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How to Build a Maze

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Mazes are fun to solve. With a little imagination, mazes can be incorporated into many different computer games. If you know how, it's a simple matter to use the computer to generate random mazes.

A traditional maze has one starting point and one finishing point. In addition, all locations inside the maze are reachable from the start, and there is one and only one path from start to finish. While it is easy to place doorways and barriers randomly inside a maze, it is more difficult to satisfy the two latter constraints. This article describes a fairly simple method that efficiently produces a random traditional maze.

The General Approach

We begin with a rectangular array. Each cell of the array is initially completely "walled in," isolated from its neighbors (see figure 1).

Secondly, we judiciously erase walls inside the array until we arrive at a structure with the following property: for *any* two cells of the array, there is only one path between them. Thus, any cell can be reached from any cell, but only by a single unique path (see figure 2). Computer-science jargon refers to such a structure as a *spanning tree*, and it is the creation of this spanning tree that is the tricky part of building a maze.

Finally, the border of the maze is broken in two places to provide a start and a finish position. Since there is a unique path between *any* two cells of the maze, there will be a unique path from start to finish. Hence, start and finish can be chosen in any convenient manner, say, at random locations on opposite sides of the maze (see figure 3).

Building the Spanning Tree

Starting with a fully "walled-up" array (see figure 1), pick a single cell in the array and call this cell the

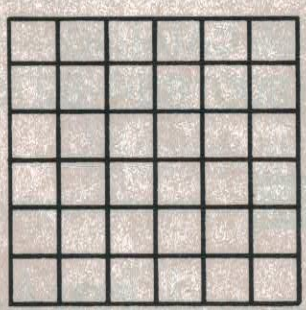


Figure 1: *The initial array from which the maze will be constructed.*

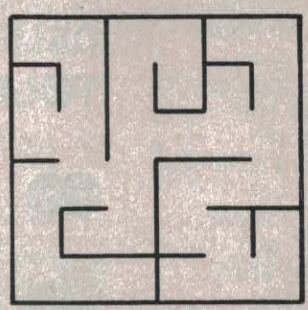


Figure 2: *One possible spanning tree for the array in figure 1.*

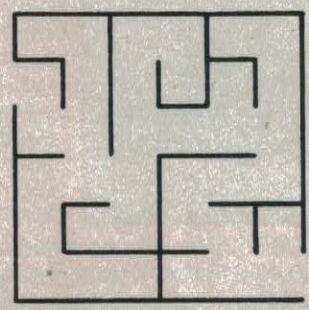


Figure 3: *The spanning tree from figure 2 with possible entry and exit points added.*

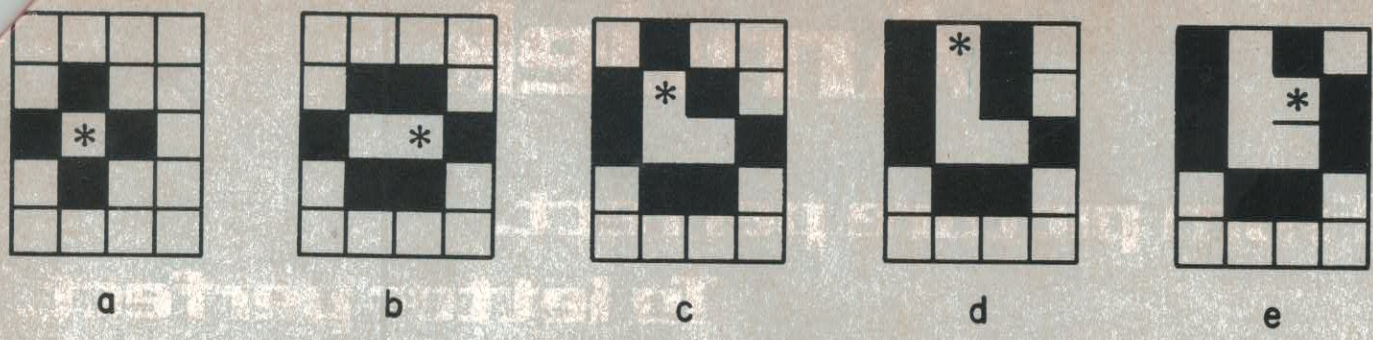


Figure 4: *Initial steps involved in building a maze. The cell added at each step is marked with an asterisk. The next cell to be added to the maze will be selected from the shaded frontier cells.*

spanning tree. Then adds cells one at a time to the spanning tree until it fills the entire array.

