Contents

1 Uncertainty

Absolute uncertainties are given to 1sf and percentage uncertainties are given to 2sf. The maximum-minimum method of calculating uncertainties is $\frac{\Delta T}{T} = \frac{T_{max}-T_{min}}{T_{max}+T_{min}}$

2 Dynamics

- Newton's Third Law: A body continues in its state of rest or uniform motion in a straight line unless a resultant external force acts on it.
- Mass: Intrinsic property of a body which resists change in motion
- Weight: Force experienced by a mass in a gravitational field
- Newton's Third Law: If a body A exerts a force on body B, then body B exerts and equal but opposite force on body A.
- Momentum of a body is defined as product of its mass, m and velocity v. $p = mv$
- Newton's Second Law: Rate of change of momentum of a body is proportional to the resultant force acting on it and in the direction of the force. $F_{net} = \frac{dp}{dt} = m\frac{dv}{dt} + v\frac{dm}{dt}$
- Apparent weight is heavier than usual during upwards acceleration but lighter than usual when accelerating downwards.
- Impulse $\Delta p = F\Delta t$ is defined as product of force F acting on an object and the time Δt for which the force acts. Impulse is the change in momentum over an interval.
- Principle of conservation of linear momentum: when bodies in a system interact, total momentum of the system remains constant provided no net external force acts on the system
- Elastic collision: total momentum and total KE are conserved
  - $u_1 - u_2 = v_2 - v_1$
  - $u_1m_1 + u_2m_2 = v_1m_1 + v_2m_2$
- Inelastic collision: total momentum conserved but total KE not conserved.

3 Forces

- Hooke's Law states that the extension of a body is proportional to the applied load if the limit of proportionality is not exceeded.
- Pressure is defined as the force per unit area, where the force is acting at a right angle to the area.
- Density of a substance is defined as its mass per unit volume.
- The force of upthrust is the vertical upwards force exerted on a body by a fluid when it is fully or partially submerged in the fluid due to the difference in fluid pressure between the upper and lower surfaces of the body.
- Archimedes principle states that the upthrust on an object is equal to the weight of the fluid displaced by the object.
- Totally submerged object
  - Object is unable to displace more fluid than its volume. If object denser than fluid, then weight of object will be greater than the upthrust, which is the weight of fluid displaced (Archimedes principle)
  - As such, object experiences a net downwards force $F_{net} = W - U > 0$ and the object accelerates downwards.
- Floating object
  - For object to be in equilibrium, $F_{net} = 0$, hence $W = U$, which is equal to the weight of fluid displaced (Archimedes principle), hence weight of object is equal to weight of fluid displaced.
  - Principle of flotation: for an object floating in equilibrium, the upthrust is equal in magnitude and opposite in direction to the weight of the object
- Viscous force is the resistance an object experiences due to a fluid when moving relative to each other
- For an object to be in equilibrium, the resultant force on the object is zero and the resultant moment on the object about any axis is zero.
  - For an object in translational equilibrium, resultant force acting on it must be zero
  - When 3 co-planar forces act on a body in equilibrium, their lines of action must either be parallel or concurrent, as the vector sum of three forces must be zero.
  - Principle of moments: for a body in rotational equilibrium, the sum of all the clockwise moments about any axis must be equal the sum of all the anticlockwise moments about the same axis
  - Moment of a force about a point is defined as the product of the force and the perpendicular distance of the line of action of the force from the pivot.
- A couple consists of a pair of equal and opposite forces whose lines of actions do not coincide. Torque of a couple is the product of one of the forces and the perpendicular distance between the forces
- The centre of gravity of a body is the point at which its whole weight appears to act. In a uniform gravitational field, center of gravity is the point where the torque of about center due to each point mass is equal. $m_1 d_1 = m_2 d_2$.
- For objects in contact, contact force R is the vector sum of normal force N and the friction f cause by motion.

4 Work, Energy, Power

- Work done by a constant force is the product of the force and the displacement in the direction of the force. $W = Fs\cos\theta$
  - Work done by a gas is $W_{gas} = p\Delta V$
- Work energy theorem states that net work done by all forces acting on a body is equal to the change in the kinetic energy of the body.
- Gravitational potential energy of an object mass m at a height h above the surface of the earth is $E_p = mgh$, where g is the acceleration of free fall near the Earth's surface, because the gravitational field is assumed to be uniform
- For a field of force, the relationship between force F and potential energy U for a one-dimensional motion is given by $F = -\frac{dU}{dx}$
- The law of conservation of energy states that energy cannot be created or destroyed. It can only be converted from one form to another.
- Power, $P = \frac{dW}{dt}$ is defined as the rate of work done or energy conversion with respect to time. If a constant force is applied and does work by moving its point of application displacement s in time t, power supplied is $P = \frac{dFs}{dt} = F\frac{ds}{dt} = Fv$.

