CONTROL SYSTEMS LABORATORY
ECE311
LAB 2: Familiarization with Equipment
and Basic Cruise Control Design

1 Purpose

The purpose of this experiment is to introduce you to the lab setup and the associated control problem. Namely, the design of a cruise control system for a cart. You will test two different types of controllers: proportional (P) and proportional-integral (PI). Furthermore, you will explore how changing the controller parameters affects the closed loop performance.
Analogously to the previous lab you will submit your pre-lab first, receive feedback and then submit all the material required for the experimental part: a lab report and all the required Simulink models. Since you will not have access to an actual physical cart, we will provide you the Simulink model `cart.slx` you will use for the experimental part.

2 Introduction

Consider a cart moving on a straight road with an unknown slope. It is assumed that the inertia of the wheels is negligible, that the friction force is proportional to the speed of the car, and that the engine imparts a force u. The schematic representation of the system is depicted below.

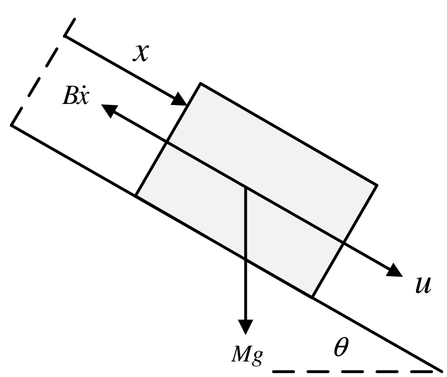

Figure 1: Schematic representation of the system

Applying Newton's law we obtain the following mathematical model of the system

$$M\ddot{x} = -B\dot{x} + u + Mg\sin\theta. \tag{1}$$

Since we are interested in controlling the speed $\dot{x}$ of the car, we rewrite the model using $v = \dot{x}$,

$$M\dot{v} = -Bv + u + Mg\sin\theta. \tag{2}$$

The force u, imparted by a DC motor, is approximately proportional to the voltage v_m applied to the motor,

$$u = K_m v_m. \tag{3}$$

The voltage v_m is the control input to the plant. The mathematical model of the cart system becomes

$$\dot{v} = -\frac{B}{M}v + \frac{K_m}{M}\left(v_m + \frac{M}{K_m}g\sin\theta\right). \tag{4}$$

Since the slope θ is unknown, the constant $\dfrac{M}{K_m}g\sin\theta$ is a disturbance acting on the plant. Letting

$$a = \frac{K_m}{M},\ b = \frac{B}{M},\ d(t) = \bar{d}\cdot\mathbf{1}(t) := \frac{M}{K_m}g\sin\theta\cdot\mathbf{1}(t), \tag{5}$$

the plant block diagram is depicted in Figure 2.

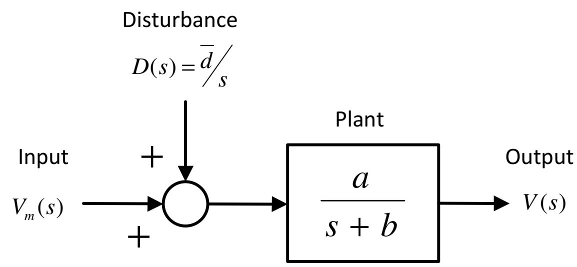

Figure 2: Block diagram of the plant

In this lab, you will familiarize yourself with the experimental setup, you will experimentally determine the constants a and b, and you'll implement basic proportional and proportional-integral controllers to regulate the car speed.
The experimental setup used in the lab is shown in Figure 3 .
The designed controller will be implemented in Simulink. For all your simulations you will use the Simulink model `cart.slx` provided to you as the additional material relative to this lab.

3 Preparation

Consider the block diagram in Figure 2 and assume that the road is flat, i.e., $\theta = 0$ and hence $D(s) = 0$. Suppose that a step voltage $v_m(t) = V_0\cdot\mathbf{1}(t)$ ($V_0 > 0$) is applied to the DC motor and that at time $t = 0$ the cart is still (i.e., $v(0) = 0$). Using the final value theorem determine $v(+\infty) = \lim_{t\to\infty} v(t)$ in terms of V_0, a, and b.

Submit this expression and its derivation in your pre-lab report.

