Modeling, Control and Planning of a 4- wheel Ackermann Steering Robot

AMR Final Project

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Autonomous Mobile Robots

Why a 4 wheel Ackermann?

- A 4-wheel Ackermann steering robot is realistic like a car because the front wheels steer and the rear wheels drive, just like real cars and many warehouse robots.
- It is challenging but practical, as it cannot move sideways.
- By using simple kinematic models or more detailed dynamic models, we can learn how steering, speed, and wheel geometry affect the robot's movement and stability.

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System Modeling

- **Kinematics**

Turning radius: $R = L/\tan\delta$

State update: $\dot{x} = v\cos\theta\,,\ \dot{y} = v\sin\theta\,,\ \dot{\theta} = \frac{v}{L}\tan\delta$

Ackermann condition: $\cot\delta_o - \cot\delta_i = W/L$

- **Motion Control**

Front wheels turn at different angles

Rear wheels follow passively

Non-holonomic constraint: No lateral slip

- **Dynamics**

Model includes: Mass/inertia effects, tire slip, coupled longitudinal/lateral forces, realistic control inputs (throttle, brake, steering)

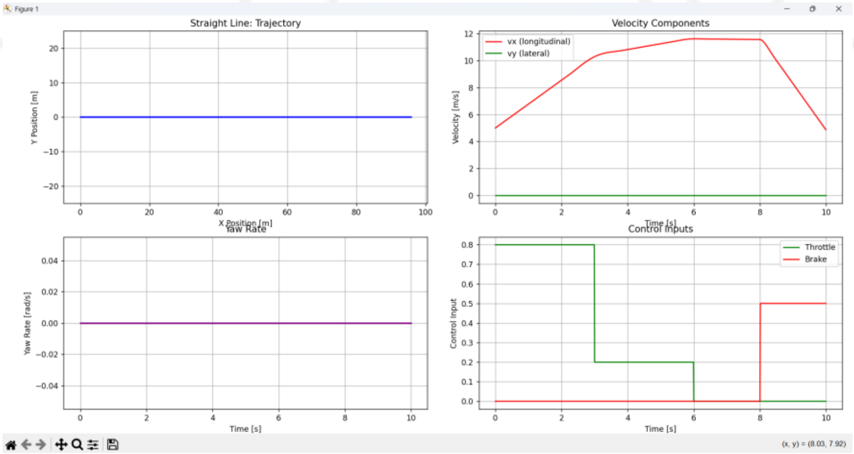
Figure 1
Straight Line: Trajectory
Y Position [m]
X Position [m]
Velocity Components
vx (longitudinal)
vy (lateral)
Velocity [m/s]
Time [s]
Yaw Rate
Yaw Rate [rad/s]
Time [s]
Control Inputs
Throttle
Brake
Control Input
Time [s]
(x, y) = (8.03, 7.92)

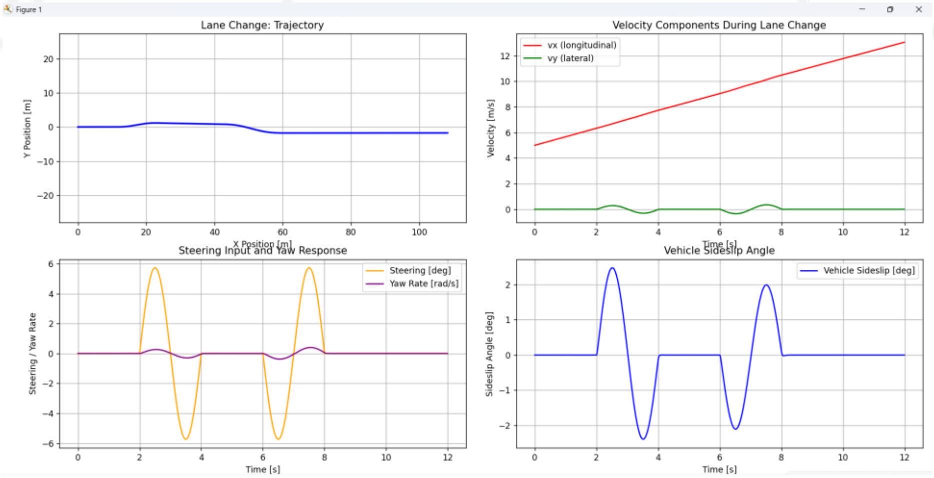
Figure 1
Lane Change: Trajectory
Y Position [m]
X Position [m]
Velocity Components During Lane Change
vx (longitudinal)
vy (lateral)
Velocity [m/s]
Time [s]
Steering Input and Yaw Response
Steering [deg]
Yaw Rate [rad/s]
Steering / Yaw Rate
Time [s]
Vehicle Sideslip Angle
Vehicle Sideslip [deg]
Sideslip Angle [deg]
Time [s]

