1. Code a numerical solution to the RP equation (1).

MATLAB was used to generate a solution to the Rayleigh-Plesset Equation. Variables were re-defined such that y_1 = R and $y_2 = \dot{R}$. This allowed the single 2nd order ODE to become 2 coupled 1^{st} order ODEs. Redefinition of variables resulted in:

$$\dot{y_1} = y_2 \qquad \dot{y_2} = \ddot{R}$$

with

$$\ddot{R} = \left[\frac{P(R,\dot{R},t)}{\rho} - \frac{3}{2}\dot{R}^2\right]\frac{1}{R}$$

where

$$P(R,\dot{R},t) = P_g^e\left(\frac{R_o}{R}\right)^{3k} + P_v - P_\infty - \frac{2\sigma}{R} - \frac{4\mu\dot{R}}{R}$$

$$P_g^e = P_\infty^e - P_v + \frac{2\sigma}{R_o}$$

$$P_\infty = P_\infty^e + P_a sin(\omega t)$$

Shown below are the codes used to obtain solutions to the RP equation

```
% This script will model the Rayleigh Plesset Equations
% Author: Dorien Villafranco
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% Requirements for ME721: Acoustic Bubble Dynamics ~ HW1
clear
clc
close all
%% Define Variables
%  rho = 998;            % kg/m^3
% P_a = 1*10^4   ;       % Acoustic Pressure amplitude (Pascals)
%  w   = 2*pi*20*10^3;   % Acoustic Driving Frequency (2*pi*20 kHz)
% Pv   = 2.33*10^3;      % Vapor Pressure (Water/Air) Boundary
%  P_inf_e = 1*10^5;     % Pressure far away from the bubble at equil ~ Pa
```

```
%  k    = 1.4;             % Polytropic Exponent of gas (air adiabitc approx.)
% sigma = 0.072;         % Surface Tension for Air/Water
% nu  = 0.001;           % Shear (dynamic) liquid viscosity for water in Pa*s
Ro  = 1*10^-5;           % Equlibrium Bubble Radius in meters
% %tspan= [0 0.0001];    % Time vector in seconds
%
t0 = 0;
tf = 0.001;
y0 = [1.5*Ro, 0]';
[t,y] = ode23(@Rayleigh2,[t0,tf],y0);
t1 = t;
% tplot = t-t0;

figure(1)
hp = plot(t,y(:,1));
grid on
set(hp,'LineWidth',2)
set(gca,'FontSize',12)
xlabel('Time (s)')
ylabel('Bubble Radius (m)')

% Question 3
% Lauterborn's Figure 2 was digitized and points importeded into data set
% format long
% LAT = dlmread('Lauterborn_Figure2.txt');
% t_lat = LAT(:,1);
% R_Ro_lat = LAT(:,2);
% figure(2)
% hp2 = plot(tplot,y(:,1)/Ro,'x');
% hold on;
% hp3 = plot(t_lat,R_Ro_lat);
% leg1 = legend('Computed Results','Lauterborn Results');
% set(leg1,'FontSize',13)
% xlim([0 17*10^-6])
% ylim([0 2])
% grid on
% set(hp2,'LineWidth',2)
