1. Code a numerical solution to the RP equation (1).

MATLAB was used to generate a solution to the Rayleigh-Plesset Equation. Variables were re-defined such that $y_1 = R$ and $y_2 = \dot{R}$. This allowed the single 2nd order ODE to become 2 coupled 1st order ODEs. Redefinition of variables resulted in:

$$\dot{y}_1 = y_2 \qquad \dot{y}_2 = \ddot{R}$$

with

$$\ddot{R} = \left[\frac{P(R, \dot{R}, t)}{\rho} - \frac{3}{2}\dot{R}^2 \right] \frac{1}{R}$$

where

$$P(R, \dot{R}, t) = P_g^e \left(\frac{R_o}{R} \right)^{3k} + P_v - P_\infty - \frac{2\sigma}{R} - \frac{4\mu\dot{R}}{R}$$

$$P_g^e = P_\infty^e - P_v + \frac{2\sigma}{R_o}$$

$$P_\infty = P_\infty^e + P_a sin(\omega t)$$

Shown below are the codes used to obtain solutions to the RP equation

```
1 % This script will model the Rayleigh Plesset Equations
2 % Author: Dorien Villafranco
3 % Department of Mechanical Engineering, Boston University
4 % Requirements for ME721: Acoustic Bubble Dynamics ~ HW1
5 clear
6 clc
7 close all
8 %% Define Variables
9 %   rho = 998;              % kg/m^3
10 % P_a = 1*10^4  ;          % Acoustic Pressure amplitude (Pascals)
11 %   w   = 2*pi*20*10^3;    % Acoustic Driving Frequency (2*pi*20 kHz
      )
12 % Pv  = 2.33*10^3;         % Vapor Pressure (Water/Air) Boundary
13 %   P_inf_e = 1*10^5;      % Pressure far away from the bubble at
      equil ~ Pa
```

```matlab
14 %   k    = 1.4;                % Polytropic  Exponent  of  gas  (air
      adiabitc  approx.)
15 % sigma = 0.072;          % Surface  Tension  for  Air/Water
16 % nu   = 0.001;           % Shear  (dynamic)  liquid  viscosity  for
      water  in  Pa*s
17 Ro   = 1*10^-5;           % Equlibrium  Bubble  Radius  in  meters
18 % %tspan= [0  0.0001];    % Time  vector  in  seconds
19 %
20 t0  =  0;
21 tf  =  0.001;
22 y0  =  [1.5*Ro,  0]';
23 [t,y]  =  ode23(@Rayleigh2 ,[t0,tf],y0);
24 t1  =  t;
25 % tplot  =  t-t0;
26
27 figure(1)
28 hp  =  plot(t,y(:,1));
29 grid  on
30 set(hp,'LineWidth',2)
31 set(gca,'FontSize',12)
32 xlabel('Time  (s)')
33 ylabel('Bubble  Radius  (m)')
34
35 % Question  3
36 % Lauterborn's  Figure  2  was  digitized  and  points  imported  into
      data  set
37 % format  long
38 % LAT  =  dlmread('Lauterborn_Figure2.txt');
39 % t_lat  =  LAT(:,1);
40 % R_Ro_lat  =  LAT(:,2);
41 % figure(2)
42 % hp2  =  plot(tplot,y(:,1)/Ro,'x');
43 % hold  on;
44 % hp3  =  plot(t_lat ,R_Ro_lat);
45 % leg1  =  legend('Computed  Results','Lauterborn  Results');
46 % set(leg1,'FontSize',13)
47 % xlim([0  17*10^-6])
48 % ylim([0  2])
49 % grid  on
50 % set(hp2,'LineWidth',2)
51 % set(hp3,'LineWidth',2)
