



## Water Resources Research

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#### Key Points:

- Use of hydrographs as functional data for the identification of representative catchment hydrograph shapes
- Establishment of flood reactivity regions using catchment-specific representative hydrograph sets
- Flood reactivity regions are similar in terms of hydrograph shapes, magnitudes, and triggering events

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## Identification of Flood Reactivity Regions via the Functional Clustering of Hydrographs

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**Abstract** Flood hydrograph shapes contain valuable information on the flood-generation mechanisms of a catchment. To make good use of this information, we express flood hydrograph shapes as continuous functions using a functional data approach. We propose a clustering approach based on functional data for flood hydrograph shapes to identify a set of representative hydrograph shapes on a catchment scale and use these catchment-specific sets of representative hydrographs to establish regions of catchments with similar flood reactivity on a regional scale. We applied this approach to flood samples of 163 medium-size Swiss catchments. The results indicate that three representative hydrograph shapes sufficiently describe the hydrograph shape variability within a catchment and therefore can be used as a proxy for the flood behavior of a catchment. These catchment-specific sets of three hydrographs were used to group the catchments into three reactivity regions of similar flood behavior. These regions were not only characterized by similar hydrograph shapes and reactivity but also by event magnitudes and triggering event conditions. We envision these regions to be useful in regionalization studies, regional flood frequency analyses, and to allow for the construction of synthetic design hydrographs in ungauged catchments. The clustering approach based on functional data which establish these regions is very flexible and has the potential to be extended to other geographical regions or toward the use in climate impact studies.

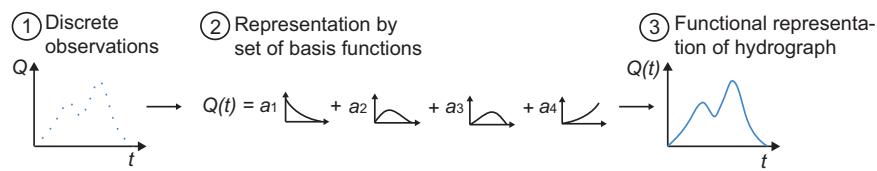
### 1. Introduction

Hydrological processes vary widely from one environment to the next (McDonnell & Woods, 2004), which causes distinct flood responses (Merz & Blöschl, 2009). It is often useful to reduce this variability by grouping catchments with similar governing processes. The establishment of regions which are similar in terms of their flood behavior is challenging but is an important first step in regionalization studies (Blöschl et al., 2013; Prinzio et al., 2011) and regional flood frequency analyses that allow for the estimation of flood frequencies in ungauged catchments (Hosking & Wallis, 1997). The often-used index flood method (Dalrymple, 1960), for instance, is based on regions consisting of hydrologically similar catchments. It uses information from sites within a given region to estimate the magnitude of extreme events corresponding to a predefined return period at a target site (Requena et al., 2017). Similarity measures typically used to delineate regions of similar hydrological behavior are physiographical (e.g., catchment area or altitude), climatological (e.g., daily rainfall statistics), and/or hydrological (e.g., mean daily flow) characteristics (see Ali et al. (2012) for an overview on variables used in previous studies). Preferably, such regions are established using physiographical or climatological catchment characteristics since these are also available for ungauged catchments (Acreman & Sinclair, 1986; Hosking & Wallis, 1997; Ilorme & Griffis, 2013), which enables the attribution of an ungauged catchment to an existing region. However, physiographical and climatological catchment similarity do not often correspond to hydrological similarity (Ali et al., 2012; Oudin et al., 2010) and even less to similarity in the flood behavior of a catchment (Merz & Blöschl, 2009). Therefore, hydrologically similar regions are often delineated based on hydrological catchment characteristics or runoff signatures (Burn & Boorman, 1992), such as the median daily flow, annual runoff coefficient, slope of the flow duration curve (Boscarello et al., 2016), seasonality indices (Castellarin et al., 2001), monthly Pardé coefficients (Hailegeorgis & Alfredsen, 2017), or statistical measures, which reflect the shape of the flood distribution (Hosking & Wallis, 1997). The latter characteristics usually focus on flood peaks and reflect only a part of

the flood behavior of a catchment. The focus on flood peaks neglects other hydrograph characteristics such as volume and shape, which are equally important for many flood risk management tasks, especially for those involving storage (Deutsche Vereinigung für Wasserwirtschaft Abwasser und Abfall, 2012; Pilgrim, 1986), and potentially provide crucial information on the flood behavior of a catchment. These hydrograph characteristics could be useful for the identification of regions similar in terms of their flood behavior since the hydrograph integrates temporal and spatial variations in water input, storage, and water processes within a catchment (Hannah et al., 2000). The goal of this study is to employ the information integrated in hydrograph shapes for the identification of regions with a similar flood behavior. We propose a catchment clustering scheme consisting of two steps that accounts for different flood mechanisms acting within a catchment. Clustering is often used as an exploratory tool to identify distinct groups so that the observations within each group are similar to each other while observations in different groups are different from each other (James et al., 2013). The clusters resulting from applying a cluster algorithm cannot be validated directly but indirectly by their interpretability and usefulness (Webster & Oliver, 2007). The first step in the clustering scheme is the identification of representative hydrograph shapes using observed flood event hydrograph shapes. The term representative is used for shapes that describe the hydrograph shape variability and potential flood responses or causative mechanisms within a catchment. Compared to existing flood process classification schemes, such as the one proposed by Merz and Blöschl (2003), no meteorological information in addition to the streamflow data are used to identify different flood event types. The second step in the clustering scheme identifies regions of catchments with a similar flood reactivity by clustering the catchment-specific sets of representative hydrograph shapes obtained in the first step.

The approach is based on the clustering of hydrograph shapes represented as functional data (FD). In contrast to classical multivariate data, FD is continuously defined and does not depend on the choice of several hydrograph characteristics, such as peak discharge, hydrograph volume, duration, or a few parameters representing the shape of a hydrograph (Yue et al., 2002), but instead uses the whole information stored in the hydrograph (Chebana et al., 2012). FD analysis is more general, flexible, and representative of the real hydrological phenomena than classical multidimensional analysis and avoids the subjective choice of a set of hydrograph characteristics (Ternynck et al., 2016). FD are conceptually defined in a continuous framework. In practice, however, they are usually observed at discrete points in time and stored in a finite-dimensional way. Hydrographs can be considered as FD since they fulfill this criterion. The first step in FD analysis is often the reconstruction of the functional form of data from discrete observations (see Figure 1 for an illustration). Most commonly, this is done by considering the data as part of a finite-dimensional space spanned by some basis functions. Alternatively, the data can be smoothed nonparametrically (Jacques & Preda, 2014). This allows the representation of an individual functional datum as a continuous function rather than as its values at particular points (Ramsay & Silverman, 2002). It has been shown in previous studies that the FD framework can be beneficial in the identification of groups of similar hydrographs over a range of temporal scales, such as yearly hydrographs (Jamaludin, 2016; Merleau et al., 2007), spring flood events (duration of 6 months) (Ternynck et al., 2016), and diurnal discharges (duration of one day) (Hannah et al., 2000).

In this study, we adapt the FD framework to cluster flood event hydrograph shapes (duration of three days) to identify catchment-specific sets of hydrograph shapes and use these sets to establish homogeneous regions in terms of flood reactivity. Well-known clustering algorithms such as hierarchical and  $k$ -means algorithms can be adapted to the case of FD (Cuevas, 2014). Jacques and Preda (2014) grouped the major approaches into four categories: (1) raw data clustering, (2) two-stage approaches which first reduce the dimension of the data and second perform clustering, (3) nonparametric clustering approaches, and (4)



**Figure 1.** Getting from discrete measurements (1) to a functional representation of a hydrograph (3) by representing the data by a set of basis functions (2).

model-based clustering approaches. An initial analysis, where each of these method categories was tested, showed that two-stage approaches were most suitable to cluster hydrographs since they resulted in meaningful clusters. We therefore focused on this type of methods.

