CPSC 513 — Assignment #2

Solutions for Question #3

Recall that, for $n \geq \mathbb{N}$, Φ_n is the (partial or total) function that is computed by the program with encoding n. The problems on this assignment concern the set

$$\text{Monotone} = \{n \in \mathbb{N} \mid \Phi_n \text{ is total and } \Phi_n(x) \leq \Phi_n(x+1) \text{ for all } x \in \mathbb{N}\}.$$

In this question you were asked to use Rice's Theorem to give another *very short* proof that Monotone is not recursive.

Solution: Notice that Monotone is the index set R_Γ of the set Γ of partially computable functions of one variable that are monotone functions.

The function $f : \mathbb{N} \to \mathbb{N}$ such that $f(n) = 0$ for all $n \in \mathbb{N}$ is certainly in Γ — it is certainly computable, since it is primitive recursive (indeed, it is one of the initial functions) and, since

$$f(x+1) - f(x) = 0 - 0 \geq 0,$$

it is also a monotone function.

On the other hand, the partial function $g : \mathbb{N} \to \mathbb{N}$ such that $g(n)$ is undefined is certainly computable too but not monotone since it is not total.

It follows by an application of Rice's Theorem that the set Monotone $= R_\Gamma$ is not a recursive set.