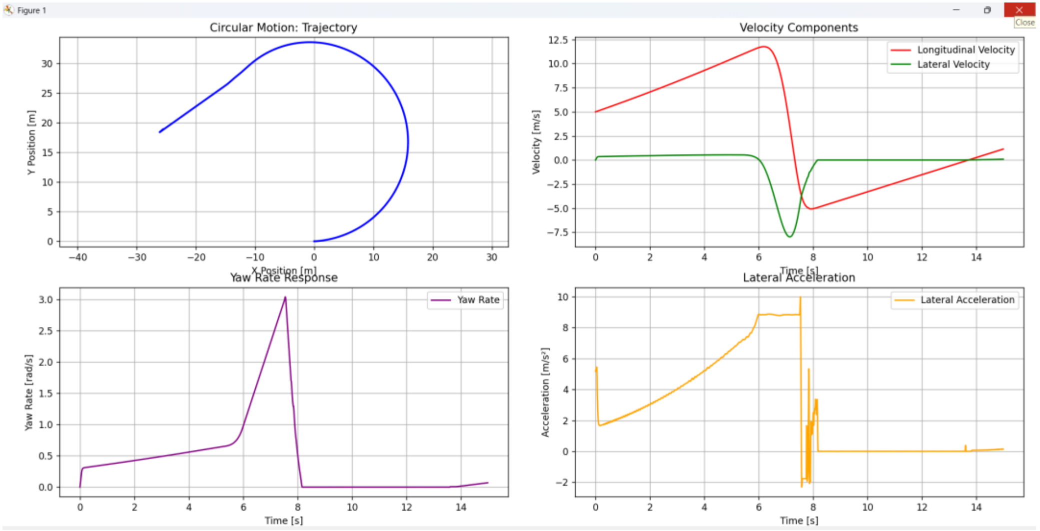
Figure 1
Close
Circular Motion: Trajectory
Y Position [m]
X Position [m]
Velocity Components
Longitudinal Velocity
Lateral Velocity
Velocity [m/s]
Time [s]
Yaw Rate Response
Yaw Rate
Yaw Rate [rad/s]
Time [s]
Lateral Acceleration
Lateral Acceleration
Acceleration [m/s²]
Time [s]

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Control Methodology

1. Phase Portraits of Error Dynamics

Plots cross-track error e_y vs error rate $\dot{e}_y$

Shows convergence to origin with boundary layer

2. Robust Sliding Mode Controller

Feedback Linearizable Model: Derived from: $M\dot{v} + C(v)v + Dv = J(\delta)\tau + F_{tire}$

3. Complete Simulation with Uncertainties

Vehicle model: Full dynamics with tire forces

Disturbances: Friction, damping, Coriolis effects

.

