CPSC 511 — Fall 2014
Solutions for Question #1 on Midterm Test

In this question you were asked to say, as precisely as you can, how the following complexity classes are related:

- DTIME(n^3): The class of languages $L \subseteq \Sigma^\star$ that can be decided by a ***one-tape*** deterministic Turing machine using a number of steps that is in $O(|x|^3)$ on input $x \in \Sigma^\star$ in the worst case.
- DTIME-$2(n)$: The class of languages $L \subseteq \Sigma^\star$ that can be decided by a ***two-tape*** deterministic Turing machine using a number of steps that is in $O(|x|)$ on input $x \in \Sigma^\star$ in the worst case.

You were also asked to mention any results that have been presented in class that help to explain how your answer is correct, and to say ***briefly*** how these results are used to do this.

Relationship:

$$\text{DTIME-}2(n) \subsetneq \text{DTIME}(n^3).$$

Explanation: It was shown during the first week of classes that a multi-tape deterministic Turing machine using $T(n)$ steps in the worst case (for an input with length n) can be simulated using a one-tape deterministic Turing machine using $O(T^2(n))$ steps in the worst case if $T(n) \in \Omega(n)$. It follows that

$$\text{DTIME-}2(n) \subseteq \text{DTIME}(n^2).$$

On the other hand, since the functions $f(n) = n^2$ and $g(n) = n^3$ are both time constructible, and

$$f(n) \in o(g(n)/(\log g(n))),$$

it follows by the ***Time Hierarchy Theorem*** that

$$\text{DTIME}(n^2) \subsetneq \text{DTIME}(n^3).$$

Thus

$$\text{DTIME-}2(n) \subsetneq \text{DTIME}(n^3)$$

as well.