Statistical Inference Project 1

August 9, 2015

Overview

In this project, the exponential distribution will be investigated and compared with the Central Limit Theorem using simulations written in R. For the purpose of this project, lambda λ will be set to 0.2 and the distribution of 40 exponentials will be investigated. The project will be conducted using 1000 simulations.

Simulations

In the following simulation, 40 exponentials are calculated and averaged. The simulation is done 1000 times.

```r
set.seed(8310) # For reproducibilty
lambda <- 0.2
number.of.simulations <- 1000
n <- 40  # distribution of 40 exponentials will be investigated.

simulation <- matrix(rexp(number.of.simulations * n, rate=lambda),
                     number.of.simulations, n)
simulation.mean <- rowMeans(simulation)  # Get mean of each row (each simulation)
```

Sample Mean versus Theoretical Mean

Requirement 1. *Show the sample mean and compare it to the theoretical mean of the distribution.*

The theoretical mean is given by the formula $1/\lambda$. Here the theoretical mean is calculated and compared with the sample mean that the simulation yielded.

```r
theoretical.mean <- 1/lambda
theoretical.mean
```

```
## [1] 5
```

```r
sample.mean <- mean(simulation.mean)
sample.mean
```

```
## [1] 5.002704
```

The theoretical mean is 5 whereas the sample mean is centered at 5.0027045.

Sample Variance versus Theoretical Variance

Requirement 2: *Show how variable the sample is (via variance) and compare it to the theoretical variance of the distribution.*

For the exponential distribution, the theoretical standard deviation σ is given by the formula $1/\lambda$ so the expected value of the variance of distribution of means is given by the formula σ^2/n or $(1/\lambda)^2/n$.

```r
theoretical.variance <- (1/lambda)^2/n
theoretical.variance
```

```
## [1] 0.625
```

```r
sample.variance <- round(var(simulation.mean), 3)
sample.variance
```

```
## [1] 0.629
```

So in the simulations run for this report, the theoretical variance is 0.625 whereas the sample variance is 0.629.

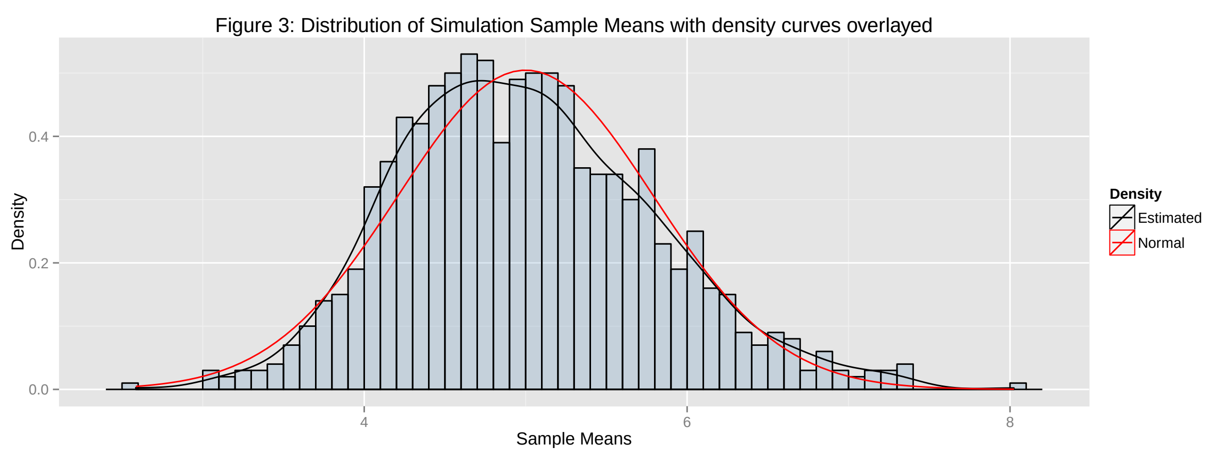
Distribution

Requirement 3: *Show that the distribution is approximately normal*

Because of the Central Limit Theorem, the means of samples should follow a normal distribution.

To determine the shape of the distribution, a density estimate curve of the data is drawn.

```r
simulation.mean.df <- data.frame(simulation.mean)  # convert to dataframe for plotting
plot <- ggplot(simulation.mean.df, aes(x = simulation.mean))
plot <- plot + geom_histogram(aes(y=..density..), color="black",
                              fill = "steelblue", binwidth = 0.1, alpha = 0.2)
plot <- plot + geom_density(aes(color = "Estimated"))
plot <- plot + stat_function(aes(color = "Normal"),
                             fun = dnorm, args = list(mean = 5, sd = sqrt(0.625)))
title <- "Figure 3: Distribution of Simulation Sample Means with density curves overlayed"
plot <- plot + ggtitle(title)
plot <- plot + ylab("Density") + xlab("Sample Means")
plot <- plot + scale_colour_manual("Density", values = c("black", "red"))
plot
```



As can be observed in Figure 3, the shape of the distribution indicates that it is approximately normal. A normal curve (in red) is overlayed for comparison.