1 Magnetar Dipole Radiation

We model magnetars similarly to pulsars and assume a magnetic dipole toy model. The dipolar magnetar field is:

$$B(\vec{r}) = \frac{3\vec{n}(\vec{m}.\vec{n}) - \vec{m}}{r^3}, \tag{1}$$

where $\vec{m}$ is the magnetic moment and $\vec{n}$ is the unit radial vector.
In analogy with Larmor's formula for electric dipole radiation, a time-dependent magnetic dipole radiates

$$\frac{dW}{dt} = -\frac{2}{3c^3} * |\ddot{\vec{m}}_\perp|^2, \tag{2}$$

where $\vec{m_\perp}$ is the component of $\vec{m}$ perpendicular to rotation axis.
Defining the angle between the rotation axis and the magnetic dipole moment as α,

$$\vec{m}_\perp = m_o \sin(\alpha) e^{-Iwt}, \tag{3}$$

so $|\ddot{\vec{m}}|_\perp^2 = {m_o}^2 \sin\alpha^2 w^4$ since $m_o = BR^3/2$ for a uniformly magnetized sphere.
It follows that

$$\frac{dW}{dt} = -\frac{{B_p}^2 R^6}{6c^3} w^4 \sin^2\alpha, \tag{4}$$

If we assume the magnetic dipole is oriented perpendicularly to the rotation axis so $\alpha = \pi/2$, the luminosity is powered by spin-down. We define the magnetar spin period $P = 2\pi/\omega$ and arrive at equation 2 from Lyons 2010:

$$L = 9.62065 * 10^{48}\, B_{p,15}^2 P_{-3}^{-4} R_6^6 \,\mathrm{erg\,s^{-1}}, \tag{5}$$

where $B_{p,15} = B_p/10^{15}$, etc.
Next we assume dipole radiation taps the rotational energy of the magnetar, so $\frac{dE_{rot}}{dt} = \frac{dW}{dt}$ where $E_{rot} = 1/2Iw^2$ so $\ddot{E}_{rot} = I\omega\ddot{\omega}$. Define a characteristic dipole spindown time τ_{dipole} as $\tau_{\mathrm{dipole}} = -\omega/\ddot{\omega}$ It follows that

$$\tau_{\mathrm{dipole}} = \frac{3c^3 I}{{B_p}^2 R^6 \omega^6}, \tag{6}$$

Then,

$$\tau_{\mathrm{dipole}} = 2051.75\ I_{45} B_{15,p}^{-2} P_{-3}^2 R_6^{-6}\,\mathrm{s}, \tag{7}$$

which is equation 3 in Lyons 2010.
Lyons assumes $P = P_o$, using initial period instead and neglecting spindown.

2 Spindown Times and Plateaus

We seek to disclaim the argument in Lyons 2009 [1] that plateau phases and spindown can be explained by the dipole radiation model of a magnetar. Following Piro Ott 2011[2] and neglecting spindown from fallback accretion, we have

$$I\dot{\Omega} = N_{\mathrm{dip}}, \tag{8}$$

where $I = .35MR^2$ and $N_{\mathrm{dip}} = -\mu^2\Omega^3/6c^3$. We solve for angular velocity as a function of time

$$\Omega = \frac{\sqrt{\frac{21}{2}} c^{3/2}\sqrt{M}R}{\sqrt{10t\mu^2 - 21c^3MR^2y}}, \tag{9}$$

Together with 4, we can solve for spindown radiation luminosity as a function of time, arriving at

$$L_{\rm dip} = \frac{147B^2c^3M^2R^{10}}{8(-21c^3MR^2y + 5/2B^2R^6t)^2}, \tag{10}$$

where we have used $\mu = BR^3/2$, the magnetic moment for a uniformly magnetized sphere, and y is a negative value related to initial period P_o by $P_o = 2\pi\sqrt{-2y}$.

Note the interesting result that luminosity may actually decrease with increasing magnetic field at a given time. A stronger field brakes the magnetar, decreasing its spin frequency as seen in 9. Since frequency comes in to the inverse 4th power, while the magnetic field comes in only to the 2nd power in 4, luminosity may indeed decreases with higher magnetic field.

