Lab 1 The Magnetic Ball Suspension System **Preparation Report** Wednesday, February 23, 2022

$$F = \frac{i^2}{y^2} \quad y = 9.8 \,\text{m/s}^2 \quad M = 1 \,\text{kg} \quad R = 3.2 \quad L = 1 \,\text{H}$$
1) State model of the system with input voltage u and output position y.
$$x = \begin{bmatrix} y \\ \dot{y} \\ i \end{bmatrix} \quad \dot{x} = f(x, u)$$

$$y = h(x, u)$$

Applying Newton's Second Law to the ball subsystem:

$$M\ddot{y} - Mg + \frac{i^2}{y^2} = 0 \implies \ddot{y} - 9.8 + \frac{i^2}{y^2} = 0$$

Applying KVL to the electromagnet subsystem:

$$\begin{aligned}
\dot{x} &= f(x, u) = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} x_2 \\ 9.8 - \frac{(x_3)^2}{(x_1)^2} \\ u - 3x_3 \end{bmatrix} \\
y &= h(x, u) = x_1
\end{aligned}$$
2) Equilibrium conditions of the system.
$$(x, u) = (\begin{bmatrix} y^*, \dot{y}^*, i^* \end{bmatrix}^T, u^*)$$

$$\begin{aligned}
To get the equilibrium point set \dot{x} = 0 \\
x_2^* &= 0 \\
9.8 - \frac{(x_3^*)^2}{(x_1^*)^2} = 0 \Rightarrow x_3^* = 3.13 y^* \\
u^* - 3x_3^* &= 0 \Rightarrow u^* = 3x_3^* = 9.39 y^*
\end{aligned}$$

Equilibrium point $\begin{pmatrix} *, * \\ \times, u \end{pmatrix} = \begin{pmatrix} [y^*, 0], 3.13y^* \end{bmatrix}^T, 9.39y^* \end{pmatrix}$

Linearization about the equilibrium condition
$$(x^*, u^*)$$
, to obtain a system $8\dot{x} = A8x + B8u$ $8y = C8x + D8u$

Define $8u = u - u^*$ $8x = x - x^* = [x_1 - y^*, x_2, x_3 - 3.13y^*]^T$

Compute $4 = \frac{1}{3x} |_{x = x^*} |_{x = u^*}$

$$\frac{df_3}{dx_1} = 0$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ \frac{19.6}{y^*} & 0 & -6.26 \\ 0 & 0 & -3 \end{bmatrix}$$

$$\frac{df_3}{dx_1} = 0$$

 $\frac{\partial f_1}{\partial x} = 0$

 $\frac{\partial f_1}{\partial u} = 0$

 $\frac{\partial f_3}{\partial f_3} = 1$

Compute
$$B = \frac{f}{fu}\Big|_{x=x^*}$$

$$\frac{f}{fu} = 0$$

$$\frac{f^2}{fu} = 0 \implies P$$

$$\Rightarrow \frac{1}{2} = \frac{3h}{3x} = 0$$

$$\frac{3h}{3x_2} = 0$$

$$D = \frac{Jh}{Ju} = 0$$

$$Linearized model$$

$$0 1 0$$

$$5\dot{x} = \frac{19.6}{y^*} 0 - 6.26$$

$$Sy = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$Sy = \begin{bmatrix} 1 & 0 \\ where & Su = u \end{bmatrix}$$

$$Sy = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$
where $Su = u$

Sinewited model
$$\begin{cases}
0 & 1 & 0 \\
\frac{19.6}{y^*} & 0 & -6.26 \\
0 & 0 & -3
\end{cases}$$

$$\begin{cases}
5x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}
\end{cases}$$
Su
$$\begin{cases}
5y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} S_{x} + 0 S_{u}$$
where
$$\begin{cases}
5u = u - u^* \\
5x = x - x^* = 0
\end{cases}$$

$$Sy = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$
where $Su = u - \frac{1}{2}$

$$Sy = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} S_{x} + 0 S_{u}$$
where $S_{u} = u - u^{*}$ $S_{x} = x - x^{*} = \begin{bmatrix} x_{1} - y^{*}, x_{2}, x_{3} - 3.13y^{*} \end{bmatrix}^{T}$ $S_{y} = y - y^{*}$

$$3) Set y^{*} = 1 \text{ and derive the open-loop transfer function } G(s) = \frac{SY(s)}{SU(s)}.$$