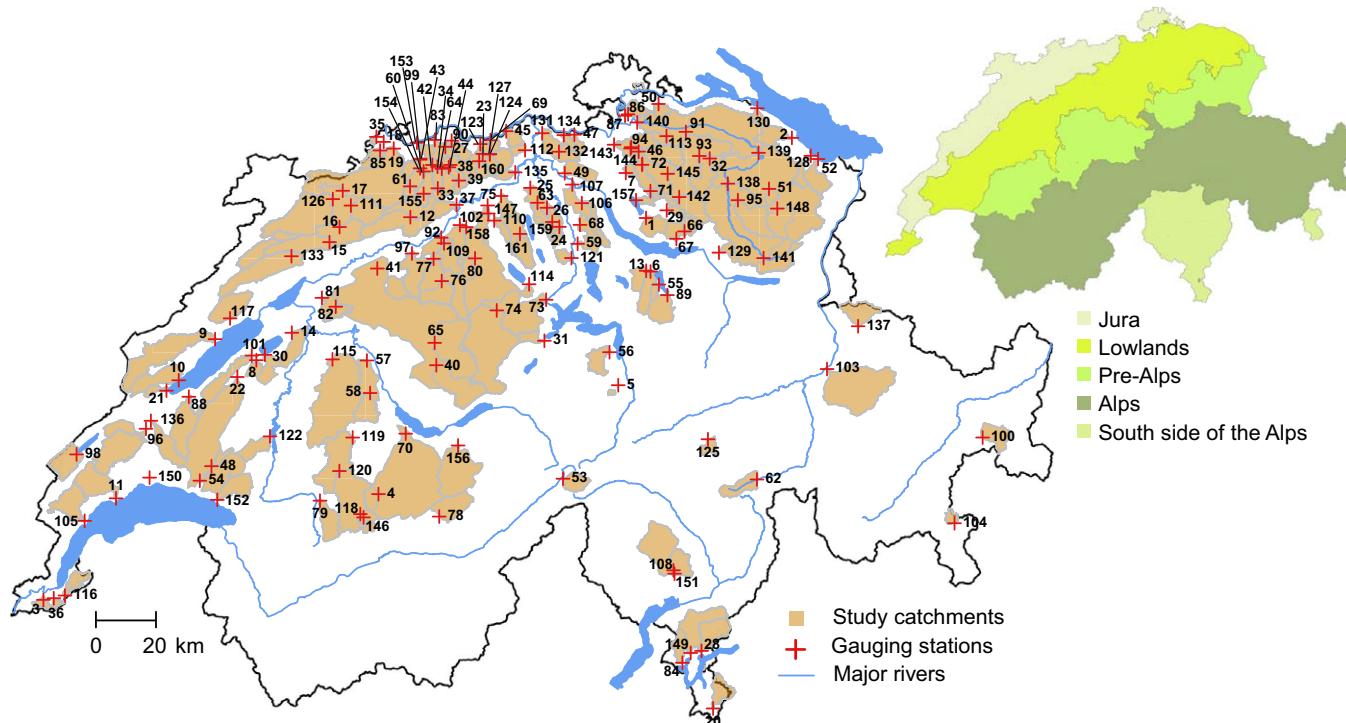
## 2. Data

### 2.1. Study Catchments

This study was performed using runoff data from 163 Swiss catchments (see Figure 2) with a wide range of catchment characteristics and flood behaviors. The selected catchments have hourly flow series of at least 20 years in duration and ranging up to 53 years. The catchments' runoff is neither significantly altered by regulated lakes upstream nor inland canals nor by urbanized areas or hydropower. The catchments are small to medium-size ( $6\text{--}1,800 \text{ km}^2$ ), situated between 400 and 2,600 m.a.s.l. (mean elevation), and either have no or only small areas with glaciers. The catchment set covers a wide range of runoff regimes and catchment characteristics and is therefore well suited for illustrating the proposed approach.

### 2.2. Flood Events

The basis for the functional hydrograph analysis was samples of flood events extracted from the runoff time series of the 163 study catchments. To sample flood events, we used a peak-over-threshold approach based on the procedure proposed by Lang et al. (1999). The threshold for the peak discharge was chosen iteratively to fulfill a target condition of four events per year on average which is a trade-off between maximizing the information content in the sample and keeping the assumption of independence between events. For each of these events, sampled according to the flood peaks, the flood volume, and hydrograph were determined over a fixed event window of 72 h. Flood events that ended in a secondary peak were removed automatically from the data set to ensure that events triggered by two independent convective precipitation events were not considered as one event. The base flow was separated from the direct flow using a recursive digital filter (Eckhardt, 2005). The resulting direct flow component of the hydrographs was then normalized so that the volume of the modified hydrographs was equal to one. This was done by dividing the ordinate of each hydrograph by the volume  $V$ . In the remainder of this paper, we refer to these



**Figure 2.** One hundred and sixty-three study catchments in Switzerland and geographical regions of Switzerland as introduced by Bundesamt für Umwelt BAFU and Eidg. Forschungsanstalt WSL (2012).

normalized hydrographs as *hydrograph shapes*. The advantage of working with normalized hydrographs is that hydrograph shapes of catchments with different sizes can directly be compared. We refer the reader to Brunner et al. (2017) for a more detailed description of the flood sampling and base flow separation procedures.

### 3. Methods

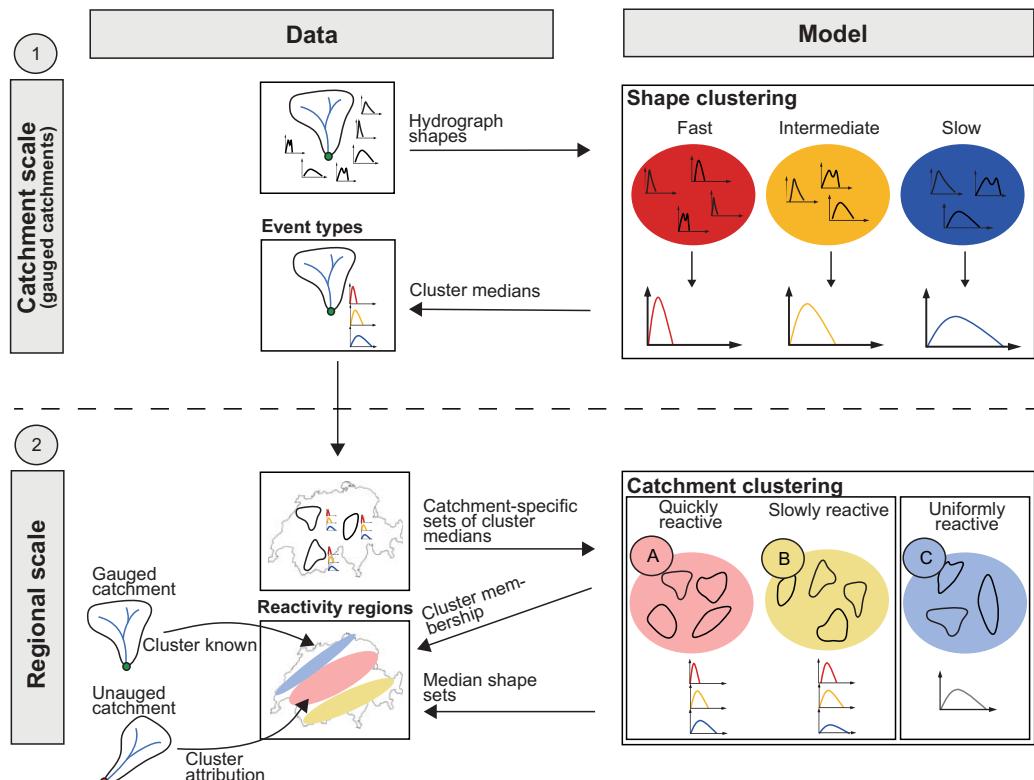
In this study, we propose a catchment clustering scheme for the establishment of regions with similar flood behaviors (for an illustration see Figure 3). The framework consists of two main parts:

1. The identification of sets of representative hydrograph shapes within individual catchments via a clustering approach in which hydrograph shapes are represented as FD. This first step allows for the characterization of the flood behavior of a catchment, that is typically influenced by several causative mechanisms (Merz & Blöschl, 2003), via several representative hydrograph shapes.
2. The establishment of regions with similar flood behaviors by clustering the catchment-specific sets of representative hydrograph shapes obtained in Step 1.

