Dynamic Airline Scheduling

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Abstract

Demand stochasticity is a major challenge for the airlines in the quest to produce profit maximizing schedules. Even with an *optimized* schedule, many flights upon departure have empty seats, while others suffer a lack of seats to accommodate passengers who desire to travel. We approach this challenge, recognizing that demand forecast quality for a particular date improves as the date approaches, by developing a *dynamic scheduling* approach that re-optimizes elements of the flight schedule during the passenger booking process. The goal is to match capacity to demand, given the many operational constraints that restrict possible assignments. We introduce flight re-timing as a dynamic scheduling mechanism and develop re-optimization models that integrate both flight re-timing and flight re-fleeting. Our re-optimization approach, re-designing the flight schedule at regular intervals, utilizes information from both revealed booking data and improved forecasts available at later re-optimizations. Experiments are conducted using data from a major airline in the United States. We investigate properties of our models and demonstrate that significant potential profitability improvements are achievable using this approach.

1 Introduction

It has been a major challenge for airlines to design a flight schedule, that is, the timetable and the corresponding fleeting on each flight leg, to match fluctuating passenger demand. The flight schedule, defining the supply side of the passenger air transportation system, is designed well in advance, typically six months to one year prior to its implementation, due to contractual and operational requirements in the industry. The design process to generate a schedule that has the maximum profit potential utilizes macro-forecasts of the economy and the airline industry, forecasts of passenger demand, estimates of average fares, and estimates of available resources, such as aircraft, personnel, and gates. The resulting schedule is published through different distribution channels. Despite the fact that scheduling decisions are made at a time when demand is highly uncertain, the flight schedule is intended to remain unchanged once published. More often than not, the published schedule fails to allocate the *optimal* number of seats, that is, capacity, to where it is needed.

During the *booking period*, that is, the time between the date the schedule is published and the departure date, airlines employ different revenue management techniques to maximize the schedule's revenue. By increasing fares on highly demanded flights to decrease low fare demands and reducing fares on flights with excess capacity to stimulate travel, revenue management techniques help to smooth demand variations. Notwithstanding these techniques, the stochastic nature of passenger demand still results in some revenue being lost due to non-optimal allocations of capacity.

To achieve the goal of balancing supply and demand, researchers have started to focus their attention on the supply side and the concept of dynamic airline scheduling was born, that is, the flight schedule is re-optimized during the booking period using improved demand forecast. Early discussions of the concept of dynamic airline scheduling can be found in Etschmaier and Mathaisel (1984) and Peterson (1986). In a survey of aircraft scheduling problems, Etschmaier and Mathaisel (1984) mention dynamic scheduling as an emerging operating philosophy, where the exact schedule could be made as the total demand situation evolves. Peterson (1986) proposes the idea of re-fleeting the schedule during the booking period to better match updated forecasts. Such re-fleeting is allowed only within the same fleet *family*, which is a set of crew-compatible aircraft types. Hence, any pilot qualified to operate one fleet type within a family is, by definition, qualified to operate all fleet types in that family. The requirement of re-fleeting within families is critical because it ensures that the crew assignment based on the initial schedule can remain intact after re-fleeting. This idea is developed, implemented and tested by Berge and Hopperstad (1993) as "Demand Driven Dispatch" (D^3). The re-fleeting problem is formulated as a multi-commodity network flow problem and heuristics are developed to solve it. In the simulation study, several planning points are set in the booking period. At each planning point, the simulator gathers incremental booking information since the last planning point. Based on cumulative bookings received before the current planning point and historical information, an updated demand forecast is generated. Aircraft are re-assigned to all flight legs using the new forecast and leg capacities are updated in the reservation system. It is assumed that booking demand follows a normal distribution truncated at zero and is specified by flight leg and fare class. It is further assumed that there are no recapture of passengers and no cancelation of booked passengers. Berge and Hopperstad (1993) evaluate the approach with computational experiments performed on a network including 22 airports, 40 aircraft representing three models from the Boeing 737 family and 244 flights per day. An improvement of 1-5% in operating profits using D^3 is reported.

Bish et al. (2004) further restrict re-fleeting to be aircraft swaps between two *swappable loops*, each of which consists of a round-trip originating and terminating at a common airport with similar time frames. Such a restriction ensures that the aircraft assigned to those routes can be swapped without violating aircraft flow balance. Two swapping strategies are analyzed, with one strategy allowing swapping only once prior to flight departure and the other allowing multiple swaps prior to departure. The conditions under which the different strategies are effective are studied.

Recently, Sherali et al. (2005) present a Demand Driven Re-fleeting (DDR) model for a single fleet family. The re-fleeting model developed is essentially the Itinerary-Based Fleet Assignment Model (see Barnhart et al. 2002) with additional constraints to limit fleeting decisions within a specific family. While the assumption of no recapture in Berge and Hopperstad (1993) is maintained, the assumption of leg-fare class-based passenger demand is relaxed and path-fare class-based passenger demand is considered. Several reformulations and partial convex hull construction mechanisms are developed, together with various classes of valid inequalities to tighten the DDR formulation. Improvements in computational speed are reported, yet the benefit of DDR is not quantified.

To the best of our knowledge, all past research in the area of dynamic scheduling relied solely on flight re-fleeting. In this paper, we introduce a new mechanism, referred to as flight re-timing, in which the timetable of the original schedule is altered slightly during the booking period to adjust the supply of seats provided in various markets. By shifting the arrival time of a flight leg inbound to a hub, and the departure time of an outbound flight from the hub, the set of feasible passenger connections may change, thereby increasing or decreasing the number of seats available in the affected markets.

A dynamic scheduling approach for airlines that integrates both flight re-fleeting (also referred to as aircraft swapping) and flight re-timing is developed. In this approach, the two dynamic scheduling mechanisms are carried out one or more times during the booking period. The goal is to adjust, for each day, the capacity provided so that it will match the particular passenger demand realizations for that day more closely. These adjustments are made well in advance, perhaps 3-4 weeks prior to implementation of the schedule, to allow sufficient time for maintenance and crew planning. Another aspect that differentiates our approach from past research is that we relax the no recapture assumption and model partial recapture in our models.

We define the contributions of our research as follows:

- We introduce a new dynamic scheduling mechanism, that of flight re-timing, and develop a schedule re-optimization model that combines both flight re-fleeting and flight re-timing. Experiments are conducted using data from a major U.S. airline.

- We demonstrate that dynamic scheduling improves profitability by 2.5-5%, or $18-36 million annually. We study the effects of forecast quality on these benefits and show that considerable benefits remain even when simple forecasts calculated from historical data are used. We also report that the full benefit of re-timing is achieved even when the number of flight legs that are re-timed is strictly restricted.

- We compare and analyze the effectiveness of flight leg re-timing and re-fleeting, our two dynamic scheduling mechanisms, when applied alone under different forecast scenarios. Flight re-timing demonstrates less sensitivity to the deterioration of forecast quality and contributes a larger portion to the potential benefit of dynamic scheduling.

- We show that benefits remain significant when dynamic scheduling is applied to weekly schedules, in which day-of-week demand variations are explicitly considered in constructing the schedules.

The remainder of this paper is organized as follows. In Section 2, we present the evolution of airline route networks and flight schedules in the U.S. to illustrate industry trend. The opportunity in a de-peaked hub-and-spoke network is discussed in Section 3. We detail our modeling architecture for dynamic airline scheduling in Section 4 with corresponding mathematical formulations in Section 5. Solution algorithm approach and computational experiences are presented in Section 6. In Sections 7 and 8, we present the setup of two case studies and the results of our computational experiments. Section 9 provides a review of important elements in this research. Finally, we conclude this discussion and summarize our findings in Section 10.

2 Airline Route Networks and Flight Schedules

In passenger air transportation, cities are connected to each other by flights. A *route network*, or *network* in short, describes how the cities are connected. *Flight schedule*, or *timetable*, describes the flight departure and arrival times in a route network. In this section, we review the evolution of airline route networks and flight schedules.

2.1 Linear Networks

Prior to deregulation in the U.S., many airlines operated over point-to-point networks in which passengers are transported directly from point of origin to point of destination without intermediate stops. This is because under regulation, there was pressure on the airlines from local communities and the Civil Aeronautics Board (CAB) to provide these direct, point-to-point services. Any airline that chose not to exercise its franchise for nonstop service in a particular market (or origin-destination pair) took the risk of the CAB revoking that airline's permission to serve that market.

Many city pairs, however, did not have sufficient demand to cover the cost of nonstop service. Therefore, cities were often added on either end of a nonstop route to create *backup markets*. Demands in backup markets were used to fill seats on the nonstop leg. Revenue from these backup markets helped to make the nonstop service economically viable. The inclusion of these backup markets resulted in an evolved network structure, referred to as as a *linear network*. In linear networks, an aircraft begins at an origin airport and makes a number of intermediate stops along its route to a destination airport. The intermediate stops are made either to refuel or to pickup and discharge passengers.

2.2 Hub-and-Spoke Networks with Banked Flight Schedules

Deregulation in the U.S. has led to significant changes in the airline route networks. After deregulation, airlines quickly adopted *hub-and-spoke* networks. In hub-and-spoke networks, airlines designate several, typically large, cities as their hubs. Nonstop flights between smaller cities are substantially reduced; instead flight services for smaller cities are provided by connecting two nonstop flight legs to and from a hub airport. The major advantage of hub-and-spoke networks compared to other network structures is the disproportionately large number of city-pairs that can be served with a given number of aircraft miles operated (see Wells and Wensveen 2004, chap. 12).

Besides providing non-stop services between hub and spoke cities, hub-and-spoke networks provide significant numbers of connecting, multiple-flight leg services between spoke cities. The result is that on a flight into or out of a hub, the airline can serve nonstop passengers between the spoke and the hub airport, and connecting passengers between two spoke cities. The consolidation of traffic leads to higher *load factors* (the portion of aircraft seating capacity that is actually sold and utilized), and, in some cases, makes it economically viable for airlines to increase frequency of flight legs in certain markets, or to operate larger aircraft with lower unit costs. The result is lower ticket costs and/or increased frequency of service for passengers.

In hub-and-spoke networks, airlines typically operate *banked schedules*, in which are a set of arriving flights occurring in a relatively short period of time, followed by a set of departing flights also occurring within a short period of time. The amount of time separating the flight arrivals and the flight departures is defined by the amount of time needed for passengers to transfer, that is, connect between arriving and departing flight legs. Figure 1 shows the departure and arrival operations of a major U.S. airline at a banked hub. In this operation, there are 11 easily identifiable banks. A positive bar corresponds to the number of arrivals in a time interval, while a negative bar corresponds to the number of departures.

Banked schedules create departure and arrival peaks at the hub, with each peak planned to last about 45-60 minutes. Peaking has associated negative economic impacts. For example, in order to process passengers and baggage during peak operations, staffing requirements at the gate, on the apron, at the ticket counter, and in baggage handling, as well as infrastructure requirements (that is, runway capacity, gate capacity, baggage handling equipment, etc.) are at a maximum. Between adjacent banks, however, there are typically 60 to 90 minutes of *quiet* time. During this period, staffing requirements are low and hence, labor, equipment and infrastructure are not fully utilized.

Banked operations, with their peak demands for infrastructure capacity, exacerbate the effects of congestion and delays. When bad weather conditions reduce airport capacity, two pronounced effects result. First, schedule delays are incurred, thereby increasing airline operating costs due to the extra crew and fuel costs when aircraft are queued to arrive or depart. Second, passenger travel times are increased, sometimes significantly if flight delays result in passengers missing their flight connections.

Yet another disadvantage of banked operations is the resulting reduction in aircraft productivity. First, because inbound flights in a common bank arrive at the hub at about the same time, aircraft operating flight legs with shorter flying times must wait at spoke cities and depart later than aircraft operating flight legs with longer flying times. This waiting

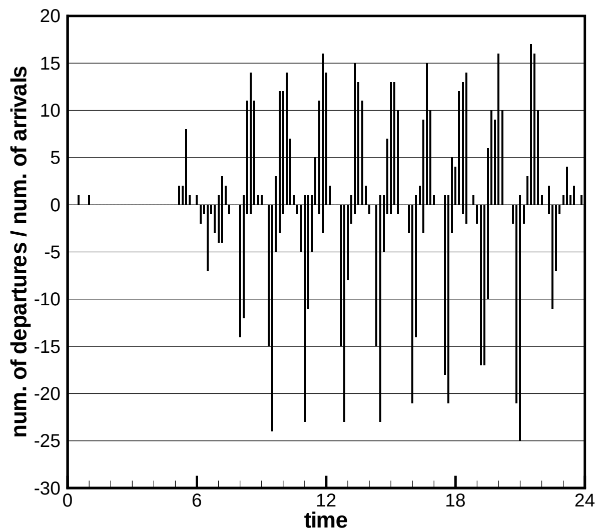

Figure 1: Departure and arrival activities at hub in a banked schedule

time is non-productive time for aircraft, and very expensive to the airlines. Second, aircraft (especially those arriving early in the bank) must sit on the ground well beyond the minimum time needed to *turn* the aircraft, that is, the minimum time needed to re-fuel, disembark and embark passengers, and service the aircraft. Moreover, because the arrival times of inbound flights in a bank are coordinated, the departure times at spoke cities might not be convenient to passengers. Similarly, because the departure times of outbound flights are coordinated, the arrival times at spoke cities again might not be convenient to passengers.

2.3 Moving Toward De-Peaked Schedules

The airline industry in the U.S. has been negatively impacted since 2001 by terrorist attacks, overcapacity, soaring fuel costs, and stiff competition. As a result, airlines have been forced to look for new approaches and strategies to achieve profitability.

Banked operations at hubs are very expensive, requiring a surplus of labor, equipment and infrastructure. *De-peaked* operations at hubs (also referred to as continuous or rolling hubs) can continue to take advantage of the hub-and-spoke network structure, but with less intensive operations at the hub and therefore less cost (Donoghue 2002; McDonald 2002). In de-peaked schedules, arrival and departure operations at the hub are smoothed, shaving the peaks and filling in the valleys of demand for resources. Flights are not coordinated in de-peaked operations to form connecting banks. Instead, the amount of time aircraft remain on the ground at the hub is not constrained by the need to provide passenger connections. Figure 2 shows the departure and arrival activities for the same airline shown in Figure 1 after the de-peaking of the same hub.

In summary, the benefits resulting from de-peaking hub operations include:

- Hub staffing can be reduced because the maximum number of arrivals and departures

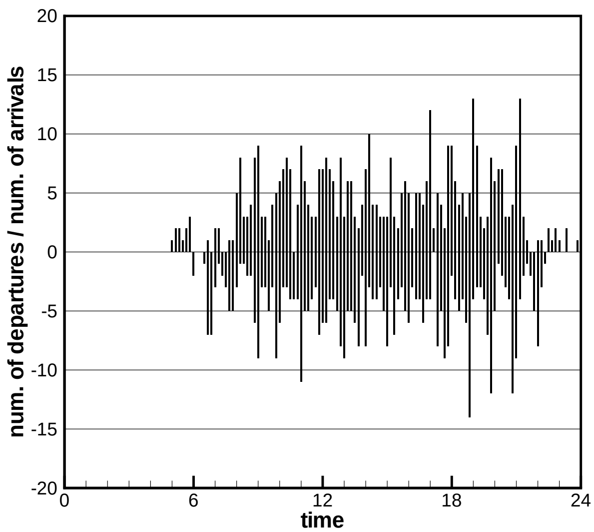

Figure 2: Departure and arrival activities at hub in a de-peaked schedule

occurring in a period of time for de-peaked operations is significantly smaller than that for peaked operations.

- Demands for infrastructure capacity, that is, airport runway and groundside capacity, gates, baggage handling equipment, etc., are similarly reduced in de-peaked operations.
- De-peaked schedules are more robust in the sense that airport capacity reductions caused by weather will have less of an impact than in the case of peaked operations with their high-levels of peak demand for capacity.
- In a de-peaked schedule, aircraft need not wait on the ground for connecting passengers. These reduced ground times for aircraft lead to increased aircraft utilization.

The major drawback of de-peaked operations is that the smoothed and spreaded-out arrivals and departures typically result in increased connection times for passengers. The effects of these increases on revenue are hard to quantify. Prior to the late 1990's, air tickets were mostly sold by travel agents using Global Distribution Systems (GDSs). Itineraries were displayed in increasing order of elapsed time, and the majority of bookings occurring in the first screen. Hence, increasing connection times, and thus, increasing elapsed travel time could displace an itinerary from being displayed in the first screen and result in significant reductions in the number of bookings for that itinerary. The adverse effect of this, however, is mitigated by recent changes in distribution: the Internet has been gaining great popularity among travelers. In 2004, more than 22% of U.S. airline tickets were sold through the Internet (Dorinson 2004). In 2005, Alaska and its sister airline, Horizon, sold 34.6% of their tickets via the airlines' website and an additional 11% of the airlines' sales came via online travel sites (Gillie 2006). Although travelers can choose to sort itineraries by schedule (departure time or elapsed time), the majority of air travel websites display search results by fare and researchers report that the fare display is most commonly used (Flint 2002). GDSs also include fare display these days and fare display becomes the most commonly used method by agencies (Flint 2002). Table 1 shows an example of the default displays of major U.S. airlines and leading online air ticket retailers. 11 out of 13 websites offer fare as the default display (or fare and schedule display simultaneously).

Website	Default Display Method
AirTran	fare and schedule simultaneously
Alaska	fare
America West	fare
American	schedule
Continental	fare
Delta	fare
Northwest	fare
Southwest	fare and schedule simultaneously
United	fare
US Airways	schedule
Expedia	fare
Orbitz	fare
Travelocity	fare

Table 1: Default display on the websites of major airlines and leading Internet air ticket retailers (obtained by visiting each website on March 6, 2006)

2.4 De-Peaking in Practice

This section discusses examples of schedule de-peaking in the airline industry. While it is not intended to be a comprehensive and thorough coverage of this topic, it does suffice the purpose of demonstrating this industry trend and its impact.