5 Circular Motion

- Angular displacement θ is the angle an object makes with respect to a reference line.
- Angular velocity ω is defined as the rate of change of angular displacement with respect to time. $v = r\omega$
- Centripetal force
  - An object in circular motion must always experience a force as its velocity is always changing, since the direction of motion is constantly changing. As such, it has to experience an acceleration and must experience a force.
  - The direction of the force acting on an object in circular motion is towards the center. As object is moving at constant speed, there must be no component of acceleration in direction of motion of object as this would increase or decrease its speed.
  - For an object in circular motion, net force provides for centripetal force.
  - Centripetal force on object $F = ma = mr\omega^2 = \frac{mv^2}{r}$
- If object in circular motion along vertical plane, GPE is constantly changing and work done has to keep KE constant. Minimum velocity is when GPE is provides minimum centripetal force, or when $mg = \frac{mv^2}{r}$
- Racing car on circular bend along track
  - Cyclists have to lean in around bend to provide fictional force f acting on bicycle to provide for centripetal force.
  - Flat horizontal track: As racing car travels faster, centripetal acceleration required to maintain the same radius $a = r\omega^2$ increases. As sideways friction provides for centripetal acceleration, sideways friction has to increase. At particular speed, force required to keep car on path exceeds maximum sideways frictional force that can be provided by track.
  - To allow the car to travel round bend relying on friction, banked track is used (incline towards center of circular path). Angle of incline θ followings $tan\theta = \frac{v_{ideal}^2}{rg}$, and no frictional force provides for centripetal acceleration.

6 Gravitational field

- Newton's law of gravitation states that two point masses attract each other with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. $F = \frac{GMm}{r^2}$
  - Note that r is considered from the center of gravity of the planet, rather than the edges of its sphere.
- A gravitational field is a region of space in which a mass placed in that region experiences a gravitational force. Gravitational field strength at a point is defined as the gravitational force experienced per unit mass at the point $g = \frac{F}{m}$
- Gravitational potential energy of a mass at a point in a gravitational field is defined as the work done by an external force in bringing the mass from infinity to that point. $U = -\frac{GMm}{r}$
  - Gravitational potential at infinity is zero. Since gravitational force is attractive in nature, to bring a mass from infinity to a point in the gravitational field, the direction of the external force is opposite to the direction of displacement of the mass. Thus negative work is done.
- Gravitational potential at a point in a gravitational field is defined as the work done per unit mass by an external force in bringing a small test mass from infinity to that point. $\phi = -\frac{GM}{r}$
- Planetary motion: given that $F_g = F_c$, find that $T^2 = \frac{4\pi^2}{GM}r^3$
- In a binary star system, gravitational force is just sufficient to provide for centripetal force and planets do not move closer to each other.
- Satellites
  - The total energy $E = E_k + E_p = \frac{GMm}{2r} - \frac{GMm}{r} = -\frac{GMm}{2r}$, hence $E_t \propto 1/r$.
  - A geostationary satellite remains at a fixed position in the sky as viewed from any location on the Earth's surface, must have orbital period same as the earth.
  - Advantages: Continuous surveillance of region under, easy for ground station to communicate as ground-based antennas can remain fixed in 1 direction, large area due to high altitude.
  - Disadvantages: Loss in signal strength, poor resolution of imaging, time-lag.

7 Oscillations

- Simple harmonic motion is the motion of a particle about a fixed point such that its acceleration is proportional to its displacement from the fixed point and is always directed towards the point.
- Angular frequency ω is defined as the rate of change of phase angle of the oscillation, and is equal to the product of 2π and its frequency.
- Amplitude x_0 is the magnitude of maximum displacement of the particle from its equilibrium position.
- Frequency f is the number of oscillations per unit time. Period T is the time taken to complete one oscillation.
- $x = x_0 \cos \omega t$, $v = -x_0 w \sin \omega t$, $a = -x_0\omega^2 \cos \omega t = -\omega^2 x$
- $v = \pm\omega\sqrt{x_0^2 - x^2}$
- Damping is the process by which energy is removed from an oscillating system.
  - Frequency of a damped system is less than undamped system, hence the period is greater.

- Light damping results in oscillations whereby amplitude decays exponentially with time.
- Critical damping results in no oscillation and returns to equilibrium position in the shortest time
- Heavy damping results in no oscillation and takes a long time to return to equilibrium position.