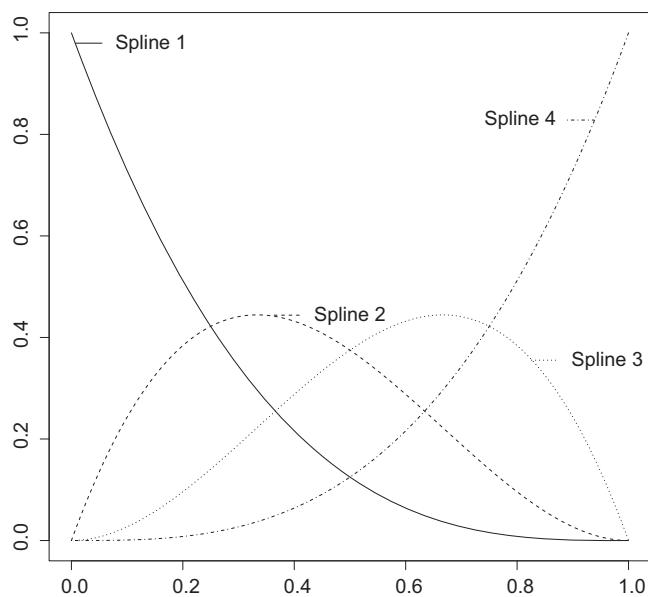
The methodology was established on the set of 163 Swiss catchments introduced above, however, its general principle is transferable to other geographical regions. The color coding used in this paper clearly distinguishes the clusters resulting from Steps 1 and 2. Results referring to the clusters established in Step 1 are displayed using saturated colors (red, orange, and blue) while those referring to the clusters established in Step 2 are displayed in pastel colors (rose, yellow, and light blue) (see Figure 3).

#### 3.1. Identification of Representative Hydrograph Shapes

The first part of this analysis focused on the identification of representative hydrograph shapes at a catchment scale. We used a clustering approach as an exploratory tool to identify the number of hydrograph shapes necessary to sufficiently describe the variability of hydrograph shapes within each of the study



**Figure 3.** Illustration of the clustering framework. The data input and output for the models are indicated for the two different parts: (1) hydrograph shape clustering, (2) catchment clustering. Hydrograph shape clustering is done on the catchment scale and catchment clustering on the regional scale.



**Figure 4.** Four B-splines of order four used for the representation of hydrograph shapes as FD.

of order four (Figure 4). B-splines of order four were sufficiently flexible to represent the hydrograph shapes under study. We did not go beyond this number to avoid unnecessary complexity. Similarly, an increase of the number of basis functions above four did not further improve the representation of hydrograph shapes as FD. We then computed the coefficients for each of the normalized hydrographs for the four B-spline bases.

The set of four coefficients per hydrograph shape was used as an input for the cluster analysis. The coefficient sets were clustered using the classical  $k$ -means algorithm. We applied the algorithm for  $k = 2$  up to  $k = 6$  number of clusters. The resulting clusters were assessed graphically for their suitability to represent the main part of the hydrograph shape variability within a catchment. In addition to this graphical assessment, we computed the silhouette widths over all catchments and hydrograph shapes (Rousseeuw, 1987) for different numbers of clusters. The silhouette width shows which hydrograph shapes lie well within their clusters and which ones lie in between clusters. The overall silhouette width provides an evaluation of clustering validity. Three clusters were found to represent hydrograph shape variability well and increasing the number of clusters did not further improve this representation (i.e., clusters with very few hydrograph shapes were built) and led to a decrease in the overall silhouette width. We decided to fix the number of clusters to three for all catchments since this allowed for their comparison across different catchments. Three clusters were a compromise between over representing hydrograph shape variability in catchments with a rather low variability and under representing variability in catchments with a rather high shape variability. Each of the three clusters consisted of a set of similar hydrograph shapes, which was summarized by their median hydrograph. We used the  $h$ -mode depth to order the hydrographs in the sample and to identify the median hydrograph within the set of similar hydrographs (Cuevas et al., 2007). The concept of data depth aims at measuring the centrality of a given curve (in our case, the hydrographs) within a group of curves and can be used to define the ranks of functional data (Fraiman & Muniz, 2001) and therewith robust estimators of a location parameter such as the median or the trimmed mean. The median hydrograph identified using the  $h$ -mode depth is actually an observed hydrograph, which would not be the case if the median hydrograph was defined by the median flow observation at each time step. The three resulting median hydrographs (one for each cluster) were said to be representative of the hydrograph shapes within the catchment. They were ranked according to their increasing time to peak to order them according to their reaction time, and were called fast, intermediate, and slow shape, respectively. The set of these three representative hydrograph shapes was used, in the following step, to describe the general flood behavior of a catchment. This set represents the variability in observed hydrograph shapes and, in the remainder of this

catchments. Contrary to classical clustering approaches, hydrograph shapes were not considered as multivariate data but expressed as FD, i.e., each hydrograph was represented as a function of time. The normalized hydrographs were projected on a set of basis functions to represent them as FD in a simple and low-dimensional framework. A basis function system is a set of known orthogonal functions. Any function can be approximated arbitrarily well by a weighted sum or by a linear combination of a sufficiently large number of functions out of such a set of basis functions. Spline functions are the most common choice of basis functions for nonperiodic FD. We used a set of B-spline functions (Data2fd from fda package in R (Ramsay et al., 2014)) as basis functions because they are able to mimic the main characteristics of hydrograph shapes (Abraham et al., 2003). A (smoothing) spline function is determined by the order of the polynomial segments and the number and placement of knots. The number of knots determines the ability of spline functions to represent sharp features in a curve and the knots can be placed such that they are denser in areas with stronger variations than in smooth areas (Höllig & Hörner, 2013). Increasing the number of splines does not always improve the fit to the data because the functional space defined by  $n$  basis or B-splines is not necessarily contained within the one defined by  $(n + 1)$  B-splines (Ramsay & Silverman, 2005). We chose a set of four B-splines

paper, we will refer to the three representative hydrograph shapes composing it as the fast, intermediate, and slow event types.

### 3.2. Establishment of Regions With Similar Flood Behaviors

The second part of this analysis focused on the regional scale to identify regions of catchments with a similar flood behavior in terms of their three representative hydrograph shapes (fast, intermediate, and slow). While these representative hydrograph shapes are quite distinct in most catchments, there are catchments where they are quite similar and a distinction between three shapes is not appropriate. The identification of regions with a similar flood behavior therefore consisted of two steps: (1) the identification of a region of catchments with a uniform flood reaction and (2) the identification of regions with similar flood reactions among the remaining catchments. The second step focused on clustering the catchments according to their three representative hydrograph shapes.

1. **Identification of a region with a uniform flood reaction:** We first identified catchments where the three representative hydrograph shapes were similar. These catchments were said to exhibit a uniform flood reaction and together built the region of uniformly reactive catchments. To identify the uniformly reactive catchments, we looked at the time to peak of the median hydrographs of the clusters obtained in the first part of the analysis. We computed the sum of the differences between the time and peak of the three representative hydrographs. The sum of the differences was found to be small in the catchments where the three hydrograph shapes were not very distinct. The threshold for the sum of the differences separating the uniformly reactive from the remaining catchments was set to a threshold of five based on visual inspection, i.e., an assessment of the similarity of three representative hydrograph shapes and the sum of differences between their times to peak. The threshold was set such that catchments where all three shapes were similar were said to be uniformly reactive and those where shapes started to differ were said to be nonuniformly reactive. The uniformly reactive catchments built their own region and were characterized by only one representative hydrograph shape.