American Airlines de-peaked operations at its Chicago O'Hare (ORD) hub in April 2002. Flint (2002) reports that after de-peaking, the number of American Airlines flights remained the same as before, but arrivals and departures were more evenly spread throughout the day. In their de-peaked schedule, American Airlines restricted the number of arriving and departing flight legs per minute to no more than one.

Mean passenger connection times increased 10 minutes from 77 minutes to 87 minutes. Average aircraft turn times, however, reduced about 5 minutes, and by about 8 minutes at the spoke stations, resulting in less non-productive ground time for aircraft and, hence, increased aircraft utilization.

Benefits to American Airlines of de-peaking ORD include:

1. Fewer aircraft and gates were used to operate the same set of flight legs. It is reported that 3 mainline jets, 2 RJs, and 4 gates were saved after de-peaking ORD (Flint 2002). These *saved* aircraft can be put to use in an expanded flight schedule, complementing the cost-saving attributes of de-peaking with revenue-gain potential.

2. On time performance was improved despite higher aircraft utilization. After spending years at the bottom of the Department of Transportation (DOT) on-time performance

scorecard, American rose to second place in the second quarter of 2002. With reductions in air traffic congestion resulting from de-peaking, block times were reduced by more than one minute at ORD, worth $4.5-5 million per year (Ott 2003).

3. Labor efficiency increased. Less people were needed to handle the same amount of work, with each individual handling more flights per shift. The intensity of work per individual, however, does not increase because the workload is more evenly spread out through the day, unlike the peaked schedule in which periods of high levels of activity are followed by periods of little activity (Flint 2002).

4. Revenues increased. American Airlines reported increased unit revenues resulting from a 1% improvement in the ratio of local (or, non-stop) to connecting traffic at ORD. Because flights are no longer coordinated to form banks at the hub, the departure and arrival times are set to convenient times for local markets, thereby attracting more non-stop passengers. Connecting revenues, however, declined as a result of longer connection times (Flint 2002).

American airlines de-peaked its Dallas/Fort-Worth (DFW) hub in November, 2002. 9 mainline jets, 2 RJs, and 4 gates were saved (Flint 2002). Because fewer gates are needed at DFW for their de-peaked schedule, American Airlines was able to move all mainline flights to Terminals A and C, both of which are on the same side of the airport. Previously, mainline flights operated in Terminals A, B, and C. In addition to the benefit to passengers of having all American Airlines flights on the same side of the airport, the airline estimates it will save at least $4.5 million annually from this consolidation (SL 2002). American Airlines subsequently de-peaked its Miami (MIA) hub in May 2004.

Besides American Airlines, Continental Airlines de-peaked its schedule at Newark (Ott 2003); United Airlines de-peaked its hub in Chicago in 2004, its hub in Los Angeles in 2005, and is expected to de-peak throughout the system, beginning with San Francisco (SFO) in the first quarter of 2006 (UAL 2006); and Delta Airlines de-peaked their Atlanta hub in January, 2005, where about 65 airplanes an hour arrive and depart throughout the day. Daily departures for Delta Airlines grew to 1,051 a day under the de-peaked schedule from 970 a day prior to de-peaking, and the number of destinations served grew to 193 from 186. After de-peaking, average passenger connection times increased by about 3 minutes, up to 77 minutes from 74 minutes, and the amount of daily flying time per jet increased by about 8 percent, with the number of daily aircraft *turns* at each of the airline's gates increasing by up to 8.5 percent (Hirschman 2004).

And, this de-peaking trend is not restricted to airlines the U.S.. Lufthansa Airlines de-peaked Frankfurt (FRA) in 2004, its biggest hub, as part of the effort to cut costs by EUR 300 million in the next two years (Flottau 2003).

3 Opportunities in a De-Peaked Hub-and-Spoke Network

In a perfectly banked schedule, all inbound and outbound flights are scheduled to allow passengers to connect between any pair of arriving and departing flights in the same bank.

Moreover, minor adjustments to flight arrival and departure times do not create, or eliminate, any connecting itineraries. In a de-peaked operation at a hub, however, minor adjustments to flight leg arrival and/or departure times can affect the set of connecting itineraries served through that hub. In fact, flight schedule re-timings can increase or decrease the supply of available seats in markets connecting at the hub. Figure 3 provides a schematic illustration of departure and arrival activities in a de-peaked hub-and-spoke network. We denote the minimum time needed for passengers to connect between flights at the hub as $MinCT$ and the maximum connection time acceptable to passengers as $MaxCT$. Note that inbound flight a cannot connect to outbound flights b and c because the associated connection times are not within the allowable limits. Re-timing flight leg b to b', however, creates a feasible connection between flight legs a and b', as shown in Figure 4. This re-timing has the effect of adding seats to the market served by flight legs a and b'. At the same time, re-timing flight b to b', however, may cause some connecting itineraries using b as the outbound flight to violate the maximum connection requirement, thereby decreasing the number of seats offered in other markets. Similarly, re-timing flight c to c' creates a feasible connection between flight legs a and c' and may cause some connecting itineraries using c as the outbound flight to violate the minimum connection time requirement, thereby decreasing the number of seats offered in other markets. The re-timing of flight legs can thus be considered a a powerful mechanism, one capable of dynamically adjusting the supply of seats to better match demands as revealed through the booking period.

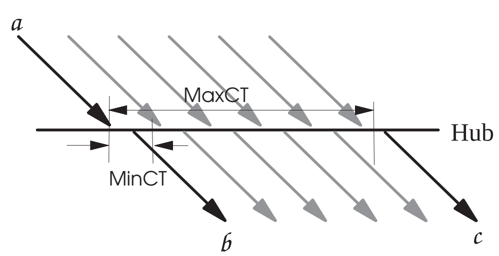

Figure 3: Original schedule

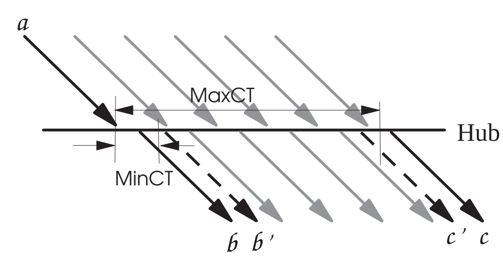

Figure 4: Re-timing creates new connecting itineraries

Flight re-timing has the added benefit that it can create more re-fleeting opportunities, as illustrated in the examples presented in Figures 5 and 6. Figure 5 depicts the original schedule, in which an Airbus A319 operates flight leg a and then flight leg c, and an Airbus A320 operates flight leg b followed by flight leg d. Suppose that more capacity is desired on flight leg d and less capacity is needed on flight leg c. The original schedule does not allow the same aircraft to operate both flight legs b and c due to insufficient turn time between the arrival of b and the departure of c. If b arrives earlier and c departs later, however, the A320 can operate b followed by c and the A319 can operate a followed by d, as depicted in Figure 6.

4 Modeling Architecture

In our dynamic scheduling approach, schedule re-optimization is performed for each departure date, thereby producing potentially different flight schedules, each of which is designed

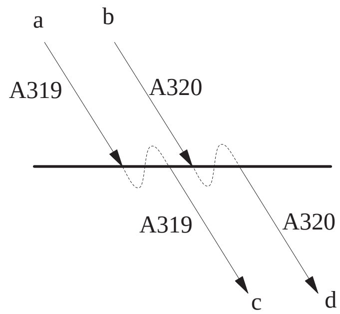

Figure 5: Original schedule

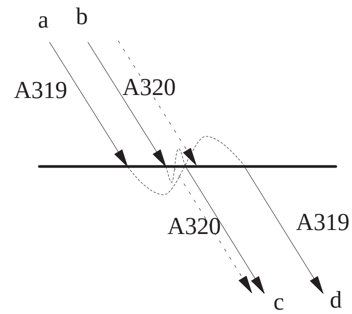

Figure 6: New schedule

to capture the individual dynamics of passenger demand for that day. We refer to the modified schedule for each day as the *new schedule* and to the schedule produced by the initial planning process as the *original schedule*. We assume that the original schedule is a *daily schedule*, that is, the same schedule is repeated each day. *Re-optimization points* are points in time during the booking period when schedule re-optimization, that is, flight leg re-timings and re-fleetings, is performed. A portrayal of our dynamic scheduling process is shown in Figure 7.

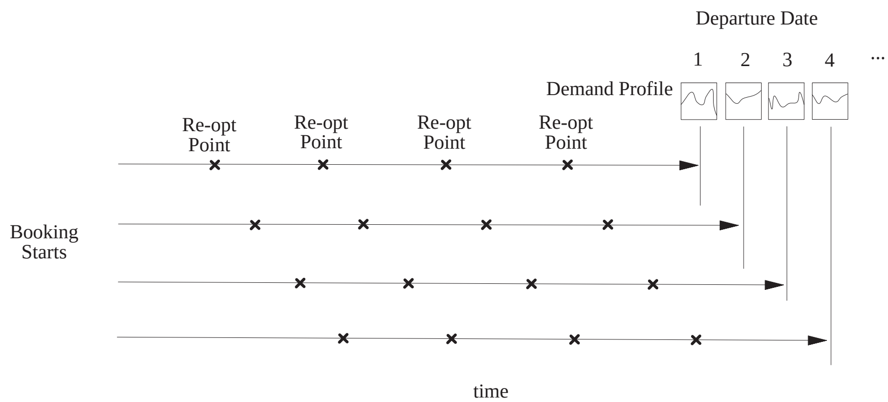

Figure 7: Dynamic scheduling process

For each day d included in the original schedule, we specify a few re-optimization points, with each re-optimization point earlier in the booking period of day d. Each schedule re-optimization results in a new schedule for a particular day, and it replaces the previous schedule, whether it is original or the result of re-optimization. At each re-optimization point for day d, three questions are answered, namely:

1. What are the current numbers of passenger bookings for each itinerary on day d?

2. What are the forecasted future itinerary bookings from the re-optimization point to day d?

3. What is the set of optimal flight leg re-timings and re-fleetings for day d given the current and forecasted future itinerary bookings?

Any solution to the third question must satisfy the following constraints:

1. Flight legs can be re-scheduled only to a time close to that of the original schedule. The set of allowable departure times for each flight leg defines that flight leg's feasible *time window*;

2. Allowable fleeting changes for a flight leg l are limited to fleet types in the same *family* as that of the original fleet assignment to l;

3. Service to passenger bookings made prior to the re-optimization point must be guaranteed in the new schedule;

4. At the start of the day, the number of aircraft of each type available at each airport is equal to the number positioned at that airport at the end of the preceding day. At the end of the day in the new schedule, the number of aircraft of each fleet type at each airport location must be no less than the number in the original schedule. Because a daily schedule re-optimization model is used, this is equivalent to constraining the number of aircraft for each fleet overnighted at each airport to be no more than that in the original schedule.

Constraints (1) limit the magnitude of schedule changes to minimize the impact to passengers who booked their itineraries before the re-optimization point and the possibility to disrupt aircraft maintenance routing and crew pairing plans if they are developed prior the last re-optimization point. Constraints (2) ensure that crew assignments remain feasible after re-fleeting. Constraints (3) guarantee service to all previously booked passengers in the new schedule. The exact meaning of "service guarantee" is explained in Section 4.1. Constraints (4) ensure that aircraft are appropriately positioned at the end of the day so that the re-optimized schedule for the next day can be implemented.

4.1 Service Guarantee to Previously Booked Passengers

In order to minimize passenger inconvenience, passengers booked before the current re-optimization point are to be accommodated on itineraries with the same flight numbers, but potentially with a slight change in flight departure and arrival times. Re-fleeting does not change the timetable, therefore passengers wouldn't even notice such changes. However, re-timing affects the flight times and could potentially make the connection times for previously booked connecting itineraries shorter than the minimum required.

For nonstop passengers booked on flights prior to re-timing, the effect is a slight deviation of time for their flights. For connecting passengers booked prior to re-timing, we make sure that the re-timing decisions do not disrupt their itineraries, that is, their new connection times after re-timing are no less than the minimum connection time. For example, suppose a passenger booked the itinerary shown in Table 2. After re-optimization, the passenger will still be traveling on flight 254 and then connecting to flight 487 as before. The only thing that changes is the flight departure and arrival times. An example of the new itinerary is shown in Table 3. For the itinerary shown in Table 3, the connection time at the hub is 30 minutes, which is still greater than the 25-minute minimum connection time. Please note that the connection times of booked connecting itineraries are allowed to exceed the maximum connection time. When flight re-timing is limited to a small magnitude, which we envision to be ±15 minutes from the original timetable, the maximum possible increase in connection time is 30 minutes.

Flight	Origin	Departure Time	Destination	Arrival Time
254	BOS	9:00am	HUB	12:00noon
487	HUB	12:45pm	LAX	1:45pm

Table 2: Itinerary prior to re-timing

Flight	Origin	Departure Time	Destination	Arrival Time
254	BOS	9:10am	HUB	12:10noon
487	HUB	12:40pm	LAX	1:40pm

Table 3: Itinerary after re-timing

4.2 Frequency and Timing of Re-Optimization Points

While outlining the concept of dynamic scheduling in the previous section, we include several re-optimization points within the booking period. The question of when to re-optimize is, itself, an optimization problem. Because the objective of our research is to provide an estimate of the potential benefits of dynamic scheduling, we include only a single re-optimization point in the booking period. Even with this simplification, the question of when to perform this one-time schedule adjustment remains. The goal in selecting the re-optimization point is to balance *flexibility to modify the schedule* and *forecast quality*. Forecast quality improves as the re-optimization point is moved later in the booking period. Flexibility to modify the schedule, however, decreases as the re-optimization point is moved later in the booking period. This difficulty stems from passenger, crew and maintenance restrictions. Later in the booking period, in order to guarantee service to the large number of passengers who have already booked, the set of feasible re-optimization decisions are substantially constrained. With respect to crews and maintenance, if the re-optimization point occurs before the crew schedules and aircraft maintenance routings are constructed, crews and maintenance requirements need not be considered in the re-optimization. If, however, the re-optimization point occurs after crew and maintenance plans have been generated, the new schedule must maintain feasibility of these plans or generate feasible alternatives.

4.3 Flow Charts

Using a single re-optimization point for each day d in the original schedule, for example 21 days prior to departure, the booking period is divided into two periods. We refer to the time period beginning at the start of the booking period and ending at the re-optimization point as *Period 1*; and the time period from the re-optimization point to day d as *Period 2*. Empirical results show that around 50% of the passengers have booked their itineraries 21 days prior to departure, providing valuable revealed demand information but leaving flexibility in the system.

In Figure 8, we depict our modeling approach for the *static* case, that is, the case in which the original flight schedule is not re-optimized. Booking limits are set for all flight legs in Period 1 to protect seats for Period 2 passenger demand. Passenger demand in Period 1 is assigned to the original schedule using a Passenger Mix Model (detailed in Section 5.3). Finally, Period 2 passenger demand is assigned to the remaining capacity on each flight leg in the original schedule, again using a passenger mix model.

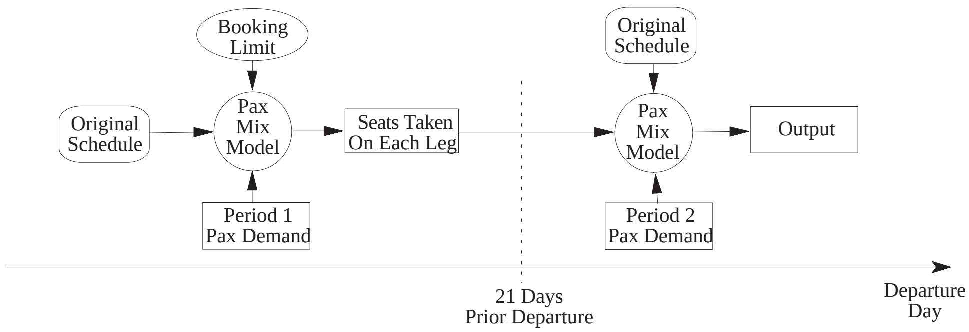

Figure 8: Static case

We illustrate our modeling approach for the dynamic scheduling case, or dynamic case, in Figure 9. A *re-optimization* module is inserted at the re-optimization point for any departure day d. The numbers of aircraft of each fleet type at each airport at the start and end of day d are computed from the original schedule. Additionally, for each flight leg, the number of seats sold in Period 1 is calculated, and the set of connecting itineraries that are booked in Period 1 are identified. These data, together with passenger demand forecasts for Period 2, are inputs to the re-optimization module that produces a new schedule to replace the original one.

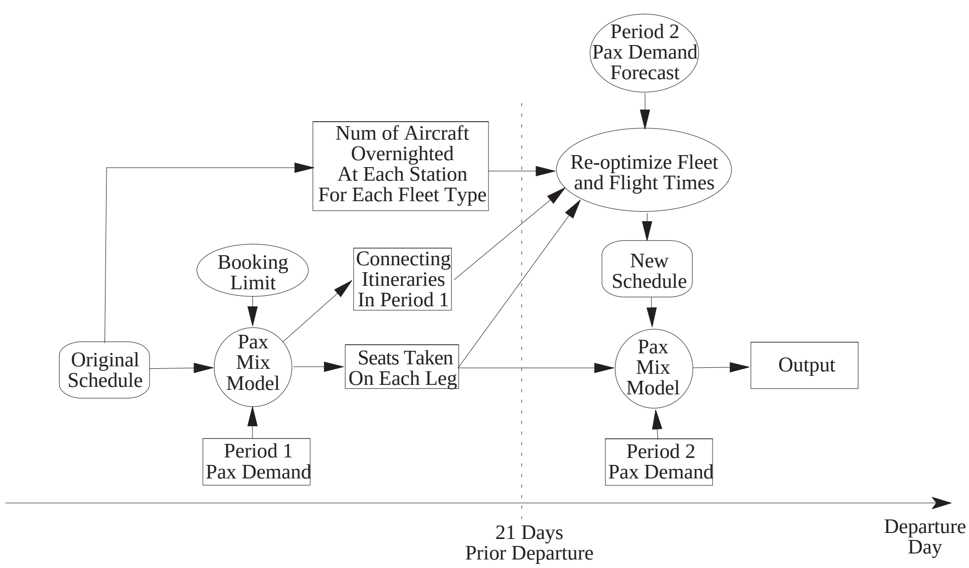

Figure 9: Dynamic scheduling case

5 Mathematical Models

In this section, we describe the network representations and the mathematical formulations of the models in our dynamic scheduling approach.