- Resonance occurs when a system responds at maximum amplitude to an external driving force. This occurs when the frequency of the driving force is equal to the natural frequency of the driven system. Resonance is useful in MRI and radios but is harmful during earthquakes.
- Natural Frequency is the frequency at which a system vibrates in the absence of external forces; high natural frequency avoids resonance from sources of external periodic driving forces.

8 Wave motion

- Progressive wave transports energy from 1 point to another in the direction of wave propagation.
- Wavelength λ is the distance between 2 consecutive points in phase. $v = f\lambda$.
- Visible light ranges from red to purple in decreasing wavelength, where $\lambda_{\text{red}} = 725nm; \lambda_{\text{purple}} = 380nm$. EM Spectrum, in order of increasing wavelength: Gamma, X-ray, Ultraviolet, Visible, Infrared, Microwave, Radio.
- Wave travelling from point source without loss of energy will have $I = \frac{P}{4\pi r^2}$ as intensity of wave distributes evenly over surface of a sphere.
- Intensity of a wave is defined as rate of transfer of energy per unit area normal to direction of propagation of wave, $I = \frac{P}{S}$, where P is power of source and S is surface area. $I \propto A^2 f^2$ since $I \propto KE$, where A and f are the amplitude and frequency respectively.
- Polarisation is a phenomenon whereby the oscillations of the wave particles in a transverse waves are restricted to one direction only and this direction is perpendicular to the direction of wave propagation. When passing through a polariser, the intensity of polarised light $I = \frac{I_0}{2}$ where I_0 is the intensity of incident unpolarised light.
- Malus' Law states that the intensity of a beam of plane-polarised light after passing through a polariser varies with the square of the cosine of the angle through which the polariser is rotated from the position that gives maximum intensity. $I = I_0 \cos^2\theta$ and $A = A_0 \cos\theta$
- Longitudinal waves have particles that oscillate about equilibrium positions in direction parallel to direction of propagation of waves.
  - Displacement is 0 at compressions and rarefactions. Compressions are regions of high air pressure, rarefactions are regions of low air pressures.
  - Direction of wave motion is towards compressions.

9 Superposition

- Principle of superposition states that when two or more waves of the same kind meet at a point in space, the resultant displacement at that point is equal to the vector sum of the displacements of the individual waves at that point.

- Interference is the superposition of two or more coherent waves (constant phase difference) to give a resultant wave whose resultant amplitude is given by the principle of superposition. Constructive interference is when waves arrive at same point with phase difference of 0 rad, maximum obtained. Destructive interference when 2 waves arrive with phase difference of π rad, or when path difference is $\frac{\lambda}{2}$, minimum is observed.

- Standing/Stationary waves in air columns or pipes: nodes refer to points of 0 amplitude while antinodes refer to points of maximum amplitude.
  - Distance between a node and an antinode is $\frac{\lambda}{4}$, and the distance between 2 consecutive nodes/antinodes is $\frac{\lambda}{2}$
  - Points in same segment (bounded by node) are in phase (all go up/down together)
  - "0^{th} overtone" known as fundamental wave.
  - In a closed pipe, i^{th} overtone has a wavelength of $\frac{4L}{2i+1}$.
  - In an open pipe, i^{th} overtone has a wavelength of $\frac{2L}{i+1}$
  - The i^{th} refers to the overtone where there are i antinodes.

- Path difference is the difference is distances that each wave travels from its source to the point the 2 waves meet.

- Diffraction is the bending of waves after passing through an aperture or round an obstacle. It is pronounced when the wavelength of the wave is of the same magnitude as the width of aperture.

- Double slit interference: i^{th} maxima occurs at $x = \frac{\lambda D}{a}$, the 1^{st} minimum having a phase difference of $\frac{\lambda}{2}$

- Diffraction grating: $d \sin\theta = n\lambda$ to locate positions of principal maxima produced by diffraction grating. Total number of maxima identified is $2n + 1$, n on each side and one as the central maxima.
  - As number of slits increases and distance between slits decreases, distance between fringes increases, fringes will be brighter, and width of fringes decreases.

- Single slit diffraction: nth dark fringe (minima) occurs at angle θ such that $\sin\theta = \frac{n\lambda}{b}$, where b is the width of the slit.
  - When b increases, $sin\theta$ increases, hence dark fringes are further apart. Peaks become wider with lower intensity/amplitude as energy spread over a greater area.

- Rayleigh's criterion: two images are just resolved when the angle of separation θ is at least $\frac{\lambda}{b}$, or the first dark fringe. When condition satisfied, objects are distinguishable as central maximum of one image falls on the first minima of other image.