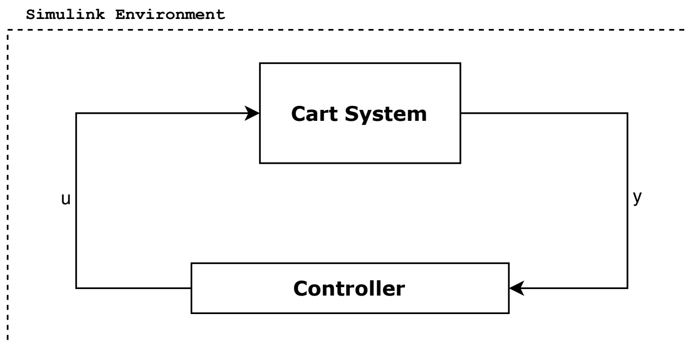

Figure 3: Block diagram of the experimental setup

4 Experiment

Be sure to include all the required Simulink models and plots in your lab report. To build the models for every section, start by opening the `cart.slx` model provided to you and save it with a different name. In this way you will always be sure that you will keep the original settings we prepared for the `cart.slx` model provided to you.

4.1 Introduction to the cart system

In this first part you will familiarize with the cart system by plotting the response to a square wave input voltage.

4.1.1 Open loop response: square wave voltage to the DC motor

In this section you will learn how to configure and setup your system. You will need to The objective is to have the cart on the rails swing back and forth when inputting a square wave. Open the plant model (the file `cart.slx` provided to you) on your simulink model. Using the `Sources/Signal Generator` block in Simulink, create a block that generates a square wave for $v_m(t)$ with amplitude 1.5 V and frequency 0.5Hz. Be sure to input the value 0 to the `offset` input. A simple way to do it is to use the `constant` block with value 0 (this corresponds to a cart system with no slope). If everything is correctly set your block diagram should look very similar to Figure 4.
Once your model is correctly set, run the simulation by clicking on the `run` button in Simulink. You should be able to see an animation showing the movement of the cart as in Figure 5. Save your model as `model_intro.slx` and include it in your submission. You will also save the position and velocity responses and include them in your lab report (Section 2.1 in the lab report).

4.1.2 Collecting data from the cart

To collect data from the proposed experiment you can use the `scope` blocks as in Figure 4 or if you prefer creating your own plot you can use the `To Workspace` block. [1]

[1] `https://www.mathworks.com/help/simulink/slref/toworkspace.html`

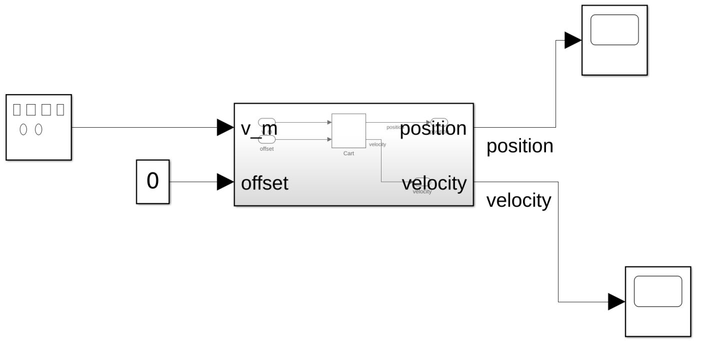

Figure 4: Schematic representation of the Matlab Simulink model

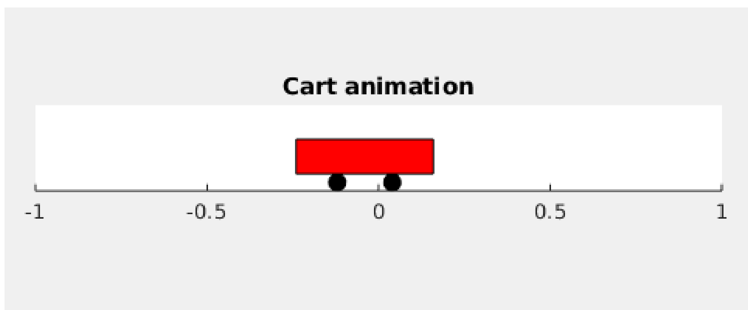

Figure 5: Cart animation

4.2 Identification of model parameters a and b

Recall that in the introduction we modelled the car as a transfer function $V(s)/V_m(s) = a/(s+b)$, where $V_m(s)$ is the Laplace transform of the input voltage signal and $V(s)$ is the Laplace transform of the cart speed signal. Before controlling the system, we need to experimentally determine the parameters a and b. This is the objective of this section. For this section start by opening the `cart.slx` model provided to you and save it with the new name `model_id.slx`. You will include this file as part of your submission.