At any point during this procedure, there will be three types of cells in the array:

- those that are already in the spanning tree
- those that are not in the spanning tree, but are immediately adjacent (horizontally or vertically) to some cell in the spanning tree (we call these cells *frontier cells*)
- all the other cells

The algorithm follows:

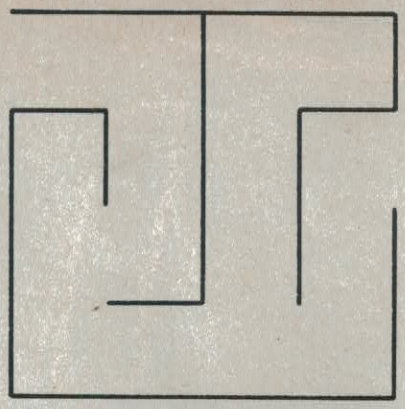
1. Choose any cell of the array and call it the spanning tree. The four cells immediately adjacent to it (fewer if it is on an edge or in a corner) thus become frontier cells.
2. Randomly choose a frontier cell and connect it to *one* cell of the current spanning tree by erasing *one* barrier. If it is adjacent to more than one cell of the spanning tree (and it could be adjacent to as many as four!), randomly choose one of them to connect it to, and erase the appropriate barrier.
3. Check the cells adjacent to the cell just added to the spanning tree. Any such cells that are not part of the spanning tree and have not previously been marked as frontier cells must now be marked as frontier cells.
4. If any frontier cells remain, go back to step 2.
5. Choose start and finish cells.

Figure 4 shows the first few steps in building a maze. In each case the array is shown as it would be just before execution of step 2 of the algorithm. Note that the newly added cell (marked by an asterisk) in figure 4e was adjacent to *two* cells in the spanning tree, yet it was connected to only one of these (the one to its left) by randomly choosing and erasing *one* barrier.

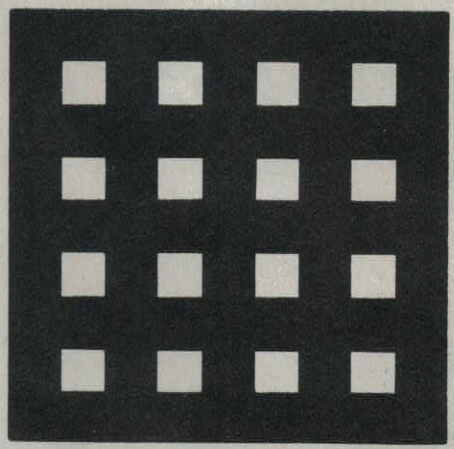
If you're mathematically inclined, it is easy to show by induction that this process results in a spanning tree. When the tree consists of a single cell, there is (vacuously) only one path between any pair of cells. As each new cell is added, it forms no new paths between cells already on the tree (since the tree is a dead end), and there is exactly one path from the new cell to any other cell (you can get out via only one cell, and from that cell there is only one path). Finally, the process ends when there are no more frontier cells (cells adjacent to the spanning tree but not in it), and this can happen only when all cells have been absorbed into the spanning tree.

Implementation Details

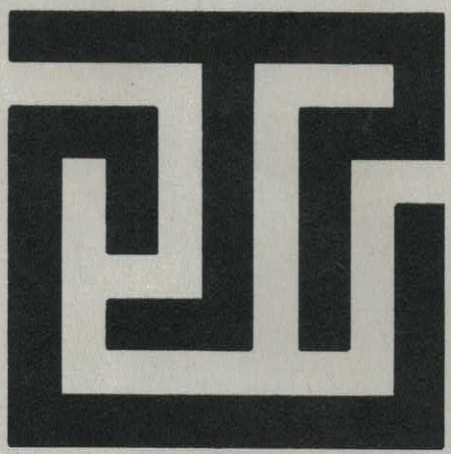
You should store in each cell of the



5a



5b



5c

Figure 5: *For an* m *by* n *maze to be displayed on a computer graphics system, a resolution of at least* 2m+1 *by* 2n+1 *must be available. The 4- by 4-maze array of figure 5a requires a graphics array of 9 by 9. The initial cells of the 4 by 4 array are shown displayed using the 9 by 9 resolution in figure 5b. The finished maze, with openings between the cells where paths exist, is shown in figure 5c.*

array a number indicating: 1. whether it is in the spanning tree, in the frontier, or in neither; and 2. if it is in the spanning tree, which of the cell's barriers have been erased. One possibility is to use—1 for frontier cells, positive numbers for cells, positive numbers for cells in the spanning tree, and 0 for all other cells.