10 Temperature and Ideal Gas

- Regions of equal temperature are in **thermal equilibrium**. Two objects are in a state of thermal equilibrium when there is no net flow of heat between the 2 objects in thermal contact.
- The mole is defined as the amount of matter containing the number of particles equal to the Avogrado's constant, $6.02 * 10^{23}$
- Asssumptions of the kinetic theory of gas
  - Gas consists of large number of molecules in random motion
  - Volume of molecules is negligible compared to the volume occupied by the gas
  - Molecules exert no intermolecular forces on one another except during intermolecular collisions
  - Collisions between molecules and the container are perfectly elastic
  - Duration of intermolecular collision is negligible compared with the duration between colisions
- Derivation of $pV = \frac{1}{3}Nm\langle c^2\rangle$
  - During a collision, $\Delta P = (-mc_x) - mc_x = -2mc_x$
  - Between collisions, molecule travels across container and back, time is $\Delta t = \frac{2d}{c_x}$
  - $F_{molecule} = \frac{\Delta P}{\Delta t} = -\frac{mc_x^2}{d}$ (By Newton's 2nd law), hence force of molecule on wall $= \frac{mc_x^2}{d}$ (Newton's 3rd law)
  - $F_{tot} = \sum \frac{mc_x^2}{d} = \frac{m}{d}\sum c_x^2 = \frac{Nm}{d}\langle c^2\rangle$
  - Applying pythagoras to 3D vector, $c^2 = c_x^2 + c_y^2 + c_z^2$, hence $\langle c_x^2\rangle = \langle c_y^2\rangle = \langle c_z^2\rangle = \frac{1}{3}\langle c^2\rangle$.
  - $P = \frac{F_{tot}}{d^2} = \frac{Nm}{d^3}\langle c_x^2\rangle = \frac{Nm}{V}\frac{1}{3}\langle c^2\rangle$
  - Hence $PV = \frac{1}{3}Nm\langle c^2\rangle$
- By letting $k = RN_A$, we have $pV = NkT$.
- $\frac{1}{3}Nm\langle c^2\rangle = NkT$, hence KE $= \frac{3}{2}kT$.

11 First Law of Thermodynamics

- The numerical value $C = \frac{Q}{\Delta T}$of the **heat capacity** of a body is the quantify of heat required to raise the temperature of the body by one degree.
- The numerical value $c = \frac{Q}{m\Delta T}$of the **specific heat capacity** of a substance is the quantity of heat required to raise the temperature of a unit mass of the substance by one degree.
- Continuous flow: steady flow of liquid/gas is passed along pipe with heavy coil. Energy Input $=$ $Pt = mc\Delta T + H =$ Energy Output, where H is energy loss to surroundings (by conservation of energy).
  - Measurements are only taken when inlet and outlet temperatures have stabilised (steady state). Specific heat capacity of the apparatus can be ignored.
  - In order to eliminate heat loss to surroundings $\frac{H}{t}$, experiment repeated with different flow rate and adjusting power of heating coil and changing power (vary current or flow rate) so that the inlet and outlet temperatures remain at same values as of first experiment (to make H constant).
- Application of kinetic model

- Melting and boiling no change in temperature: At melting point molecules vibrate violently that attractive forces cannot hold them together, lattice structure collapses and solid undergoes phase change. Energy supplied to solid, called latent heat of fusion does not cause increase in temperature because energy is used to **overcome attractive forces** between atoms and **cause lattice structures to break**. Constant during boiling as heat supplied is used to overcome attractive forces between molecules and **break bonds completely** (latent heat of vaporisation).
  - SLH of vaporisation > fusion: to melt solid, work must be done to separate **some** of the molecules against mutual attractions so the structure no longer has any rigidity. When liquid vaporises, **all** remaining bonds must be broken → Melting breaks relatively fewer bonds.
  - Volume occupied in gaseous state > liquid state, gas needs to do more work against external/atm pressure to accomplish this. Energy required by gas to do work during expansion.
  - Why does cooling effect accompany vaporisation: Evaporation result of exchange of energy between molecules. Kinetic theory suggests that molecules of liquid are at continual motion and make frequent collisions, where **some molecules gain and others lose energy**. If molecule near surface gains enough energy, able to overcome attractive forces of molecules below it, resulting in decrease in the average kinetic energy of the remaining molecules.
  - Increase rate of evaporation:
    * Increase liquid surface area
    * Increase temperature (average kinetic energy) of all molecules
    * Cause a draught to remove vapour molecules before they return to liquid
    * Reducing air pressure above liquid, decreasing probability of vapour molecule rebounding off air molecule.

- Specific latent heat of fusion/vaporisation is the quantity of heat required to convert unit mass from solid-liquid/liquid-gas **without any change of temperature**

- The **internal energy U of a system** is the sum of the kinetic energy, due to the random motion of the molecules, and the potential energy, associated with the intermolecular forces in the system.
  - Does **not** include those due to bulk movement or position of the whole system.