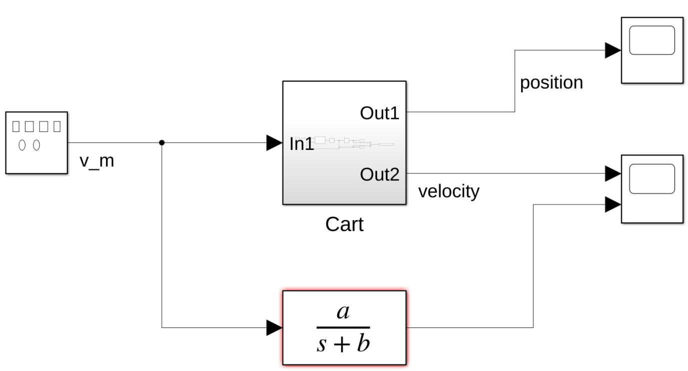

Figure 6: Schematic representation of the Matlab Simulink model

1. The detailed steps to build the new model are listed below.

   - Open another copy of the `cart.slx` Simulink model and the Library browser. Keep the input to the `offset` port at 0 as in Figure 4.
   - Using the `Continuous/Transfer Function` block, create a system with transfer function $\frac{a}{s+b}$. This block represents the mathematical model of the plant assuming that the disturbance is zero, that is, assuming that the cart track is horizontal. The objective here is to experimentally determine the values a and b.
   - Using the `Sources/Signal Generator` block in Simulink, create a block that generates a square wave for $v_m(t)$ with amplitude 1.5 V and frequency 0.5Hz.
   - Connect both the output of the transfer function block and the output of the cart to a Scope (found in the `Sinks` library).

   In this way you will be able to directly compare the velocity output of the cart and the transfer function models on the same scope.

2. We choose the initial guesses for a and b in the system model as 0.5 and 5 respectively.

3. Run the Simulink model to obtain the corresponding results for the considered a and b. Save the resulting plot and include it in your lab report.

4. Look at the experimental velocity and determine as accurately as you can its total variation over the half period. We'll denote such variation by $\triangle v$.

5. Notice that over the half period under consideration, the signal $v_m(t)$ performs a step of amplitude $V_0 = 3V$. We'll make the approximation to consider the signal v_t to be in steady state at the end of the half period. So we deduce that, in response to an input step of amplitude $3V$, the plant output has a total variation of $\triangle v$. Using the formula you found in your lab preparation and the value $\triangle v$ you just found, find a relationship between a and b. Specifically, find an expression of the type $a = f(b)$.

6. It should be now clear that for any choice of b, setting $a = f(b)$ guarantees that the steady state value of the model output approximately coincides with that of the actual plant output. Now you'll tune b to make sure that the transients coincide as much as possible.

   Keep the value of b you were using earlier, and set $a = f(b)$ in Simulink. Run the simulation and verify that the steady-state values of the model and actual plant outputs coincide. Include this simulation result in your lab report.

7. Now try to increase b. Don't forget, every time you modify b, you must also update $a = f(b)$ in Simulink. Run the simulation to see if the new value of b yields better results. Keep tuning b until you minimize the discrepancy between the actual plant and model outputs.

   Once you have found reasonable estimate save Simulink model `model_id.slx` with the final values of a and b you estimated. Report your findings in the report (Section 2.2 in the lab report) together with the plot that shows that your final choices of the parameters actually approximate the cart response. You'll use the values of a and b you just found in Lab 3.

4.3 Proportional Control

Now you'll implement a proportional controller to regulate the speed of the cart. A proportional controller is a controller of the form

$$v_m(t) = Ke(t), \tag{6}$$

where $e(t) := r(t) - v(t)$ is called the tracking error. This is the difference between the reference signal $r(t)$ and the actual plant output $v(t)$. In the cart experiment, $r(t)$ represents a desired velocity profile for the car, while $v(t)$ represents the actual cart speed. Do the following steps for this section.