 $G(s) = C(sI-A)^{-1}B + D$

$$Sy = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$
where $Su = u - u$

$$\frac{\partial f_1}{\partial u} = 0$$

$$\frac{\partial f_2}{\partial u} = 0$$

$$\frac{\partial f_3}{\partial u} = 1$$

$$\frac{\partial f_4}{\partial u} = 1$$

$$\frac{\partial f_1}{\partial u} = 0$$

$$\frac{\partial f_2}{\partial u} = 0 \implies B$$

$$\frac{\partial f_3}{\partial u} = 1$$

$$Compute C = \frac{\partial h}{\partial x}\Big|_{\substack{x = x \\ u = u}}^{x = x}$$

$$\frac{\partial f_3}{\partial u} = 1$$
Compute $C = \frac{\partial h}{\partial x}\Big|_{\substack{x=x^*\\u=u^*}}$

$$\frac{\partial h}{\partial x_1} = 1 \qquad \frac{\partial h}{\partial x_2} = 0 \qquad \frac{\partial h}{\partial x_3} = 0$$
Compute $D = \frac{\partial h}{\partial u}\Big|_{\substack{x=x^*\\u=u^*}}$

$$\frac{Jh}{Jx_1} = 1 \qquad \frac{Jh}{Jx_2} = 0 \qquad \frac{Jh}{Jx_3} = 0 \qquad \Rightarrow r \qquad C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

$$Compute \qquad D = \frac{Jh}{Ju} \Big|_{\substack{x = x^* \\ u = u^*}}$$

$$D = \frac{Jh}{Ju} = 0$$

$$Linewized model$$

$$\begin{cases} x = x^* \\ x = u^* \end{cases}$$

$$\frac{\partial f^2}{\partial x_2} = 0 \qquad \frac{\partial f^2}{\partial x_3} = -\frac{1}{2}$$

$$\frac{\partial f^3}{\partial x_2} = 0 \qquad \frac{\partial f^3}{\partial x_3} = -3$$

$$\frac{\partial f_1}{\partial x_2} = 1$$

$$\frac{\partial f_2}{\partial x_3} = 0$$

$$\frac{\partial f_2}{\partial x_2} = -2 \times 0$$

$$\frac{\partial f_3}{\partial x_2} = -3$$

$$\frac{y-y^*}{y^*} = 0$$

$$\frac{y-y^*}{x_3} = 0$$

$$\frac{y-y^*}{x_3} = 0$$

$$\frac{f_1}{f_2} = 0$$

$$\frac{f_1}{f_3} = 0$$

Define
$$\int u = u - u^*$$
 $\int x = x - x^* = \left[x_1 - y^*, x_2, x_3 - 3.13y^* \right]$ $\int y = y - y^*$

Compute $A = \frac{3f}{Jx} \Big|_{x = x^*}$

$$\frac{3f_1}{Jx_1} = 0$$

$$\frac{3f_2}{Jx_1} = 2(x_3^*)^2(x_1^*)^{-3} = 2(3.13y^*)^2(y^*)^{-3} = \frac{19.6}{y^*}$$

$$\frac{3f_2}{Jx_2} = 0$$

$$\frac{3f_2}{Jx_3} = -2x_3^* = -6.26y^*$$

$$\frac{f_1}{f_2} = 0$$

$$\frac{f_1}{f_2} = 0$$

$$\frac{f_2}{f_2} = -2 \times 3$$

$$\frac{2}{3} = 0$$

$$\frac{1}{3} = -2 \times 3$$

$$\frac{f_1}{x_3} = 0$$

$$\frac{2}{x_3} = -2 \times 3$$

$$\frac{f_1}{x_3} = 0$$

$$\frac{z}{z_3} = -2 \times z^*$$

$$\frac{z}{z_3} = -3$$

$$\frac{f_1}{x_3} = 0$$

$$\frac{z}{x_3} = -2x_3$$

$$\frac{z}{x_3} = -3$$

$$Sy = y - y^*$$

$$G(s) = \frac{sY(s)}{sU(s)}$$

$$y^* = 1 \text{ and derive the open-loop transfer function } G(s) = \frac{8Y(s)}{8U(s)}.$$

$$y^* = 1 \text{ and derive the open-loop transfer function } G(s) = \frac{8Y(s)}{8U(s)}.$$

$$y^* = 1 \text{ and derive the open-loop transfer function } G(s) = \frac{8Y(s)}{8U(s)}.$$

$$y^* = 1 \text{ and derive the open-loop transfer function } G(s) = \frac{8Y(s)}{8U(s)}.$$