- In an ideal gas, $U = E_K = \frac{3}{2}nRT = \frac{3}{2}NkT = \frac{3}{2}pV$

- The First Law of Thermodynamics states that the increase in the internal energy of a system is equal to the sum of the heat supplied Q to the system and the work done W on the system, and the internal energy of a system depends only on its state. $\Delta U = Q + W$

- Pressure-Volume graph
  - Work done W is equal to area under P-V graph, or $p\Delta V$
  - Isobaric process: p is constant (horizontal graph).
  - Isovolumetric process: V constant, $W = 0, \Delta U = Q$.
  - Isothermal process: No change in temperature. $\Delta U = 0$, since $U = \frac{3}{2}NkT$ and internal energy is only dependent on temperature. Hence $Q + W = 0$.
  - Adiabatic process: $Q = 0$. Occurs when system is insulated or process is fast. $Q = 0, \Delta U = W$. $\frac{pV}{T}$ constant.
  - Cyclic Process $\Delta U_{cyc} = 0$ since original state.
    * Magnitude of work done by a cyclic process is equal to the area enclosed by the graph.
    * If cyclic process is anticlockwise on P-V graph, work done on gas is positive, $W > 0$. Thus $Q < 0$ and heat transferred out of system.
    * If cyclic process is clockwise, work done on gas is negative, $W < 0$, and $Q > 0$.

12 Electric Fields

- Electric field strength $E = \frac{F}{q}$ at a point is the electric force exerted per unit positive charge placed at a point
- Electric field lines:
  - Originate from positive charges and terminate on negative charges
  - Density of field lines indicates magnitude of field strength
  - Field lines do not cross
  - Draw at least 3 lines above and 3 line below
- Coulomb's law: Force $F = \frac{1}{4\pi\epsilon_0}\frac{Qq}{r^2}$ between 2 point charges propotional to the product of the charges and inversely proportional to the square of the distance between them. Hence, $E = \frac{F}{q}$ $= \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$.
  - Point charge: Charges must be uniformly distributed and not concentrated on the outer surfaces of spheres.
  - Force is a vector while potential energy is a scalar
- Electric potential energy of a charge at a point in an electric field is defined as the work done by an external force in bringing the charge from infinity to that point. Potential energy due to a point charge is . $F_E = -\frac{dU}{dr}$, hence $U = \frac{1}{4\pi\epsilon_0}\frac{Qq}{r}$.
- Electric potential at a point is the work done per unit positive charge by an external force in brining a small test chagre from infinity to that point. $V = \frac{U}{q} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$. $V = -\int Edt.$
- 2 equally but oppositely charged parallel plates produce a uniform electric field between them. Field strength has same direction and magnitude at any point. Acceleration would be $\frac{F}{m} = \frac{Eq}{m}$. E can be found from ΔV potential difference as $|E| = \frac{|\Delta V|}{d}$.
  - Electrons attracted toward more positive plate
- Equi-potential line: Perpendicular to electric field at every point, movement along line has no work done.
  - Lines must not intersect
  - Uniform field - straight lines parallel to plates
  - Point charge - circular
- Conducting field: field strength 0 inside field but can be considered as point charge outside sphere.

13 Current of electricity

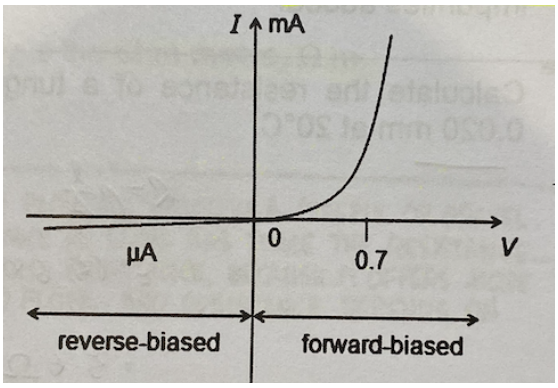
- Electric current $I = \frac{dQ}{dt}$ (A or Cs^{-1}) is the rate of flow of charge
- Total charge current-carrying conductor $I = nAv_Dq$, where n is number density of charge carriers, A is cross-sectional area, q charge and v_D is drift velocity. Current is constant through any metal wire even if area changes (drift velocity counteracts change in area).
- The potential difference V (V or JC^{-1}) between 2 points in a circuit is defined as the amount of electrical energy per unit charge that is converted to other forms of energy when charge passes from one point to another. Current flows from point of higher potential to point of lower potential.
- Electromotive force (emf) of a source is the amount of electrical energy per unit charge that is converted from other forms of energy when charge passes through the source.

