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Thermodynamics of Circumbinary Disks
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1 Introduction

We consider the evolution of a circumbinary disks. We do not yet allow for a dead zone in the disk.

The thermal timescale is much smaller than the viscous timescale in a circumbinary disk, and so we approximate the temperature in equilibrium.

2 Thermodynamics

We allow for heating of the disk due to viscous dissipation, irradiation from the binary, and dissipation of density waves due to a tidal torque.

Assuming thermal equilibrium, we have, at low optical depths, $\tau \ll 1$,

$$\tau \sigma T_m^4 = F_{tid} + F_\nu + \tau F_{irr} \,, \tag{1}$$

and at high optical depths, $\tau \gg 1$,

$$\sigma T_m^4 = F_{tid} + \frac{3\tau}{4} F_\nu + F_{irr} \,. \tag{2}$$

Extrapolated to all optical depths,

$$\sigma T_m^4 = \left(1 + \frac{1}{\tau}\right) F_{tid} + \left(\frac{3\tau}{4} + \frac{1}{\tau}\right) F_v + F_{irr} \,, \tag{3}$$

where T_m the disk midplane temperature. The optical depth is defined as $\tau = \frac{1}{2}\kappa\Sigma$, where κ is the opacity. We will assume a power-law for the opacity,

$$\kappa = \kappa_o T^\beta \,. \tag{4}$$

We will use the prescription in Bell & Lin 2004 for opacity.

The tidal heating term per surface is

$$F_{tid} = \frac{1}{2} \left(\Omega_b - \Omega\right) \Lambda \Sigma \,, \tag{5}$$

where $\Omega_b = \sqrt{GM/a^3}$ is the binary angular velocity, with a the binary semimajor axis, $\Omega = \sqrt{GM/r^3}$ is the Keplerian velocity, M is the total mass of the binary, and Λ is the torque due to the binary. At radius $r > h(r) + a$,

$$\Lambda = f \frac{q^2 GM}{a} \left(\frac{a}{r-a}\right)^4 \,, \tag{6}$$

where q is the binary mass fraction and f is a correction factor that fixes the inner disk truncation radius. Note that Martin 2013 fix $f = 1$, which places their disk inner edge further out.

The disk height $h(r)$ is given by

$$\frac{h}{r} = \left(\frac{k\,T\,r}{GM\mu}\right)^{\frac{1}{2}} \,, \tag{7}$$

by assuming hydrostatic equilibrium for a gas-pressure dominated disk. The heating due to irradiation from the binary is

$$F_{irr} = \frac{1}{2} \frac{L_c}{4\pi r^2} \zeta \,, \tag{8}$$

where ζ is the incidence angle of radiation from the binary onto the disk at a height ηh. Far from the binary, we can approximate ζ as

$$\zeta = r\frac{d}{dr}\left(\frac{\eta h}{r}\right) . \tag{9}$$

The heating due to viscous dissipation is

$$F_\nu = \frac{3}{8\pi}\frac{F_J\Omega}{r^2} = \frac{9}{8}\Omega^2\nu\Sigma , \tag{10}$$

where the second equality comes from the definition of the angular momentum flux $F_J = 3\pi\nu\Sigma l$, with $l = \Omega r^2$, the specific angular momentum. (*adjust for dead zone*) The viscosity is expressed as

$$\nu = \alpha\frac{c_s^2}{\Omega} \tag{11}$$

where the sound speed is $c_s = \sqrt{\frac{P}{\rho}}$ and the viscosity is for a gas-pressure dominated disk

$$v = \alpha\frac{k_B T}{\mu\Omega} . \tag{12}$$

3 Disk Evolution

We can now evaluate the disk evolution equation by expressing viscosity as a function of surface density Σ and radius r.

The disk evolution equation is

$$\frac{\partial\Sigma}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left[3r^{1/2}\frac{\partial}{\partial r}\left(r^{1/2}\nu\Sigma\right) - \frac{2\Lambda\Sigma r^{3/2}}{(GM)^{1/2}}\right] . \tag{13}$$

4 Conservation Equations

The equation for mass conservation is

$$M = \int 2\pi r\Sigma\, dr , \tag{14}$$

and for momentum conservation,

$$L = \int 2\pi r\Sigma\, r^2\Omega\, dr . \tag{15}$$

We express the evolution equation in a conservative form, in terms of Σ rather than F_J. This will facilitate verifying conservation every time step of the numerical procedure.

5 Initial and Boundary Conditions

We will set $\Sigma = 0$ for our inner boundary condition. Our outer boundary condition is irrelevant because our solution will never reach it, so we can likewise set $\Sigma = 0$ at our outer boundary condition.

We can derive a more general inner boundary condition with the hypothesis $\partial\dot{M}/\partial l = 0$, or equivalently,

$$\frac{\partial}{\partial l}\left(3r^{1/2}\frac{\partial}{\partial r}\left(r^{1/2}\nu\Sigma\right) - \frac{2\Lambda\Sigma r^{3/2}}{(GM)^{1/2}}\right) = 0 . \tag{16}$$

The binary torque Λ vanishes to 0 at the inner edge. If we further assume that the accretion rate $\dot{M}$ is 0, we get that Σ must be a constant, which we can fix to 0 and return our inner boundary condition above.

I will add initial surface density and temperature profiles soon. For now, we are using a delta function for the surface density.

6 Numerical Method

We will use FiPy to solve Eq. 13. The package supports partial differential equations with a nonlinear diffusion term and an advection term.