Technical University of Berlin
Faculty II - Mathematics and Natural Sciences
Institute of Mathematics
Dr. D. Peschka, A. Selahi

Numerical Mathematics II for Engineers

Homework Assignment 4

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by **Group 5**

Kagan Atci	338131	Physical Engineering, M.Sc.
Navneet Singh	380443	Scientific Computing, M.Sc.
Riccardo Parise	412524	Scientific Computing, M.Sc.
Daniel V. Herrmannsdoerfer	412543	Scientific Computing, M.Sc.

Exercise 1

Considered is the Laplace operator $Lu = -\Delta$ in $\Omega = (0, L)^2$ and $u \in C^4(\bar{\Omega})$.

a) Let $\underline{x} = [x_1, x_2]^T \in \Omega_h$. Applying the Laplace operator in the form of finite difference on $u(\underline{x})$ takes place by differentiating $u(\underline{x})$ with respect to x_1 and x_2 one at a time. The D^-D^+ stencil is employed as the 2nd order finite difference operator with one neighbor on either side at the distance h.

$$\frac{\partial^2 u}{\partial x_1^2} = D^-D^+u(x_1, \,\cdot\,) = \frac{u(x_1+h, x_2) - 2u(x_1, x_2) + u(x_1-h, x_2)}{h^2} + \mathcal{O}(h^2) \tag{1}$$

$$\frac{\partial^2 u}{\partial x_2^2} = D^-D^+u(\,\cdot\,, x_2) = \frac{u(x_1, x_2+h) - 2u(x_1, x_2) + u(x_1, x_2-h)}{h^2} + \mathcal{O}(h^2) \tag{2}$$

With remainders neglected in Equations (1) and (2), the Laplace operator can be approximated by adding both equations

$$\Delta u \approx \frac{u(x_1+h, x_2) + u(x_1, x_2+h) - 4u(x_1, x_2) + u(x_1-h, x_2) + u(x_1, x_2-h)}{h^2}. \tag{3}$$

Since u is a four times continuously differentiable function in $\bar{\Omega}$ and the remainder is $\mathcal{O}(h^2)$, it holds $||f_h - L_hR_hu||_h \in \mathcal{O}(h^2)$ as $h \to 0$, where f_h is the right hand side of the BVP, L_h the finite difference matrix and R_h the restriction operator within

$\bar{\Omega}$. Hence, Equation (3), also called as the 5-point stencil, is considered as a 2nd order consistent approximation of Δu.

b)

c)

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d)

Exercise 2

a)

b)

Exercise 3

a)

b)

c)

d)