$$y^* = 1 \text{ and derive the open-loop transfer function } G(s) = \frac{8Y(s)}{8U(s)}.$$

$$y^* = 1 \text{ and derive the open-loop transfer function } G(s) = \frac{8Y(s)}{8U(s)}.$$

$$y^* = 1 \text{ and derive the open-loop transfer function } G(s) = \frac{8Y(s)}{8U(s)}.$$

$$y^* = 1 \text{ and derive the open-loop transfer function } G(s) = \frac{8Y(s)}{8U(s)}.$$

$$y^* = 1 \text{ and derive the open-loop transfer function } G(s) = \frac{8Y(s)}{8U(s)}.$$

$$y^* = 1 \text{ and derive the open-loop transfer function } G(s) = \frac{8Y(s)}{8U(s)}.$$

$$y^* = 1 \text{ and derive the open-loop transfer function } G(s) = \frac{8Y(s)}{8U(s)}.$$

$$y^* = 1 \text{ and derive the open-loop transfer function } G(s) = \frac{8Y(s)}{8U(s)}.$$

$$y^* = 1 \text{ and derive the open-loop transfer function } G(s) = \frac{8Y(s)}{8U(s)}.$$

$$y^* = 1 \text{ and derive the open-loop transfer function } G(s) = \frac{8Y(s)}{8U(s)}.$$

$$y^* = 1 \text{ and derive the open-loop transfer function } G(s) = \frac{8Y(s)}{8U(s)}.$$

$$y^* = 1 \text{ and derive the open-loop transfer function } G(s) = \frac{8Y(s)}{8U(s)}.$$

$$y^* = 1 \text{ and derive the open-loop transfer function } G(s) = \frac{8Y(s)}{8U(s)}.$$

$$y^* = 1 \text{ and derive the open-loop transfer function } G(s) = \frac{8Y(s)}{8U(s)}.$$

$$y^* = 1 \text{ and derive the open-loop transfer function } G(s) = \frac{8Y(s)}{8U(s)}.$$

$$y^* = 1 \text{ and derive the open-loop transfer function } G(s) = \frac{8Y(s)}{8U(s)}.$$

$$y^* = 1 \text{ and } G(s) = \frac{8Y(s)}{8U(s)}.$$

$$y^* = 1 \text{ and } G(s) = \frac{8Y(s)}{8U(s)}.$$

$$y^* = 1 \text{ and } G(s) = \frac{8Y(s)}{8U(s)}.$$

$$y^* = 1 \text{ and } G(s) = \frac{8Y(s)}{8U(s)}.$$

$$y^* = 1 \text{ and } G(s) = \frac{8Y(s)}{8U(s)}.$$

$$y^* = 1 \text{ and } G(s) = \frac{8Y(s)}{8U(s)}.$$

$$y^* = 1 \text{ and } G(s) = \frac{8Y(s)}{8U(s)}.$$

$$y^* = 1 \text{ and } G(s) = \frac{8Y(s)}{8U(s)}.$$

$$y^* = 1 \text{ and } G(s) = \frac{8Y(s)}{8U(s)}.$$

$$y^* = 1 \text{ and }$$

Poles and zeros of
$$G(s)$$

Zeros = None

Poles = $(s^2-19.6)(s+3)=0$
 $P_1 = -3$
 $P_{2,3} = \frac{\pm \sqrt{4.19.6}}{2} = \pm \sqrt{19.6} = \pm 4.427$

A) Corresponding impulse-response function $g(t) = \int_{-1}^{1} (G(s)) ds$ the linearized system.

 $G(s) = \frac{r_1}{s-4.427} + \frac{r_2}{s+4.427} + \frac{r_3}{s+3}$

 $r_1 = \frac{-6.26}{(s+4.427)(s+3)}\Big|_{s=4.427} = -0.1$

 $r_2 = \frac{-6.26}{(s-4.427)(s+3)} \bigg|_{s=-4.427} = 0.5$

 $r_{J} = \frac{-6.26}{\left(s^{2} - 196\right)} \bigg|_{s = -7} = 0.6$

 $G(s) = \frac{-6.26}{(s^2-19.6)(s+3)} = \frac{-6.26}{s^3+3s^2-19.6s-58.8}$

$$g(t) = (-0.1e^{4.427t} + 0.5e^{-4.427t} + 0.6e^{-3t}) \cdot u(t)$$
 where $u(t)$ is the step function
Sketch of the response of $g(t)$