5.1 Terminology

To facilitate our discussion, we define the following terms. A *flight leg* is a nonstop trip of an aircraft from an origin airport to a destination airport (one take-off and one landing). An *itinerary* consists of a specific sequence of scheduled flight legs in which the first leg originates from the origin airport at a particular time and the final leg terminates at the final destination airport at a later time. We model a round trip itinerary as two distinct one-way itineraries. The number of intermediate cities traversed is called the *number of stops* in this itinerary. A *non-stop itinerary* consists of only one flight leg, which originates from the origin and terminates at the destination. A *connecting itinerary* is an itinerary that has one or more stops. Although there do exist connecting itineraries with more than one intermediate stop, it is very rare. In the scope of our research, we assume all connecting itineraries have exactly one stop.

5.2 Network Representations

A *time-space network* is a mechanism designed for modeling dynamic networks. Nodes in a time-space network are associated with both time and place, and arcs represent scheduled movements between locations, or remaining at the same location for some period of time.

To achieve schedule re-optimization, we create two tailored time-space networks: an *aircraft flow network* and a *passenger flow network*.

The aircraft flow network is used to model the flow of aircraft over a flight schedule, with a different flow network created for each fleet type. Each node in fleet k's network corresponds either to the departure time of a flight leg f, or its arrival time plus the minimum amount of time needed to turn a type k aircraft at the arrival station of leg f. Each arc in fleet k's network is classified as either a *flight arc* or a *ground arc*. Flight arcs represent scheduled flight legs, while *ground arcs* represent an aircraft's ability to remain on ground at the same place over time. A *wrap-around arc* is a ground arc that connects the first and last node at an airport station. The *count line* is an arbitrarily chosen point of time that is used to count the number of aircraft needed to operate a given flight schedule.

The passenger flow network is used to model the flow of passengers over a fleeted flight schedule. Each node in the passenger flow network corresponds either to the departure time, or the arrival time of leg f. Each arc is classified as either a *flight arc* or a *connection arc*. Flight arcs again represent scheduled flight legs, but *connection arcs* represent a passenger's ability to connect between two flight legs. Feasible passenger itineraries correspond to paths in this network; and flight arc capacities are determined by the type of aircraft assigned to the flight leg represented by that arc.

5.3 Passenger Mix Model

Given the set of *unconstrained passenger demand*, that is, the number of passengers wishing to travel, over its flight schedule with assigned fleet types, an airline's objective is to maximize their revenues by accommodating as many high fare passengers as possible. For some flight legs, unconstrained demand exceeds supply and passengers must be *spilled* either to another itinerary offered by the airline or a different airline, or to another transportation mode. When passenger demand exceeds seat capacity, the objective of the airline is to spill low fare passengers or passengers that can be easily recaptured on alternative itineraries offered by the airline.

Various difficulties arise in modeling and solving this problem of revenue maximization. Network effects, booking orders, itinerary cancellations by passengers, and the stochasticity of unconstrained demands all contribute to make this a very complex problem. In existing research, as well as the research presented here, some (but not all) of these complicating aspects are addressed. The complexities that are captured tend to be dictated by the methodology employed. For example, revenue management techniques are effective in modeling booking orders and stochasticity, while mathematical programming approaches are well-suited to model network interactions and the associated trade-offs.

Soumis et al. (1981) uses a mathematical programming model to assign passengers to itineraries. Dissatisfaction costs are assigned to unattractive itineraries and spill costs are assigned to overloaded itineraries. Glover et al. (1982) propose a minimum cost network flow model with side constraints to determine which passenger to spill. They refer to this problem as the *passenger mix problem*, which is solved by *Passenger Mix Models (PMMs)*. Other network approaches for PMMs are detailed in Dror et al. (1988), Phillips et al. (1991), Farkas (1995). Previous approaches assume either *no recapture*, that is, a passenger that cannot be accommodated on their preferred itinerary is not served by the airline; or *perfect recapture*, where each passenger is assumed to be indifferent to all itineraries serving his/her desired origin-destination city pair. Kniker (1998) represents one of the first efforts to model the more realistic case of *partial recapture* in which only a percentage of passengers who cannot be accommodated on their desired itineraries are willing to accept an alternative itinerary. These percentages, or *recapture rates*, vary for each alternative itinerary depending on its quality (that is, non-stop vs. connecting, time-of-day of departure and arrival, etc.). Hence, in this approach, passengers spilled from their desired itinerary p are assumed to be recaptured onto an alternative itinerary r at a recapture rate of b_p^r. Before we present the model by Kniker (1998), we introduce the following additional notation:

Data

P	:	set of all itineraries indexed by p or r.
L	:	set of flight legs in the flight schedule indexed by l.
$SEAT_l$	:	number of seats available on flight leg $l \in L$.
D_p	:	unconstrained demand for itinerary $p \in P$, that is, the number of passengers wishing to travel on itinerary p.
δ_p^l	=	$\begin{cases} 1, & \text{if itinerary } p \text{ traverses flight leg } l; \\ 0, & \text{otherwise.} \end{cases}$
$fare_p$	:	average fare for itinerary p.
$A(p)$	:	set of alternative itineraries for passengers requesting itinerary p.

b_p^r	:	fraction of passengers spilled from itinerary p, and recaptured by itinerary $r \in A(p)$. $b_p^r \in [0,1), \forall r \in A(p), p \in P$ and $\sum_{r\in A(p)} b_p^r < 1$.

Decision Variables

x_p^r	:	the number of passengers requesting itinerary p who are served by itinerary $r \in A(p)$.
y_p	:	the number of passengers requesting itinerary p who are spilled and not recaptured by the airline.

The key-path formulation of PMM with partial recapture (Kniker (1998)) is as follows: maximize

$$\sum_{p\in P} fare_p(D_p - \sum_{r\in A(p)} x_p^r - y_p) + \sum_{p\in P}\sum_{r\in A(p)} fare_r b_p^r x_p^r$$

Subject to:

$$\sum_{r\in A(p)} x_p^r + y_p \le D_p, \forall p \in P \tag{1}$$

$$\sum_{p\in P} \delta_p^l (D_p - \sum_{r\in A(p)} x_p^r - y_p) + \sum_{p\in P}\sum_{r\in A(p)} \delta_r^l b_p^r x_p^r \le SEAT_l, \forall l \in L \tag{2}$$

$$x_p^r \ge 0, \forall r \in A(p), p \in P \tag{3}$$

$$y_p \ge 0, \forall p \in P \tag{4}$$

Recognizing that $\sum_{p\in P} fare_p D_p$ is a constant, the objective can be re-written as

$$\sum_{p\in P}\sum_{r\in A(p)} \left(fare_r b_p^r - fare_p\right) x_p^r + \sum_{p\in P}(-fare_p)y_p.$$

Empirically the term $\left(fare_r b_p^r - fare_p\right)$ is usually negative, therefore in order to maximize the value of the objective function, x_p^r is pushed toward zero, if possible. The same is true for y_p. The result is that Constraints (1) are often redundant. This fact is exploited and a constraint generation approach is employed to solve the model.

Kniker (1998) points out that optimal PMM solutions can vary quite significantly, depending on the recapture rates used; and moreover, recapture rates are difficult to estimate. Due to PMM's sensitivity to recapture rate estimates, we adopt a simplifying approach in which we assume perfect recapture between selected subsets of itineraries, defined for each passenger. A passenger's subset includes all itineraries that: 1) depart within the passenger's window of acceptable departure times; and 2) provide a service level, measured as the number of stops in an itinerary, that is as good as that requested by the passenger. This subset of itineraries for each passenger define a *market*, with *average fare* equal to that of the itineraries in the subset. Recapture rates are 0 between itineraries in a market and itineraries serving this origin-destination city pair but not contained in the market. Before we present our PMM model, we introduce additional notation:

Data

M	:	set of markets.
$fare_m$	:	average fare in market $m \in M$.

D_m	:	unconstrained demand in market $m \in M$.
$R(m)$	:	set of itineraries serving market $m \in M$.
δ_{mr}^l	=	$\begin{cases} 1, & \text{if itinerary } r \in R(m) \text{ in market } m \in M \text{ traverses flight leg } l \in L; \\ 0, & \text{otherwise.} \end{cases}$

Decision Variables

x_{mr}	:	number of passengers assigned to any itinerary $r \in R(m)$ in market $m \in M$.

Our PMM model is:
maximize

$$\sum_{m \in M} fare_m \sum_{r \in R(m)} x_{mr}$$

subject to:

$$\sum_{r \in R(m)} x_{mr} \leq D_m, \forall m \in M \tag{5}$$

$$\sum_{m \in M} \sum_{r \in R(m)} \delta_{mr}^l x_{mr} \leq SEAT_l, \forall l \in L \tag{6}$$

$$x_{mr} \geq 0, \forall m \in M, \forall r \in R(m) \tag{7}$$

5.4 Re-Optimization Model

The work of Berge and Hopperstad (1993), Bish et al. (2004), and Sherali et al. (2005) are most pertinent to our research in a dynamic scheduling context. Because the re-optimization model applies re-fleeting and re-timing simultaneously, a brief review of re-fleeting and re-timing in other areas of airline scheduling is presented below.

5.4.1 Literature Review

Research in the area of re-fleeting models and algorithms has focused largely on recovery from irregular operations, for instance, Jarrah et al. (1993), Rosenberger et al. (2002), Thengvall et al. (2000), and Yu and Luo (1997). Apart from these research, Talluri (1996) and Jarrah (2000) study models and algorithms to re-fleet a schedule that can be applied in dynamic scheduling.

Talluri (1996) addresses the problem of aircraft swapping in the context of both schedule planning and schedule recovery. Algorithms are presented to answer the following question: given a balanced and fleeted daily flight schedule, how do we change the fleet type assigned to a flight leg from Type A to Type B, while requiring the smallest number of changes to the existing assignment. A flight network involving only Type A and Type B aircraft is constructed. Then the flight arcs and ground arcs corresponding to Type B are reversed. Each arc is assigned unit cost and a shortest paths algorithm is used to find the solution. The algorithm is able to find solutions quickly for problems involving two fleet types and the required swapping of fleet types on a single flight leg. Revenue and cost information are not considered in the algorithm.

Jarrah (2000) uses multi-type, multi-leg, re-fleeting models to modify planned fleet assignments incrementally. Several modules are defined, namely: 1) a popping module that can be used to find the least costly way to remove a specified number of aircraft from the schedule; 2) a change-of-gauge module that can be used to replace a user-specified number of Type A equipment with Type B equipment; 3) a swapping module that can be used to swap fleet assignments within a user-specified group of fleet types to maximize incremental profit; 4) a utilization module that can be used to adjust block time between two fleet types while maximizing incremental profit; and 5) a balancing module that can be used to find the most profitable set of swaps that will balance an *imbalanced schedule*, that is, a schedule in which the aircraft balance constraints are violated. A set of near-optimal solutions are determined by iteratively solving the model, with cuts added to the model at each iteration to preclude solutions from being re-generated. Let S represent the current solution; and x_i^k equal to 1 if fleet type k is assigned to leg i, and equal to 0 otherwise. The added constraints are of the form $\sum_{(i,k)\in S} x_i^k \leq |S| - 1$. The flight schedule is assumed to be fixed and passenger demands and associated fares are assumed to be flight leg specific.

Although flight re-timing has not been employed in dynamic scheduling approaches, it has been incorporated into schedule planning extensively. Levin (1971) is the first to propose a scheduling and fleet routing model with time windows. Time windows were created around the original scheduled flight departure times, with departures allowed to occur at discrete intervals within the time window. Later, Bandet (1994) and Desaulniers et al. (1997) present formulations for the fleet assignment and aircraft routing problem with time windows.

Recently, Rexing et al. (2000) present a generalized fleet assignment model to schedule flight departures and assign aircraft types to flight legs simultaneously. Two solution approaches are presented: one, the Direct Solution Approach (DST), is designed for speed and simplicity; while the other, the iterative solution approach (IST), is tailored to address large-scale problems in which memory management is an issue. In every test scenario, they produce fleet assignments with significantly lower costs than the basic fleet assignment model without time windows. Moreover, the approach is used to achieve more productive schedules for aircraft, indicating potential reductions in the number of aircraft needed to operate the flight schedule.

Klabjan et al. (2002) apply the idea of time windows to crew scheduling and develop a crew scheduling model that yields solutions with significantly lower planned costs than those obtained from conventional models. Lan et al. (2006) apply flight re-timing techniques to schedule planning models to reduce the number of occurrences of passengers missing their flight connections.

5.4.2 Model Statement

Our schedule re-optimization model is a generalization of models presented in Berge and Hopperstad (1993) and Bish et al. (2004) with enhanced modeling aspects. The major distinction is that in our model, *flight copies* are created and scheduled near the original flight departure time to allow for flight leg re-timing. In addition, we relax the assumption of leg demand independence and consider passenger recapture. Before introducing our schedule re-optimization model, we introduce the following additional notation:

Data and Parameters .

Π	:	set of fleet types.
S	:	set of cities.
G^{π}	:	set of ground arcs in fleet $\pi \in \Pi$'s network.
$FML(\pi)$	:	set of fleet types in the same family as $\pi \in \Pi$.
D_m^F	:	demand forecast for market m.
$fare_m^F$	:	forecasted average fare for demand in market m.
$C(l)$	:	set of flight copies for flight leg $l \in L$.
$\langle l, k \rangle$	:	copy $k \in C(l)$ of flight leg $l \in L$.
$c_{lk\pi}$	:	cost to fly $\langle l, k \rangle$ with aircraft type $\pi \in \Pi$, where $k \in C(l), l \in L$.
c_{π}	:	fixed cost to have one additional aircraft of type $\pi \in \Pi$.
N^{π}	:	nodes in flight network of fleet type $\pi \in \Pi$.
n^{π}	:	number of aircraft available for fleet type $\pi \in \Pi$.
BKD_l	:	number of seats already booked on flight leg $l \in L$ before re-optimization.
${y_{\pi}^{i}}^{0}$	:	number of aircraft overnighted at city $i \in S$ for fleet type π in the original schedule.
π_l^0	:	the fleet type used on leg $l \in L$ in the original schedule.
T	:	set of time interval at the hub, indexed by t.
MAX^{at}	:	maximum number of aircraft arrivals at the hub in interval $t \in T$.
MAX^{dt}	:	maximum number of aircraft departures from the hub in interval $t \in T$.

$$\zeta_{g\pi}^{i} = \begin{cases} 1, & \text{arc } g \in G^{\pi} \text{ is a wrap around arc at city } i \in S; \\ 0, & \text{otherwise.} \end{cases}$$

$$\overline{\alpha}_{lk\pi}^{i} = \begin{cases} 1, & \text{if } \langle l, k \rangle \text{ in fleet } \pi\text{'s network begins at node } i \in N^{\pi}; \\ -1, & \text{if } \langle l, k \rangle \text{ in fleet } \pi\text{'s network terminates at node } i \in N^{\pi}; \\ 0, & \text{otherwise.} \end{cases}$$

$$\widehat{\alpha}_{g\pi}^{i} = \begin{cases} 1, & \text{if ground arc } g \in G^{\pi} \text{ begins at node } i \in N^{\pi}; \\ -1, & \text{if ground arc } g \in G^{\pi} \text{ terminates at node } i \in N^{\pi}; \\ 0, & \text{otherwise.} \end{cases}$$

$$\overline{\beta}_{lk\pi} = \begin{cases} 1, & \text{if c in fleet } \pi\text{'s network crosses the count line;} \\ 0, & \text{otherwise.} \end{cases}$$

$$\widehat{\beta}_{g\pi} = \begin{cases} 1, & \text{if ground arc } g \in G^{\pi} \text{ crosses the count line;} \\ 0, & \text{otherwise.} \end{cases}$$

$$\gamma_{lk}^{at} = \begin{cases} 1, & \text{if } \langle l, k \rangle \text{ arrives at the hub during interval } t \in T; \\ 0, & \text{otherwise.} \end{cases}$$

$$\gamma_{lk}^{dt} = \begin{cases} 1, & \text{if } \langle l, k \rangle \text{ departs from the hub during interval } t \in T; \\ 0, & \text{otherwise.} \end{cases}$$

Decision Variables

$$f_{lk\pi} = \begin{cases} 1, & \text{fleet } \pi \in \Pi \text{ is used to fly flight copy} \langle l, k \rangle, \text{ where } k \in C(l), l \in L; \\ 0, & \text{otherwise.} \end{cases}$$

$y_{g\pi}$	:	number of aircraft on ground arc $g \in G^{\pi}$.
z_{π}	:	number of aircraft used for fleet type π.

