Limit sketches and presentability

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Categories

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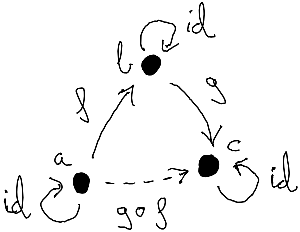

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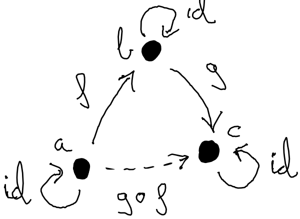
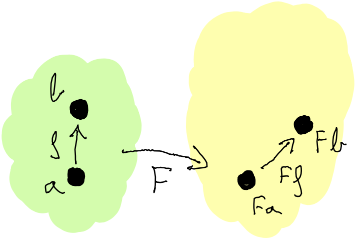


Functors and natural transformations



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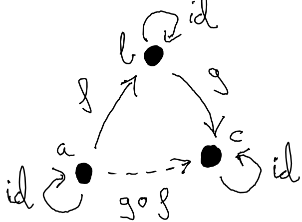

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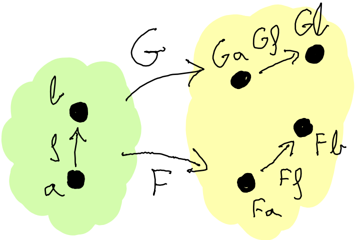

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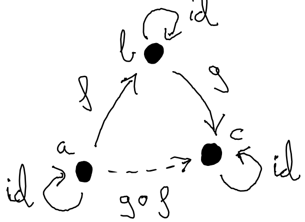

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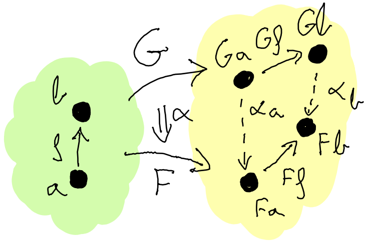

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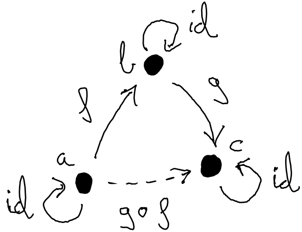

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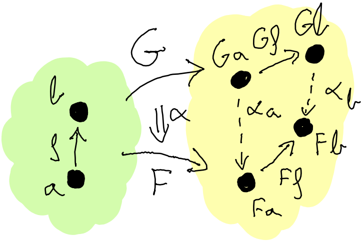

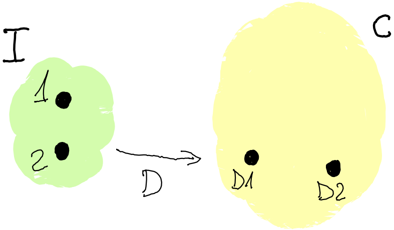

Cones and limits

- Diagram $D : I \to C$

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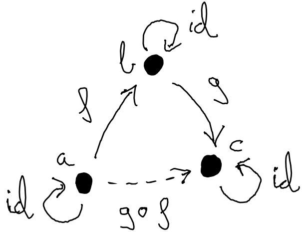

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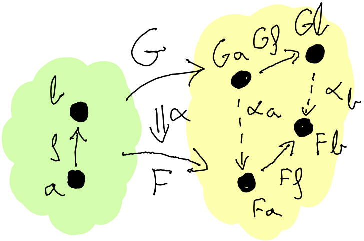

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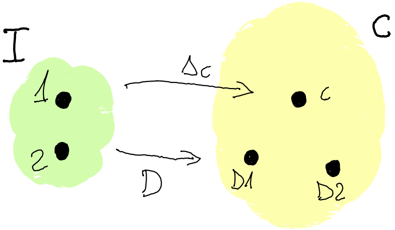

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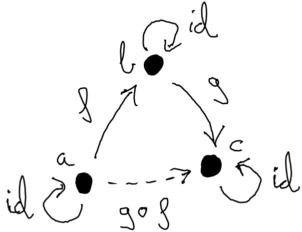

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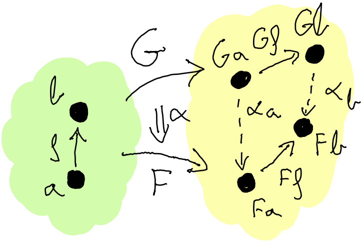