% set(hp3,'LineWidth',2)
% set(gca,'FontSize',13)
% xlabel('Time (s)')
% ylabel('R/Ro')

% Question 4
```

```
clear t
tf = 0.001;
[T,Y] = ode45(@Runge_Kutta, [t0,tf],y0);
figure(3)
hp4 = plot(T,Y(:,1)+Ro);
set(gca,'FontSize',12)
set(hp4,'LineWidth',2)
xlabel('Time (s)')
ylabel('Bubble Radius (m)')
grid on


% Quesiton 5
 rho = 998;              % kg/m^3
 P_a = 1*10^2   ;        % Acoustic Pressure amplitude (Pascals)
 w   = 2*pi*20*10^3;     % Acoustic Driving Frequency (2*pi*20 kHz)
 Pv  = 2.33*10^3;        % Vapor Pressure (Water/Air) Boundary
 P_inf_e = 1*10^5;       % Pressure far away from the bubble at equil ~ Pa
 k   = 1.4;              % Polytropic Exponent of gas (air adiabitc approx.)
 sigma = 0.072;          % Surface Tension for Air/Water
 nu  = 0.001;
 tspan= [0:0.000001:0.001];    % Time vector in seconds

w_sq = (3*k*P_inf_e)/(rho*Ro^2);
damp = (2*nu)/(sqrt(rho*Ro^2*3*k*P_inf_e));
M = ((1-w.^2/w_sq)).^2 + (2*damp*w.^2/w_sq)^(-0.5);
phi = -atan((2*damp*w/w_sq.^0.5)/(1-(w.^2/w_sq)));
y_approx = -P_a/(rho*Ro*w_sq)*M*sin(w*tspan+phi);

figure(4)
plot(tspan,y_approx)

figure(5)
hp10 = plot(t1,y(:,1),'x');
hold on;
hp20 = plot(T,Y(:,1)+Ro);

hp30 = plot(tspan,y_approx+Ro);
xlim([0 0.001])
set(gca,'FontSize',14)
grid on
leg2 = legend('Equation 1','Equation 3','Equation 5');
xlabel('Time (s)')
ylabel('Bubble Radius (m)')
```

```
set(leg2,'FontSize',14)
set(hp10,'LineWidth',1)
set(hp20,'LineWidth',1)
set(hp30,'LineWidth',2)
```

```
function Rdot2 = Rayleigh(t,y)
rho = 998;          % kg/m^3
P_a = 0   ;         % Acoustic Pressure amplitude (Pascals)
w = 2*pi*20*10^3;  % Acoustic Driving Frequency (2*pi*20 kHz)
Pv= 2.33*10^3;     % Vapor Pressure (Water/Air) Boundary
P_inf_e=1*10^5;    % Pressure far away from the bubble at equil(Pa)
k = 1.4;            % Polytropic Exponent of gas (air adiabitc)
sigma = 0.072;     % Surface Tension for Air/Water
nu  = 0.001;       % Shear(dynamic) liquid viscosity for water(Pas)
Ro  = 1*10^-5;     % Equlibrium Bubble Radius in meters
Rdot = y(2);
R = y(1);
% Pressure in the gas @ equlibrium
P_ge = P_inf_e - Pv + (2*sigma/Ro);

% Pressure far away from the bubble
P_inf = P_inf_e + P_a.*sin(w*t);
P = P_ge*(Ro/R)^(3*k) + Pv - P_inf - (2*sigma/R) - (4*nu*Rdot/R);
Rdot2 = [y(2); (P/rho - (3/2)*y(2).^2)*(1./y(1))];
```