The objective of the re-optimization model is to maximize future revenue less operating cost and fixed cost:

$$\sum_{m\in M}\sum_{r\in R(m)} x_{mr} fare^F_m - \sum_{l\in L}\sum_{k\in C(l)}\sum_{\pi\in\Pi} c_{lk\pi} f_{lk\pi} - \sum_{\pi\in\Pi} z_\pi c_\pi$$

The set of constraints are:

$$\sum_{k\in C(l)}\sum_{\pi\in\Pi} f_{lk\pi} = 1, \forall l \in L \tag{8}$$

$$\sum_{r\in R(m)} x_{mr} \leq D^F_m, \forall m \in M \tag{9}$$

$$\sum_{m\in M}\sum_{r\in R(m)} \delta^{lk}_{mr} x_{mr} \leq \sum_{\pi\in\Pi} f_{lk\pi}(CAP_\pi - BKD_l), \forall l \in L, k \in C(l) \tag{10}$$

$$\sum_{l\in L}\sum_{k\in C(l)} f_{lk\pi}\overline{\alpha}^i_{lk\pi} + \sum_{g\in G^\pi} y_{g\pi}\widehat{\alpha}^i_{g\pi} = 0, \forall i \in N^\pi, \pi \in \Pi \tag{11}$$

$$\sum_{l\in L}\sum_{k\in C(l)} f_{lk\pi}\overline{\beta}_{lk\pi} + \sum_{g\in G^\pi} y_{g\pi}\widehat{\beta}_{g\pi} = z_\pi, \forall \pi \in \Pi \tag{12}$$

$$z_\pi \leq n^\pi, \forall \pi \in \Pi \tag{13}$$

$$\sum_{l\in L}\sum_{k\in C(l)} \gamma^{at}_{lk} \sum_{\pi\in\Pi} f_{lk\pi} \leq MAX^{at}, \forall t \in T \tag{14}$$

$$\sum_{l\in L}\sum_{k\in C(l)} \gamma^{dt}_{lk} \sum_{\pi\in\Pi} f_{lk\pi} \leq MAX^{dt}, \forall t \in T \tag{15}$$

$$f_{lk\pi'} = 0, \forall l \in L, k \in C(L), \pi' \notin FML(\pi^0_l) \tag{16}$$

$$\sum_{g\in G^\pi} y_{g\pi}\zeta^i_{g\pi} \leq y^{i^0}_\pi, \forall i \in S, \forall \pi \in \Pi \tag{17}$$

$$f_{lk\pi} \in \{0,1\}, \forall l \in L, k \in C(l), \pi \in \Pi \tag{18}$$

$$x_{mr} \geq 0, \forall m \in M, r \in R(m) \tag{19}$$

$$y_{g\pi} \geq 0, \forall g \in G^\pi, \pi \in \Pi \tag{20}$$

$$z_\pi \geq 0, \forall \pi \in \Pi \tag{21}$$

Constraints (8) ensure that each flight leg is covered exactly once. Constraints (9) are passenger flow constraints limiting the number of passengers transported in each market to the value of that market's unconstrained demand. Constraints (10) limit the number of future passenger bookings to the remaining number of available seats. Constraints (11) ensure aircraft flow balance. Constraints (12) and (13) ensure that the number of aircraft of each fleet type used to operate the schedule is no more than that used in the original schedule. Constraints (14) and (15) limit the number of departure and arrival activities in each time interval to the maximum allowable. Constraints (16) enable re-fleeting within families. Constraints (17) enforces the requirement that aircraft are positioned as in the original schedule, at the start and end of each day. Constraints (18), (19), (20) and (21) are constraints on possible variable values. In the next section, we describe constraints to guarantee service to previously booked passengers.

5.4.3 Service Guarantee to Booked Passengers

All seats sold in Period 1 (prior to the re-optimization point) are *protected* during schedule re-optimization, that is, all nonstop passengers can be served in the new schedule on their original, although possibly re-timed, flight legs. To ensure that connecting passengers have sufficient time to connect between re-timed flights, we add constraints limiting the connection time for booked itineraries to be at least the minimum allowable. Let Q denote the set of connecting itineraries that are booked in Period 1. For the pair of flight legs $(l_1, l_2) \in Q$, let $CT(\langle l_1, k_1\rangle, \langle l_2, k_2\rangle)$ be the connection time between copy k_1 of flight leg l_1 and copy k_2 of flight leg l_2. For a given $(l_1, l_2) \in Q$, let $C(l_1, l_2) = \{(k_1, k_2) | CT(\langle l_1, k_1\rangle, \langle l_2, k_2\rangle) \geq minCT, \forall k_1 \in C(l_1), \forall k_2 \in C(l_2)\}$. Assume we create $2a$ flight copies and index the flight copies as shown in Figure 10. The connection time between copy $+1$ of l_1 and copy 0 of l_2 is greater than $MinCT$, therefore the flight copy pair $(+1, 0) \in C(l_1, l_2)$. The connection time between copy $+a$ of l_1 and copy -a of l_2 is smaller than $MinCT$, therefore the flight copy pair $(+a, -a) \notin C(l_1, l_2)$.

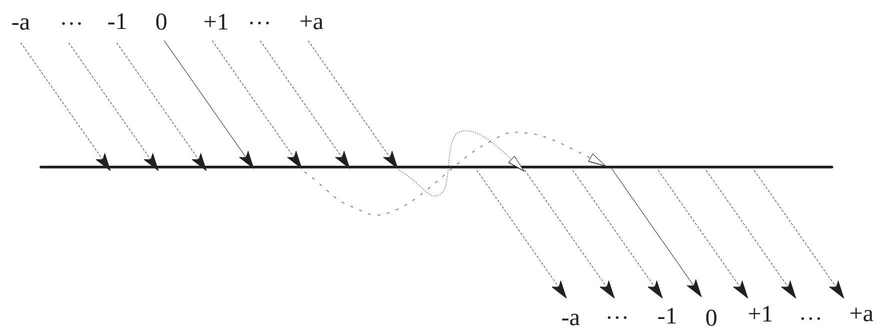

Figure 10: Flight copy indices

We introduce new binary decision variables $u_{l_1 k_1}^{l_2 k_2}$ for each $(l_1, l_2) \in Q$, and each $(k_1, k_2) \in C(l_1, l_2)$, where

$$u_{l_1 k_1}^{l_2 k_2} = \begin{cases} 1, & \text{if the connection formed by } \langle l_1, k_1\rangle \text{ and } \langle l_2, k_2\rangle \text{ is feasible after} \\ & \text{re-optimization;} \\ 0, & \text{otherwise.} \end{cases}$$

The following constraints ensure that, after schedule re-optimization, all leg pairs in Q can still form feasible connecting itineraries:

$$\sum_{(k_1,k_2)\in C(l_1,l_2)} u_{l_1 k_1}^{l_2 k_2} = 1, \forall (l_1, l_2) \in Q, \text{ and} \tag{22}$$

$$\sum_{\pi\in\Pi} f_{l_1 k_1 \pi} + \sum_{\pi\in\Pi} f_{l_2 k_2 \pi} \geq 2u_{l_1 k_1}^{l_2 k_2}, \forall (l_1, l_2) \in Q, (k_1, k_2) \in C(l_1, l_2). \tag{23}$$

Constraints (22) ensure that among the feasible connections between copies of flight legs l_1 and l_2, exactly one must be enabled in the new schedule. Constraints (23) guarantee that if the connection formed by $\langle l_1, k_1\rangle$ and $\langle l_2, k_2\rangle$ is in the new schedule, aircraft are assigned to $\langle l_1, k_1\rangle$ and $\langle l_2, k_2\rangle$.

Alternatively, instead of ensuring that at least one flight copy pair $(k_1, k_2) \in C(l_1, l_2)$ is enabled in the new schedule, it is also possible to forbid the selection of copy pairs not in $C(l_1, l_2)$. To achieve this goal, we only need the following constraints:

$$\sum_{\pi \in \Pi} f_{l_1 k_1 \pi} + \sum_{\pi \in \Pi} f_{l_2 k_2 \pi} \leq 1, \forall (l_1, l_2) \in Q, (k_1, k_2) \notin C(l_1, l_2). \tag{24}$$

Constraints (24) are preferred to constraints (22) and (23) because: 1) they introduce no new binary variables; and 2) they require the addition of fewer constraints to the schedule re-optimization model, as detailed in the following proposition.

Proposition 1 *There are fewer constraints of the form (24) than of the form (23).*

Proof: For each flight leg, equal numbers of flight copies are created, with departure times evenly spaced, both earlier and later around the departure time of the original flight leg. Assume there are $2a$ copies, indexed as shown in Figure 10. Consider two flight legs in the original schedule and let CT be the connection time between them. Denote the time displacement between flight copies as d, and let x be the copy index of l_1 and y be the *copy index* of l_2. We compute the connection times $CT(\angle l_1, x\rangle, \langle l_2, y\rangle) = CT - (x + y)d$ between copies of the inbound and outbound flight legs, as shown in Table 7. Rows represent the copy index of the inbound flight leg, while columns represent the copy index of the outbound flight leg.

Index	-a	···	-1	0	+1	···	+a
-a	CT	···	CT-(a-1)d	CT-ad	CT-(a+1)d	···	CT-2ad
···	···	···	···	···	···	···	···
-1	CT+(a-1)d	···	CT	CT-d	CT-2d	···	CT-(a+1)d
0	CT+ad	···	CT+d	CT	CT-d	···	CT-ad
+1	CT+(a+1)d	···	CT+2d	CT+d	CT	···	CT-(a-1)d
···	···	···	···	···	···	···	···
+a	CT+2ad	···	CT+(a+1)d	CT+ad	CT+(a-1)d	···	CT

Table 7: Connection times between flight copies

For all connections in the table's lower-left triangle including the diagonal, connection times are greater than CT. Each of these entries, then, represent a constraint in Constraints (23). Let

$$m = \underset{b}{argmax}\{b | CT - bd \geq MinCT, b \in \overline{\mathbb{N}^-}\},$$

then the total number of constraints in Constraints (24) is

$$p = \frac{1}{2}(2a + 1 - m)(2a - m),$$

and the total number of constraints in Constraints (23) is

$$q = (2a + 1)^2 - \frac{1}{2}(2a + 1 - m)(2a - m).$$

We have:

$$\begin{aligned} q - p =& (2a+1)^2 - (2a+1-m)(2a-m) \\ \geq& (2a+1)^2 - (2a+1)(2a-m) \\ \geq& (2a+1)(2a+1-2a+m) \\ \geq& (2a+1)(m+1) \\ >& 0. \end{aligned}$$

■

We, thus, design our re-optimization model to include Constraints (24) and Constraints (8) through (21).

6 Solution Approach and Computational Experiences

The passenger mix model and the re-optimization model are both implemented in C using ILOG CPLEX 9.0. Computational experiments are conducted on a workstation equipped with one Intel Pentium 4 2.8 GHz processor and 1 GB RAM.

For a typical problem in our experiments, the passenger mix model has around 3,000 rows, 9,000 columns, and 20,000 non-zeros after CPLEX preprocessing (see ILOG 2003, p. 322-324). The resulting formulation is solved easily, requiring only seconds of computation time, using the dual-simplex optimization routine in the CPLEX callable library.

The re-optimization model is solved by calling the mixed integer programming optimization routine in the CPLEX callable library. To improve tractability of our branch-and-bound algorithm for the schedule re-optimization model, we replace branching based on variable dichotomy with branching based on Type I Special Ordered Sets for Constraints (8) and assign CAP_π as weights to each variable $f_{lk\pi}$ (see ILOG 2003). Consider one node in the branch-and-bound tree, and suppose that variables in the cover constraint for flight leg l_0, that is,

$$\sum_{k \in C(l_0)} \sum_{\pi \in \Pi} f_{l_0 k \pi} = 1,$$

take value $f^*_{l_0 k\pi}$ $(\forall \pi \in \Pi, k \in C(l_0))$, and some of these values are fractional. We compute $\overline{w} = \sum_{k \in C(l_0)} \sum_{\pi \in \Pi} f^*_{l_0 k \pi} CAP_\pi$ and partition the set of binary variables $f_{l_0 k \pi}$ $(\forall \pi \in \Pi, k \in C(l_0))$ into two groups, $G_1 = \{f_{l_0 k\pi} | CAP_\pi \leq \overline{w}, k \in C(l_0), \pi \in \Pi\}$ and $G_2 = \{f_{l_0 k \pi} | CAP_\pi > \overline{w}, k \in C(l_0), \pi \in \Pi\}$. On one branch in our special ordered set branching strategy, we impose the following restriction: $\sum_{f_{l_0 k\pi} \in G_1} f_{l_0 k \pi} = 1$; and on the second branch, we require: $\sum_{f_{l_0 k\pi} \in G_2} f_{l_0 k\pi} = 1$. Hane et al. (1995) illustrate the superiority of branching based on Type I special ordered sets over that of branching on individual variables for fleet assignment problems (although Hane et al. apply a slightly different method to create G_1 and G_2). The sizes and solution times of the re-optimization model for a typical problem in our experiments are reported in Table 8. Instances A and B correspond to the two forecast scenarios introduced in Section 7.1.

	Instance A	Instance B
Num. of rows	16,731	21,262
Num. of columns	389,689	767,763
Num. of nonzeros	1,108,637	2,242,589
Num. of nodes searched	7,250	3,250
Solution time (hours)	10	20
Optimality gap	1.88%	2.34%

Table 8: Problem sizes and solution times

7 Case Study 1: Daily Schedules

In this section, we demonstrate the potential impact of dynamic airline scheduling, using data obtained from a major U.S. airline. The airline operates a hub-and-spoke network with a banked schedule, with about 1000 flights serving about 100 cities daily.

As a first step of our case study, we transform the banked hub-and-spoke schedule into a de-peaked daily hub-and-spoke schedule using a deterministic mathematical model (see Jiang 2006). Seven copies of each flight leg are created at -30, -20, -10, 0, +10, +20, and +30 minutes offset from the leg's departure time in the banked schedule. Our model's objective is to maximize profit while satisfying flight cover constraints, aircraft balance constraints, aircraft count constraints, and de-peaking constraints (constraints that limit the number of departure and arrival activities per minute). We use the resulting de-peaked schedule as our original schedule, over which we carry out all experiments detailed in this section. The original schedule is a daily de-peaked schedule with about 300 flights originating from the major hub and the same number of flights arriving at that hub.

As shown in Table 9, the airline operates 7 fleets with two fleet families, namely {JET2,JET3} and {RJ1,RJ2}. During re-optimization, seven flight copies are created each at -15, -10, -5, 0, +5, +10, + 15 minutes offset from the original flight times.

Fleet Type	Capacity
JET1	190
JET2	150
JET3	124
JET4	132
RJ1	86
RJ2	50
RJ3	37

Table 9: Fleet composition and capacity

7.1 Assumptions on Unconstrained Demand and Forecast Quality

Past research in this field has assumed that unconstrained passenger demands conform to certain probabilistic distributions. Berge and Hopperstad (1993) assume that the mean and standard deviation of unconstrained demand is specified by flight leg and fare class in terms of independent normal distributions truncated at zero. In Bish et al. (2004), leg demand is considered as independent normal random variables.

The assumption that leg demands are independent is doubtful as is shown in Kniker (1998) and is inappropriate in our experiments, especially when we emphasize capturing connecting passengers through flight re-timing. However, it is difficult if not impossible to define the join probability distribution function of unconstrained market demands. Instead of making assumptions on the stochastic behavior of unconstrained market demands and drawing samples from the joint distribution for our experiments, we obtain unconstrained market demands for each individual day by running an unconstraining algorithm on *observed bookings*. Such unconstrained market demands in each day are treated as a sample from the joint market demand distribution function.

In our case study, we examine data detailing the airline's operations for one week indexed as Day 1 (a Sunday) to Day 7 (a Saturday). The cumulative demand curves for the 7 days and the average cumulative demand curve are shown in Figure 11, while cumulative demand curves as a fraction of total demand are depicted in Figure 12. At 21 days prior to departure, about $45 \sim 50\%$ of total passenger demand has been realized. The shape of the curve reflects the booking behavior of passengers on each day: a flat curve indicates that a large portion of the passengers book well in advance; while a steep curve indicates that a larger portion of the passengers book close to departure. Day 1 (a Sunday) and Day 7 (a Saturday) have flatter curves because a large fraction of passengers traveling during the weekend are leisure travelers who tend to book early (Figure 12).

We study the effects of demand uncertainty on the quality of the solutions generated using our dynamic scheduling approach. We develop bounds on the potential benefits of dynamic scheduling by conducting experiments in which demand forecasts are achieved under one of two scenarios, namely:

1. the *perfect information* scenario in which future passenger demands are assumed to be known with certainty. Profit improvements estimated under this scenario provide an upper bound on the potential impact of our dynamic scheduling approach; and

2. the *historical average* scenario in which future passenger demands are estimated using a simple approach that averages historical demand data. Because airlines typically utilize more sophisticated, and hence more accurate, forecasting techniques, the impacts of dynamic scheduling estimated under this scenario provide a lower bound on the true impacts.

The quality of forecasts that are generated using average historical demand is shown in Figure 13. Unconstrained demand in Period 2 is depicted on the x-axis and the corresponding Period 2 demand forecasts derived using historical demand data are depicted on the y-axis. For perfect forecasts, all dots in these figures lie on the diagonal.

While Figure 13 provides a good visualization of forecast quality, we develop the following metrics to evaluate forecast quality. We begin by letting D_m^t be the true value of future demand in any market m and D_m^f be the corresponding forecast. We define *deviation* for market m as $D_m = D_m^f - D_m^t$; *absolute deviation* for market m as $|D_m|$; *relative deviation* for market m as $RD_m = \frac{(D_m^f - D_m^t)}{D_m^t}$; and *absolute relative deviation* for market m as $|RD_m|$. Positive deviation in a market indicates that forecasted demand is an overestimate of true passenger demand, while negative deviation indicates that true demand is underestimated. Absolute deviation measures the distance between estimated and true demand.

We partition markets into 8 groups based on their true market demand. The range of demands in each group are as follows: 0, (0,5], (5,10], (10,20], (20, 30], (30, 40], (40, 50], and above 50. In Figure 14, we show the average absolute deviation within each market group. Larger average absolute deviation values are observed as market size grows. In Figure 15, we show the average deviation within each market group. Notably, the average deviation for markets where $D_m^t = 0$ is always positive. Moreover, forecasts based on historical averages consistently overestimate demand for Day 7 (Figure 15). We depict the average absolute relative deviation within each market group in Figure 16; and the average relative deviation within each market group in Figure 17. When $D_m^t = 0$, relative deviation and absolute relative deviation are undefined and are therefore not reported.