2. **Identification of regions with similar flood reactions:** In most catchments, the hydrograph shape variability could only be well represented by three hydrograph shapes: a fast, an intermediate, and a slow hydrograph. However, a fast hydrograph in one catchment was not necessarily a fast hydrograph in another catchment. Contrarily, catchments showed differences in their general runoff reaction time. To identify regions of catchments with a similar reactivity, we applied clustering to their sets of representative hydrograph shapes. Catchments with similar sets of representative hydrograph shapes were said to be of similar reactivity and similar climate conditions. Working with normalized shapes allowed for the identification of catchments with a similar reactivity independent of their catchment size. For the clustering, the three representative hydrographs per catchment were again expressed as FD and we again computed coefficients for four B-splines of order four. Similar to Step 1, neither increasing the number of spline bases nor their order further improved the clustering results, which was also confirmed by the overall silhouette width. The four coefficients for each of the representative hydrograph shapes were stacked together to form a vector of  $4 \times 3 = 12$  coefficients. We applied hierarchical clustering instead of  $k$ -means clustering to allow for nonelliptical clusters (Gordon, 1999). As an objective function, we used Ward's minimum variance criterion, which minimizes the total within-cluster variance (Ward, 1963). The hierarchical clustering tree was symmetrical which suggested cuts at either  $k = 2$  or  $k = 4$ . We found that two clusters were sufficient to distinguish catchments with a generally fast reaction (quickly responding fast, intermediate, and slow events) from catchments with a generally slow reaction (delayed fast, intermediate, and slow events). This was confirmed by the overall silhouette width, which was highest for  $k = 2$  and decreased with an increase of the number of clusters.

This second part of the analysis divided the study domain into three regions with a similar reactivity, which we herein refer to as *reactivity regions*. A region where catchments showed a fast reaction to rainfall events, a region where catchments showed a slow reaction to rainfall events, and a region where catchments were characterized by a relatively uniform reaction to precipitation events.

### 3.3. Event Condition Analysis

The hydrograph clustering approaches proposed in this study first divided the events observed in a catchment into three event types and second, divided the catchments into three regions of similar reactivity based on hydrograph shapes, without considering event magnitudes or pre-event conditions. A direct

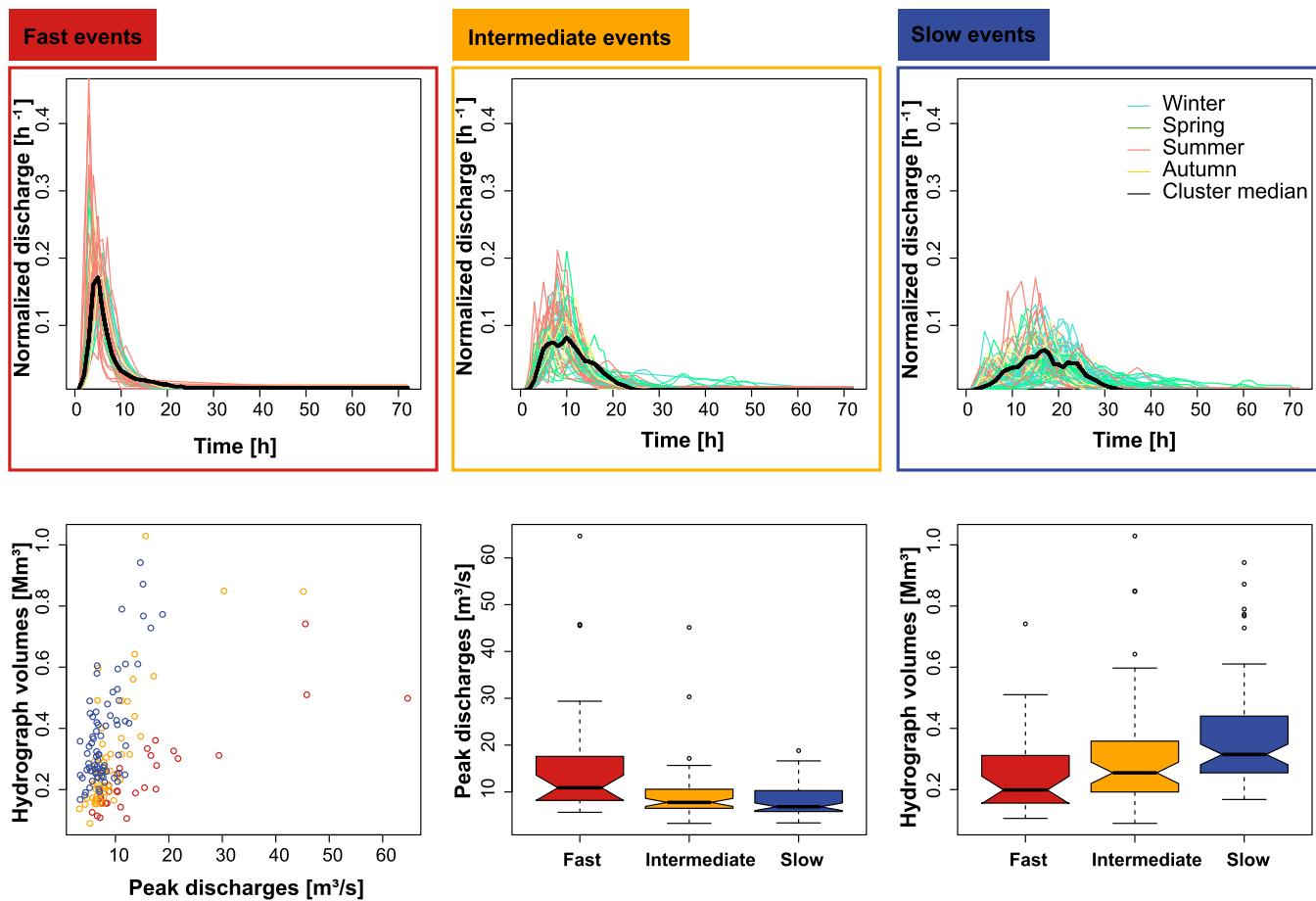
validation of the results is neither possible for the event type clusters nor the reactivity regions since a cluster analysis is exploratory (James et al., 2013) and the “true” cluster memberships are not known. The validity of clusters was therefore indirectly assessed by their suitability to form meaningful clusters in relation to hydrometeorological conditions. Event types might be distinct in their triggering precipitation or antecedent wetness conditions. We looked at the precipitation events triggering the individual flood events and at their antecedent wetness conditions to investigate the link between event type, region, and event conditions. Event precipitation and antecedent wetness were computed using hourly gridded precipitation data. We used the grid-data product CombiPrecip provided by MeteoSwiss (2013), which was computed using a geostatistical approach combining rain-gauge measurements and radar estimates and is available from the year 2005 at a spatial resolution of 1 km. Continuous precipitation time series from 2005 to 2014 were obtained for each catchment by simply averaging the precipitation of the individual grid-cells lying within the catchment. These time series were used to compute the current precipitation index ( $I_{CP}$ ), which reflects the current catchment wetness. It is defined as a continuous function of precipitation, which accumulates on rainy days and exponentially decays during the periods of no rainfall with a recession coefficient of 0.9 (Smakhtin & Masse, 2000). We then defined the antecedent wetness condition prior to a flood event as the  $I_{CP}$  at the beginning of the event. The time series were also used to compute two characteristics related to the event triggering precipitation: total precipitation amount and maximum hourly precipitation intensity. The precipitation data related to the sampled flood events was defined over a window of a maximum of 84 h starting 12 h before the onset of the flood event and ending with the flood event. The three characteristics, antecedent wetness, total event precipitation, and maximum hourly event precipitation, were used to analyze differences in the triggering mechanisms of flood events belonging to the three different event types occurring in the three regions with similar flood behavior.

## 4. Results

### 4.1. Identification of Representative Hydrograph Shapes

The identification of clusters of representative hydrograph shapes within a catchment was based on a functional representation of the hydrograph shapes with respect to four B-spline basis functions. The coefficients of these B-splines allowed for the separation of fast, intermediate, and slow events within a catchment (see Figure 5 showing the Lanete-Huttwil as an example catchment). The fast events were characterized by rather steep rising and falling limbs of the hydrographs. Intermediate events typically also showed quite steep rising limbs while the falling limbs were less steep than the ones observed for the fast events. Slow events were typically characterized by both elongated rising and falling limbs. The fast events were clearly related to a season of occurrence, namely, summer (red curves in the upper plot of Figure 5) while intermediate and slow events occurred throughout the seasons. The clusters established via the hydrograph shape also had a meaning in terms of event magnitude even though they have been established based on the normalized hydrograph shape information only (see the boxplots of peak discharges and hydrograph volumes in the lower plot of Figure 5). Events in the fast event cluster were generally characterized by high peak discharges but low hydrograph volumes while events in the slow event cluster showed high volumes but low peak discharges. The magnitudes of intermediate events lay in between those of the fast and slow events. This general pattern was visible in most catchments of the data set even though it was not always as clear as in the case of the Lanete-Huttwil catchment (Figure 5).

