CPSC 511 — Fall 2014
Solutions for Question #5 on Midterm Test

In this question you were asked to state and prove Savitch's Theorem, and to use this to prove that

$$\text{PSPACE} = \text{NPSPACE}$$

Savitch's Theorem states that if $S(n)$ is a space constructible function such that $S(n) \geq \log n$ then

$$\text{NSPACE}(S(n)) \subseteq \textbf{\textit{SPACE}}(S(n)^2).$$

This is stated and proved as Theorem 8.5, for the case that $S(n) \geq n$, in Section 8.1 of the textbook. The result is generalized to include the case that $S(n) \geq \log n$ at the end of Section 8.2, on page 351.

Since every deterministic Turing machine is easily converted into a nondeterministic Turing machine that uses the same space, it should be clear that

$$\text{PSPACE} \subseteq \text{NPSPACE}.$$

Suppose now that $L \subseteq \Sigma^\star$ is a language such that $L \in$ NPSPACE. Then, since

$$\text{NPSPACE} = \bigcup_{k \geq 1} \text{NSPACE}(n^k),$$

$L \in$ NSPACE(n^k) for some integer $k \geq 1$. It now follows by Savitch's Theorem that $L \in$ SPACE(n^{2k}). Now, since $2k$ is also a positive integer and

$$\text{PSPACE} = \bigcup_{k \geq 1} \text{SPACE}(n^k),$$

it follows that $L \in$ PSPACE as well. Since L was arbitrarily chosen from NPSPACE, it follows that

$$\text{NPSPACE} \subseteq \text{PSPACE},$$

so that PSPACE $=$ NPSPACE, as claimed.