- The resistance $R = \frac{V}{I}$ (Ω) of a circuit component is the ratio of potential difference across it to the current flowing through it. Ohm's law states that the potential difference across a conductor is proportional to the electric current passing through it provided temperature is constant.

- Resistance of a uniform conductor $R = \rho\frac{l}{A}$. Resistivity ρ measured in Ωm.

- Light dependent / temperature dependent resistor: resistance decreases with increasing brightness/temperature.

- I-V characteristics
  - Ohmic conductors: Temperature constant, resistance remains constant. Graph is a straight-line graph through the origin where R is equal to the gradient.
  - Diodes: Allows current to flow in 1 direction only. Reverse bias, current very small; forward bias, current increases very rapidly (low resistance).



  - Non-Ohmic behaviour:
    * Current increase, more charge carriers move through conductors, more collisions with lattice ions. This increases the temperature of the conductor.
    * Resistance: Frequency of collision between lattice ions and electrons increase, movement of charge carriers slow down.
    * Conductivity more significant (Thermistor): Temperature increases, more electrons acquire energy to break free from atoms, number of charge carriers increase significantly. Conductivity increases more than resistivity, ratio $\frac{V}{I}$ decreases at high current.
    * Resistance more significant (Filament Lamp): Electrons already free and mobile at room temperature, as temp increases, no appreciable increase in number of conducting free electrons.

- $P = \frac{dW}{dt} = IV$ (W or Js^{-1}). This can also be expressed as $W = QV$, where Q is net amount of charge and W is electrical energy converted to other forms of energy.

- Internal resistance of source
  - When no current flow ($I = 0$), pd across internal resistance and $V = V_R = E$.
  - When current I flows, pd across internal resistance is Ir and terminal pd is $V = V_R = E - Ir$. Graph of I against V has y-intercept at E and gradient of $-r$.
  - $E = I(r + R)$. Note that r is considered infinite when no current is flowing.
  - Power transferred to external load $P_R = I^2R = \frac{E^2R}{(r+R)^2}$. This is maximised when $R = r$, or when the resistance of load is equal to the internal resistance of the source.

14 Electromagnetism

- A magnetic field is a region of space in which a moving charge or a charge carrying-conductor or any ferromagnetic object will experience a magnetic force when placed in it.
- Infinite straight wire: right hand grip where fingers in direction of magnetic field and thumb in direction of conventional current. $B = \frac{\mu_0 I}{2\pi r}$
- Circular coil and solenoid: Fingers in direction of current and thumb gives magnetic field direction. $B = \frac{\mu_0 NI}{2r}$ where N is number of turns of coil.
- Long solenoid: $B = \mu_0 nI$, where n is the coils per unit length.
  - Magnetic flux desnity at ends of a coil/solenoid is half of the magnetic flux at the centre of the coil/solenoid
- The magnetic flux density B/T of a magnetic field is numerically equal to the force per unit length acting on a long straight conductor carrying a unit current at right angles to the magnetic field. $F = BIL$.
- Current between current-carrying conductors: like currents attract and unlike currents (opposite directions) repel. Force per unit length $\frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi d}$ where I is currents and d is the distance between parallel wires.
- Force on a moving charge: Moving charge is a current hence experiences force in magnetic field. $F = BIL = B\frac{q}{t}L = Bqv$ when perpendicular. $F = Bqv\sin\theta$. Conventional current, current flows same direction as a proton and opposite from an electron.
- Motion of charged particle: Work done on charged partile is zero as magnetic force F is perpendicular to motion of particle, hence magnetic force on moving charge provides for centripetal force. $F = Bqv = \frac{mv^2}{r}$, hence $B = \frac{mv}{rq}$.
- Charged particle enter magnetic field at angle, then $v\cos\theta$ in direction of field leads to forward motion and $v\sin\theta$ leads to circular motion perpendicular to magnetic field.
  - Helix motion
  - Pitch of helix is forward motion travelled per vertical period.
- Magnetic and electric fields: Fields must be perpendicular in orientation in order to cancel out. $Bqv = qE$, hence $v = \frac{E}{B}$.

15 Electromagnetic Induction

- Magnetic flux $\phi/Wb = BAcos\theta$ is the product of an area and the component of magnetic flux density perpendicular to the area.
- Magnetic flux linkage $\Phi/Wb = NBAcos\theta$ of a coil is the product of magnetic flux through the coil and the number of turns in a coil.
- Faraday's law of electromagnetic induction $emf = -\frac{d\Phi}{dt}$ states that induced emf is proportional to the rate of change of magnetic flux linkage.
  - When there is a change in magnetic flux linkage, by Faraday's Law, a current is induced.
  - When there is change in motion of magnet, North/South pole is induced to oppose motion.
- Lenz' Law states that direction of induced emf is such as to cause effects to oppose the change producing it.
  - When there is change in magnetic flux linkage, by Lenz' Law, the induced current in the loop must flow in direction so as to produce a magnetic field in the same direction as applied magnetic field.