52 % set(gca,'FontSize',13)
53 % xlabel('Time  (s)')
54 % ylabel('R/Ro')
55
56 % Question  4
```

```matlab
57  clear t
58  tf = 0.001;
59  [T,Y] = ode45(@Runge_Kutta, [t0,tf],y0);
60  figure(3)
61  hp4 = plot(T,Y(:,1)+Ro);
62  set(gca,'FontSize',12)
63  set(hp4,'LineWidth',2)
64  xlabel('Time (s)')
65  ylabel('Bubble Radius (m)')
66  grid on
67
68
69  % Quesiton 5
70   rho = 998;              % kg/m^3
71   P_a = 1*10^2   ;        % Acoustic Pressure amplitude (Pascals)
72   w   = 2*pi*20*10^3;     % Acoustic Driving Frequency (2*pi*20 kHz)
73   Pv  = 2.33*10^3;        % Vapor Pressure (Water/Air) Boundary
74   P_inf_e = 1*10^5;       % Pressure far away from the bubble at
        equil ~ Pa
75   k   = 1.4;              % Polytropic Exponent of gas (air adiabitc
        approx.)
76   sigma = 0.072;          % Surface Tension for Air/Water
77   nu  = 0.001;
78   tspan= [0:0.000001:0.001];   % Time vector in seconds
79
80  w_sq = (3*k*P_inf_e)/(rho*Ro^2);
81  damp = (2*nu)/(sqrt(rho*Ro^2*3*k*P_inf_e));
82  M = ((1-w.^2/w_sq)).^2 + (2*damp*w.^2/w_sq)^(-0.5);
83  phi = -atan((2*damp*w/w_sq.^0.5)/(1-(w.^2/w_sq)));
84  y_approx = -P_a/(rho*Ro*w_sq)*M*sin(w*tspan+phi);
85
86  figure(4)
87  plot(tspan,y_approx)
88
89  figure(5)
90  hp10 = plot(t1,y(:,1),'x');
91  hold on;
92  hp20 = plot(T,Y(:,1)+Ro);
93
94  hp30 = plot(tspan,y_approx+Ro);
95  xlim([0 0.001])
96  set(gca,'FontSize',14)
97  grid on
98  leg2 = legend('Equation 1','Equation 3','Equation 5');
99  xlabel('Time (s)')
100 ylabel('Bubble Radius (m)')
```

```matlab
101  set(leg2,'FontSize',14)
102  set(hp10,'LineWidth',1)
103  set(hp20,'LineWidth',1)
104  set(hp30,'LineWidth',2)
```

```matlab
 1  function Rdot2 = Rayleigh(t,y)
 2  rho = 998;          % kg/m^3
 3  P_a = 0   ;         % Acoustic Pressure amplitude (Pascals)
 4  w = 2*pi*20*10^3;   % Acoustic Driving Frequency (2*pi*20 kHz)
 5  Pv= 2.33*10^3;      % Vapor Pressure (Water/Air) Boundary
 6  P_inf_e=1*10^5;     % Pressure far away from the bubble at equil(Pa)
 7  k = 1.4;            % Polytropic Exponent of gas (air adiabitc)
 8  sigma = 0.072;      % Surface Tension for Air/Water
 9  nu  = 0.001;        % Shear(dynamic) liquid viscosity for water(Pas)
10  Ro  = 1*10^-5;      % Equlibrium Bubble Radius in meters
11  Rdot = y(2);
12  R = y(1);
13  % Pressure in the gas @ equlibrium
14  P_ge = P_inf_e - Pv + (2*sigma/Ro);
15
16  % Pressure far away from the bubble
17  P_inf = P_inf_e + P_a.*sin(w*t);
18  P = P_ge*(Ro/R)^(3*k) + Pv - P_inf - (2*sigma/R) - (4*nu*Rdot/R);
19  Rdot2 = [y(2); (P/rho - (3/2)*y(2).^2)*(1./y(1))];
```