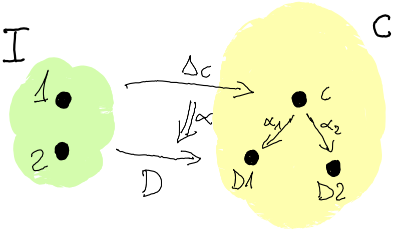

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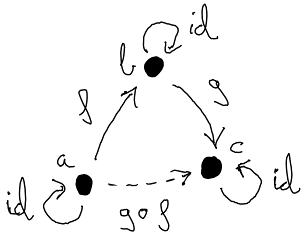

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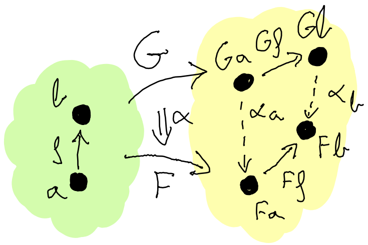

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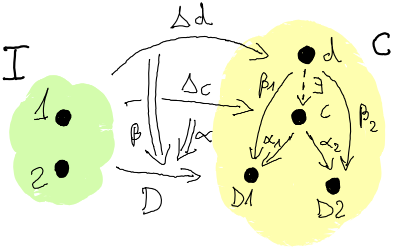

- Diagram $D : I \to C$
- Cone $\alpha : \Delta c \Rightarrow D$
- α is a limit if for all $d \in C$

  $\mathrm{Hom}(d, c) \cong \mathrm{Cones}(D, d)$

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Examples. Set, Grp, sSet, . . . **Non example.** Top

Limit sketches

A **limit sketch** (Bastiani and Ehresmann 1972) is a pair (A, C) of a small category A and a set of cones C over A.

Example. Let A be the small category generated by the square (a).

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$$A \xrightarrow{\quad \mathbf{F} \quad} \mathbf{Set}$$

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A model $\mathbf{F}$ of the sketch $(A, \{c\})$ is a pullback of sets

Representation theorem

Theorem (Adamek and Rosicky 1994)

The following are equivalent:

(i) *Presentable categories.*

(ii) *Limit-sketchable categories.*

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Goal

$$\text{Presentable } \infty\text{-categories} \overset{?}{\simeq} \text{Limit-sketchable } \infty\text{-categories}$$

Plan

Presentable ∞-categories

Limit ∞-sketches

Representation theorem

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Informal higher categories

A **higher category** has objects and:

- ☑ n-morphisms between $(n-1)$-morphisms for all $n \geq 1$,
- ☑ Composition, identities and associativity of n-morphisms weakly up to a $(n+1)$-morphism for all $n \geq 1$.

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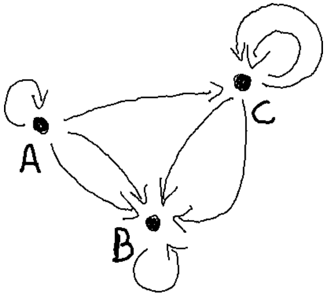

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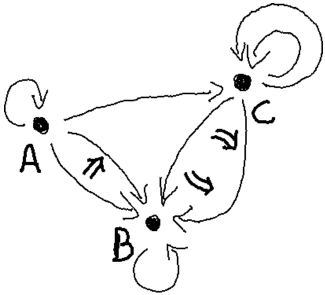

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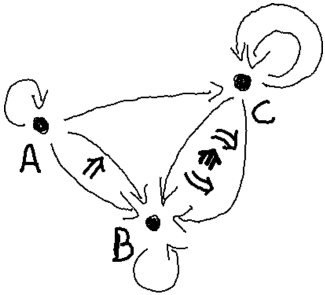

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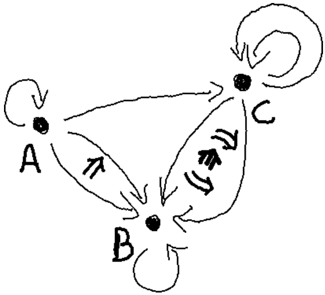

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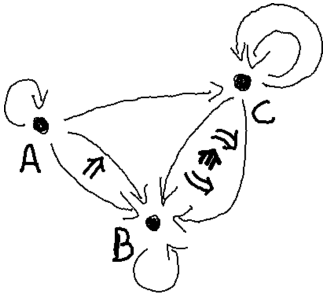

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- ∞-**category** $:= (\infty, 1)$-category
- ∞-**groupoid** $:= (\infty, 0)$-category

Limits and colimits

Let $\mathcal{C}$ be an ∞-category, and I be a small ∞-category. Given any object $x \in \mathrm{Obj}(\mathcal{C})$, the **constant diagram** $\Delta x : I \to \mathcal{C}$ sends all objects of I to x, and all higher morphisms to higher identities.