2.1 Assumptions

We correct for anistropic emission using expression 5 of Lyons 2009 [1].

$$E_{\rm beam} = (1 - \cos\theta_b)E_{\rm iso}, \tag{11}$$

where θ_b is the beam's opening angle, and we assumed that this does not change with time. Thus the analogous correlation holds for luminosity.
I have neglected K correction thus far. Lastly, I have assumed $M = 1.4M_\odot$, $R = 10^6$ cm unless noted otherwise.
The assumption of $1.4M_\odot$ for the magnetar mass seems troublesome since the noncanonical model demands magnetar collapse to a black hole to shut off the light curve, but $1.4M_\odot$ is a lower limit on NS mass - we don't expect collapse to a BH. However, the requisite near breakup spin may prevent an intermediate magnetar from forming, allowing immediate NS collapse to BH. Could a similar breakup instability lead to BH collapse for these low mass NS?
I have used Ned Wright's Java Cosmology Calculator for Standard Cosmological Model to arrive at luminosity distances.
The figures below are light curves using data from Swift for LGRB 101225A in the .3-10 keV bandpass.

3 K-correction

I K-correct into the X-ray bandpass in the frame of the magnetar as follows. (include note on comoving vs luminosity distance) The spectral indices Γ are available from SWIFT. The spectral index β (define eqn) is simply $\Gamma - 1$ and the K-corrected luminosity is then:

$$L_{[.3-10keV]} = \frac{4\pi f_{[.3-10keV]}}{d_L^2}(1+z)^{-1+\beta} \tag{12}$$

4 Analysis

We will consider 2 checks on the magnetar model for noncanonical light curves: 1), does the timescale of the plateau agree with physical constraints from dipole spindown, and can the steep cutoff also be explained by the dipole model?

4.1 Possible Scenarios Involving a Magnetar as GRB Central Engine

- We can have a plateau and apparent cutoff explained entirely by the magnetar dipole curvature. For instance, see GRB 060607 4.

- We can have accretion induced collapse to a blackhole for a variety of magnetar initial masses, from $1.5 - 2.4\, M_{\odot}$ and a variety of accretion parameters $.1 - 10$.

- We can have spindown collapse, where centrifugal support is no longer able to sustain the magnetar even in the case of no accretion.

  We may initially have a hypermassive, $> 2.5 M_{\odot}$, differentially rotating magnetar. However, bar mode, magnetic, and other instabilities will redistribute angular momentum to make the magnetar rigidly rotating within the first second. Thus the above collapse arguments still apply (citation needed).

- We may also have no collapse and see an indefinite plateau, or immediate collapse and thus no plateau (find examples).

References

[1] N. Lyons, P.T. O'Brien, B. Zhang, R. Willingale, E. Troja, et al. Can X-Ray Emission Powered by a Spinning-Down Magnetar Explain Some GRB Light Curve Features? 2009.

[2] Anthony L. Piro and Christian D. Ott. Supernova Fallback onto Magnetars and Propeller-Powered Supernovae. *Astrophys.J.*, 736:108, 2011.