2. Perform the following sanity checks on your code:

i. Equilibrium: Set $P_a = 0$; $\dot{R}(t{=}0) = 0$; $R(t{=}0) = R_o$ and run the code with these initial conditions for at least $t = 0.001s$. Plot $R(t)$; is it stable?
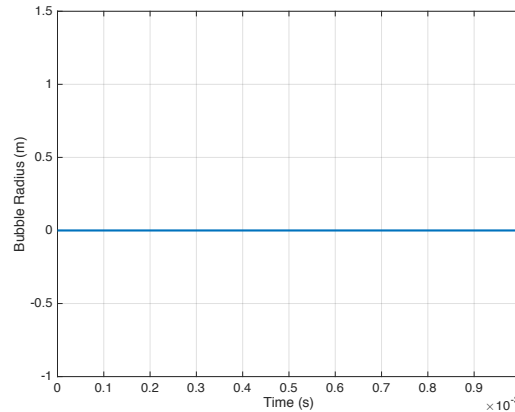

Figure 1: Equilibrium Solution for the Rayleigh Plesset Equation

ii. Ring down: Set run your code for each initial condition. Plot $R(t)$ does it decay to R0? What is the frequency of the ringing? How does it compare with the linear natural frequency.
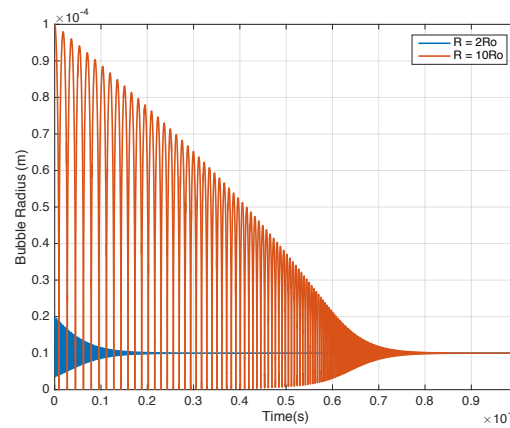

Figure 2: Ring Down test for Rayleigh Plesset Code

iii. Step drive: Set $\dot{R}(t=0) = 0$; R(t=0) = R_0; $P_\infty = P_\infty^e + P_c H(t - t_o)$ where P_c is 1×10^5 Pa and H(t) is the unit step function (Heaviside). Plot R(t) does it make sense?
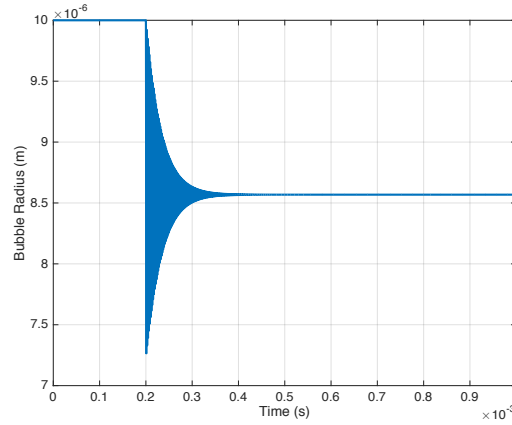

Figure 3: Step Drive test for Rayleigh Plesset Code

3. Benchmark your code against a previously published solution, using their parameter values. You may choose anything nontrivial* in the literature of the past 50 years, but be careful to compare results of the same equation at the same parameters, since there are a variety of Rayleigh-Plesset equations.

I used the parameters laid out in the paper by Lauterborn. The figure below shows the comparison of the results. Not sure why they do not align properly. It may be an incorrect use of his parameters in the papers or a sign difference in the equation - will double check later and update.
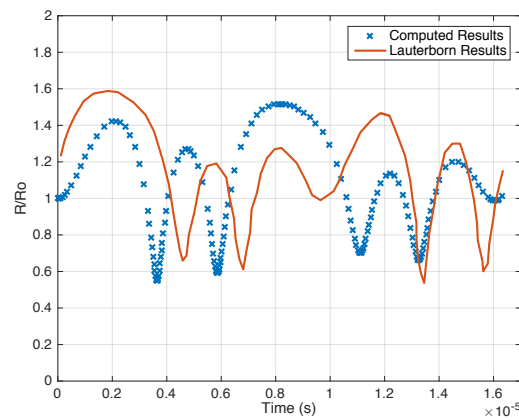

