

Ejercicios

Pruebe las siguientes relaciones

1. $X^2 = Y^2 = Z^2 = I$

2. $H = \frac{1}{\sqrt{2}}(X + Z)$



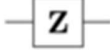
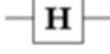
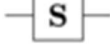
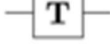
3. $X = HZH$

4. $Z = HXH$

5. $-1Y = HYH$

6. $S = T^2$

7. $-1Y = XYX$

Operator	Gate(s)	Matrix
Pauli-X (X)	 \oplus	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
Pauli-Y (Y)		$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$
Pauli-Z (Z)		$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
Hadamard (H)		$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
Phase (S, P)		$\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$
$\pi/8$ (T)		$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$

$$\textcircled{1} \quad \underline{X^2 = Y^2 = Z^2 = \underline{I}}$$

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$X^2 = X'X' = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 \cdot 0 + 1 \cdot 1 & 0 \cdot 1 + 1 \cdot 0 \\ 1 \cdot 0 + 0 \cdot 1 & 1 \cdot 1 + 0 \cdot 0 \end{pmatrix} = \boxed{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}$$

$$Y^2 = Y'Y' = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \begin{pmatrix} 0 \cdot 0 + (-i) \cdot i & -i \cdot 0 + (-i) \cdot 0 \\ i \cdot 0 + 0 \cdot i & i \cdot (-i) + 0 \cdot 0 \end{pmatrix}$$

$$\begin{cases} i \times -i = -i \cdot i \\ \text{por def.} = -1 \end{cases}$$

$$= \boxed{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}$$

$$Z^2 = Z'Z' = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 \cdot 1 + 0 \cdot 0 & 0 \cdot 1 + 0 \cdot (-1) \\ 0 \cdot 1 + (-1) \cdot 0 & 0 \cdot 0 + (-1) \cdot (-1) \end{pmatrix} = \boxed{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}$$

$$\boxed{Id = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}$$

Probado.

$$\textcircled{2} \quad H = \frac{1}{\sqrt{2}} (X + Z)$$

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$\begin{aligned} \boxed{\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}} &= \frac{1}{\sqrt{2}} \left(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right) \\ &= \frac{1}{\sqrt{2}} \begin{pmatrix} 0+1 & 1+0 \\ 1+0 & 0+(-1) \end{pmatrix} = \boxed{\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}} \end{aligned}$$

$$\textcircled{5} \quad -1Y = HYH$$

$$-1 \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \underbrace{\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}}_1 \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$\underline{\begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \underline{\begin{pmatrix} i & -i \\ -i & -i \end{pmatrix}}$$

$$= \frac{1}{\sqrt{2}} \begin{pmatrix} i & -i \\ -i & -i \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} i & -i \\ -i & -i \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 0 & 2i \\ -2i & 0 \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 2i \\ -2i & 0 \end{pmatrix} \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 2i \\ -2i & 0 \end{pmatrix} = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$$

= ?

Probada

Regla de Born

¡Importante!

$$\text{Probability}(x) = |\text{amplitude}(x)|^2$$

- Se utiliza para determinar la probabilidad de medir un estado cuántico en un valor particular.
- Se usa para realizar predicciones sobre el comportamiento de sistemas cuánticos

Ejemplo:

Tenemos un qubit en el estado $|0\rangle$. Podemos aplicar la compuerta Hadamard y tenemos:

$$H|0\rangle = |+\rangle \quad |+\rangle \rightarrow \text{estado superpuesto}$$

$$\text{podemos expresar } |+\rangle \text{ como } |+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

1. La probabilidad de medir el estado $|0\rangle$ se calcula como el coeficiente de $|0\rangle$ al cuadrado en superposición.

$$P(|0\rangle) = |\langle 0|+\rangle|^2 = \frac{(\langle 0|0\rangle + \langle 0|1\rangle)}{2} = \frac{1}{2}$$

$$P(|1\rangle) = |\langle 1|+\rangle|^2 = \frac{(\langle 1|0\rangle + \langle 1|1\rangle)}{2} = \frac{1}{2}$$