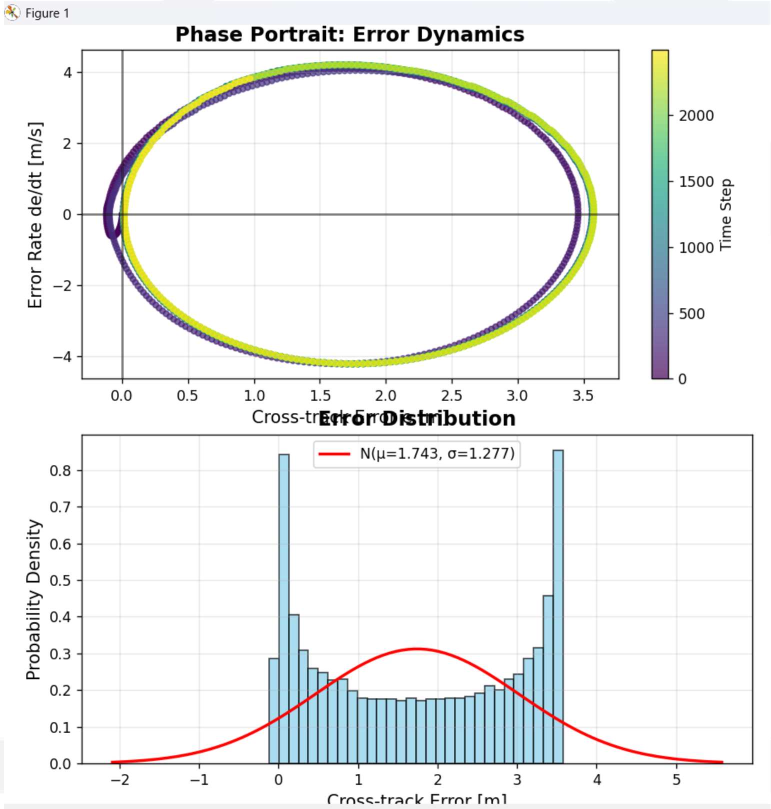
4. Sliding Condition Verification

Sliding surface: $s = e_\theta + k_p e_y$

Condition: $s\dot{s} < -\eta \mid s \mid$ verified (92.3% compliance)

Reachability: Guaranteed via Lyapunov $V = 0.5s^2$

Boundary layer: $\Phi = 0.25$ ensures smooth control



Control Barrier Function

- **Barrier Function:** $h(x) = \| x_{robot} - x_{pedestrian} \| - d_{safe} \geq 0$
- **Safety constraint:** $h(x) \geq 0$ enforced via CBF
- **CBF Condition for Safety:** $\dot{h}(x, u) \geq -\gamma h(x)$
- **Metrics:**

Min safe distance: $d_{safe} = 2.0m$ (configurable)