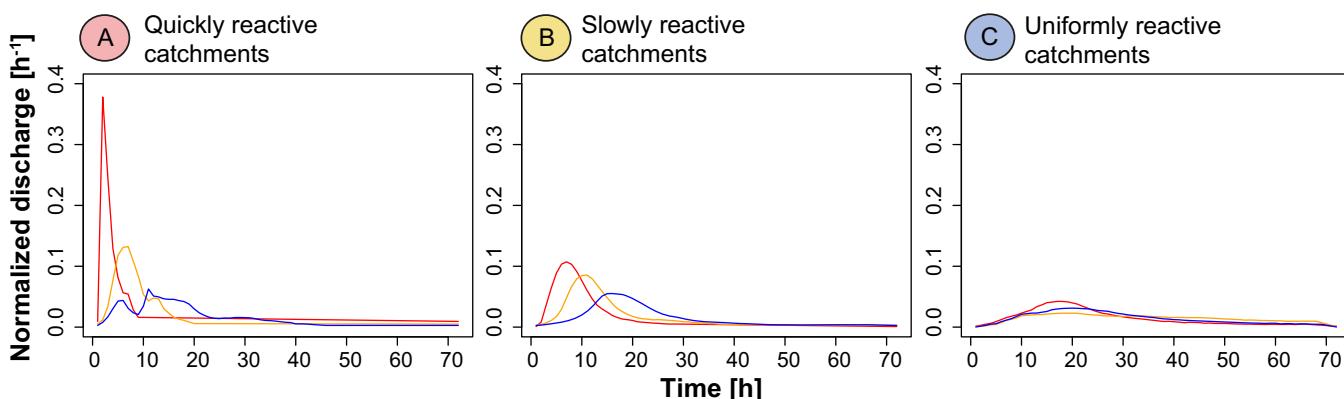
The three median hydrographs of the fast, intermediate, and slow event clusters together built the catchment-specific set of representative hydrograph shapes (see Figure 6 for three examples). While this set consisted of three distinct hydrograph shapes in most catchments, one hydrograph was per definition (see section 3.2) sufficient to describe the runoff reaction in the uniformly reactive catchments (see Figure 6c representing the Birs-Münchenstein). On the contrary, the remaining catchments showed different types of runoff reactions. However, a fast hydrograph in one catchment (e.g., Figure 6b representing Arbogne-Avenches) would rather represent a slow hydrograph in another catchment (e.g., Figure 6a illustrating Alp-Einsiedeln). We concluded that catchments can be generally characterized by different event-reaction times since fast, intermediate, and slow hydrographs looked differently in different catchments.



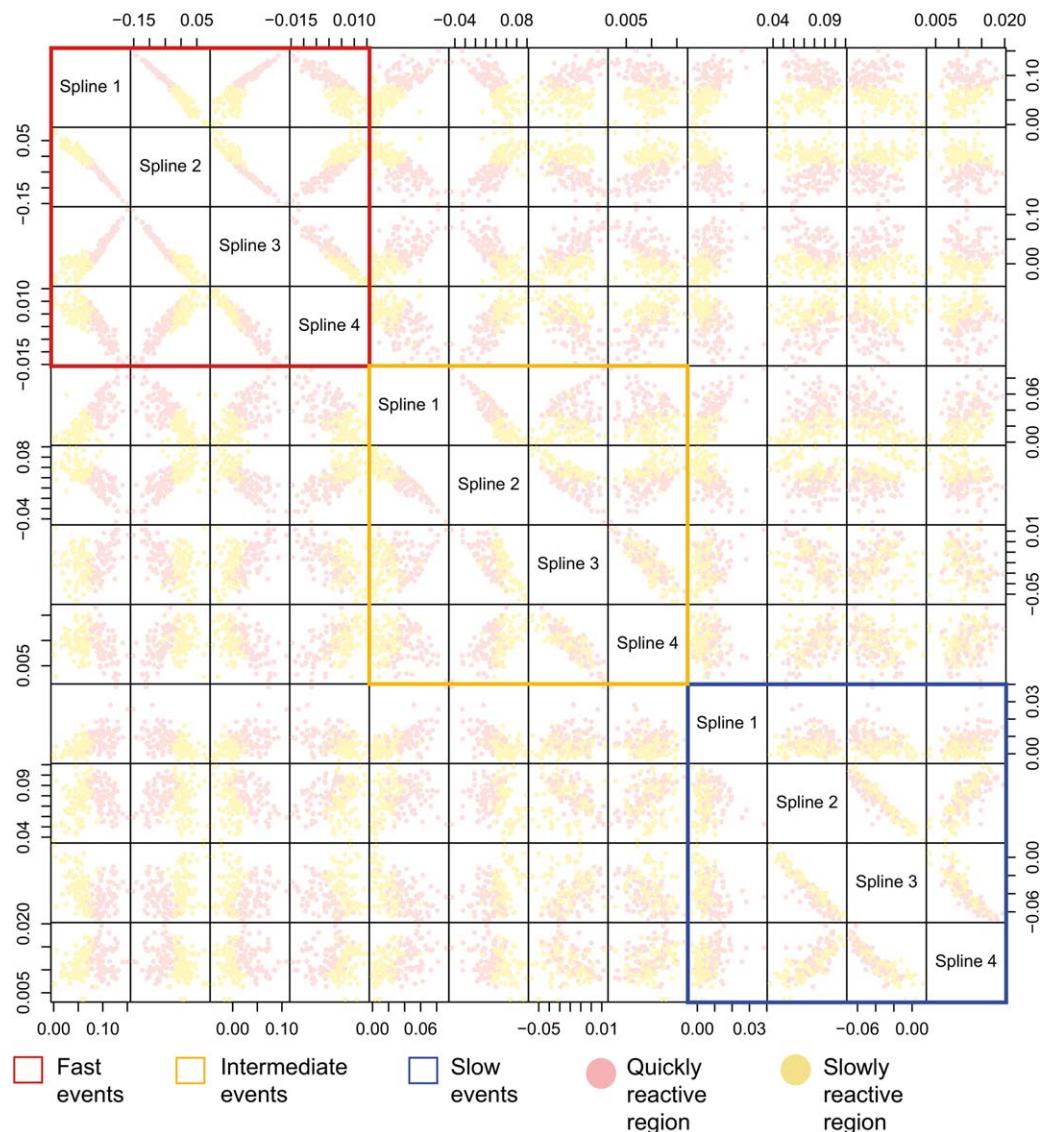
**Figure 5.** Three clusters of hydrograph shapes for the Langete-Huttwil catchment. (top) The events belonging to the fast, intermediate, and slow clusters are displayed according to season using four colors. The median hydrograph shape per cluster is indicated in black. (bottom) The characteristics of the events belonging to the three clusters in terms of the magnitudes of peak discharges and hydrograph volumes are displayed in a scatterplot and as boxplots. The notches of the boxplots indicate the confidence intervals around the medians.

