Group 5 NumMat II Ex1

23. Oktober 2019

1 Discussion and Analyze of PDEs

1.1 Relevant PDEs

1.2 Navier-Stokes

The Navier-Stokes-Equation describes the motion of the viscous fluid substances and is expressed for compressible fluid as

$$\rho(\partial_t u + u \cdot \nabla u) = -\nabla p + \mu \nabla^2 u + f \tag{1}$$

with ρ the density, u velocity vector, p pressure, and μ kinematic viscosity of the fluid. Equation (1) is expressed in homogenous form by setting $f = 0$ as follows

$$\rho(\partial_t u + u \cdot \nabla u) + \nabla p - \mu \nabla^2 u = 0 \tag{2}$$

For $u(t, x) = (u_0 x_2(H - x_2), 0)^T$ with $u_0 \in \mathbb{R}$, $x = (x_1, x_2) \in \Omega = \mathbb{R} \times (0, H)$, and $t \in (0, \infty)$, the partial differentiations result

$$\frac{\partial u}{\partial t} = (0, 0)^T \tag{3}$$

$$\nabla u = (0, 0)^T \tag{4}$$

$$\nabla^2 u = (0, 0)^T \tag{5}$$

since u is not t-dependent and u_1 and u_2 are not effected by x_1 and x_2, respectively. Equations 3, 4 and 5 show that u is a twice differentiable function, satisfying the homogenous Navier-Stokes PDE with a boundary condition in a domain $\Omega \in \mathbb{R}^2$, which is reffered to as the classical solution for second order PDEs.

For the given conditions, Equation (2) can be expressed as

$$\nabla p = 0 \tag{6}$$

This can be referred to a 2D-flow model in a tube with a width of H at any certain height, which is observed along the gravity axis. Therefore, the pressure in the domain Ω is described as $p = const. \in [0, \infty)$.

2 Linear Algebra