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Cocones and **colimit cocones** are defined as cones and limit cones in the opposite ∞-category.

Accessibility

Let κ denote a regular cardinal and $\mathcal{C}$ an ∞-category.

- An ∞-category $\mathcal{K}$ is κ-**filtered** if, for every κ-small ∞-category I, every diagram $D : I \to \mathcal{K}$ admits a cocone $\alpha : D \Rightarrow \Delta x$.

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An ∞-category $\mathcal{C}$ is **accessible** if it is locally small and there is a regular cardinal κ such that:

- ☑ $\mathcal{C}$ admits κ-filtered colimits.
- ☑ There is some essentially small sub-∞-category of κ-compact objects which generates $\mathcal{C}$ under κ-filtered colimits.

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Definition

An ∞-category is **presentable** if it is accessible and cocomplete.

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Example

(a) The ∞-category of homotopy types $\mathcal{S}$ is presentable.

(b) Any ∞-topos is presentable.

(c) The nerve of any presentable 1-category is presentable.

(d) If $\mathcal{A}$ is a small ∞-category and $\mathcal{C}$ is a presentable ∞-category, then $\mathrm{Fun}(\mathcal{A}, \mathcal{C})$ is presentable.

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Let $\mathcal{C}$ be a complete ∞-category. A functor $F : \mathcal{K} \to \mathcal{C}$ is a **model** of a limit ∞-sketch $\mathcal{T} = (\mathcal{K}, \mathfrak{L})$ in $\mathcal{C}$ if it sends each cone in $\mathfrak{L}$ to a limit cone in $\mathcal{C}$.

$$\mathrm{Mod}(\mathcal{T}, \mathcal{C}) \coloneqq \infty\text{-category of models of } \mathcal{T} \text{ in } \mathcal{C}$$
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We say that an ∞-category is **limit ∞-sketchable** (or **essentially ∞-algebraic**) if it is equivalent to the ∞-category of models of some limit ∞-sketch.

Examples: ∞-algebraic theories

An ∞-**algebraic theory** (or ∞-**Lawvere theory**) is a small ∞-category with finite products. A **model** (or **algebra**) for an ∞-algebraic theory $\mathcal{A}$ is a functor $\mathcal{A} \to \mathcal{S}$ that preserves products.

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Theorem (Rosicky 2007 and Lurie 2009)

The ∞-category of models of an ∞-algebraic theory is presentable.

Examples: Internal precategories

Let $\mathcal{C}$ be a complete ∞-category, $\mathcal{A}$ be the nerve of Δ^{op}, and c_n be the cone with apex and diagram for all $n \in \mathbb{N}$:

$$
\begin{tikzcd}
& & & & {[n]} \arrow[dlll, dashed, bend right] \arrow[dl, dashed] \arrow[drrr, dashed, bend left] \arrow[ddllll, dashed, bend right] \arrow[ddll, dashed, bend right] \arrow[dd, dashed] \arrow[ddrr, dashed, bend left] \arrow[ddrrrr, dashed, bend left] & & & & \\
& {[0]} \arrow[dl] \arrow[dr] & & {[0]} \arrow[dl] \arrow[dr] & & & & {[0]} \arrow[dl] \arrow[dr] & \\
{[1]} & & {[1]} & & {[1]} & \overset{(n)}{\cdots} & {[1]} & & {[1]}
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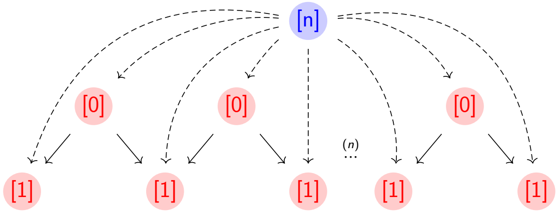
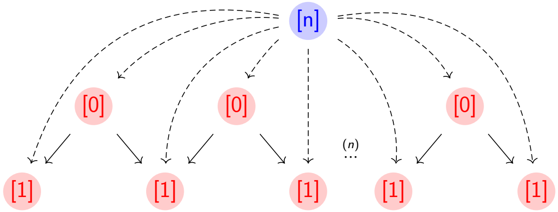