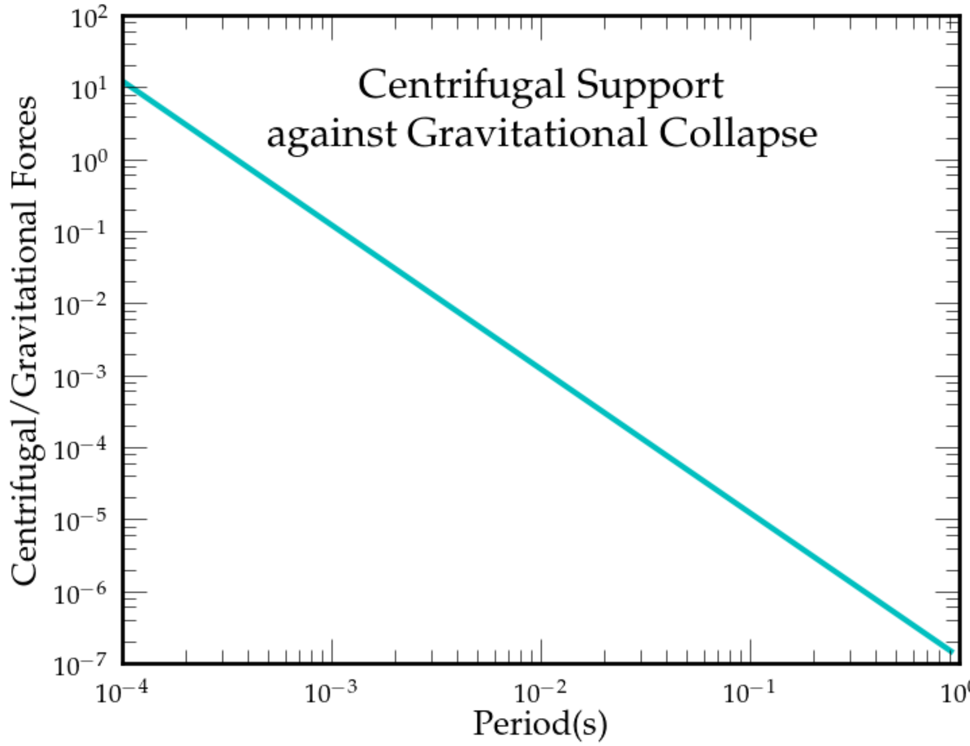

Figure 1

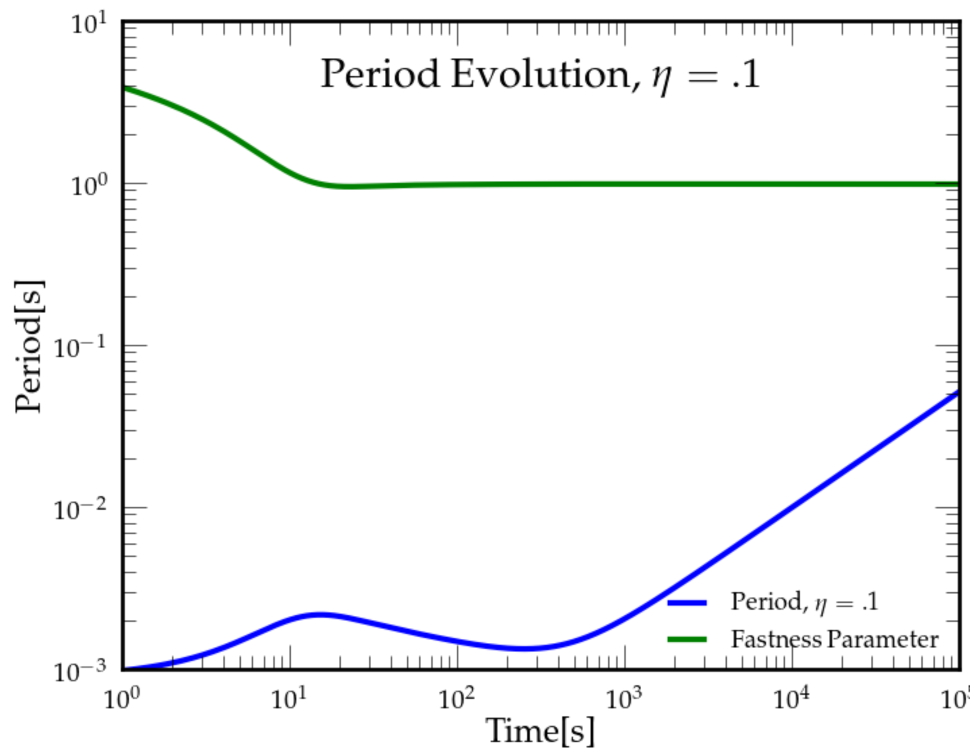

Figure 2: Period evolution of a $10^{15}\,G$, $2.4M_{\odot}$ magnetar. If it survives to 10s without collapse, the magnetar will be able to power a plateau 10^{3} G. This calculation accounts for accretion with η characterizing the explosion energy of the associated supernovae. Note that we are in the propeller regime since the fastness parameter > 1.

