CPSC 511/611 — Assignment #1
The Cobham-Edmonds Thesis

1 About This Assignment

This assignment can be completed by groups of up to two students in CPSC 511; students in CPSC 611 are to complete it individually.

The assignment is due by 11:59 pm on Tuesday, October 14. A printed copy of answers for written questions should be submitted by each group (for CPSC 511) or student (for CPSC 611) using the drop boxes on the second floor of the Mathematical Sciences building. If you typeset your submission then one student in your group can also submit an electronic copy (as a PDF file) using the assignment dropbox in D2L. However, a printed copy is still required if you do this!

2 Problems To Be Solved

1. Define a ***two-dimensional*** Turing machine to be a Turing machine, each of whose "tapes" is actually a two-dimensional ***grid*** (infinite in one direction, along each dimension):

   - The symbols of this machine's input string

     $$\omega = \sigma_1\sigma_2\ldots\sigma_n \in \Sigma^\star$$

     are initially stored in the cells in columns $1 - -n$ of row 1, and every other cell initially stores a blank.
   - Possible directions of movements for each tape head are now $\mathtt{L}$ (Left), $\mathtt{R}$ (Right), $\mathtt{S}$ (Stay), $\mathtt{U}$ (Up) or $\mathtt{D}$ (Down).

   Show that if $f : \mathbb{N} \to \mathbb{N}$ is a time-constructible function, and $L \subseteq \Sigma^\star$ is a language that is decided using a two-dimensional Turing machine, taking at most $f(|\omega|)$ steps on input

$\omega \in \Sigma^\star$, then there exists a (standard multi-tape) Turing machine that decides L using $O(f^2(|\omega|) \log f(|\omega|))$ steps on input $\omega \in \Sigma^\star$ in the worst case.

It follows from this that the complexity class $\mathcal{P}$ is unchanged if *this* model of computation is used to define it, as well.

2. The "Time Speedup Theorem" that was presented in class applied to k-tape Turing machines for $k \geq 2$, but required an increase in the number of tapes from one to two when $k = 1$.

   Recall that, for functions $f, g : \mathbb{N} \to N$, $f \in o(g)$ if f grows, asymptotically, *strictly slower* than g: For every constant $\epsilon > 0$ there exists a natural number N such that $f(n) \leq \epsilon \cdot g(n)$ for all $n \in N$ such that $n \geq N$.

   Prove the following "Speedup Theorem" for one-tape Turing machines: If $f : \mathbb{N} \to \mathbb{N}$ is a function such that $n^2 \in o(f)$, there exists a one-tape Turing machine deciding L that uses at most $f(|\omega|)$ steps for every input ω, and ϵ is a constant such that $\epsilon > 0$, then there exists another one-tape Turing machine that decides L using at most $\epsilon \cdot f(|\omega|)$ steps on every input string ω such that $n = |\omega|$ is sufficiently large.