CPSC 511 — Midterm Test

November 12, 2014

Name: ____

Please **DO NOT** write your ID number on this page.

Instructions:

1. Answer all questions in Part A and any TWO questions in Part B.
2. Answer questions in the space provided. You should not need more space than this, but you may continue an answer on pages for unanswered questions in Part B or the back of the last page if you need to.
3. Point form answers are acceptable if complete enough to be understood.
4. No aids are allowed.
5. This test is out of 45.

Duration: 75 minutes

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Part A: Answer ALL Questions.

Question	Score	Available
1		5
2		10
3		10
Total:		25

Part B: Answer TWO Questions.

Question	Score	Available
4		10
5		10
6		10
7		10
Total:		20

Total:

Question	Score	Available
Part A		25
Part B		20
Total:		45

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Part A: Answer ALL Questions.

(5 marks) 1. Consider the following complexity classes:

- DTIME(n^3): The class of languages $L \subseteq \Sigma^\star$ that can be decided by a ***one-tape*** deterministic Turing machine using a number of steps that is in $O(|x|^3)$ on input $x \in \Sigma^\star$ in the worst case.
- DTIME-$2(n)$: The class of languages $L \subseteq \Sigma^\star$ that can be decided by a ***two-tape*** deterministic Turing machine using a number of steps that is in $O(|x|)$ on input $x \in \Sigma^\star$ in the worst case.

Say ***as precisely as you can*** how these complexity classes are related. Mention any results that have been presented in class that help to explain why your answer is correct and say ***briefly*** how these results are used to do this.

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(10 marks) 2. Let $G = (V, E)$ be an undirected graph. Recall that a subset $S \subseteq V$ of the vertices in G is a ***clique*** if there is an edge between u and v for ***every*** pair of distinct vertices $u, v \in S$.

Now consider the following ***decision problems.***

k-Clique

Instance:	An undirected graph $G = (V, E)$ and a positive integer k such that $k \leq \|V\|$
Question:	Does there exist a clique with size at least k in G?

Half-Clique

Instance:	An undirected graph $G = (V, E)$ such that $\|V\| = 2n$ for some positive integer n
Question:	Does there exist a clique with size at least $n = \|V\|/2$ in G?

Use the fact that the ***k-Clique*** problem is $\mathcal{NP}$-complete to prove that the ***Half-Clique*** problem is $\mathcal{NP}$-complete as well.

Note: Please state and use ***reasonable*** assumptions about the encodings of undirected graphs, and the operations on graphs that are carried out efficiently, that are needed to solve this problem.

Hint: Consider some process that *doubles* the number of vertices in a graph in a useful way. ***If you get stuck and cannot see how to do this*** then, for part marks, you should solve the following easier problem instead: Use the fact that the ***Half-Clique*** problem is $\mathcal{NP}$-complete to prove that the ***k-Clique*** problem is $\mathcal{NP}$-complete.

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3. Consider ***log-space mapping reductions.***

(3 marks) (a) Give the definition of a log-space mapping reduction.

(7 marks) (b) Sketch a proof that log-space mapping reductions are ***transitive:*** That is, if $L_1 \subseteq \Sigma_1^\star$, $L_2 \subseteq \Sigma_2^\star$ and $L_3 \subseteq \Sigma_3^\star$, then **if**

- L_1 is log-space mapping reducible to L_2, and
- L_2 is log-space mapping-reducible to L_3,

then L_1 is log-space mapping reducible to L_3 as well.

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Part B: Answer Any TWO Questions.

(10 marks) 4. Prove that $\mathcal{NP} \subseteq$ EXPTIME.

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(10 marks) 5. State and prove ***Savitch's Theorem.*** Then explain how this implies that

$$\text{PSPACE} = \text{NPSPACE}.$$

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(10 marks)

6. Recall that PATH is the language of encodings of a directed graph $G = (V, E)$ and pair of vertices $s, t \in V$ such that there is a path from s to t in G.

   Sketch a proof that PATH is $\mathcal{NL}$-complete (with respect to log-space mapping reductions).

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(10 marks) 7. Give the definition of a ***space-constructible function***. Then sketch a proof that if $S(n)$ is a space-constructible function and ϵ is any ***positive*** constant, then ***if***

- there is a deterministic Turing machine deciding a language $L \subseteq \Sigma^\star$ using space at most $S(|x|)$, on input $x \in \Sigma^\star$, in the worst case

then

- there is a deterministic Turing machine deciding L using space at most $\lceil \epsilon \cdot S(|x|) \rceil$ on input $x \in \Sigma^\star$, in the worst case, as well.

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