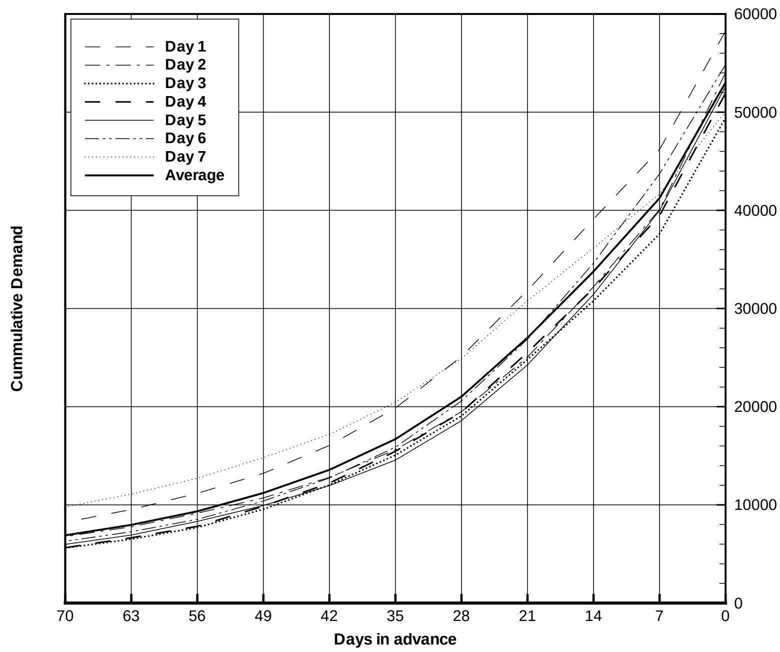

Figure 11: Cumulative demand

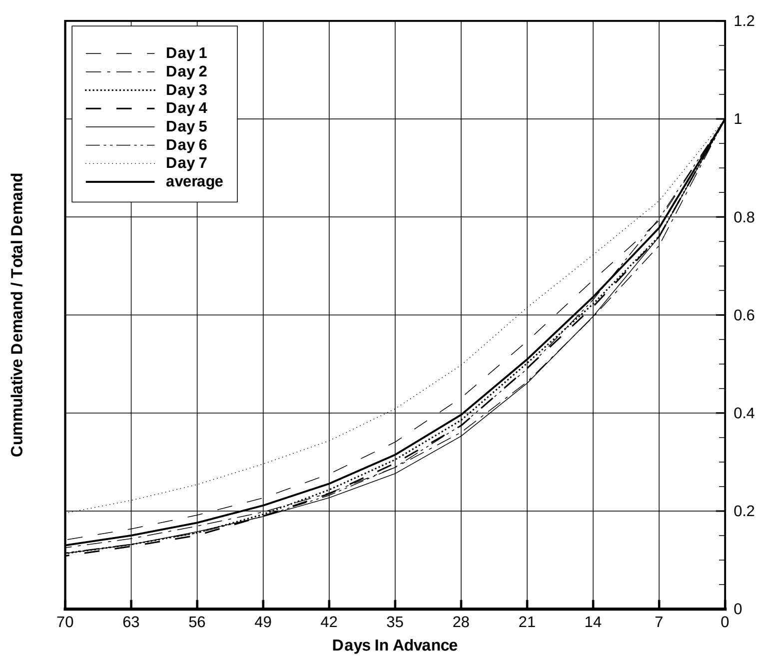

Figure 12: Cumulative demand as a fraction of total demand

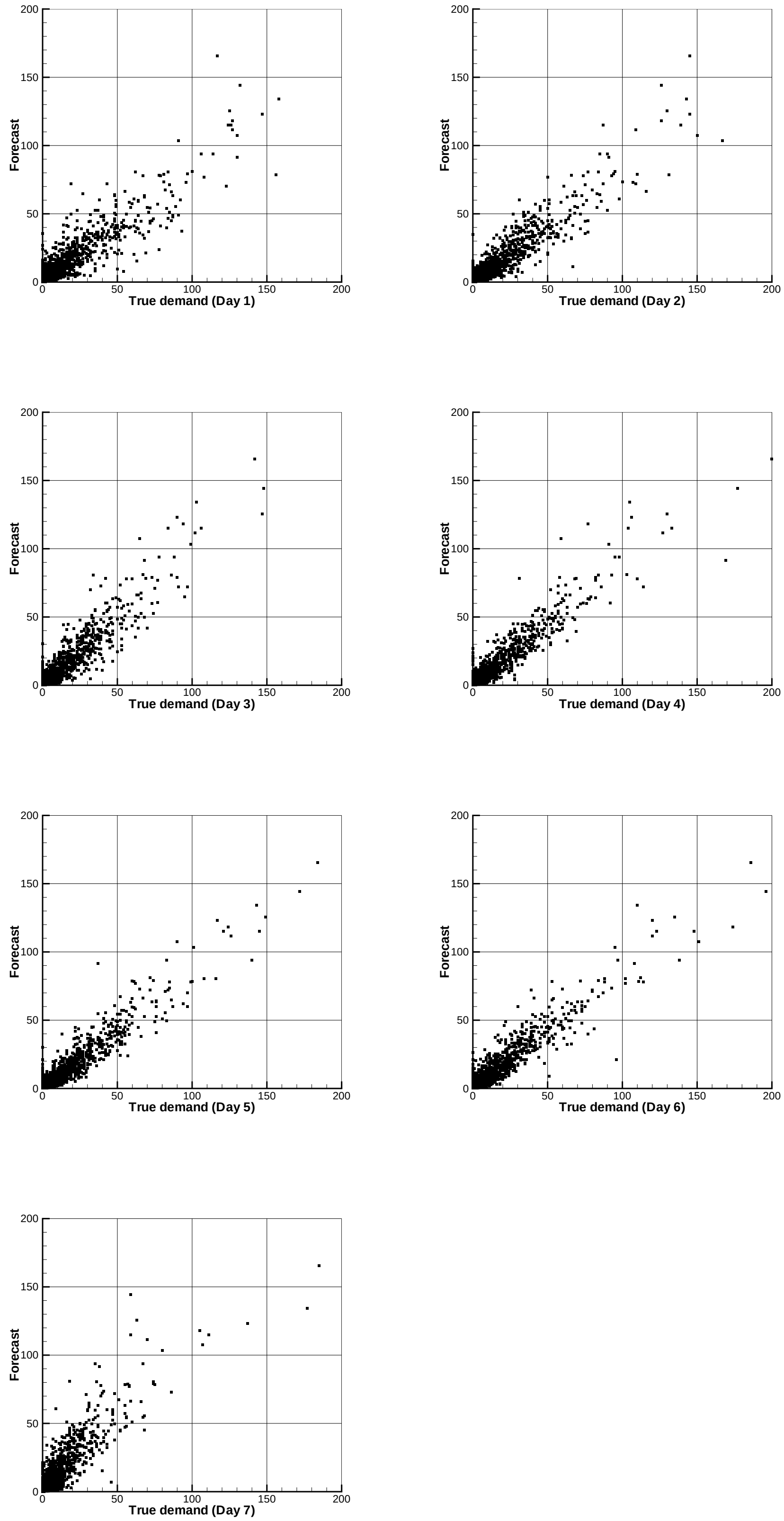

Figure 13: Forecast quality (scatter plot)

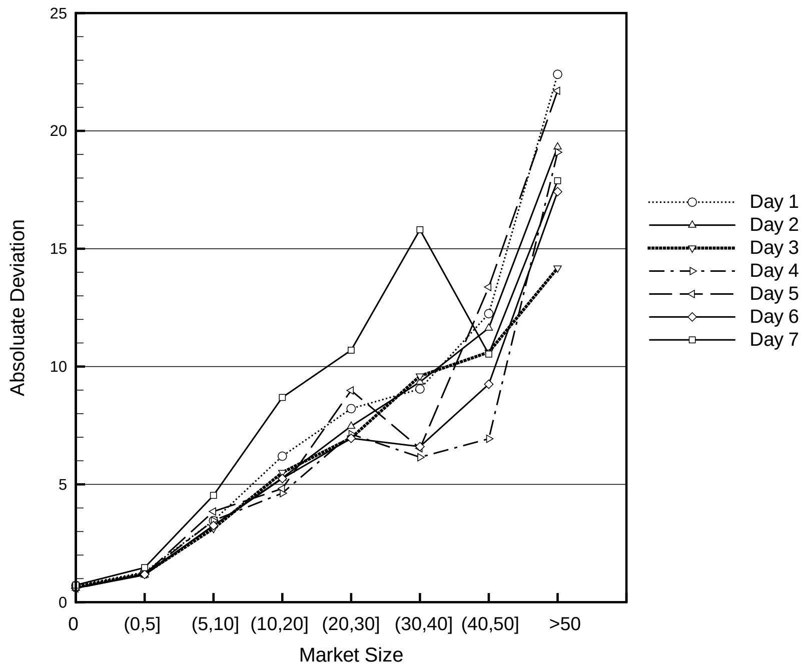

Figure 14: Average absolute deviation in each market group

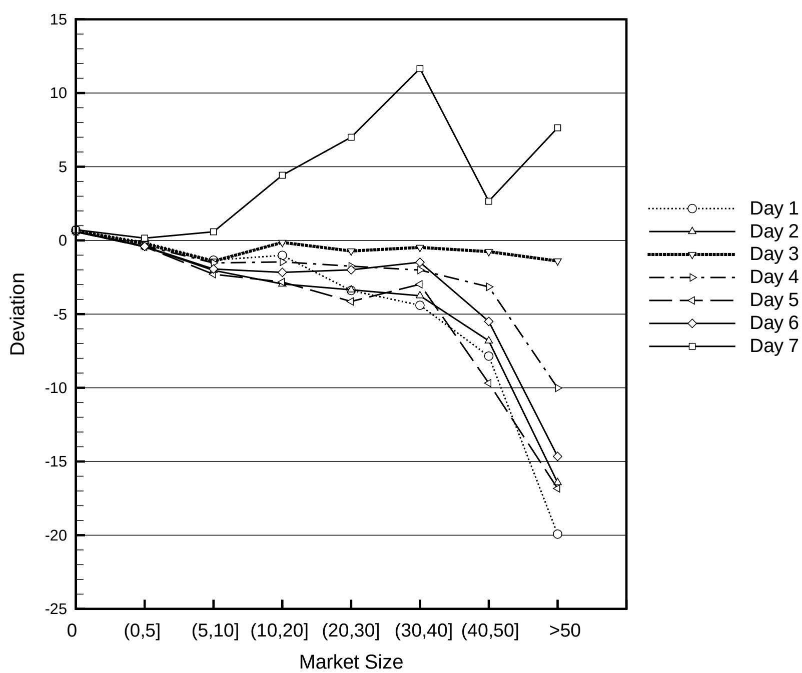

Figure 15: Average deviation in each market group

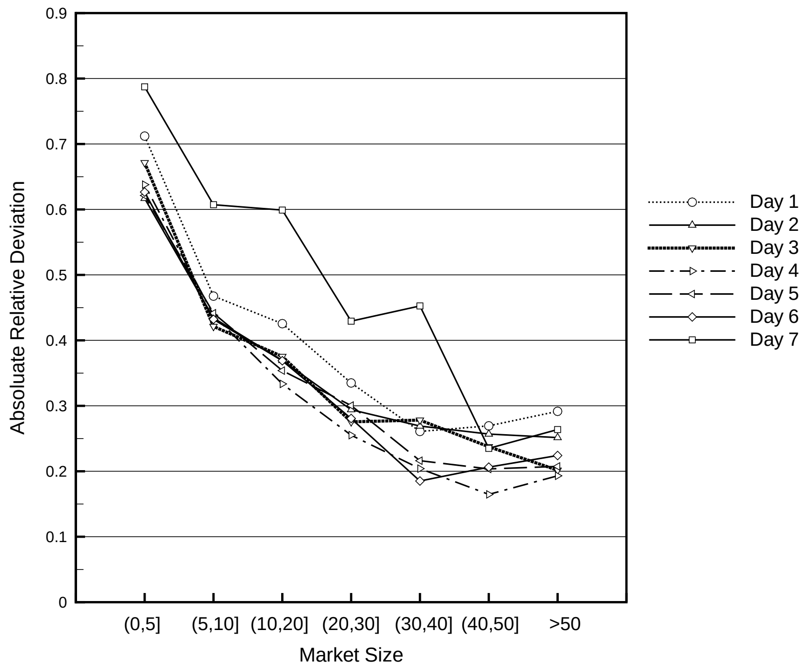

Figure 16: Average absolute relative deviation in each market group

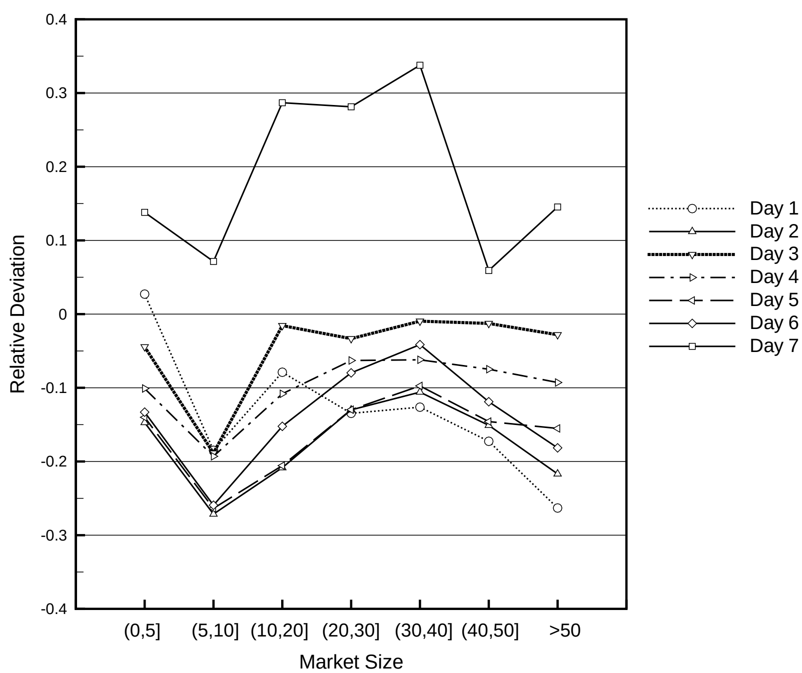

Figure 17: Average relative deviation in each market group

7.2 Results

In this section, we present the results of our experiments. We refer to the scenario with perfect information regarding future demands as *Forecast A*, and the scenario with forecast of future demands estimated using historical averages as *Forecast B*. Unless otherwise specified, when we refer to *profit increase*, *increase in revenue*, *increase in number of passengers* and similar metrics, we are measuring the change over the static case.

7.2.1 Profitability

Table 10 shows revenues, costs, and profits for the static case and the dynamic scheduling cases under Forecasts A and B, and reports the corresponding percentage changes relative to the static case. Under Forecast A, revenues increase 1-2% in each of the 7 days, while operating costs do not change significantly. The result is a 4-8% increase in profitability, or $70-140k daily. The average profit increase is 5.26%, or $99k daily ($36 million annually). This result, achieved with perfect demand forecasts, constitutes an upper bound on the profitability gains achievable with dynamic scheduling. More modest improvements in revenue and profit are observed under Forecast B, specifically, a 2.64%, or $50k increase in average daily profit ($18 million annually) is observed. Arguably, the sophisticated forecasting tools employed by airlines result in demand forecasts that are of better quality than those used in Forecast B. This result achieved with crude demand estimates, then, represents a lower bound on the profitability potential of dynamic scheduling.

	Day 1	**Day 2**	**Day 3**	**Day 4**	**Day 5**	**Day 6**	**Day 7**	**Average**
				Static				
Revenue	9,058,867	8,018,606	6,870,656	7,363,064	7,946,805	8,333,631	7,176,026	7,823,951
Cost	5,929,789	5,929,789	5,929,789	5,929,789	5,929,789	5,929,789	5,929,789	5,929,789
Profit	3,129,079	2,088,817	940,868	1,433,276	2,017,016	2,403,842	1,246,238	1,894,162
				Dynamic scheduling under Forecast A				
Revenue	9,217,867	8,136,941	6,934,930	7,446,343	8,037,549	8,457,232	7,263,078	7,927,706
	1.76%	1.48%	0.94%	1.13%	1.14%	1.48%	1.21%	1.33%
Cost	5,952,668	5,941,897	5,922,260	5,919,607	5,931,148	5,937,127	5,933,309	5,934,002
	0.39%	0.20%	-0.13%	-0.17%	0.02%	0.12%	0.06%	0.07%
Profit	3,265,199	2,195,045	1,012,670	1,526,736	2,106,401	2,520,105	1,329,769	1,993,704
	4.35%	5.09%	7.63%	6.52%	4.43%	4.84%	6.70%	5.26%
Profit incr.	136,120	106,227	71,803	93,461	89,385	116,263	83,531	99,541
				Dynamic scheduling under Forecast B				
Revenue	9,138,582	8,053,907	6,903,108	7,399,164	7,991,578	8,403,273	7,269,020	7,879,805
	0.88%	0.44%	0.47%	0.49%	0.56%	0.84%	1.30%	0.71%
Cost	5,929,631	5,923,936	5,916,015	5,936,996	5,934,338	5,935,877	5,972,766	5,935,651
	0.00%	-0.10%	-0.23%	0.12%	0.08%	0.10%	0.72%	0.10%
Profit	3,208,952	2,129,972	987,094	1,462,168	2,057,239	2,467,397	1,296,254	1,944,154
	2.55%	1.97%	4.91%	2.02%	1.99%	2.64%	4.01%	2.64%
Profit incr.	79,873	41,154	46,226	28,893	40,223	63,554	50,016	49,991

Table 10: Daily operating results under two forecast scenarios (in dollars)

7.2.2 Comparison Between Re-Timing and Re-Fleeting

The two dynamic scheduling mechanisms, flight leg re-timing and flight leg re-fleeting, are examined and compared in this section. Specifically, we study the profit contributions of each, and compare the relative magnitude of their contributions.

Table 11 shows the results under Forecast A for dynamic scheduling (that is, re-fleeting and re-timing both), re-fleeting only, and re-timing only. Let $P^A(s)$ be the profit increase using mechanism s under Forecast A. The first observation is that $P^A(\text{re-timing + re-fleeting}) > P^A(\text{re-timing}) + P^A(\text{re-fleeting})$ for all days. The two mechanisms, then, are synergistic and achieve greater profit gains than the sum of those achieved by each mechanism individually.