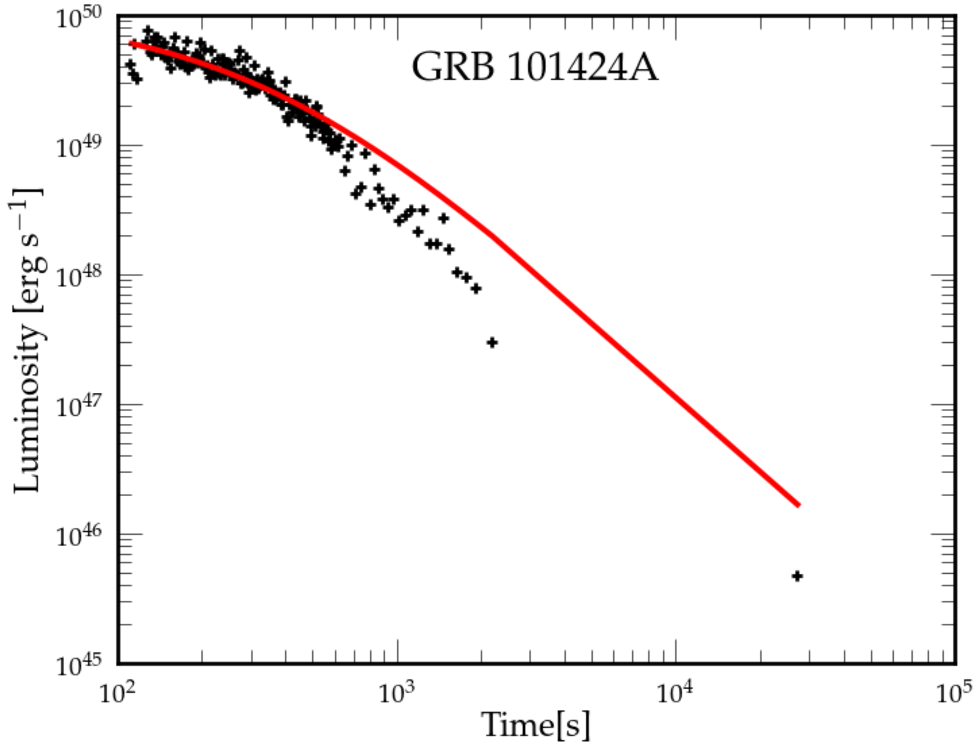

Figure 3: GRB 101424A $8.06 * 10^{15}$ G, $2.5M_{\odot}$ 3° beam, 4 ms initial period best fit

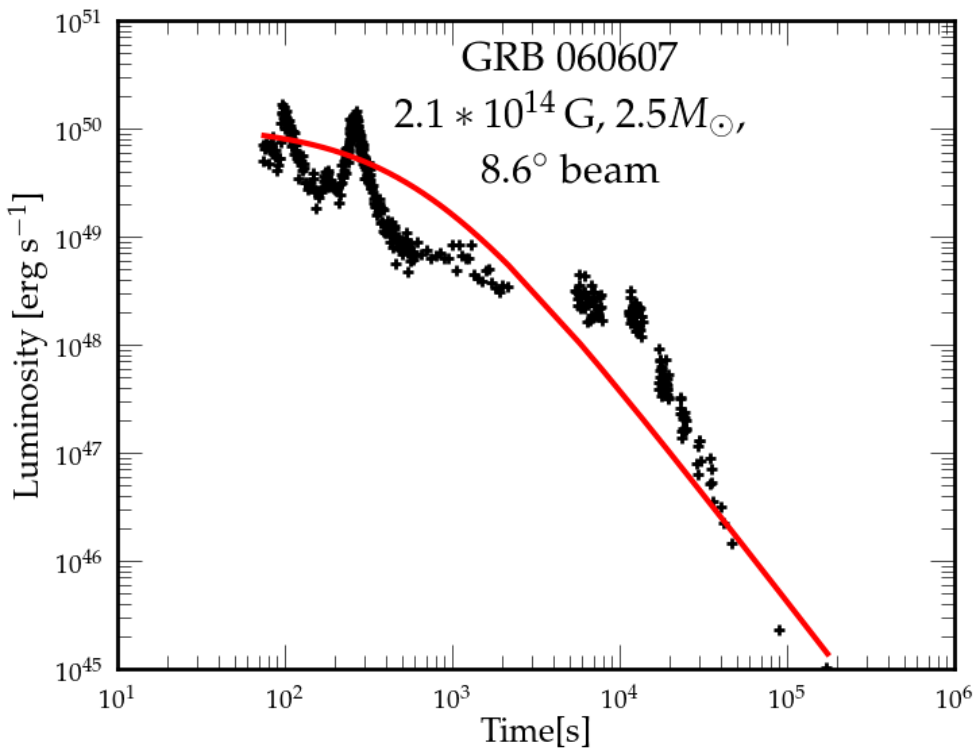

Figure 4: GRB 060607 $2.1 * 10^{14}$ G, $2.5M_{\odot}$, 8.6° beam, 1 ms initial period

