David Veloz Solórzano 163678

1.1. P.D. ~~$p_i^T A p_j = 0$~~ ~~$\forall i \neq j$~~ Si $p_i^T A p_j = 0 \quad \forall i \neq j \wedge$ A es sim. y posit. def. $\Rightarrow p_i$ son l.i.

Dem. Supongamos que $\exists \; \alpha_i$ .7. $\sum \alpha_i p_i = 0$

donde α_i no son todos cero.

$$\sum \alpha_i p_i = 0 \iff \sum \alpha_i A p_i = 0 \iff \cancel{\sum \alpha_i p_j^T A p_i = 0}$$

$\alpha_j p_j^T A p_j = 0 \wedge$ como A es posit. def.

$\Rightarrow p_j^T A p_j > 0$ lo que implica que

$\alpha_j = 0$

$\therefore \{p_i\}$ es l.i. $\blacksquare$

~~Pta~~

1.2 P.D. $\forall x_0 \in \mathbb{R}^n$ $\{x_k\} \to x^*$ donde x^* es la solución optima en ~~el~~ maximo n iteraciones

Dem Como $\{p_i\}$ ~~es~~ son l.i. $\Rightarrow$ $span\{p_i\} \geq \mathbb{R}^n$.

Por lo que podemos expresar $x^* - x_0$ de la sig. forma.

$$x^* - x_0 = \sigma_0 p_0 + \sigma_1 p_1 + \ldots + \sigma_{n-1} p_{n-1}$$

para algunos escalares σ_k.

Esto si y solo si

$$\underline{p_k^T A (x^* - x_0)} = p_k^T A (\sigma_0 p_0 + \sigma_1 p_1 + \ldots + \sigma_{n-1} p_{n-1})$$

y como $\quad p_i^T A p_j = 0 \quad \forall\, i \neq j$

$$\Rightarrow \quad \sigma_k = \frac{p_k^T A (x^* - x_0)}{p_k^T A p_k}$$

y como en el método del grad. conjugado $x_{k+1} = x_k + \alpha p_k$ $\wedge$ el $\min(\frac{1}{2}x^T A x - b^T x) = Ax - b = r(x) \Rightarrow (r_k = A x_k - b)$

teremos que $\qquad$ usando que $x_k = x_0 + \alpha_0 p_0 + \ldots + \alpha_{k-1} p_{k-1}$

$$\alpha_k = \frac{-r_k^T p_k}{p_k^T A p_k} \quad \wedge$$

concluimos que $p_k^T A (x_k - x_0) = 0$

$$\therefore \quad p_k^T A (x^* - x_0) = p_k^T A (x^* - x_k) = -p_k^T R_k$$

donde $\sigma_k = \alpha_k$ y se converge en maximo n iteraciones ∎

2.1

La segunda cond. fuerte de Wolfe es:

$$|\nabla f(x_K+\alpha_K p_K)^T p_K| \leq c\,|\nabla f(x_K)^T p_K| \qquad 0<c<1$$

$$\Longleftrightarrow \quad \nabla f(x_K+\alpha_K p_K)^T p_K \geq -c\,|\nabla f(x_K)^T p_K|$$

como p_K es una dirección de descenso

$$\Longleftrightarrow \quad \nabla f(x_K+\alpha_K p_K)^T p_K = c\,\nabla f(x_K)^T p_K$$

por teorema

$$\Longleftrightarrow \quad \nabla f(x_K+\alpha_K p_K)^T p_K - \nabla f(x_K)^T p_K = (c-1)\,\nabla f(x_K)^T p_K > 0$$

pues $0<c<1$

~~Definamos~~

$\wedge$ Como $y_K = \nabla f(x_K+\alpha_K p_K)^T - \nabla f(x_K)^T$

$\wedge$ $s_K = \alpha_K p_K$

tenemos que

$$\nabla f(x_K+\alpha_K p_K)^T \alpha_K p_K - \nabla f(x_K)^T \alpha_K p_K = (c-1)\,\nabla f(x_K)^T \alpha_K p_K > 0$$

$$= (c-1)\,\nabla f(x_K)^T s_K > 0$$

$$\Longleftrightarrow \quad (\nabla f(x_K+\alpha_K p_K)^T - \nabla f(x_K)^T)\, s_K$$

$$\Longleftrightarrow \quad (y_K)^T s_K > 0$$

$$\Longleftrightarrow \quad s_K^T y_K > 0 \qquad \blacksquare$$

2.2

$$B_{k+1} H_{k+1} = B_k - \frac{B_k s_k s_k^T B_k}{s_k^T B_k s_k} + \frac{y_k y_k^T}{y_k^T s_k} \left(H_{k+1} \right)$$

~~donde~~ ~~where~~ $H_{k+1} y_k = s_k$

$\iff \quad y_k = (H_{k+1})^{-1} s_k$

$$\Rightarrow B_{k+1} H_{k+1} = B_k (H_{k+1}) - \frac{B_k s_k s_k^T B_k}{s_k^T B_k s_k} (H_{k+1}) + \frac{\left((H_{k+1})^{-1} s_k\right)\left((H_{k+1})^{-1} s_k\right)^T}{\left((H_{k+1})^{-1} s_k\right)^T s_k}$$

$y_k^T y_k$

$$\Rightarrow B_{k+1} H_{k+1} = \left(B_k H_{k+1} - \cancel{\frac{B_k s_k s_k^T B_k}{s_k^T B_k s_k} H_{k+1}} \right) + I$$

$\rightarrow 0$

$\therefore$ B_{k+1} y H_{k+1} son inversas.