	Day 1	**Day 2**	**Day 3**	**Day 4**	**Day 5**	**Day 6**	**Day 7**	**Average**
				Static				
Revenue	9,058,867	8,018,606	6,870,656	7,363,064	7,946,805	8,333,631	7,176,026	7,823,951
Cost	5,929,789	5,929,789	5,929,789	5,929,789	5,929,789	5,929,789	5,929,789	5,929,789
Profit	3,129,079	2,088,817	940,868	1,433,276	2,017,016	2,403,842	1,246,238	1,894,162
				Dynamic scheduling				
Revenue	9,217,867	8,136,941	6,934,930	7,446,343	8,037,549	8,457,232	7,263,078	7,927,706
	1.76%	1.48%	0.94%	1.13%	1.14%	1.48%	1.21%	1.33%
Cost	5,952,668	5,941,897	5,922,260	5,919,607	5,931,148	5,937,127	5,933,309	5,934,002
	0.39%	0.20%	-0.13%	-0.17%	0.02%	0.12%	0.06%	0.07%
Profit	3,265,199	2,195,045	1,012,670	1,526,736	2,106,401	2,520,105	1,329,769	1,993,704
	4.35%	5.09%	7.63%	6.52%	4.43%	4.84%	6.70%	5.26%
Profit incr.	136,120	106,227	71,803	93,461	89,385	116,263	83,531	99,541
				Re-fleeting only				
Revenue	9,109,981	8,056,714	6,879,294	7,392,490	7,971,557	8,365,499	7,213,944	7,855,640
	0.56%	0.48%	0.13%	0.40%	0.31%	0.38%	0.53%	0.41%
Cost	5,940,955	5,934,028	5,925,018	5,932,752	5,931,862	5,927,903	5,941,608	5,933,447
	0.19%	0.07%	-0.08%	0.05%	0.03%	-0.03%	0.20%	0.06%
Profit	3,169,027	2,122,686	954,275	1,459,737	2,039,695	2,437,596	1,272,336	1,922,193
	1.28%	1.62%	1.43%	1.85%	1.12%	1.40%	2.09%	1.48%
Profit incr.	39,948	33,869	13,408	26,462	22,679	33,754	26,098	28,031
				Re-timing only				
Revenue	9,116,276	8,065,954	6,912,824	7,413,155	7,973,981	8,375,223	7,219,623	7,868,148
	0.63%	0.59%	0.61%	0.68%	0.34%	0.50%	0.61%	0.56%
Cost	5,929,731	5,929,673	5,929,673	5,929,673	5,929,789	5,929,673	5,929,615	5,929,689
	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Profit	3,186,545	2,136,281	983,151	1,483,483	2,044,192	2,445,551	1,290,008	1,938,459
	1.84%	2.27%	4.49%	3.50%	1.35%	1.74%	3.51%	2.34%
Profit incr.	57,466	47,464	42,284	50,207	27,176	41,709	43,770	44,297

Table 11: Comparison between re-fleeting and re-timing under Forecast A (in dollars)

Table 12 shows the results under Forecast B for dynamic scheduling, re-fleeting only, and re-timing only. Let $P^B(s)$ be the profit increase using mechanism s under Forecast B. $P^B(\text{re-timing + re-fleeting}) > P^B(\text{re-timing}) + P^B(\text{re-fleeting})$ is no longer true. In fact, in Days 2 and 4, $P^B(\text{re-timing + re-fleeting}) < P^B(\text{re-timing})$. However, the average profit improvement under dynamic scheduling is still greater than the sum of that under re-fleeting only and that under re-timing only. Moving from Forecast A to Forecast B, $P^B(\text{re-time}) \leq P^A(\text{re-time})$[1] and $P^B(\text{re-fleet}) \leq P^A(\text{re-fleet})$ are observed. These results are as expected given the deterioration in forecast quality in Forecast B compared with Forecast A.

Table 13 shows the ratio of the profit increase under Forecast B to that under Forecast A, that is, $P^B(s)/P^A(s)$ for some s. We conclude that re-fleeting is more sensitive to forecast quality because $P^B(\text{re-fleeting})/P^A(\text{re-fleeting})$ is significantly smaller than $P^B(\text{re-timing})/P^A(\text{re-timing})$. When re-fleeting is applied alone under Forecast B, there are two days (Days 2 and 3) when profits actually decline.

[1] $P^B(\text{re-time}) > P^A(\text{re-time})$ by 0.05% for Day 2, which is because we set the optimality GAP in CPLEX to 0.1%.

	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Average
				Static				
Revenue	9,058,867	8,018,606	6,870,656	7,363,064	7,946,805	8,333,631	7,176,026	7,823,951
Cost	5,929,789	5,929,789	5,929,789	5,929,789	5,929,789	5,929,789	5,929,789	5,929,789
Profit	3,129,079	2,088,817	940,868	1,433,276	2,017,016	2,403,842	1,246,238	1,894,162
				Dynamic scheduling				
Revenue	9,138,582	8,053,907	6,903,108	7,399,164	7,991,578	8,403,273	7,269,020	7,879,805
	0.88%	0.44%	0.47%	0.49%	0.56%	0.84%	1.30%	0.71%
Cost	5,929,631	5,923,936	5,916,015	5,936,996	5,934,338	5,935,877	5,972,766	5,935,651
	0.00%	-0.10%	-0.23%	0.12%	0.08%	0.10%	0.72%	0.10%
Profit	3,208,952	2,129,972	987,094	1,462,168	2,057,239	2,467,397	1,296,254	1,944,154
	2.55%	1.97%	4.91%	2.02%	1.99%	2.64%	4.01%	2.64%
Profit incr.	79,873	41,154	46,226	28,893	40,223	63,554	50,016	49,991
				Re-fleeting only				
Revenue	9,083,826	8,014,105	6,861,061	7,361,464	7,953,547	8,356,088	7,193,072	7,831,880
	0.28%	-0.06%	-0.14%	-0.02%	0.08%	0.27%	0.24%	0.10%
Cost	5,933,199	5,931,027	5,920,451	5,920,567	5,932,015	5,930,458	5,943,515	5,930,176
	0.06%	0.02%	-0.16%	-0.16%	0.04%	0.01%	0.23%	0.01%
Profit	3,150,627	2,083,078	940,610	1,440,897	2,021,532	2,425,629	1,249,557	1,901,704
	0.69%	-0.27%	-0.03%	0.53%	0.22%	0.91%	0.27%	0.40%
Profit incr.	21,548	(5,739)	(258)	7,622	4,516	21,787	3,319	7,542
				Re-timing only				
Revenue	9,113,048	8,067,139	6,909,084	7,396,036	7,972,271	8,359,889	7,214,381	7,861,693
	0.60%	0.61%	0.56%	0.45%	0.32%	0.32%	0.53%	0.48%
Cost	5,929,789	5,929,789	5,929,789	5,929,673	5,929,673	5,929,731	5,929,673	5,929,731
	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Profit	3,183,259	2,137,351	979,295	1,466,364	2,042,599	2,430,158	1,284,709	1,931,962
	1.73%	2.32%	4.08%	2.31%	1.27%	1.09%	3.09%	2.00%
Profit incr.	54,180	48,533	38,428	33,088	25,583	26,316	38,471	37,800

Table 12: Comparison between re-fleeting and re-timing under Forecast B (in dollars)

	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Average
P^A(re-fleeting)	39,948	33,869	13,408	26,462	22,679	33,754	26,098	28,031
P^B(re-fleeting)	21,548	-5,739	-258	7,622	4,516	21,787	3,319	7,542
P^B(re-fleeting)/P^A(re-fleeting)	53.9%	-16.9%	-1.9%	28.8%	19.9%	64.5%	12.7%	26.9%
P^A(re-timing)	57,466	47,464	42,284	50,207	27,176	41,709	43,770	44,297
P^B(re-timing)	54,180	48,533	38,428	33,088	25,583	26,316	38,471	37,800
P^B(re-timing)/P^A(re-timing)	94.3%	102.3%	90.9%	65.9%	94.1%	63.1%	87.9%	85.3%

Table 13: The ratio of profit increase under Forecast B to that under Forecast A when each mechanism is applied alone

Examining Table 11 and Table 12, it is observed that P^A(re-timing) $>$ P^A(re-fleeting) and P^B(re-timing) $>$ P^B(re-fleeting) for all days. The profit increase when only re-timing is allowed is greater than that when only re-fleeting is allowed. This is because: 1) all flights can have the potential to be re-timed; however, not all flight legs can be re-fleeted; 2) under re-fleeting, the set of possible connecting itineraries remain unchanged. Re-timing can create new itineraries to increase the schedule's ability to serve a broader set of passenger demands, thus improving the ability to capture more high fare passengers and/or capture more passengers to fill empty seats.

In Table 14, we report the statistics about increases in revenue and number of passengers when re-timing and re-fleeting are applied alone under Forecast A. Results for Forecast A are presented because they are not complicated by the effect of forecast quality on re-timing and re-fleeting decisions, and thus clearly demonstrating the behavior of the two mechanisms. The first two rows show the increase in the numbers of nonstop passengers and in nonstop revenues for each day compared to the static case. The next two rows show the increase in the numbers of connecting passengers and in connecting revenues in each day compared to the static case. We see that re-timing primarily captures connecting passengers.

The rest of the rows in this table analyze connecting passengers in detail. When re-timing is allowed, all connecting itineraries can be categorized into three groups:

- Itinerary Group A: these connecting itineraries are infeasible in the original schedule, but become feasible after re-timing;
- Itinerary Group B: these connecting itineraries are feasible in the original schedule, but become infeasible after re-timing;
- Itinerary Group C: these connecting itineraries are feasible before and after re-timing.

Rows 5 and 6 in this table report the numbers and associated revenues of connecting passengers traveling on Itinerary Group A after re-timing in each day. Rows 7 and 8 report the numbers and associated revenues of connecting passengers traveling on Itinerary Group B in the static case in each day. These passengers are not able to travel on the re-timed schedule and therefore are spilled or recaptured on alternative itineraries. Rows 9 and 10 report the increase in the numbers and associate revenues of connecting passengers traveling on Itinerary Group C. It is seen that because flight re-timing is able to create new itineraries to reach a larger set of passenger demands, it is able to change the revenue composition of connecting passengers and serve passengers with high revenue gains. On average, a larger amount of revenue from connecting passengers traveling on Itinerary Group A are captured than are lost through eliminating itineraries in Itinerary Group B and spilling passengers in Itinerary Group C. Markets with high revenue potential are served by reducing or eliminating service to markets with low revenue potential.

Another interesting result deserving attention is that $P^B(\text{re-time}) > P^A(\text{re-fleet})$ for all days except Day 6, implying that the profit improvement when re-timing alone is applied under Forecast B is larger than that when re-fleeting alone is applied under Forecast A.

In summary, from our experiments, we find that re-timing is less sensitive than re-fleeting to forecast quality; and re-timing contributes more than re-fleeting to the potential benefits of dynamic scheduling.

	Day 1		Day 2		Day 3		Day 4		Day 5		Day 6		Day 7		Average	
	Re-fleet only	Re-time only	Re-fleet only	Re-time only	Re-fleet only	Re-time only	Re-fleet only	Re-time only	Re-fleet only	Re-time only	Re-fleet only	Re-time only	Re-fleet only	Re-time only	Re-fleet only	Re-time only
Incr. in num. of nonstop pax	221	98	154	18	49	22	80	27	156	-12	76	22	32	6	110	26
Incr. in nonstop rev.	34,705	12,531	17,040	115	4,247	1,242	10,087	3,299	16,883	-766	7,639	3,879	4,857	-946	13,637	2,765
Incr. in num. of connecting pax	55	202	146	265	61	204	120	262	36	174	146	222	190	215	108	221
Incr. in connecting rev.	16,409	44,877	21,069	47,233	4,391	40,926	19,339	46,792	7,870	27,942	24,229	37,714	33,061	44,542	18,052	41,432
Num. of connecting pax on Itin. A	-	775	-	957	-	714	-	866	-	861	-	897	-	640	-	816
Associated rev.	-	155,813	-	174,496	-	127,481	-	159,286	-	148,771	-	165,141	-	121,638		150,375
Num. of connecting pax on Itin. B	-	334	-	358	-	403	-	383	-	376	-	391	-	244	-	355
Associated rev.	-	63,602	-	62,182	-	60,674	-	63,370	-	70,359	-	69,192	-	38,875		61,179
Change in num. of connecting pax on Itin. C	-	-238	-	-334	-	-107	-	-220	-	-311	-	-284	-	-181	-	-239
Associated rev.	-	-47,334	-	-65,081	-	-25,881	-	-49,124	-	-50,469	-	-58,236	-	-38,221	-	-47,764

Table 14: Passenger and revenue (in dollars) statistics under Forecast A when re-fleeting and re-timing are applied alone

7.2.3 Sensitivity to the Number of Re-Timed Flights

In our dynamic scheduling approach, schedule adjustments are made at each re-optimization point and finalized after the last re-optimization. In our case, the last (and the only) re-optimization point is 21-days prior to the departure day. Because schedule changes can raise many operational concerns (as detailed in Section 9), our goal is to determine a balance that minimizes the number of schedule changes while maximizing the benefits of changes. We examine the sensitivity of our results as we limit the number of allowed schedule changes, particularly the number of re-timed flights. We do not investigate the sensitivity of the solution quality to limits on the number of fleeting changes because re-fleeting within fleet families requires fewer operational changes.

Experiments are conducted under Forecast A when the number of re-timed flights is constrained. The number of re-fleeted flights is unrestricted. In Figure 18, we show the profit increase as a function of the total number of re-timed flights. In Figure 19, we show the profit achieved as a fraction of that achievable when all flight legs are allowed to be re-timed. No incremental profit increase is observed when the number of re-timed flights is allowed to exceed 300, therefore the maximum value on the x-axis of these figures is set to be 300. All curves are concave functions of the number of re-timed flights. The case when the number of re-timed flights is restricted to zero is equivalent to the re-fleeting only case. We observe that marginal profit improvements decrease as the number of re-timed flights increases; and profit improvement is nearly fully realized when the number of re-timed flights is 100. Hence, even re-timing a moderate number of the flight legs allows us to reap nearly all of the potential benefits.

Table 15 shows the profit increase when we limit the number of re-timed flights to 100 under Forecast B. The profit increases associated with limiting the number of re-timed flights are comparable to those when there is no limit. Interestingly, in Days 5, 6 and 7, larger profit improvements are achieved when limits are placed on the number of re-timed flights. It shows that when forecast is imperfect, having less re-optimization flexibility does not necessarily lead to less profit increase. Had we conducted the experiments with the number of re-timed flights restricted between 0 and 100, we do not expect to see concave functions similar to those shown in Figure 18 and Figure 19.

	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Average
Static case								
Revenue	9,058,867	8,018,606	6,870,656	7,363,064	7,946,805	8,333,631	7,176,026	7,823,951
Cost	5,929,789	5,929,789	5,929,789	5,929,789	5,929,789	5,929,789	5,929,789	5,929,789
Profit	3,129,079	2,088,817	940,868	1,433,276	2,017,016	2,403,842	1,246,238	1,894,162
Dynamic scheduling under Forecast B (No limit on num. of re-timed flights)								
Revenue	9,138,582	8,053,907	6,903,108	7,399,164	7,991,578	8,403,273	7,269,020	7,879,805
Cost	5,929,631	5,923,936	5,916,015	5,936,996	5,934,338	5,935,877	5,972,766	5,935,651
Profit	3,208,952	2,129,972	987,094	1,462,168	2,057,239	2,467,397	1,296,254	1,944,154
Profit incr.	2.55%	1.97%	4.91%	2.02%	1.99%	2.64%	4.01%	2.64%
Dynamic scheduling under Forecast B (Num. of re-timed flights $\leq$ 100)								
Revenue	9,146,525	8,052,977	6,897,193	7,400,498	7,991,952	8,408,739	7,256,829	7,879,245
Cost	5,964,639	5,933,586	5,918,713	5,948,519	5,927,238	5,929,069	5,959,104	5,940,124
Profit	3,181,886	2,119,391	978,480	1,451,980	2,064,714	2,479,670	1,297,725	1,939,121
Profit incr.	1.69%	1.46%	4.00%	1.30%	2.36%	3.15%	4.13%	2.37%

Table 15: Profit increase when limiting the number of re-timed flights under Forecast B

7.2.4 Properties of New Connecting Itineraries

One of the benefits of flight re-timing is to create new connecting itineraries serving markets with greater than expected demand. In Figure 20, we show two types of itineraries that re-timing can create. In the figure, passengers cannot connect between the inbound flight and outbound flights a and d because connection times are either too short or too long. We refer to these as *infeasible connections*. When flight leg a is re-timed to b and d is re-timed to c, two new connections are created. We classify connecting itineraries enabled by schedule re-optimization as either *Type I connecting itineraries* or *Type II connecting itineraries*. Type I connecting itineraries are those that were infeasible prior to re-optimization because their connection times were less than the *minimum* allowed. Type II connecting itineraries are those whose connection times before re-optimization exceeded the *maximum* allowed. Passengers traveling on Type I (or Type II) connecting itineraries are called Type I (or Type II) passengers. It is more desirable to create Type I itineraries to serve Type I passengers, because Type II itineraries have much longer connection times and their salability is poor. In Table 16, we show the number of Type I and Type II passengers and the average connection time for each type under Forecast A and Forecast B. Although no explicit incentive is given in our re-optimization model to create Type I itineraries and accommodate Type I passengers, Table 16 shows that about 80% of the passengers traveling on newly created itineraries are of Type I under both forecast scenarios, with an average connecting time of 30-34 minutes. The reason why the majority of passengers are Type I passengers is because when protecting connecting itineraries booked by previous passengers, inbound flights are more likely to be moved earlier, while outbound flights are more likely to be moved later. When an inbound flight is moved earlier, it creates Type I connecting itineraries; when an outbound flight is moved later, it also creates Type I connecting itineraries. The consequence is that more Type I itineraries are created and therefore, we serve more Type I passengers.