Figure 4: Comparison of Computational Results with those obtained by Lauterborn

7

4. Recall the linear EOM we derived in class for R(t) = R_o + y(t), y << R_o

$$\ddot{y} + \left(\frac{4\mu}{\rho R_o^2}\right)\dot{y} + \frac{1}{\rho R_o^2}\left(3k\left(P_\infty^e - P_v + \frac{2\sigma}{R_o}\right) - \frac{2\sigma}{R_o}\right)y = \frac{-P}{\rho R_o}sin(\omega t)$$

i. Code a numerical integration solution to (3)

```
1  function dy = Runge_Kutta(t,y)
2
3  rho = 998;              % kg/m^3
4  P_a = 0   ;            % Acoustic Pressure amplitude (Pascals)
5  w   = 2*pi*20*10^3;    % Acoustic Driving Frequency (2*pi*20 kHz)
6  Pv  = 2.33*10^3;       % Vapor Pressure (Water/Air) Boundary
7  P_inf_e = 1*10^5;      % Pressure far away from the bubble at equil
            ~ Pa
8  k   = 1.4;             % Polytropic Exponent of gas (air adiabitc
      approx.)
9  sigma = 0.072;         % Surface Tension for Air/Water
10 nu  = 0.001;           % Shear (dynamic) liquid viscosity for water
         in Pa*s
11 Ro  = 1*10^-5;          % Equlibrium Bubble Radius in meters
12 dy = zeros(2,1);
13 dy(1) = y(2);
14 dy(2) = -P_a/(rho*Ro).*sin(w*t)-((4*nu)/(rho*Ro^2))*y(2)-(1/(rho*
      Ro^2))*...
15     (3*k*(P_inf_e - Pv + (2*sigma/Ro)) - (2*sigma/Ro))*y(1);
```

ii. Benchmarks Works for all similar cases. See comparison in Problem 5

5. For the linearized EOM in problem 4, we can approximate the natural frequency as:

$$\omega_o^2 = \frac{3kP_\infty^e}{\rho R_o^2}$$

and the damping ratio as:

$$\xi = \frac{2\mu}{(\rho R_o^2 3kP_\infty^2)^{0.5}}$$

with the usual solution to sine forcing given as:

$$y(t) = \frac{-P_a}{\rho R_o \omega_o^2} M(\omega) sin(\omega t + \phi(\omega))$$
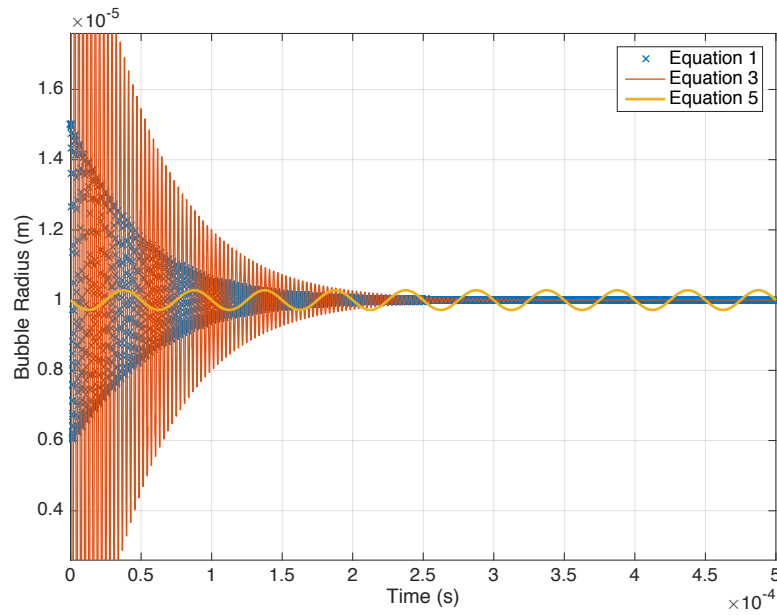
i. Compare results of all there developments of R(t)

Figure 5: Comparison of all developments of R(t)