Lab 1 The Magnetic Ball Suspension System **Preparation Report** Wednesday, February 23, 2022

Electromagnet

F

M

Steel ball

Mg

$$F = \frac{i^2}{y^2}$$
 $g = 9.8 \text{ m/s}^2$
 $M = 1 \text{ kg}$
 $R = 3.2$
 $L = 1 \text{ H}$

1) State model of the system with input voltage
$$u$$
 and output position y .

$$x = \begin{bmatrix} y \\ y \\ i \end{bmatrix} \qquad x = f(x, u)$$

$$y = h(x, u)$$
Applying Newton's Second Law to the ball subsystem:

My - Mg +
$$\frac{i^2}{y^2} = 0$$
 \Rightarrow y - 9.8 + $\frac{i^2}{y^2} = 0$ 1)

Applying KVL to the electromagnet subsystem:

 $u = iR + 2\frac{di}{dt} \Rightarrow u = 3i + \frac{di}{dt}$ 2

From 1 & 2

$$x = \begin{bmatrix} y \\ \dot{y} \\ \dot{z} \end{bmatrix} \Rightarrow \begin{cases} x_1 = y \\ x_2 = \dot{y} \end{cases} \quad \dot{x} = \begin{cases} \dot{x}_1 = \dot{y} \\ \dot{x}_2 = \ddot{y} \end{cases} \quad \dot{x} = \begin{cases} \dot{x}_1 = \dot{x}_2 \\ \dot{x}_2 = 9.8 - \frac{(x_3)^2}{(x_1)^2} \\ \dot{x}_3 = u - 3x_3 \end{cases}$$

$$\dot{x} = f(x, u) = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} x_2 \\ 9.8 - \frac{(x_3)^2}{(x_1)^2} \\ u - 3x_3 \end{bmatrix}$$

$$y = h(x, u) = x_1$$

$$2) \text{ Equilibrium conditions of the system.}$$

2) Equilibrium conditions of the system.

$$(x, u) = ([y^*, y^*, i^*]^T, u^*)$$

To get the equilibrium point set $x = 0$
 $x_2^* = 0$
 $9.8 - \frac{(x_3^*)^2}{(x_1^*)^2} = 0 \implies x_3^* = 3.13 y^*$
 $u^* - 3x_3^* = 0 \implies u^* = 3x_3^* = 9.39 y^*$

* *

Equilibrium point
$$(x, u) = ([y^*, 0, 3.13y^*]^T, 9.39y^*)$$

Linearization about the equilibrium condition (x, u) to obtain a system $8x = A8x + B8u$
 $8y = C8x + D8u$

Define $8u = u - u^*$
 $8x = x - x^* = [x_1 - y^*, x_2, x_3 - 3.13y^*]^T$
 $8y = y - y^*$

Compute $A = \frac{4}{1} |_{x = x^*}$

Compute
$$A = \frac{3f}{f_{x}}\Big|_{x=x^{*}}$$

$$\frac{3f_{1}}{f_{x_{1}}} = 0$$

$$\frac{3f_{1}}{f_{x_{2}}} = 1$$

$$\frac{3f_{1}}{f_{x_{3}}} = 0$$

$$\frac{3f_{2}}{f_{x_{1}}} = 2(x_{3}^{*})^{2}(x_{1}^{*})^{-3} = 2(3.13y^{*})^{2}(y^{*})^{-3} = \frac{19.6}{y^{*}}$$

$$\frac{3f_{2}}{f_{2}} = 0$$

$$\frac{3f_{3}}{f_{3}} = 0$$

$$\frac{3f_{3}}{f_{3}} = 0$$

$$\frac{3f_{3}}{f_{3}} = 0$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ \frac{19.6}{y^*} & 0 & -6.26 \\ 0 & 0 & -3 \end{bmatrix}$$

$$Compute \quad B = \frac{f}{fu} \Big|_{x=u^*} x = x^*$$

$$\frac{f}{fu} = 0$$

$$\frac{f^2}{fu} = 0 \implies P$$

 $\frac{\partial f_1}{\partial u} = 0$

 $\frac{\partial f_3}{\partial f_3} = 1$

$$\frac{\partial f}{\partial u} = 0$$

$$\frac{\partial f^{2}}{\partial u} = 0$$

$$\Rightarrow B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\frac{\partial f^{3}}{\partial u} = 1$$

$$Compute C = \frac{\partial h}{\partial x} \Big|_{\substack{x = x \\ u = u}}^{x = x}$$

$$C = \frac{Jh}{Jx} \Big|_{x}$$

$$\frac{Jh}{Jx_{2}} = 0$$

$$h$$

 $G(s) = C(sI-A)^{-1}B + D$

Poles and zeros of G(s) Zeros - None

Poles $\rightarrow (s^2 - 19.6)(s+3) = 0$

 $G(s) = \frac{-6.26}{(s^2-19.6)(s+3)} = \frac{-6.26}{s^3+3s^2-19.6s+58.8}$

 $P_1 = -3$ $P_{2,3} = \frac{\pm \sqrt{4.19.6}}{2} = \pm \sqrt{19.6} = \pm 4.427$

 $G(s) = \frac{\Gamma_1}{s-4.427} + \frac{\Gamma_2}{s+4.427} + \frac{\Gamma_3}{s+7}$

 $r_1 = \frac{-6.26}{(s+4.427)(s+3)}\Big|_{s=4.427} = -0.1$

 $r_2 = \frac{-6.26}{(s-4.427)(s+3)} \bigg|_{s=-4.427} = 0.5$

Sketch of the response of g (t)