		Forecast A			Forecast B		
		Num. of pax	Pct.	Avg. conn. time	Num. of pax	Pct.	Avg. conn. time
Day 1	Type I	544	85.5%	32.4	514	74.6%	33.5
	Type II	92	14.5%	175.2	175	25.4%	144.7
Day 2	Type I	370	92.2%	31.0	782	76.5%	34.3
	Type II	31	7.8%	175.4	240	23.5%	150.9
Day 3	Type I	316	84.3%	30.5	570	73.9%	34.4
	Type II	59	15.7%	175.9	201	26.1%	164.3
Day 4	Type I	277	78.7%	31.0	581	83.4%	33.1
	Type II	75	21.3%	174.9	116	16.6%	174.2
Day 5	Type I	534	84.4%	31.9	651	84.1%	33.6
	Type II	99	15.6%	176.5	123	15.9%	175.6
Day 6	Type I	449	85.1%	31.3	394	76.5%	33.7
	Type II	78	14.9%	174.4	121	23.5%	170.5
Day 7	Type I	237	64.6%	31.4	674	91.9%	32.8
	Type II	130	35.4%	174.5	60	8.1%	174.4

Table 16: Statistics on Type I and Type II passengers ($MinCT = 25$ minutes and $MaxCT =$ 180 minutes)

7.3 Quality of the Original Schedule

As stated in the beginning of Section 7, the original schedule used in our experiments is generated by an optimization model. In this section we investigate the quality of the original schedule and analyze whether the benefits of dynamic scheduling is resulted from a poorly designed original schedule, particularly, a poorly designed timetable.

We examine flight re-timing decisions under perfect information for the 7 days. Perfect information is used because re-timing decisions under perfect information indicate there may be better flight times for those flight legs. If a flight leg is re-timed at least 5 out of 7 days, it is classified as frequently re-timed. If the re-timing decisions for a flight leg occurs in the same direction (that is, earlier or later than the original schedule) for all days, these re-timing decisions are called *consistent*. When a flight leg is frequently and consistently re-timed, it is likely that this flight is poorly scheduled in the original schedule. The benefits of employing dynamic scheduling techniques for this flight leg could be minimal if this flight leg had been optimally scheduled in the original schedule. In Table 17, we report the re-timing decisions for the frequently re-timed flight legs in our experiments. In the first row, we report that Flight leg 185 is postponed by 10 minutes on 6 out of the 7 days and by 15 minutes in one day. Flight leg 185 is clearly frequently and consistently re-timed, and hence, it might be poorly positioned in the original schedule. If, however, a flight leg is frequently, but not consistently re-timed (for example, Flight legs 50 and 216), we do not view it as poorly placed in the original schedule. Instead, variations in daily demand require it to be scheduled differently on a day-to-day basis.

Table 17 shows the 22 frequently re-timed flights, that is, re-timed at least 5 times. Of these 22 flights, 7 are moved inconsistently. In the worst case, then, the schedule for 15 flights, or less than 2% of the flight legs, in the original schedule is *not optimal*. Given that changes to one flight leg's schedule impacts the set of feasible schedules for a set of other flight legs, it is likely that alternative original schedules will similarly have some small percentage of flight legs that will be consistently rescheduled under dynamic scheduling approaches. We conclude, then, that our potential is very limited to improve the original schedule.

Flight #	Flight Detail					-15	-10	-5	0	+5	+10	+15	consistent?
185	SJC	1220	→	HUB	1410						6	1	
470	MCI	900	→	HUB	951	5	2						
610	YYC	725	→	HUB	937					1		6	
68	HUB	1655	→	SJC	1844				1			6	
171	HUB	1436	→	DFW	1856				1	1	4	1	
234	BUR	1525	→	HUB	1650	6			1				
263	HUB	1425	→	ABQ	1635				1		1	5	
455	STL	830	→	HUB	951	4	1		1	1			N
512	HUB	1439	→	MSP	1953				1		4	2	
600	HUB	755	→	BUR	918			6	1				
645	SAN	2000	→	HUB	2122				1	2	2	2	
50	HUB	925	→	ATL	1622		1	3	2	1			N
168	DTW	755	→	HUB	900				2	5			
195	LAS	650	→	HUB	804	4			2		1		N
216	SNA	1000	→	HUB	1119		1	3	2		1		N
220	SNA	1200	→	HUB	1320				2	2		3	
235	HUB	1812	→	ABQ	2022	1	1		2			3	N
326	LAS	1156	→	BOS	2012	3			2	2			N
417	IAH	600	→	HUB	644	5			2				
559	PDX	700	→	HUB	935	1			2			4	N
605	SNA	1715	→	HUB	1837	4		1	2				
709	EUG	555	→	HUB	825	5			2				

Table 17: Re-timing decisions for frequently re-timed flights

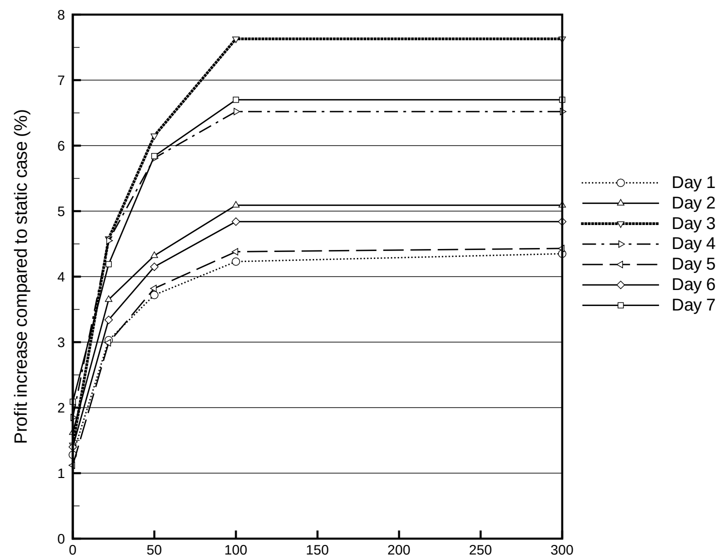

Figure 18: Profit increase as a function of total number of re-timed flights (Forecast A)

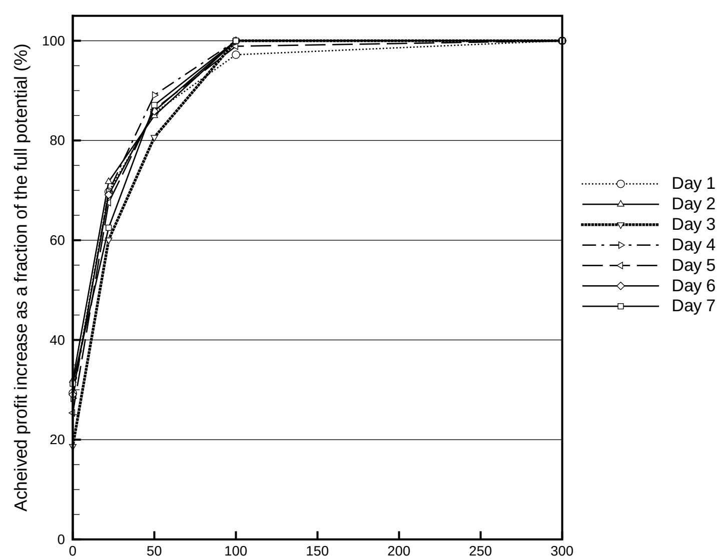

Figure 19: Achieved profit increase as a function of total number of re-timed flights (Forecast A)

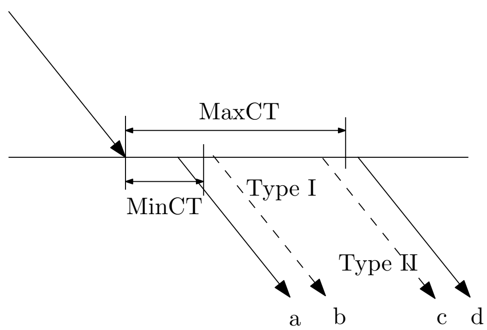

Figure 20: Two types of new connecting itineraries

8 Case Study 2: Weekly Schedules

In the previous section, a daily repeatable schedule is assumed in which the same flight schedule is repeated each day. The advantage of this assumption is reduced computational requirements and ease of implementation operationally. Given this daily schedule, dynamic scheduling can then be applied to adjust each day's schedule in response to passenger demand variations. The question we pose in this section is whether or not a weekly schedule, that is, one in which a different schedule is designed for each day of the week, is adequate to capture demand variations and eradicate the need for dynamic scheduling.

8.1 Schedule Generation

In Figure 21, we show the variations in daily flight load factors for a major U.S. airline in a 4-week period, depicting the variability in daily passenger demands. For each day, the mean

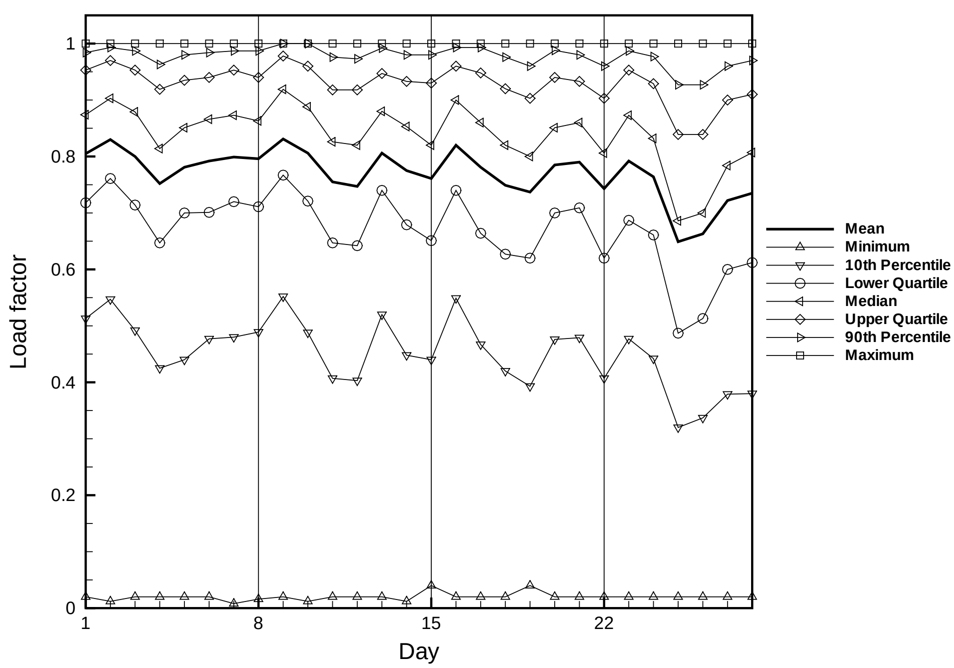

Figure 21: Daily mean load factor and quantiles of the load factor histogram for a major U.S. airline in a 4-week period. Days 1, 8, 15, and 22 correspond to Saturdays

load factor and several representative quantiles of the load factor histogram are shown. It is seen that the variations in load factors follow a weekly pattern. Clearly, higher revenues can be realized by flying a different schedule tailored to each day of the week, that is, a weekly flight schedule. A weekly schedule can be developed by expanding the daily time-lines of the time-space network to weekly time-lines. A weekly schedule design model, however, with its extended size, is much harder to solve than a daily schedule design model. In this research, we do not tackle the difficult problem of weekly schedule design, but rather generate a schedule that incorporates our knowledge about day-of-week demand variations and by solving the daily schedule design model (see Jiang 2006) for each day of the week, using average demands for that day. We generate a Sunday schedule based on average Sunday demands, a Monday schedule based on average Monday demands, $\cdots$, and a Saturday schedule based on average Saturday demands. These seven daily schedules do not constitute a weekly schedule because each day's schedule is constructed independently and there is no guarantee that aircraft balance will be preserved from one day to the next. Notwithstanding this, the resulting daily schedules provide an approximation (albeit an overly optimistic one) of the ability of a weekly schedule to capture day-of-week demand variations. If dynamic scheduling can still improve the economics of daily schedules constructed in this manner, it should have at least as great a benefit when applied to a weekly schedule.

In our analysis, we select a specific day of week, namely, Monday, and develop a daily schedule based on average Monday demand. Then, we evaluate the effects of applying dynamic scheduling to seven Mondays (indexed as $W1$, $W2$, $\cdots$, $W7$) in seven consecutive weeks. Although we do not perform tests for the remaining days of the week, we expect that the results would be similar.

8.2 Unconstrained Demand and Forecast Quality

We treat unconstrained demand and forecast quality in the same way as detailed in Section 7.1. In Figure 22, we depict the average cumulative demand curve and the cumulative demand curves for the 7 experimental Mondays. We observe that there are variations in total unconstrained demand, despite the fact that these seven days are all the same day of the week. In Figure 23, we show cumulative demand curves as a fraction of total demand in each day. Not surprisingly, we note that the curves are more similar in Figure 23 than those for all days of one week in Figure 12.

Similar to our previous case study, two forecast scenarios are considered: 1) the perfect information scenario; and 2)the historical average scenario. In Figures 24 through 28, we show the quality of the forecasts generated from average historical demand. It is worth noting that the shapes of the curves in Figures 25 through 28 are more uniform than their counterparts in the previous case study (Figures 14 through 17).

Examining Figures 26 and 28, all curves are below zero for market group (0,5], (5,10], and (40,50] and above 50. It may seem odd, because there should be at least one curve above zero, however, this is because the historical average demands are averaged over twelve Mondays, but we only show seven of them in the figures, that is, the we only show the seven curves corresponding to the seven Mondays we perform experiments on.

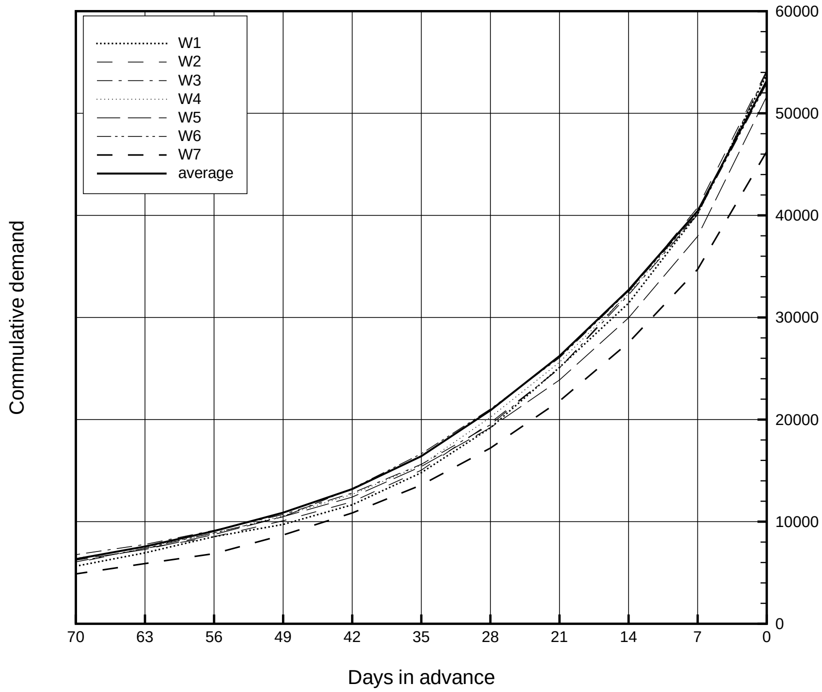

Figure 22: Cumulative demand curves

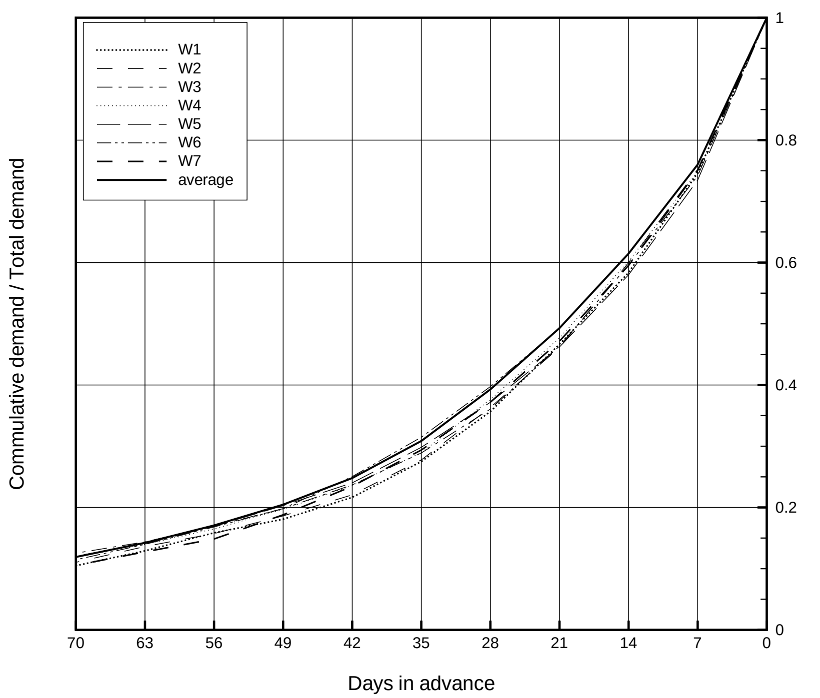

Figure 23: Cumulative demand curves as a fraction of total demand

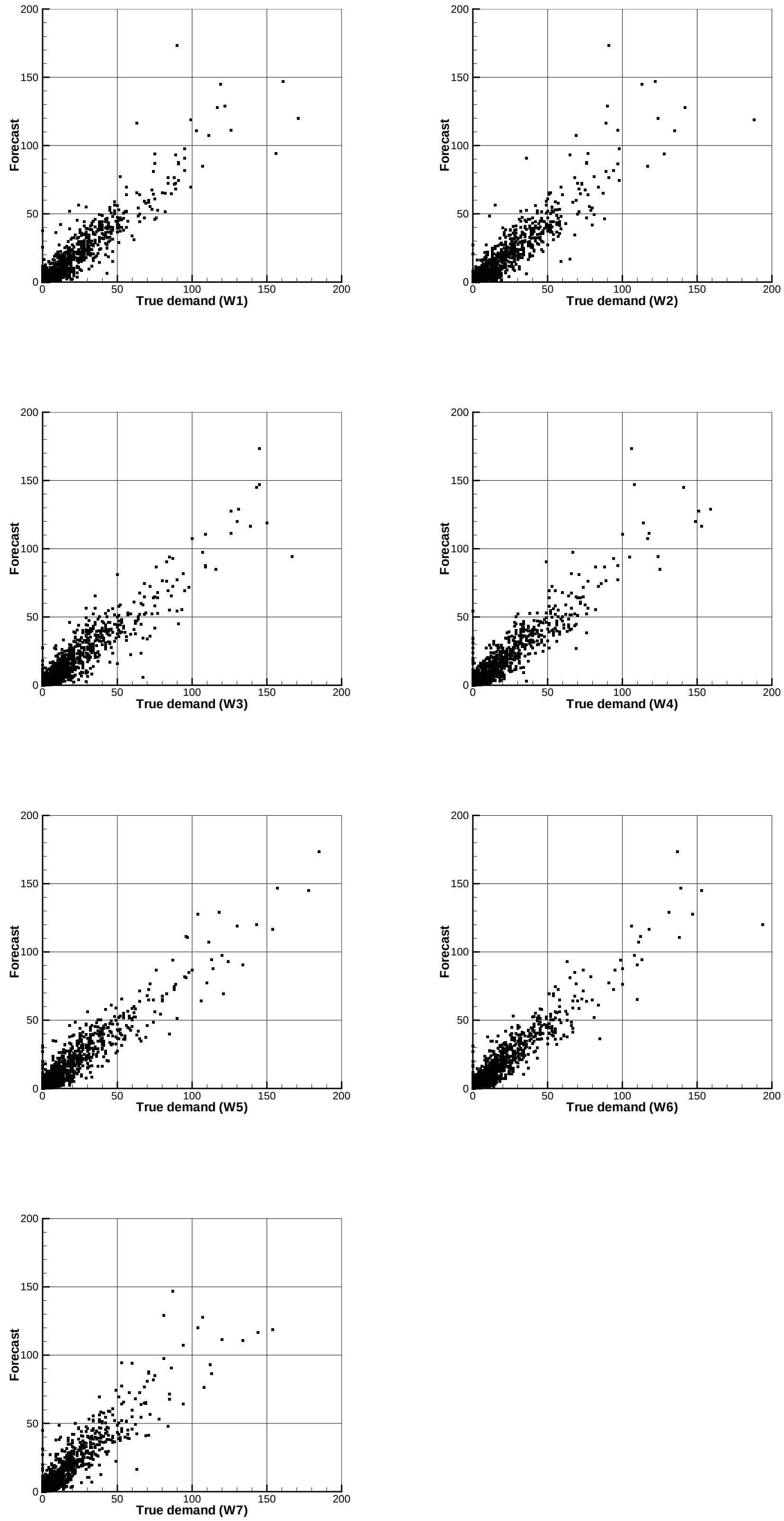

Figure 24: Forecast quality (scatter plot)