any cell of the spanning tree is open to at least one other cell, I suggest the following encoding: start with 0 in each cell, add 1 if the barrier on the right is erased; add 2 if the barrier below is erased; add 4 if the barrier on the left is erased, and add 8 if the barrier above is erased. The result will be a number from 1 to 15 that specifies exactly which combination of barriers has been erased. (Decoding this number shouldn't be too hard if you work with binary numbers.) Note that when you erase a barrier between two cells you will have to add the appropriate numbers to each of them.

The minor exception mentioned above is the initial cell of the spanning tree, immediately after step 1 of the algorithm (see figure 4a). Since it is the first, it is not yet open to any other cell. Give it the value 16 (or 100, or 1984, if you prefer) so that it will be positive, and subtract this number out again in step 5.

Now that the array representation has been settled, let's discuss efficient implementation of the algorithm. In step 2 a frontier cell was randomly chosen. To prevent bumbling around in the array, you must keep a list of those cells. This can be simply accomplished by storing the indices of the n cells of the frontier (each of which is specified by a row number and a column number) in the first n locations of two arrays, R (row numbers) and C (column numbers). A frontier cell can be quickly chosen by randomly choosing a k less than or equal to n, and using the cell whose indices are given by $R(k)$, $C(k)$.

Since the order of the n frontier cell locations in arrays R and C is not important, the following code suffices to remove the chosen cell k:

$$R(k) = R(n)$$
$$C(k) = C(n)$$
$$n = n-1$$

```
6a  XXXXXXXXX
      X   X
    XXX X XXX
    X X X X
    X X X X X
    X   X X X
    X XXX X X
    X       X
    XXXXXXXXX

6b  XXXXXXXXXXXXXXXXX
              XX      XX
    XXXXXX    XX  XXXXXX
    XX  XX    XX  XX
    XX  XX    XX  XX  XX
    XX        XX  XX  XX
    XX  XXXXXX    XX  XX
    XX                XX
    XXXXXXXXXXXXXXXXX
```

Figure 6: *The maze of figure 5 as it might appear as printed output, with each maze-array element represented by space characters or X characters. One space or X is used in 6a; two spaces or Xs are used in 6b.*

When this frontier cell is added to the spanning tree, some of the cells adjacent to it (those having a zero value) become new frontier cells, and their locations must be inserted into the R and C arrays. Adjacent cells with value −1 are already frontier cells and already have their locations recorded in the R and C arrays; they must not be inserted again.

Finally, how large should arrays R and C be? For an m by n array, analysis shows that in the worst case $(2/3)\ mn$ locations will be required, but practical experience shows that $3(m+n)$ is almost always enough. However, if you use the latter figure there is a slight probability that the program will fail.

Concluding Remarks

While we have discussed building a maze, nothing has been said regarding how to display it. That depends entirely on your particular hardware and software; the answers are different for the display screen of a Commodore PET than for that of an Apple II, and different again for a character printer.

To display a maze on a screen with graphics capabilities, the following scheme is appropriate. For an m by n maze, you need to be able to display at least $2m+1$ points vertically and $2n+1$ points horizontally—the "cells" will be those points at the intersection of even-numbered rows with even-numbered columns (see figures 5a through 5c). Maze building on the screen proceeds exactly as in figures 1 through 3, except that the walls are necessarily thicker.

To print a maze out, the same general scheme is used with, say, "X" characters for walls and blanks for paths (see figure 6). Of course, you can't erase an X once it is printed, so it will be necessary to build the entire maze internally before printing it. Then you can decipher and print the maze one row at a time.

As a final note, if you are an aficionado of hexagonal grids, the maze algorithm is easily modified for other than rectangular grids. Implementation may be a bit messy—but then, implementation is always messy. ■