#### 4.2. Establishment of Regions With Similar Flood Behaviors

The catchment-specific sets of representative hydrograph shapes were used to establish regions with similar flood behaviors. The 11 catchments with a set of very similar hydrograph shapes were said to belong to a uniformly reactive cluster (see Figure 7c). The remaining catchments were then used to establish regions



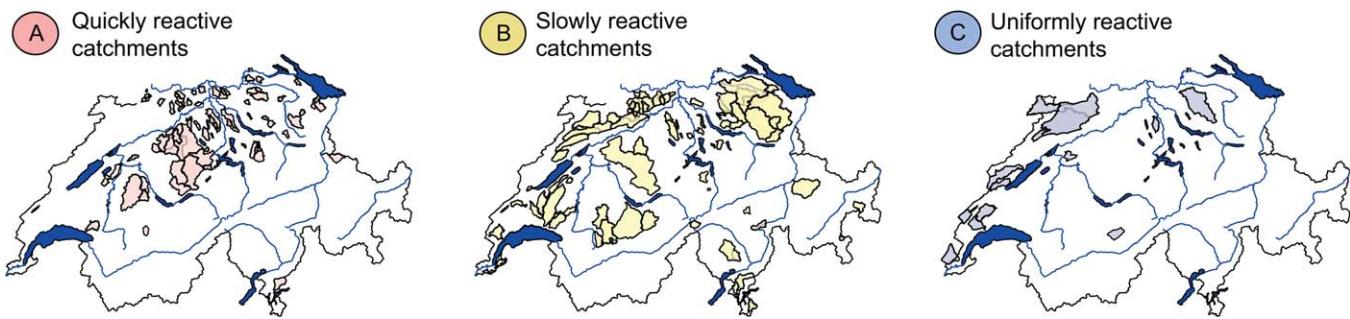
**Figure 6.** Fast, intermediate, and slow median hydrograph shapes for three catchments: (a) Alp-Einsiedeln (quickly reactive region), (b) Arbogne-Avenches (slowly reactive region), and (c) Birs-Münchenstein (uniformly reactive region).



**Figure 7.** Catchments belonging to the (a) quickly reactive, (b) slowly reactive, and (c) uniformly reactive regions.

of catchments with similar flood behaviors. The clustering of the catchment-specific event sets resulted in two clusters with distinct flood event-reaction times: 71 quickly reactive catchments (see Figure 7a) and 81 slowly reactive catchments (see Figure 7b). The quickly reactive catchments were mainly located in the Swiss Plateau, slowly reactive catchments in the Jura mountains, the Alpine region, and the Swiss Plateau, and uniformly reactive catchments mainly in the Jura mountains.

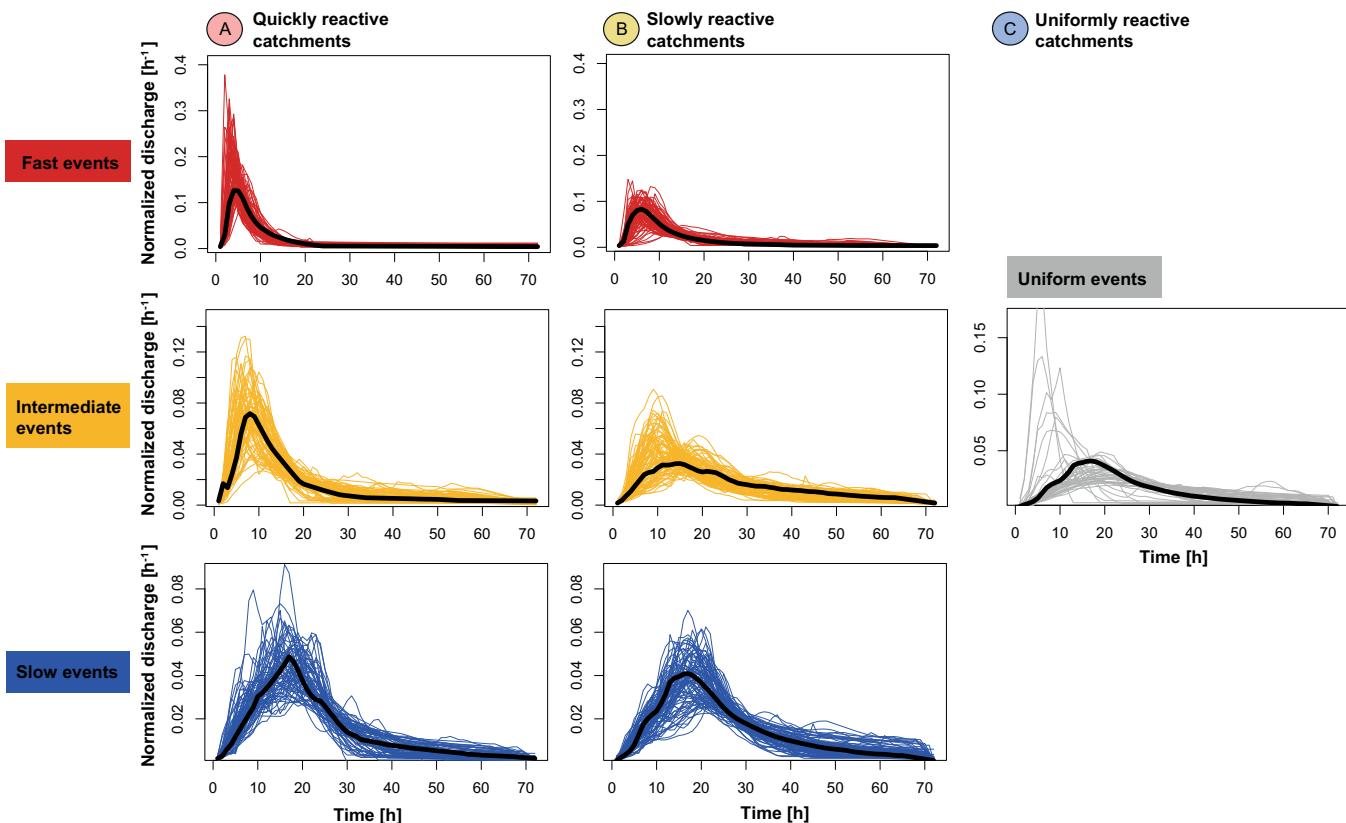
At a regional level, catchment-specific sets of representative hydrographs were distinct between catchments of a generally quick and a generally slow flood runoff reaction. The B-spline coefficients of these catchment-specific sets indicated which hydrograph features were important in distinguishing quickly reactive from slowly reactive catchments. Figure 8 shows the coefficients of the four B-splines for the representative hydrographs of the catchments in the quickly and slowly reactive region for the three event type classes fast, intermediate, and slow. The coefficients for Splines 1–4 (see Figure 4) of the representative fast, intermediate, and slow hydrographs were different for catchments in the quickly reactive and catchments in the slowly reactive region (Figure 8). These differences were the most expressed for the fast events and less expressed for the intermediate and slow events. In summary, quickly reactive catchments were characterized by hydrograph shapes that can be described as a sum of Splines 1 and 3 with a high coefficient and