2. Perform the following sanity checks on your code:

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i. Equilibrium: Set $P_a = 0$; $\dot{R}$(t=0) = 0; R(t=0) = R_o and run the code with these initial conditions for at least t = 0.001s. Plot R(t); is it stable?

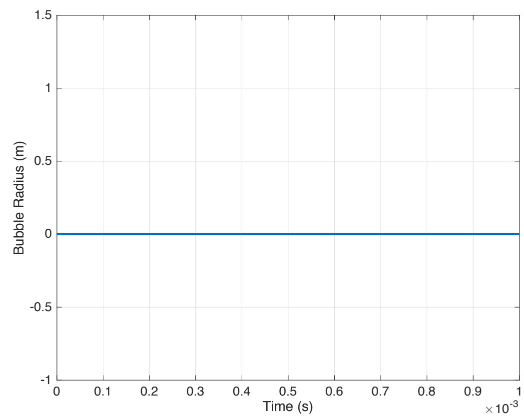

Figure 1: Equilibrium Solution for the Rayleigh Plesset Equation

ii. Ring down: Set run your code for each initial condition. Plot R(t) does it decay to R0? What is the frequency of the ringing? How does it compare with the linear natural frequency.

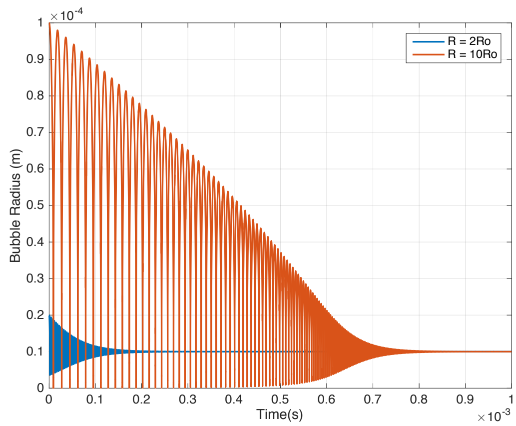

Figure 2: Ring Down test for Rayleigh Plesset Code

iii. Step drive: Set $\dot{R}$(t=0) = 0; R(t=0) = R_0; $P_\infty = P_\infty^e + P_c H(t - t_o)$ where P_c is 1x10^5 Pa and H(t) is the unit step function (Heaviside). Plot R(t) does it make sense?

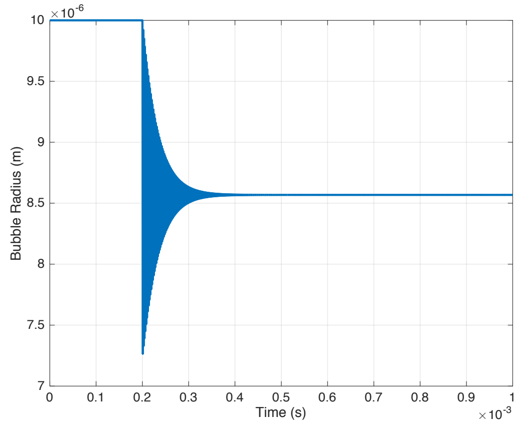

Figure 3: Step Drive test for Rayleigh Plesset Code

3. Benchmark your code against a previously published solution, using their parameter values. You may choose anything nontrivial* in the literature of the past 50 years, but be careful to compare results of the same equation at the same parameters, since there are a variety of Rayleigh-Plesset equations.

I used the parameters laid out in the paper by Lauterborn. The figure below shows the comparison of the results. Not sure why they do not align properly. It may be an incorrect use of his parameters in the papers or misinterpretation of his constants used. Nonetheless, the general shape of the solution seems to be present.

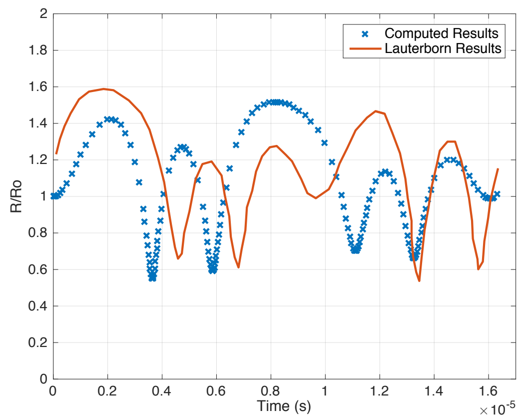

Figure 4: Comparison of Computational Results with those obtained by Lauterborn

4. Recall the linear EOM we derived in class for R(t) = R_o + y(t), y << R_o

$$\ddot{y} + \left(\frac{4\mu}{\rho R_o^2}\right)\dot{y} + \frac{1}{\rho R_o^2}\left(3k\left(P_\infty^e - P_v + \frac{2\sigma}{R_o}\right) - \frac{2\sigma}{R_o}\right)y = \frac{-P}{\rho R_o} sin(\omega t)$$

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i. Code a numerical integration solution to (3)

```
function dy = Runge_Kutta(t,y)

rho = 998;              % kg/m^3
P_a = 0   ;         % Acoustic Pressure amplitude (Pascals)
w   = 2*pi*20*10^3;    % Acoustic Driving Frequency (2*pi*20 kHz)
Pv  = 2.33*10^3;       % Vapor Pressure (Water/Air) Boundary
P_inf_e = 1*10^5;      % Pressure far away from the bubble at equil ~ Pa
k   = 1.4;             % Polytropic Exponent of gas (air adiabitc approx.)
sigma = 0.072;         % Surface Tension for Air/Water
nu  = 0.001;           % Shear (dynamic) liquid viscosity for water in Pa*s
Ro  = 1*10^-5;           % Equlibrium Bubble Radius in meters
dy = zeros(2,1);
dy(1) = y(2);
dy(2) = -P_a/(rho*Ro).*sin(w*t)-((4*nu)/(rho*Ro^2))*y(2)-(1/(rho*Ro^2))*...
    (3*k*(P_inf_e - Pv + (2*sigma/Ro)) - (2*sigma/Ro))*y(1);
```

