

CPSC 511/611 — Assignment #3

Space Complexity

1 About This Assignment

This assignment can be completed by groups of up to two students in CPSC 511; students in CPSC 611 are to complete it individually.

The assignment is due by 11:59 pm on Wednesday, November 19. A printed copy of answers for written questions should be submitted by each group (for CPSC 511) or student (for CPSC 611) using the drop boxes on the second floor of the Mathematical Sciences building. If you typeset your submission then one student in your group can also submit an electronic copy (as a PDF file) using the assignment dropbox in D2L. However, a printed copy is still required if you do this!

Problems To Be Solved

1. Recalling the definition of **deterministic finite automata**

$$M = (Q, \Sigma, \delta, q_0, F)$$

as needed (from Chapter 1 of the textbook), and working with a “reasonable” encoding of these automata over an alphabet

$$\Sigma_{DFA} = \{q, s, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9(,), \{, \}, , \},$$

complete the following question.

- (a) Prove that if $M_1 = (Q_1, \Sigma, \delta_1, q_{0,1}, F_1)$ and $M_2 = (Q_2, \Sigma, \delta_2, q_{0,2}, F_2)$ are two deterministic finite automata with the same alphabet Σ , $|Q_1| = n_1$ and $|Q_2| = n_2$, then either M_1 and M_2 are equivalent (that is, they have the same language) or there exists some string $\omega \in \Sigma^*$ whose length is at most $n_1 \times n_2$ such that *exactly* one of M_1 and M_2 accept this string.

Hint: Modify the proof of the “Pumping Lemma for Regular Languages.”

- (b) Let $\text{EQ}_{\text{DFA}} \subseteq \Sigma_{\text{DFA}}^*$ be the language of encodings of ordered pairs of deterministic finite automata, M_1 and M_2 , with the same alphabet, such that the languages of M_1 and M_2 are the same.

Prove that $\text{EQ}_{\text{DFA}} \in \mathcal{NL}$.

Hint: Use the result of part (a) — and remember that $\mathcal{NL} = \text{co-}\mathcal{NL}$.

- (c) Finally, prove that EQ_{DFA} is **complete** for $\text{co-}\mathcal{NL}$ with respect to log-space mapping reductions.