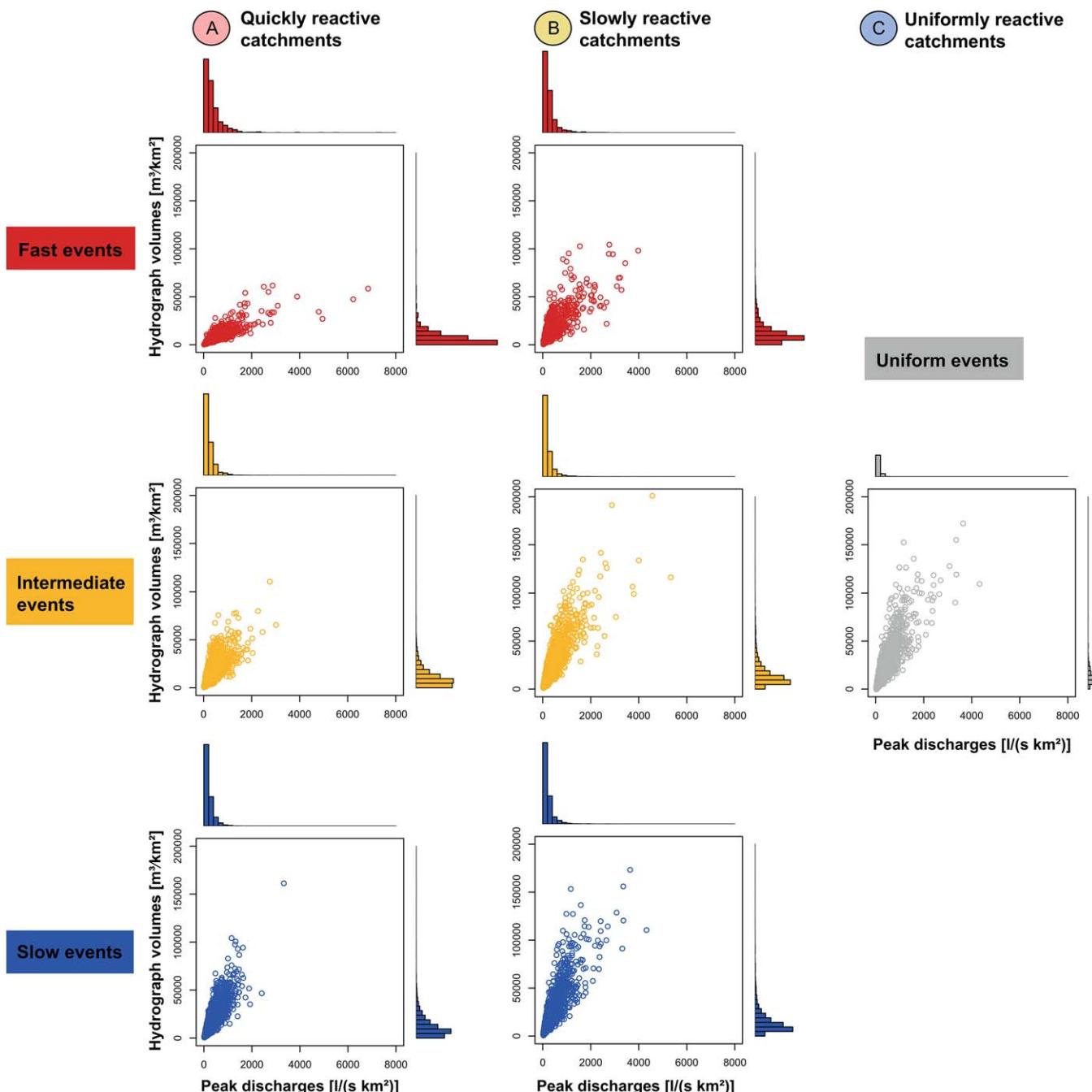
**Figure 8.** Coefficients for Splines 1–4 (see Figure 4) for the two reactivity regions quick and slow. The spline coefficients are given for fast, intermediate, and slow events separately. Scatterplots within the colored boxes (red, orange, and blue) indicate the relationship of the Splines 1–4 within one event type while the scatterplots outside the colored boxes indicate relationships between splines across different event types.

Splines 2 and 4 with a low coefficient. The opposite was the case for hydrographs found in catchments belonging to the slowly reactive region.

The shape clusters and their medians were representative for their region (Figure 9). The catchments in the quickly reactive and the slowly reactive regions (see Figure 7) could be characterized by a set of representative hydrograph shapes while the catchments in the uniformly reactive region were sufficiently described by one hydrograph shape. Catchments in the quickly reactive region were characterized by hydrograph sets with a steeper recession limb than the catchments in the slowly reactive region. Figure 10 shows that the reactivity regions not only differed in terms of their representative hydrograph shapes but also in terms of hydrograph magnitudes. Events occurring in the quickly reactive catchments were characterized by



**Figure 9.** Representative hydrograph shapes of all catchments in a reactivity region ((a) quickly reactive, (b) slowly reactive, and (c) uniformly reactive). The median hydrograph shapes per region and event class (fast (red), intermediate (orange), and slow (blue)) are indicated in black. The scales of the y axis are the same per event type but not across event types.

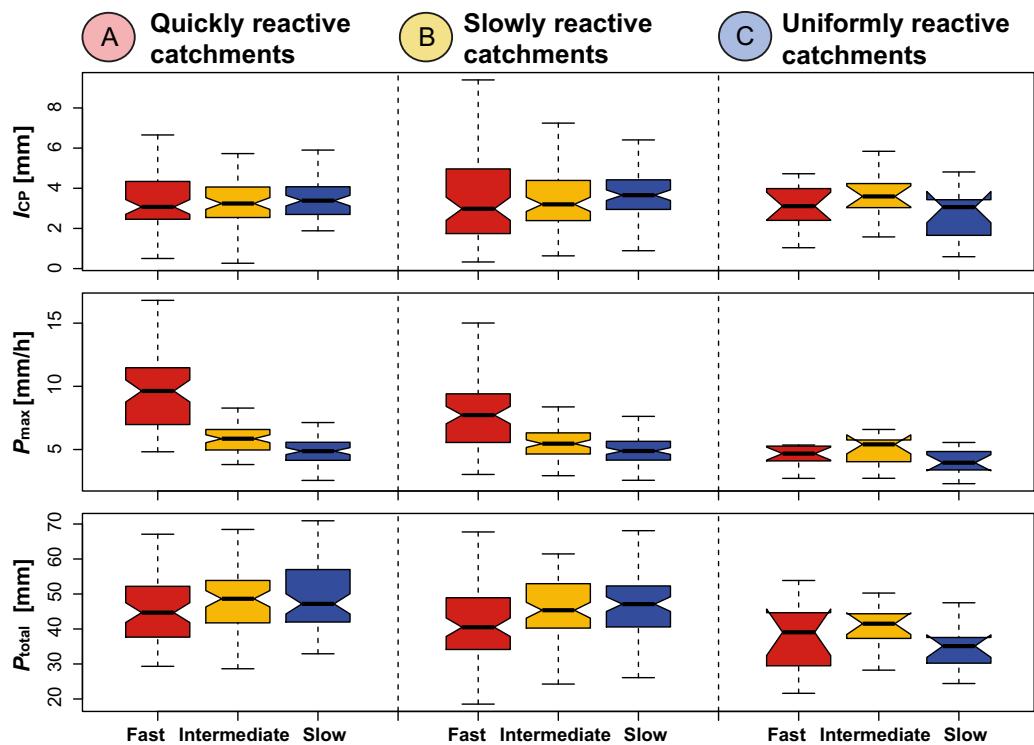


**Figure 10.** Specific peak discharges ( $\text{l}/(\text{s km}^2)$ ) and hydrograph volumes ( $\text{m}^3/\text{km}^2$ ) for the event types in the three reactivity regions ((a) quickly reactive, (b) slowly reactive, and (c) uniformly reactive). The two variables are jointly displayed as a scatter plot and separately in a histogram. The scales of the histograms and scatter plots are the same across all event types and reactivity regions.

rather high peak discharges compared to flood volumes while events occurring in the slowly reactive catchments showed rather high volumes compared to peak discharges.

#### 4.2.1. Event Conditions

The events occurring in the three regions and belonging to the three event types fast, intermediate, and slow not only showed different runoff characteristics (magnitudes and shapes) but differed in their pre-event conditions (see section 3.3) in the form of their triggering precipitation events and much less expressed in antecedent wetness (Figure 11). The flood events occurring in the three regions clearly differed in terms of their triggering precipitation. Flood events occurring in the quickly reactive region were



**Figure 11.** Event conditions for the three event types: fast, intermediate, and slow in the three regions (a) quickly reactive, (b) slowly reactive, and (c) uniformly reactive: antecedent wetness ( $I_{CP}$ ), maximum hourly precipitation intensity ( $P_{max}$ ), and total event precipitation ( $P_{total}$ ). The notches of the boxplots indicate the confidence intervals around the medians.

characterized by higher maximum hourly precipitation intensities than catchments in the slowly and uniformly reactive regions. Within the regions, fast events showed clearly higher hourly precipitation intensities than intermediate and slow events. The triggering precipitation events for the floods occurring in the three regions also differed in terms of their total precipitation amount. Precipitation events triggering floods in the quickly reactive region had generally higher precipitation amounts than precipitation events triggering floods in the slowly and uniformly reactive regions. Within one region, fast events showed slightly lower triggering precipitation amounts than intermediate and slow events. Note that the differences of maximum hourly precipitation intensities between event types within the uniformly reactive region were much less distinct than the ones in the quickly and slowly reactive regions. This indicates that a distinction between the three event types does indeed not make sense in the uniformly reactive region, also from a meteorological point of view. The antecedent wetness conditions differed only slightly between regions and event types. The differences in triggering precipitation amounts and intensities highlight that a subdivision of floods into three event types is meaningful and that the three regions have a hydrometeorological meaning.