ii. Benchmarks Works for all similar cases. See comparison in Problem 5

5. For the linearized EOM in problem 4, we can approximate the natural frequency as:

$$\omega_o^2 = \frac{3kP_\infty^e}{\rho R_o^2}$$

and the damping ratio as:

$$\xi = \frac{2\mu}{(\rho R_o^2 3kP_\infty^2)^{0.5}}$$

with the usual solution to sine forcing given as:

$$y(t) = \frac{-P_a}{\rho R_o \omega_o^2} M(\omega) sin(\omega t + \phi(\omega))$$

---

i. Compare results of all there developments of R(t)

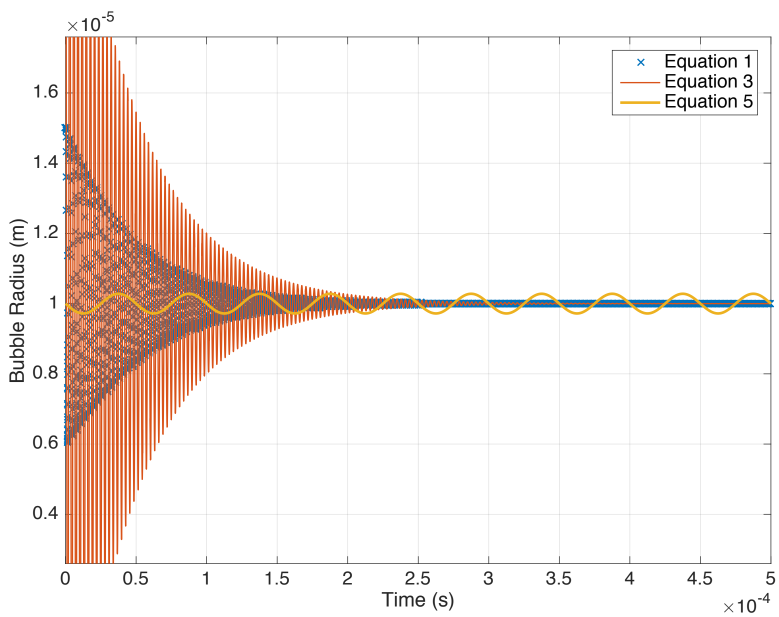

Figure 5: Comparison of all developments of R(t)

Due to the form of the solution based on equation 3 having a highly oscillating form during the first couple steps of the ring-down, the solutions do not, for all the cases explored agree within 10%. However, it can be said that after about $1x10^{-4}$ seconds in time, the solutions do agree within 10% of each other. The final value of the bubble radius for all cases also shows minimal difference. It is certain that they are indeed arriving at the same bubble radius after long progression in time, but do not necessarily agree within the aforementioned time span. The figure below shows the results of the code when the initial radius was changed to 10^{-6} m.

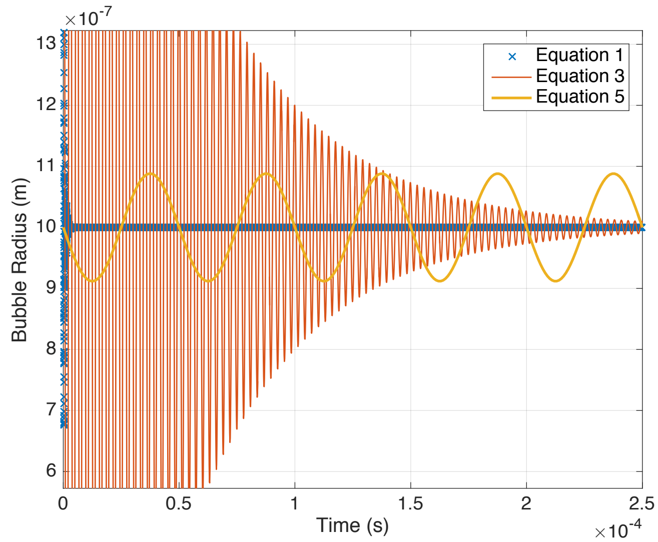

Figure 6: Comparison of all developments of R(t) for initial radius of 10^-6 m