Establish the Sears Function

Von Karaman and Sears (1938) analyzed the problem of a thin-airfoil moving through a sinusoidal vertical gust field. The gust was considered as an upwash velocity that is uniformly convected by the free stream. The forcing function was considered to be:

$$w_g(x,t) = sin\left(w_g t - \frac{w_g x}{V}\right) \tag{1}$$

$$w_g(x,t) = sinw_g tcos\left(\frac{w_g x}{V}\right) - cosw_g tsin\left(\frac{w_g x}{V}\right) \tag{2}$$

There were two cases of interested. First, if the gust was referenced to the airfoil's leading edge the equation simply became $w_g(t) = sinw_g t$. Second, if the gust is referenced to the mid-chord, then x = b = c/2 and the forcing function becomes $w_g(t)cosk_g sinw_g t - sink_g cosw_g t$which is the equivalent to a phase shift. In the original work of Sears, the mid-chord was used and the lift coefficient was written as:

$$C_l = 2\pi\left(\frac{w_0}{V}\right)S(k_g) \tag{3}$$

where $S(k_g)$ is known as the Sear's function. The gust encounter frequency is given by

$$k_g = \frac{2\pi V}{\lambda_g} \tag{4}$$

where λ_g is the wavelength of the gust. The figure below outlines the development of the Sear's problem.

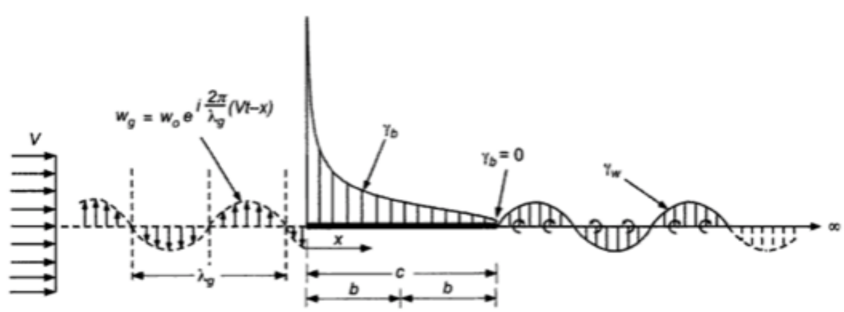

Figure 1: Model of thin airfoil encountering a sinusoidal vertical gust (Sear's Problem)

To suit our problem at hand we will redefine the wave number k as:

$$k = \frac{wc}{2U} \qquad [dimensionless] \tag{5}$$

where w is the frequency (2πf), c/2 is the half chord length and U is the freestream velocity. The Sears function can be expressed in terms of Hankel Functions as shown below:

$$S(k) = \frac{2}{\pi k(H_0^2(k) - iH_1^2(k))} \qquad [dimensionless] \tag{6}$$

From the Sear's function, the analytical lift response of the flat plate to a passing vortex (Blade Vortex Interaction Problem) can be described as:

$$L = \rho\Gamma\frac{c}{2}e^{-kh}\Big(-iS(k)\Big) \tag{7}$$

To match the desired units for Lift as a function of frequency and x_2 with k being assumed to being dimensionless, the following equation can be defined:

$$L(w, x_2) = \rho\Gamma 2R_3\Big(\frac{c}{2}\Big)e^{-k_1x_2}\Big(-iS(k)\Big) \qquad \left[\frac{FT}{L}\right] \tag{8}$$

Now if the Lift is made a function of k_1 instead of w,

$$L(k_1, x_2) = \rho U_\infty\Gamma 2R_3\Big(\frac{c}{2}\Big)e^{-k_1x_2}\Big(-iS(k)\Big) \qquad \left[\frac{FL}{L}\right] \tag{9}$$

Changing dependency from x_2 to k_2:

$$L(k_1, k_2) = \frac{1}{2\pi}\int_{-\infty}^{\infty} L(k_1, x_2)e^{-ik_2x_2}dx_2 \tag{10}$$

Substituting the expression for $L(k_1, x_2)$ into the above equation, the result is:

$$L(k_1, k_2) = \rho\Gamma U_\infty\Big(\frac{c}{2}\Big)\frac{2R_3}{2\pi}\Big(-iS(k_1)\Big)\frac{2k_1}{k_1^2 + k_2^2} \qquad \left[\frac{FL^2}{L}\right]?? \tag{11}$$