## 5. Discussion

### 5.1. Identification of Representative Hydrograph Shapes

The approach proposed for the clustering of hydrograph shapes using FD is simple to apply and avoids the reduction of flood hydrographs to a few hydrograph characteristics, as done when applying standard multivariate clustering techniques. The results presented above showed that the hydrograph clustering approach based on functional data allows for the identification of three representative hydrograph shape classes within individual catchments. These classes are not only distinct in their event shapes but also their event magnitudes and triggering mechanisms. The first class identified consists of fast events with steep rising and falling limbs caused by precipitation events with a high intensity but rather low total amounts. These events mainly occur in summer, which in previously developed classification schemes correspond to flash

floods (Diezg & Weingartner, 2007; Merz & Blöschl, 2003; Sikorska et al., 2015). They are characterized by rather high peak discharges and low hydrograph volumes. The second event type class was composed of the intermediate events that were characterized by rather steep rising but elongated falling limbs and jointly high peak discharges and hydrograph volumes. Such events occur throughout the year and are triggered by precipitation events with medium intensity and amount. The third event class consisted of slow events with both elongated rising and falling limbs. These events typically showed high volumes but low peak discharges which is related to the precipitation events triggering them which are of low intensity but high total amount. The link between the triggering mechanism of the flood events and their shape and magnitude indicates that a hydrograph clustering approach based on functional data, as proposed in this study, is able to build event classes that are hydrologically and meteorologically meaningful. Eventually, the three event types could even be linked to hydrometeorological patterns as suggested by Nied et al. (2014, 2017).

### 5.2. Establishment of Regions With Similar Flood Behaviors

Our results point out that catchment-specific sets of representative hydrograph shapes are useful in establishing regions of catchments with similar flood behaviors. Catchments with a set of three very similar hydrograph shapes, which could not be easily separated into fast, intermediate, and slow events, built the uniformly reactive region. The events occurring in the catchments belonging to this region were generally characterized by rather elongated rising and falling limbs, high volumes, and low peak discharges. The catchments belonging to this region lie mainly in the Jura but also in the Alps (see Figure 2). They are characterized by karstic geology with permeable rock and a low network density. These catchment properties lead to a rather attenuated runoff reaction that leads to similar flood events independent of the triggering mechanism and antecedent wetness. Catchments with a generally fast runoff response, independent of the event type, built the quickly reactive region. Events occurring in this region were characterized by rather high peak discharges but rather low hydrograph volumes for all three event types. The catchments belonging to the quickly reactive region mainly lie in the Swiss Plateau. They were characterized by rather impermeable rocks (based on geological map of Switzerland (Bundesamt für Statistik, 2003)) and a high network density (based on the river network of Switzerland (Swisstopo, 2017)). These characteristics contributed to a fast runoff reaction. Finally, catchments with a generally slow or delayed runoff response formed the slowly reactive region. The events occurring in these catchments generally showed higher volumes but lower peak discharges compared to events occurring in the catchments of the quickly reactive region. Compared to the catchments in the quickly reactive region, they have more permeable rocks and lower network densities, which lead to an attenuation of the runoff events.

The distinct catchment characteristics of quickly, slowly, and uniformly reactive catchments allow for the establishment of a classification rule using tree-based algorithms such as random forest, bagging, or boosting (James et al., 2013), which can be used to assign ungauged catchments to one of the reactivity classes. As a consequence, the reactivity regions could provide a useful basis for the regional estimation of hydrological model parameters, regional frequency analyses, the construction of design hydrographs in ungauged catchments, and other applications.

### 5.3. Perspectives

The clustering approach based on functional data proposed in this study can be adjusted to new regions or contexts. Here we discuss four potential fields of future application: (1) application in other geographical regions and on other temporal scales, (2) use of reactivity regions in design hydrograph construction, (3) application in a climate impact context, and (4) application in a fuzzy clustering setting.

1. **Application in other geographical regions and on other temporal scales:** The functional approach to cluster flood hydrograph shapes for the identification of catchment-specific sets of hydrograph shapes and the subsequent clustering of these sets to identify reactivity regions has been developed and tested based on a set of 163 Swiss catchments with hourly flow series. The applicability of the approach is not limited to this geographical region but can be applied in other regions with similar data availability and catchments with similar physiographical and hydrological characteristics. However, its applicability might be limited to humid catchments since dry catchments were not included in the data set. The applicability of the approach is neither limited to the use of hourly flow series but could be used on daily series if the catchments and their reaction times are larger than in Swiss

catchments. The number of B-splines used for the transformation of hydrograph shapes into FD might have to be increased in the case of more irregular flood hydrograph shapes than those observed in medium-size Swiss catchments. In addition, the number of shape clusters necessary to represent the hydrograph shape variability within a catchments might need to be adjusted. And finally, the number of reactivity regions might need to be adapted according to the variability of runoff and flood regimes in the region of interest.

2. **Use of reactivity regions in design hydrograph construction:** One potential field of application of the reactivity regions is design hydrograph construction in ungauged catchments. For this, the design hydrograph construction procedure proposed by Brunner et al. (2017) for gauged catchments can be adapted and combined with an index flood approach (Dalrymple, 1960; Hosking & Wallis, 1997). The design hydrograph construction approach is based on a bivariate flood frequency analysis, which considers the dependence between peak discharges and hydrograph volumes, and models the hydrograph shape via a probability density function (Yue et al., 2002). Its combination with an index flood approach, where bivariate quantiles are estimated using pooled event data from several catchments, allows for an extension to ungauged catchments. The data pools can be formed by peak discharges and hydrograph volumes corresponding to the events belonging to the three event type classes within a reactivity region. The design variable estimates for peak discharges and hydrograph volumes of each of these classes can then be used together with the representative hydrograph shape of the corresponding event class to construct synthetic design hydrographs for a certain return period. An ungauged catchment can be attributed to one of the reactivity classes via a classification rule. The construction of synthetic design hydrographs in ungauged catchments can subsequently be based on the data of the reactivity class it is assigned to.
3. **Application in a climate impact context:** Under climate change, new flood types might develop and existing types might change (Köplin et al., 2014; Turkington et al., 2016) since different atmospheric circulation patterns favor different flood types (Nied et al., 2017). The hydrograph clustering approach proposed here might be useful in a climate impact context for the detection of new flood types and change in the proportion of occurrence of different flood types. This is important for future flood risk management since the flood type is closely linked to the spatial flood extent, temporal flood progression, and flood magnitudes (Nied et al., 2014).
4. **Application in a fuzzy clustering setting:** Rao and Srinivas (2006) pointed out that most catchments only partly resemble other catchments in the region they have been assigned to. This is also the case when using the clustering approach proposed in this study and the assignment of a catchment to one group or another is not necessarily straightforward. Similarly, Sikorska et al. (2015) showed that flood events usually show partial memberships to more than one flood type. The clustering approach based on functional data proposed here could be transferred into a fuzzy setting (Rao & Srinivas, 2006; Sikorska et al., 2015; Tokushige et al., 2007) that would allow for catchments and hydrograph shapes with partial or distributed memberships to more than one region or event type.

## 6. Conclusions

Flood hydrograph shapes contain a wealth of information that can be used for the identification of regions with a similar flood behavior but is insufficiently exploited by traditional multivariate clustering procedures. FD analysis makes much better use of the valuable information stored in a hydrograph and was found to be a useful tool for both clustering hydrograph shapes and the identification of flood reactivity regions. The clustering of hydrograph shapes within a catchment showed that three hydrograph shapes are sufficient to describe the hydrograph shape variability within a catchment. The sets of representative hydrograph shapes within a catchment were successfully used to establish regions with a similar flood behavior not only in terms of hydrograph shape but also hydrograph magnitude regarding peak discharges and hydrograph volumes. The clustering approach for hydrograph shapes and its use for the identification of flood reactivity regions has many potential fields of application. On the one hand, the reactivity regions could be used in regionalization studies, regional flood frequency analyses, and for design hydrograph construction in ungauged basins. On the other hand, the clustering approach could be applied to other geographical and climatological regions, transferred to a fuzzy clustering setting allowing for several partial group memberships, or be used in a climate impact context. The clustering approach for hydrograph shapes is a flexible

and promising approach and has the potential to exploit the process information stored in flood hydrograph shapes.

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