E(s)$ (assuming $R(s) = 0$)

$$C(s) = K\left(1 + \frac{1}{T_I \cdot s}\right)$$

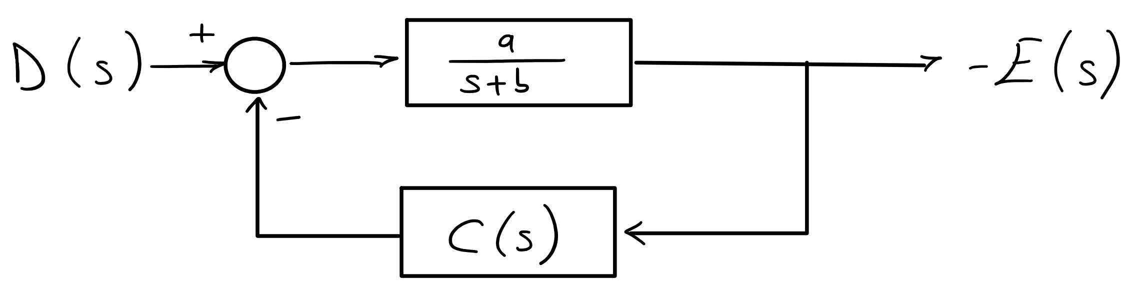


$$E(s) \cdot C(s) \cdot \frac{a}{s+b} = V(s)$$

$$E(s) = R(s) - V(s) \quad \Rightarrow \quad V(s) = R(s) - E(s)$$

$$E(s) \cdot C(s) \cdot \frac{a}{s+b} = R(s) - E(s)$$

$$\boxed{\frac{E(s)}{R(s)} = \frac{1}{C(s)\frac{a}{s+b} + 1}}$$



$$\boxed{\frac{E(s)}{D(s)} = \frac{-\frac{a}{s+b}}{1 + C(s)\frac{a}{s+b}}}$$

2. Let $R(s) = \dfrac{\bar{v}}{s}$ and recall that $D(s) = \dfrac{\bar{d}}{s}$. Using superposition, write the expression for $E(s)$ in the closed loop system.

$$\boxed{E(s) = \frac{\bar{v}}{s} \, \frac{1}{C(s)\frac{a}{s+b} + 1} + \frac{\bar{d}}{s} \, \frac{-\frac{a}{s+b}}{1 + C(s)\frac{a}{s+b}}}$$

3. Applying the final value theorem to $E(s)$, show that if $\lim_{x \to \infty} e(t)$ exists, then it must be zero. You have thus shown that a PI controller is capable of meeting SPEC1.

$$\lim_{x \to \infty} e(t) = \lim_{s \to 0} sE(s) = \lim_{s \to 0} \left[\bar{v} \, \frac{1}{C(s)\frac{a}{s+b} + 1} + \bar{d} \, \frac{-\frac{a}{s+b}}{1 + C(s)\frac{a}{s+b}} \right]$$

$$= \lim_{s \to 0} \frac{\bar{v} - \bar{d}\,\frac{a}{s+b}}{K\left(\frac{1}{T_I \cdot s}\right)\frac{a}{s+b} + 1} = \underline{\underline{0}}$$