1. Open another copy of the `cart.slx` Simulink model and the Library browser. Keep the input to the `offset` port at 0 as in Figure 4. You will save your model as `model_p.slx` and once complete, include it in your submission.

2. Set up your Simulink model as in Figure 7. This time, the input voltage to the cart is $v_m(t) = K(r(t) - v(t)) = Ke(t)$.

3. The velocity reference is set to a square wave form with the amplitude of $0.2\ m/s$ and frequency of $0.5\ Hz$. Use the same signal generator block you used in the previous step.

4. Start with a prortional gain $K = 5$. Do you expect the error to asymptotically tend to 0? Can you explain why, using the concepts learned in class?

5. Using the concepts learned in class (e.g. final value theorem) try to predict what would be the effect of increasing the controller gain K.

6. Now, verify your hypothesis with the help of simulations. Increase the controller gain[2], and run the system again. Repeat this operation a few times and save your simulation results to include in your report. Do the simulations results match your expectations?

   Report all your findings in the Lab Report (Section 2.3).

4.4 Proportional-Integral Control

You'll now implement a proportional-integral controller. A PI controller is an enhancement of a PI controller and has the form

$$v_m(t) = Ke(t) + \frac{K}{T_I} \int_0^t e(\tau) d\tau, \tag{7}$$

[2]Usually, for a physical implementation, the hardware limitations of your control system will limit the maximum controller gain you can use. In this case we ask you not to usea controller gain higher than 100 since it will make the simulation unstable.

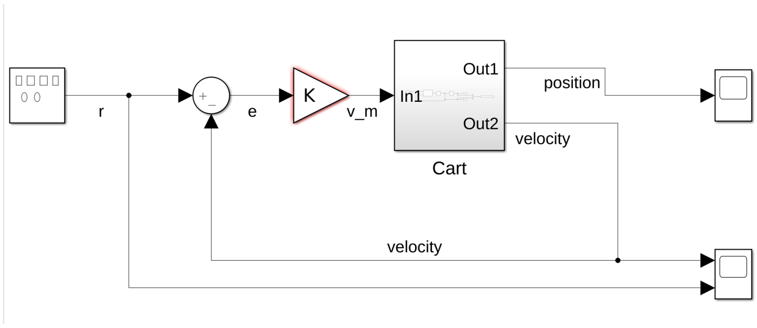

Figure 7: Schematic representation of the Matlab Simulink model

where K and T_I are two positive design constants. Taking the Laplace transform, we find that a PI controller has a transfer function (from e to v_m),

$$K + \frac{K}{T_I s} = K(\frac{T_I s + 1}{T_I s}) \tag{8}$$

Do the following steps for this section.

1. Open another copy of the `cart.slx` Simulink model and the Library browser. Keep the input to the `offset` port at 0 as in Figure 4. You will save your model as `model_pi.slx` and once complete, include it in your submission.

2. As in the previous experiment, the velocity reference is set to a square wave form with the amplitude of 0.2m/s and frequency of 0.5Hz.

3. Set $T_i = 0.07$ and $K = 2$. There are two ways to implement your PI controller:

   - Properly use an integrator block and a proportional block;
   - Use a transfer function block implementing directly (8).

   For this exercise you will have to build your own schematics, instead of using the schematics proposed in this lab instructions for the other exercises.

4. Following the same procedure described in the previous sections, run the Simulink simulation to observe the obtained experimental results. Report the plot representing the displacement and the velocity of the cart.

5. Include the plot for $T_i = 0.07$ and $K = 2$.

6. Next, keep T_i constant and start increasing K. Simulate your system for different values of K and record your observations: what's the effect of increasing K?

7. How does the performance of the P and PI controllers compare?

   Report all your findings in the Lab Report (Section 2.4).

5 Submission

For this lab you are required to submit the following material:

1. Pre-lab report (first deadline);
2. Lab report;
3. Lab files:
   - `model_intro.slx`
   - `model_id.slx`
   - `model_p.slx`
   - `model_pi.slx`