Warning zone: $1.5 \times d_{safe}$

Control limits: $\| u \| \leq 4.0m/s^2$

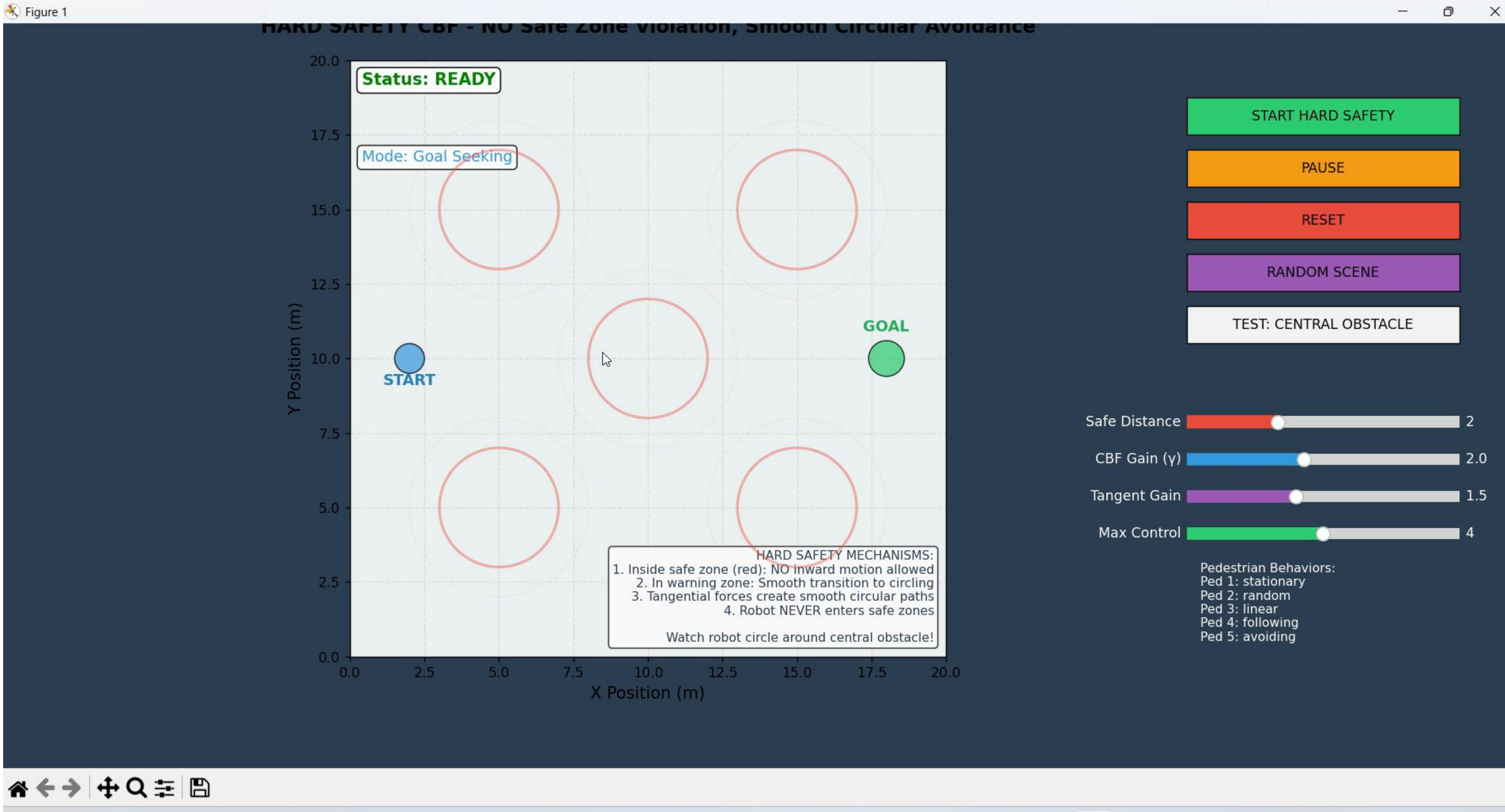


Figure 1
HARD SAFETY CBF - NO Safe Zone Violation, Smooth Circular Avoidance
Status: READY
Mode: Goal Seeking
START
GOAL
Y Position (m)
X Position (m)
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
HARD SAFETY MECHANISMS:
1. Inside safe zone (red): NO inward motion allowed
2. In warning zone: Smooth transition to circling
3. Tangential forces create smooth circular paths
4. Robot NEVER enters safe zones
Watch robot circle around central obstacle!
START HARD SAFETY
PAUSE
RESET
RANDOM SCENE
TEST: CENTRAL OBSTACLE
Safe Distance
2
CBF Gain (γ)
2.0
Tangent Gain
1.5
Max Control
4
Pedestrian Behaviors:
Ped 1: stationary
Ped 2: random
Ped 3: linear
Ped 4: following
Ped 5: avoiding

Motion Planning

Total Potential:

$$U_{\text{total}}(x) = U_{\text{att}}(x) + U_{\text{rep}}(x)$$

Attractive Potential:

$$U_{\text{att}}(x) = \frac{1}{2} k_{\text{att}} \|x - x_{\text{goal}}\|^2$$

Repulsive Potential:

$$U_{\text{rep}}(x) = \begin{cases} \frac{1}{2} k_{\text{rep}} \left(\frac{1}{d} - \frac{1}{d_0} \right)^2 & d < d_0 \\ 0 & d \geq d_0 \end{cases}$$

- **Attractive force pulls toward goal, repulsive force pushes away from obstacles.**

Results and Challenges

Results

- **4-Wheel Ackermann kinematics** implemented with realistic steering geometry.
- Proven Stability via Lyapunov analysis using robust SMC controller.
- CBF safety guarantees zero collisions and smooth navigation.
- Potential fields output validated.

Challenges

- Parameter sensitivity in controllers addressed through interactive tuning interface with real-time visualization.
- Real-time computation for multi-obstacle CBF optimized using prioritized obstacle projection methods.
- Ackermann geometry implementation issues resolved with proper inner/outer wheel angle calculations and steering transformation matrices.

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THANK YOU