Situation	Magnetic flux (ϕ)	emf
Moving Rod	$\phi = BLx$	$emf = Blv$
Rotating Coil (AC Generator)	$\phi = BA\cos\theta$	$emf = NBA\omega\sin\omega t$
Rotating Disc		$emf = BAf = \frac{Br^2\omega}{2}$

16 Alternating currents

- An alternating current varies periodically with time in magnitude and direction.
- Peak current I_0 is the amplitude of the current
- The RMS value of the alternating current or voltage is that value of a direct current or voltage that would produce thermal energy at the same rate in a resistor.
- Sinusoidal current: rms value $I_{rms} = \frac{I_0}{\sqrt{2}}$, $P_{\text{rms}} = \frac{P_0}{2}$
- Ideal Transformer
  - Assuming that no energy loss, no flux leakage $\rightarrow$ magnetic flux ϕ through each turn is same in both the primary and secondary coils.
  - $\phi = \frac{\epsilon_P}{N_P} = \frac{\epsilon_S}{N_S}$
  - No energy dissipation: $I_P V_P = I_S V_S$
- Causes of energy loss
  - Joule Heating of copper wires, $P_{lost} = I^2 R_{wire} = I^2 \frac{\rho L}{A}$ (thick copper wire, low resistance)
  - Magnetic flux leakage, when not all magnetic flux is used to do useful work. Presence of iron core increases magnetic flux and reduces leakage between both coils, minimised if both coils made to occupy same space by placing primary and secondary coils in concentric manner.
  - Hysteresis Loss as energy used in process of magnetising iron and reversing magnetism every time current reverses heats up iron core. (Soft iron, high values of permeability which can be easily magnetised and demagnetised
  - Eddy current losses, when induced emf causes small circulating currents, which lead to power loss in the form of thermal energy (loss of mechanical energy). Can be minimised through the use of a laminated core, and large electrical surface resistance confines eddy currents to each individual lamina and eddy currents and heating effect greatly reduced.
- Diode
  - Ideal diode has infinite resistance in reverse bias and zero resistance in forward bias.
  - Real diode has minimal leakage current in reverse bias and a threshold voltage after which $I \propto V$

17 Quantum Physics

- The Photoelectric effect is a process in which electrons are emitted from a metal surface when electromagnetic radiation of sufficiently high frequency is incident on the surface.
- A Photoelectron is an electron emitted from the surface of a material due to the incident electromagnetic radiation.
- The stopping potential V_s is the negative potential of collector with respect to emitter which prevents the most energetic photoelectrons from reaching the collector and hence results in zero photocurrent.
  - When collector sufficiently positive wrt E, every single electron is attracted to C. Ammeter read constant saturated/maximum photocurrent i_0. Rate of emission of $e^- = \frac{i_0}{e}$.
  - Sufficiently negative wrt E, most energetic photoelctrons do not have sufficient energy to reach C. Maximum KE of $e^- = eV_s$.
- A photon is a quantum of electromagnetic energy. Energy in each quantum $E = hf = h\frac{c}{\lambda}$, where h is the Planck's constant. When there are N electrons, $E_{total} = Nhf = Nh\frac{c}{\lambda}$
- The work energy function Φ of a metal is defined as minimum amount of energy necessary for an electron to escape from the surface of a material. It is constant for a given metal.
- Einstien's Photoelectric equation states that photon energy is the sum of work function energy and maximum kinetic energy of proton. Hence $hf = \Phi + \frac{1}{2}m_e v_{max}^2$. Can also be expressed $hf = \Phi + eV_s$.
- Threshold Frequency: Minimum frequency of EM radiation below which no emission of photoelectron occurs.
  - $KE_{max} = hf - \Phi$. KE of most energetic photoelectrons increases linearly with frequency but is independent of intensity.
  - Intensity of light, rate of incident photons on metal, more photons and more collisions leads to more photoelectrons but with the same energy.
  - At threshold frequency f_0, $KE = 0$, $hf_0 = \Phi$
- Insignificant time lag
  - Photoelectron emitted if gains enough energy from photon. All photon energy delivered immediately to electron in a single collision.
- Graphical Representation
  - Current-Intensity Graph
    * Rate of emission of electrons $\propto$ rate of incident protons. Current $i \propto I$.
    * $I = \frac{P}{A} = \frac{E}{tA} = \frac{N}{t}\frac{hf}{A}$. I determined by value of $\frac{N}{t}$ as well as hf.
    * $P = \frac{dE}{dt} = \frac{dN_p}{dt} \cdot hf$
    * Increased intensity of incident radiation, greater number of photons strike metal surface per second, greater number of photoelectrons can be emitted per unit time.
  - Current - Potential Graph
    * Positive V_{CE}: All photoelectrons collected, current constant at i_0. $i_0 = e\frac{dN_p}{dt}$, higher intensity higher graph.
    * Stopping potential V_s is the same across intensities. Below V_s, no current.
    * Higher intensity beam produces more electrons per unit time (proportional). $i_0 = e\frac{dN}{dt}$, increases $\frac{dN}{dt}$.