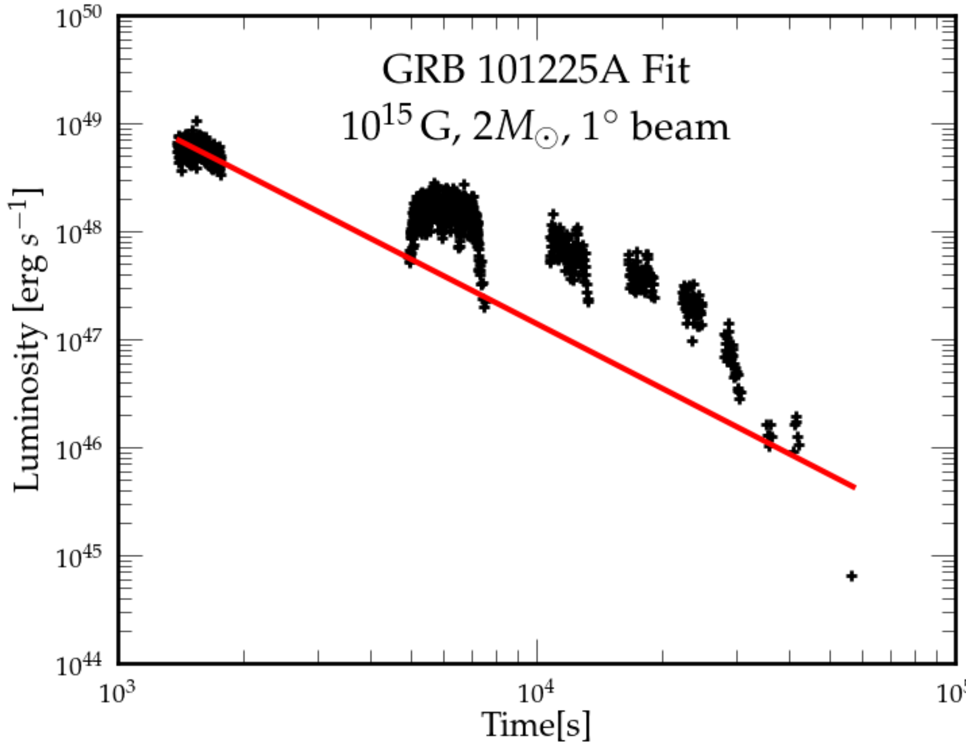

Figure 5: GRB 101424A $1 * 10^{15}$ G, $2.0 M_{\odot}$ 1° beam, 1 ms initial. Note both the lack of fit and the extended duration of the plateau which cannot be centrifugally supported as seen in figure 2

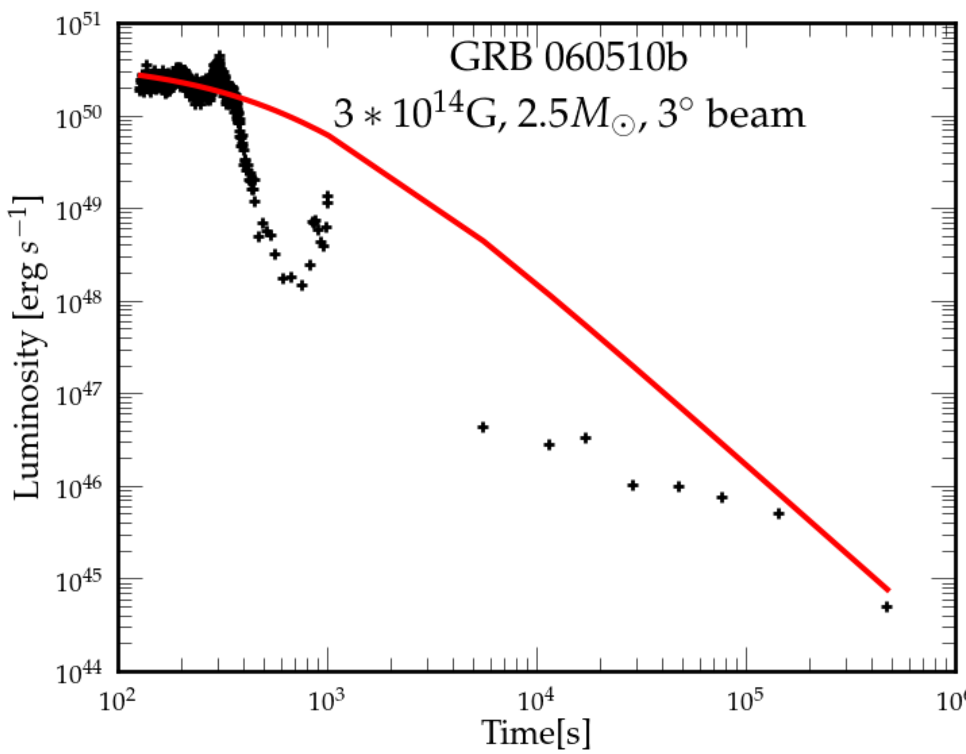

Figure 6: GRB 060510b $3 * 10^{14}$ G, $2.5 M_{\odot}$, 3° beam, 1 ms initial period. Note that the dipole law cannot explain the early onset steep decay.