

# Stats

## Assumptions

- Have to answer **in context of question**
- Binomial Distribution
  - Trials are independent
  - Probability of success remains constant (not affected by results of other trial)
- Random Samples
  - Each element in population has **equal chance** of being selected
  - Selections are **independent** of each other (selection of 1 element does not affect the chance of any other element being selected)

## Discrete Random Variables

- Variance of discrete random variable  $X$  is a measure of **expected spread of a random variable about its mean**. It is defined as  $Var(X) = E(X - \mu)^2 = E(X^2) - [E(X)]^2$
- Standard deviation defined as  $\sigma = \sqrt{Var(x)}$
- $Var(aX \pm bY) = a^2 Var(X) + b^2 Var(Y)$
- $Var(X + b) = Var(X)$  since horizontal shift does not affect variance

## Normal Distribution

- Standardisation: Covert to  $Z \sim N(0, 1)$  where  $Z = \frac{X - \mu}{\sigma}$
- $aX \pm bY \sim N(a\mu_1 \pm b\mu_2, a^2\sigma_1^2 + b^2\sigma_2^2)$

## Sampling

- Sample mean  $\bar{x} = \frac{\sum x}{n}$  is an **unbiased estimate of  $\mu$**
- Unbiased estimate of  $\sigma^2$  is  $s^2 = \frac{n}{n-1} \left[ \frac{\sum (x - \bar{x})^2}{n} \right] = \frac{1}{n-1} \left[ \sum x^2 - \frac{(\sum x)^2}{n} \right]$ . Is equal to product of  $\frac{n}{n-1}$  and sample variance.
- Sample mean  $\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$ , Sample sum  $\sum X = n\bar{X} \sim N(n\mu, n\sigma^2)$
- **Central Limit Theorem**: Since  $n = 50 > 30$  is large, by Central Limit Theorem,  $n \sim N()$  **approximately**

## Hypothesis Testing

- **Level of Significance** of a hypothesis test  $\alpha$  is defined by probability of rejecting  $H_0$  when (given that)  $H_0$  is true
- Test statistic  $\bar{X}$  is random variable used to make decision whether to reject  $H_0$  by measuring degree of agreement between sample and null hypothesis.
- **Critical Region** refers to range of sample mean (variable), (context), such that there is sufficient evidence at  $\alpha\%$  significance level to conclude that the mean (variable) is not  $\mu$  (variable).
- **p-value** is the probability of observing a sample, a test statistic value "at least as extreme as" the observed value computed under the assumption the null hypothesis is true. It is the smallest level of significance at which  $H_0$  can be rejected.
- z-test: Since  $p\text{-value} = p \leq \alpha$ , we reject/accept  $H_0$  and conclude there is sufficient/insufficient evidence at  $\alpha\%$  level of significance, (that the mean is not  $\mu_0$  / to support the claim that mean is not  $\mu_0$ )