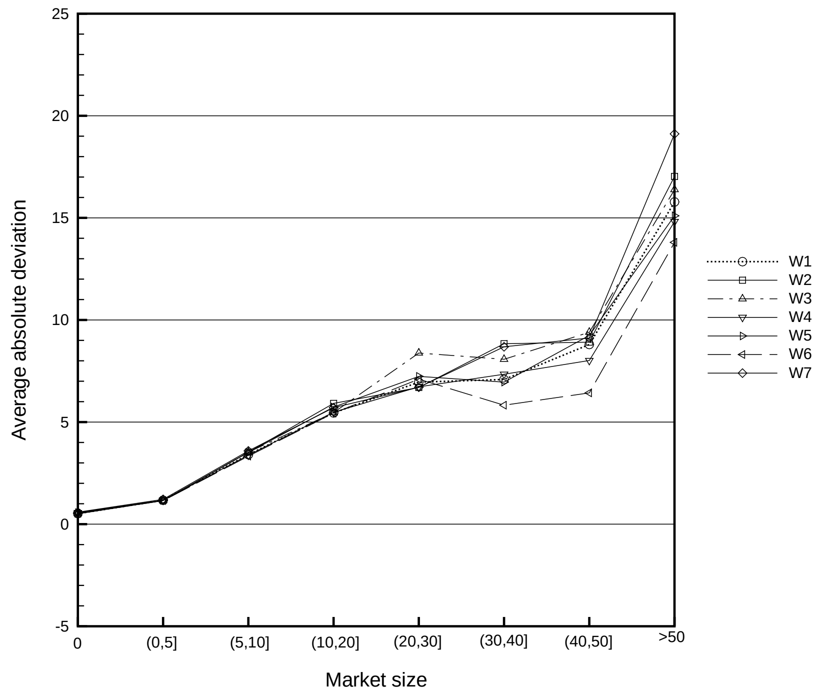

Figure 25: Average absolute deviation in each market group

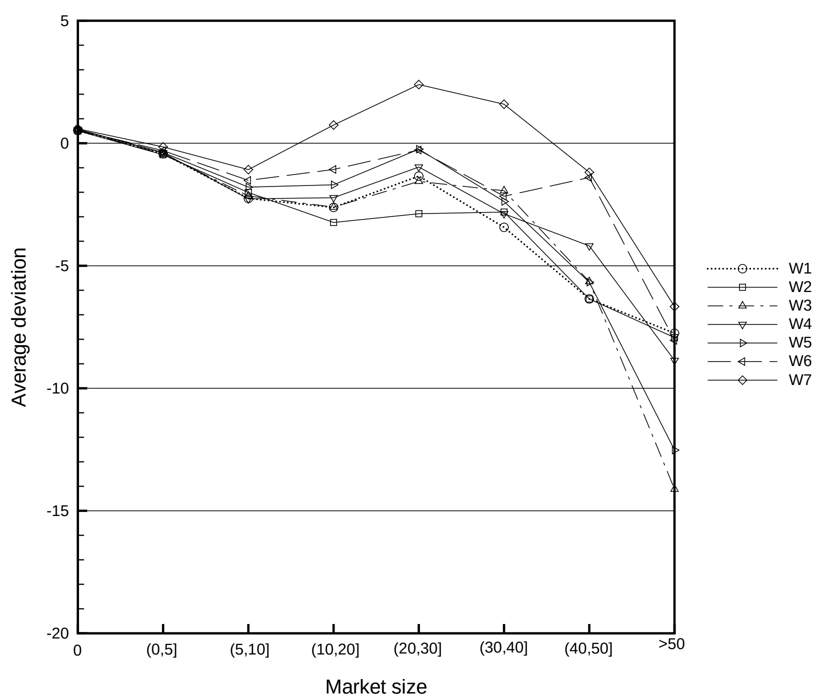

Figure 26: Average deviation in each market group

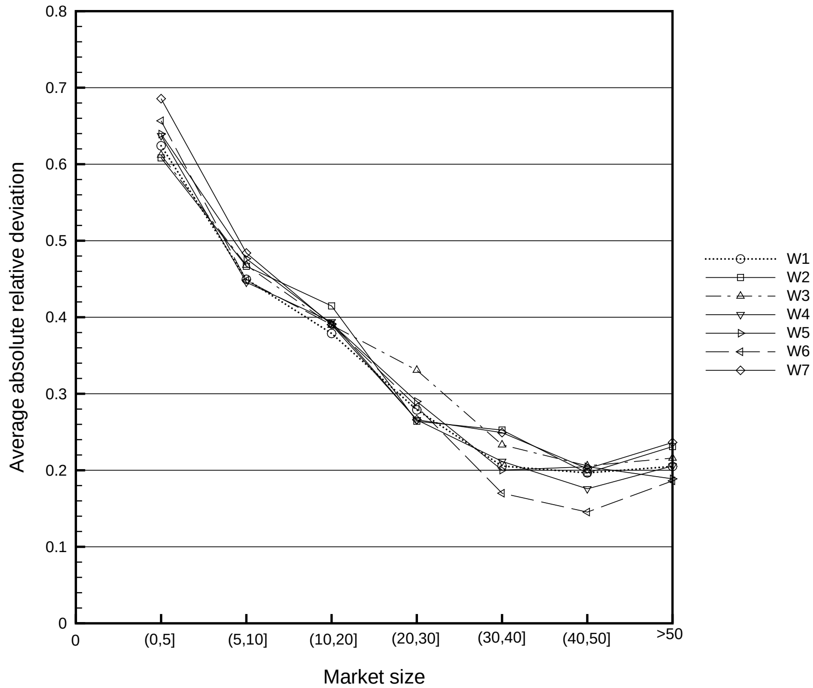

Figure 27: Average absolute relative deviation in each market group

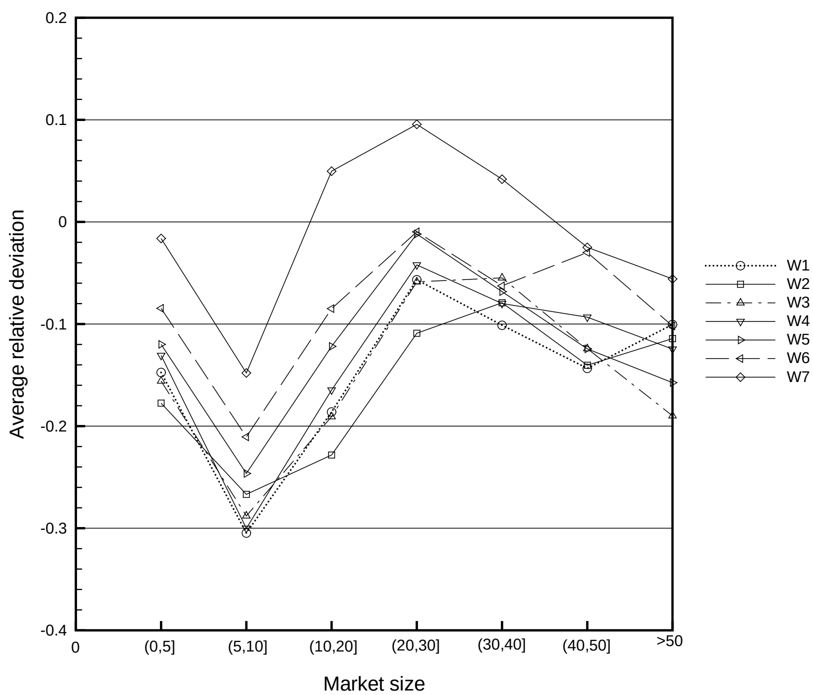

Figure 28: Average relative deviation in each market group

8.3 Results

In this section, we present the results of our experiments. Based on the results presented in Section 7, we limit the number of re-timed flights to 100 in all of the following experiments.

In Table 18, we show, for each day, the revenues, costs, and profits associated with the static schedule, and the dynamic scheduling cases under the two forecast scenarios. The benefit of dynamic scheduling remains significant. Under Forecast A, revenue goes up by 1.41%, costs increase by 0.28%, and profit improves by 4.97%, or $92k daily ($33 million annually) on average. Under Forecast B, revenue increases by 0.77%, costs increase by 0.29%, and profit improves by 2.28%, or $42k daily ($15 million annually). We believe that it is likely that the sophisticated forecasting engines used by airlines will result in demand forecasts of better quality than Forecast B, and hence, the results under Forecast B should represent a lower bound on the profitability potential of dynamic scheduling when applied to this particular day of week under weekly schedules.

	W1	**W2**	**W3**	**W4**	**W5**	**W6**	**W7**	**Average**
				Static				
Revenue	7,825,855	7,758,300	8,006,721	8,079,302	7,778,746	7,178,135	7,729,064	7,765,160
Cost	5,905,362	5,905,362	5,905,362	5,905,362	5,905,362	5,905,362	5,905,362	5,905,362
Profit	1,920,493	1,852,938	2,101,359	2,173,940	1,873,385	1,272,774	1,823,702	1,859,799
				Dynamic scheduling under Forecast A				
Revenue	7,939,360	7,863,389	8,122,944	8,203,797	7,878,834	7,272,242	7,839,553	7,874,303
	1.45%	1.35%	1.45%	1.54%	1.29%	1.31%	1.43%	1.41%
Cost	5,918,295	5,924,856	5,925,398	5,932,377	5,912,754	5,917,284	5,923,882	5,922,121
	0.22%	0.33%	0.34%	0.46%	0.13%	0.20%	0.31%	0.28%
Profit	2,021,065	1,938,532	2,197,547	2,271,420	1,966,081	1,354,958	1,915,672	1,952,182
	5.24%	4.62%	4.58%	4.48%	4.95%	6.46%	5.04%	4.97%
Profit incr.	100,572	85,594	96,188	97,480	92,697	82,185	91,970	92,384
				Dynamic scheduling under Forecast B				
Revenue	7,879,739	7,820,891	8,049,311	8,114,287	7,848,838	7,249,730	7,809,586	7,824,626
	0.69%	0.81%	0.53%	0.43%	0.90%	1.00%	1.04%	0.77%
Cost	5,915,842	5,918,301	5,915,032	5,915,032	5,922,767	5,920,097	5,949,482	5,922,364
	0.18%	0.22%	0.16%	0.16%	0.29%	0.25%	0.75%	0.29%
Profit	1,963,897	1,902,590	2,134,279	2,199,255	1,926,072	1,329,634	1,860,104	1,902,262
	2.26%	2.68%	1.57%	1.16%	2.81%	4.47%	2.00%	2.28%
Profit incr.	43,405	49,652	32,920	25,315	52,687	56,860	36,402	42,463

Table 18: Daily operating results under two forecast scenarios (in dollars)

8.4 Quality of the Original Schedule

We present the re-timing decisions of frequently re-timed flight legs in Table 19. 27 flight legs are frequently re-timed, among which 3 are not consistently re-timed. Compared to Table 17, we have slightly more flight legs frequently re-timed in this experiment, but the number still represents only about 2% of the total number of flight legs. For reasons stated in the previous section, we believe that our estimated profit improvements result principally from enhancements possible through dynamic scheduling rather than from an inadequate original schedule.

Flight #	Flight detail					-15	-10	-5	0	5	10	15	Consistent?
271	LAS	1210	→	HUB	1324	7							
276	HUB	1738	→	DEN	2025					2	2	3	
512	HUB	1439	→	MSP	1953						3	4	
167	LAS	2232	→	DTW	2927				1	1	3	2	
174	ONT	720	→	HUB	834				1	3	3		
185	SJC	1220	→	HUB	1410				1		3	3	
339	HUB	1346	→	ONT	1453				1		6		
606	HUB	1955	→	TUS	2037		4	2	1				
719	HUB	1258	→	FAT	1432	2		1	1	1	2		N
748	SBA	1220	→	HUB	1353	6			1				
50	HUB	925	→	ATL	1622		2	3	2				
111	PHL	1335	→	HUB	1525			5	2				
215	HUB	805	→	SNA	924	4	1		2				
216	SNA	1000	→	HUB	1119			4	2		1		N
231	HUB	1804	→	ORD	2327				2		3	2	
270	HUB	1002	→	LAS	1115	1		4	2				
293	LAS	1146	→	SMF	1317	2	3		2				
355	HUB	830	→	MCO	1543				2			5	
378	ONT	710	→	HUB	822	2	3		2				
455	STL	810	→	HUB	931		5		2				
479	PDX	1906	→	LAS	2113		5		2				
571	LAS	0	→	GEG	222	2	1	2	2				
604	HUB	1431	→	SNA	1546				2		2	3	
627	SLC	1850	→	HUB	1928		5		2				
637	HUB	1940	→	FAT	2115				2	3		2	
783	HUB	1625	→	BIL	1955				2			5	
814	YUM	710	→	HUB	810	2	1		2		1	1	N

Table 19: Re-timing decisions for frequently re-timed flights

9 Effects of Dynamic Scheduling on Aircraft Maintenance Routing, Crew Scheduling, and Passenger Itineraries

Changes to a published schedule impact several elements of the airline's operations, including aircraft maintenance routing, crew schedules, and passenger itineraries. The objective of aircraft maintenance routing is to develop a *maintenance feasible* route for each aircraft, that is, a sequence of flight legs visiting maintenance stations at regular intervals to ensure adequate opportunities for maintenance checks. Similarly, the goal of crew scheduling is to develop for each crew member, a sequence of flight legs to be operated by that crew. This sequence must satisfy numerous work rules defined by government agencies and collective bargaining agreements between labor and the airline. If maintenance routes and crew schedules are developed prior to the re-optimization point, changes in flight departure and arrival times can alter aircraft turn times and crew connection times, rendering the aircraft routes and crew schedules infeasible. Moreover, changes in aircraft fleeting, while having no impact on crew schedules, can require changes in the routes of the swapped aircraft to ensure that all affected aircraft have sufficient opportunities for maintenance. Although we do not conduct experiments to quantify the effects of dynamic scheduling on crews and maintenance routing, Berge and Hopperstad (1993) report that determining revised, feasible aircraft maintenance routing solutions after schedule changes is typically not difficult. If altering aircraft routing and crew scheduling is too onerous or expensive, the re-optimization point can be moved earlier to precede the release of the aircraft and crew plans. It is worth noting that airlines, who desire the ability to alter plans closer to the date of departure, are now moving toward delaying the release of aircraft routing and crew pairing plans.

Notification of schedule changes is necessary for nonstop passengers whose flights are moved earlier and for connecting passengers whose flights departing the origin city are moved earlier. This can be accomplished in several different ways, for example, automated emails, automated text messages, and automated phone calls over conventional telephone networks or using voice-over-internet protocol (VoIP).

10 Summary

Dynamic airline scheduling provides a way to manage capacity dynamically to match fluctuating passenger demand. In this research, we introduce a new mechanism for dynamic scheduling, namely, flight re-timing. We develop an optimization model that combines both flight re-fleeting and flight re-timing. We conduct experiments that demonstrate the significant benefit of dynamic scheduling and the synergy between flight re-fleeting and re-timing. The estimated profit increase is $36 million annually when perfect forecast is used in re-optimization. When imperfect forecast is used, the estimated profit increase remains significant at $18 million annually. Sensitivity analysis is conducted on the number of re-timed flights per day. It shows that the full benefit is achieved when a moderate number of flights (10% of the total number of flights) are re-timed.

The performance of flight re-timing and flight re-fleeting are compared when applied alone. It is shown that flight re-timing constitutes a larger portion of the total benefit than flight re-fleeting. We also find that flight re-timing is less sensitive to forecast quality. When forecasts are imperfect, flight re-timing realizes 85% of the benefit achieved when forecasts are perfect.

A side study is performed assuming a weekly schedule is in place to account for day-of-week demand variations. Dynamic scheduling continues to improve schedule profitability when a weekly schedule is applied. On average, the estimated profit increase is $32 million when forecasts are perfect and $15 million when forecasts are imperfect.

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