 $r_{J} = \frac{-6.26}{\left(s_{-}^{2}196\right)} \bigg|_{s_{-}=7} = 0.6$

Linearized model
$$S\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ \frac{19.6}{y^{**}} & 0 & -6.26 \\ 0 & 0 & -3 \end{bmatrix} S_{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} S_{u}$$

Linearized model
$$\begin{cases}
0 & 1 & 0 \\
\frac{19.6}{y^*} & 0 & -6.26 \\
0 & 0 & -3
\end{cases}$$

$$\begin{cases}
8x + 0 \\
1
\end{cases}$$

$$\begin{cases}
4x + 0 \\
5x + 0 \\
5x$$

Linearized model
$$\begin{cases}
0 & 1 & 0 \\
\frac{19.6}{y^*} & 0 & -6.26 \\
0 & 0 & -3
\end{cases}$$

D=
$$\frac{Jh}{Ju}$$
=0

Linearized model

[0 1 0]

$$= \frac{Jh}{Jx} \Big|_{x=x^*}$$

$$\frac{-h}{x_2} = 0 \qquad \frac{Jh}{Jx_3} = 0$$

$$\int_{x_2}^{x_2} \frac{Jh}{Ju} \Big|_{x=x^*}$$

$$\int_{u=u^*}^{x=x^*}$$

$$= \frac{Jh}{Jx} \Big|_{\substack{x = x \\ u = u}}^{x = x}$$

$$\frac{h}{dx} = 0 \qquad \frac{Jh}{Jx_3} = 0$$

$$0 = \frac{Jh}{Ju} \Big|_{\substack{x = x \\ u = u}}^{x = x}$$

$$= \frac{Jh}{Jx} \Big|_{x=x^*}$$

$$\frac{h}{dx} = 0 \qquad \frac{Jh}{Jx_3} = 0$$

$$= \frac{Jh}{Ju} \Big|_{x=x^*}$$

$$= \frac{Jh}{Ju} \Big|_{x=x^*}$$

Compute
$$C = \frac{1}{Jx} \Big|_{x=x^*}^{x=x^*}$$

$$\frac{Jh}{Jx_1} = 1 \qquad \frac{Jh}{Jx_2} = 0 \qquad \frac{Jh}{Jx_3} = 0$$
Compute $D = \frac{Jh}{Ju} \Big|_{x=x^*}^{x=x^*}$

$$D = \frac{Jh}{Ju} = 0$$

Compute
$$C = \frac{Jh}{Jx}\Big|_{\substack{x=x^*\\u=u^*}}$$

$$\frac{Jh}{Jx_1} = 1 \qquad \frac{Jh}{Jx_2} = 0 \qquad \frac{Jh}{Jx_3} = 0$$
Compute $D = \frac{Jh}{Ju}\Big|_{\substack{x=x^*\\u=u^*}}$

$$D = \frac{Jh}{Ju} = 0$$

$$\frac{\partial f}{\partial u} = 1$$

$$Compute \quad C = \frac{\partial h}{\partial x} \Big|_{x=x^*} = 1$$

$$\frac{\partial h}{\partial x_1} = 1 \quad \frac{\partial h}{\partial x_2} = 0 \quad \Rightarrow \quad C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

$$Compute \quad D = \frac{\partial h}{\partial u} \Big|_{x=x^*} = 0$$

$$D = \frac{\partial h}{\partial u} = 0$$

where $S_{u} = u - u^{*}$ $S_{x} = x - x^{*} = [x_{1} - y^{*}, x_{2}, x_{3} - 3.13y^{*}]'$ $S_{y} = y - y^{*}$

3) Set $y^* = 1$ and derive the open-loop transfer function $G(s) = \frac{gY(s)}{gu(s)}$.

4) Corresponding impulse-response function $g(t) = \int_{-\infty}^{\infty} (G(s)) ds$ the linearized system.

 $g(t) = (-0.1e^{4.427t} + 0.5e^{-4.427t} + 0.6e^{-3t}) \cdot u(t)$ where u(t) is the step function

 $= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} s & -1 & 0 \\ -19.6 & s & 6.26 \\ 0 & 0 & s+3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{(s^2 - 19.6)(s+3)} \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} s(s+3) & s+3 & -6.26 \\ 19.6(s+3) & s(s+3) & -6.26s \\ 0 & 0 & s^2 - 19.6 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$=$$
 $\begin{bmatrix} \mathcal{J} & \mathcal{C} \end{bmatrix}$

$$\frac{\partial f_1}{\partial x_2} = 1$$

$$\frac{\partial f_2}{\partial x_3} = 0$$

$$\frac{\partial f_2}{\partial x_3} = -2$$

$$\frac{\partial f_3}{\partial x_2} = 0$$

$$\frac{\partial f_3}{\partial x_3} = -3$$

$$\frac{f_1}{f_3} = 0$$

$$\frac{f_2}{f_3} = -2 \times 3$$

$$= -3$$

$$\frac{f_1}{f_2} = 0$$

$$\frac{z}{z} = -2 \times 1$$

$$\frac{f_1}{x_3} = 0$$

$$\frac{1-y^{*}}{1-y^{*}} = 0$$

$$\frac{f_1}{f_2} = 0$$

$$\frac{f_1}{f_2} = 0$$

$$\frac{y-y^*}{x_3} = 0$$

$$\frac{z}{z_3} = -2 \times 3$$

$$\frac{f_1}{f_2} = 0$$

$$\frac{f_2}{f_3} = -2 \times 3$$

$$\frac{f_1}{f_3} = 0$$

$$\frac{2}{3} = -2 \times 3$$

$$\frac{f_1}{f_{x_3}} = 0$$

$$\frac{f_2}{f_2} = -2 \times 3$$

$$y - y^*$$

$$\frac{f_1}{x_3} = 0$$