    - ∗ Higher frequency: Stopping potential V_s increases with frequency. More energetic photons liberate electrons with higher KE_{max}. Greater stopping potential $V_s = \frac{hf}{e} - \frac{\Phi}{e}$.
  - Stopping Potential - Frequency (Across metals)
    - ∗ Slope of line same since slope given $V_s = \frac{h}{e}f - \frac{\Phi}{e}$.
    - ∗ Dotted lines extrapolation of graph since no photo-emission below threshold frequency.

- Wave-particle duality - theory where matter and waves have particle-like and wave-like characteristics
  - De Broglie wavelength is the wavelength associated with wave-like properties of a particle is $\lambda = \frac{h}{p} = \frac{h}{mv}$, where p is the momentum.
  - For particles with mass (not photons): $KE = \frac{p^2}{2m} \to p = \sqrt{2mE} = \sqrt{2meV}$
- Line Spectrums
  - Bohr's model: Only certain allowed orbits in atom, with each corresponding to fixed energy state. Atom can only radiate energy when electron transits from more energetic state to lower energy state. Energy emitted as one quantum (photon) of radiation, where $E_i - E_f = hf$.
  - Ground state refers to lowest energy level ($n = 1$) in which the atom is the most stable.
  - Ionisation energy of atom is minimum energy required to remove an electron completely from atom. Transition from ground state $n = 1$ to infinite level (electron no longer bound to atom) $n = \infty$.
  - Electron can gain energy from 2 sources:
    - ∗ Particle collision: colliding particle transfer part of all of its energy, must be sufficient for orbital electron to transit up to higher energy level but need not match difference in energy levels of atom.
    - ∗ Photon: For orbital electron to gain energy from photon, photon energy must be exactly equal to energy difference, and photon will be absorbed by electron.
  - Emission light spectrum: Gas heated or bombarded by electrons, electrons excited to higher energy levels. Remain there momentarily before de-exciting to lower levels, emitting photons with energy corresponding to energy difference between 2 energy levels. Discrete energy levels, hence energy differences also discrete. Results in lights of these frequency being present, coloured lines on a dark background.
  - Absorption light spectrum: White light passes through cool gas, energies of photos must equal energy difference between 2 energy levels. Excited atom will return to lower energy state by emitting photons of energies equal to energy difference. However, emissions occur in all directions and have lower intensities. Light of these frequencies appear to be missing from spectrum, line spectrum consists of dark lines on a bright coloured background.
  - Different gases result in different unique patterns.
- X-Ray Spectrum: Material bombarded with electrons and photons are released
  - Continuous spectrum: Electromagnetic radiation produced whenever highly charged electrons are suddenly decelerated, continuous as electrons hitting metal target are slowed down to different extent.
  - Minimum wavelength: all kinetic energy of one electron is given up in a single collision to produce a single x-ray photon. Hence $eV = hf = h\frac{c}{\lambda + \text{min}}$
  - Smaller minimum wavelength, use stronger pd to increase V.
  - Characteristic X-rays: Sufficient energy transferred when accelerated electron collides into electron of target atom in ground state shell (K-shell, L-shell and M-shell for $n = (1, 2, 3)$). Vacancy in K-shell filled by electron de-exciting from other shell, X-ray photon produced. Hence $hf = \frac{hc}{\lambda} = E_n - E_1$.

  - K_α photon when electron from shell $n = 2$, K_β photon when electron from shell $n = 3$. Intensity of K_α X-ray greater than K_β X-ray as 2^{nd}-shell is closer to 1^{st}-shell. Greater probability that vacancy in 1^{st}-shell filled by electron in 2^{nd}-shell than 3^{rd}-shell.
  - Characteristic peaks not detected when energy from electron is not sufficient to knock electrons out of inner shell.

- Heisenberg Uncertainty Principle states that if a measurement of position of particle is made with uncertainty Δx and a simultaneous measurement of its x-component of momentum is made with uncertainty Δp_x, the product of the 2 uncertainties can never be smaller than h.