Then $\mathcal{T} = (\mathcal{A}, \{c_n \mid n \in \mathbb{N}\})$ is a limit ∞-sketch, and a model $F : \mathcal{A} \to \mathcal{C}$ is a simplicial object in $\mathcal{C}$ such that

$$F_n \xrightarrow{\sim} F_1 \times_{F_0} F_1 \times_{F_0} \cdots \times_{F_0} F_1. \qquad \textit{(Segal condition)}$$

Examples: Internal precategories

Let $\mathcal{C}$ be a complete ∞-category, $\mathcal{A}$ be the nerve of Δ^{op}, and c_n be the cone with apex and diagram for all $n \in \mathbb{N}$:



Then $\mathcal{T} = (\mathcal{A}, \{c_n \mid n \in \mathbb{N}\})$ is a limit ∞-sketch, and a model $F : \mathcal{A} \to \mathcal{C}$ is a simplicial object in $\mathcal{C}$ such that

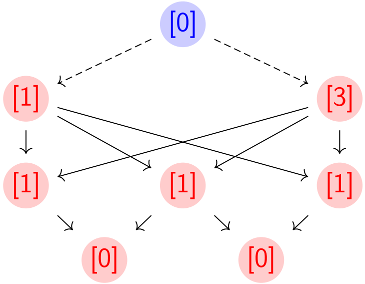
$$F_n \xrightarrow{\sim} F_1 \times_{F_0} F_1 \times_{F_0} \cdots \times_{F_0} F_1. \qquad \textit{(Segal condition)}$$

$$\mathrm{Mod}(\mathcal{T}, \mathcal{C}) \simeq \textbf{Internal precategories}$$

$$\mathrm{Mod}(\mathcal{T}) \simeq \textbf{Segal spaces}$$

Examples: Internal univalent categories

Let $\mathcal{A}$ be as before, $\mathfrak{L}_S$ be the set of cones of the previous sketch, and d_n be the cone with apex and diagram for all $n \in \mathbb{N}$:



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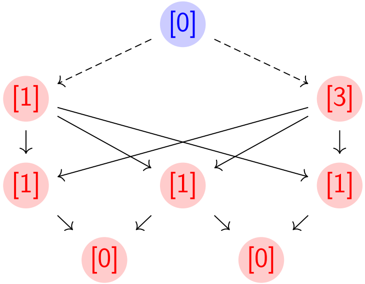

Then $\mathcal{T} = (\mathcal{A}, \mathfrak{L}_1 \cup \{d_n \mid n \in \mathbb{N}\})$ is a limit ∞-sketch, and a model $F : \mathcal{A} \to \mathcal{C}$ is an internal precategory in $\mathcal{C}$ such that

$$\begin{tikzcd}
F_0 \arrow[r] \arrow[d] \arrow[dr, phantom, very near start, "\lrcorner"] & F_3 \arrow[d] \\
F_1 \arrow[r] & F_1 \times_{F_0}^{d_1,d_1} F_1 \times_{F_0}^{d_0,d_0} F_1
\end{tikzcd}$$

Examples: Internal univalent categories

Let $\mathcal{A}$ be as before, $\mathfrak{L}_S$ be the set of cones of the previous sketch, and d_n be the cone with apex and diagram for all $n \in \mathbb{N}$:

$$\begin{tikzcd}
 & & {[0]} \arrow[dll, dashed] \arrow[drr, dashed] & & \\
{[1]} \arrow[d] \arrow[drr] \arrow[drrrr] & & & & {[3]} \arrow[d] \arrow[dllll] \arrow[dll] \\
{[1]} \arrow[dr] & & {[1]} \arrow[dl] \arrow[dr] & & {[1]} \arrow[dl] \\
 & {[0]} & & {[0]} &
\end{tikzcd}$$

Then $\mathcal{T} = (\mathcal{A}, \mathfrak{L}_1 \cup \{d_n \mid n \in \mathbb{N}\})$ is a limit ∞-sketch, and a model $F : \mathcal{A} \to \mathcal{C}$ is an internal precategory in $\mathcal{C}$ such that

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\end{tikzcd}$$

$$\mathrm{Mod}(\mathcal{T}, \mathcal{C}) \simeq \textbf{Internal univalent categories}$$

$$\mathrm{Mod}(\mathcal{T}) \simeq \textbf{Complete Segal spaces}$$

Plan

Presentable ∞-categories

Limit ∞-sketches

Representation theorem

Representation theorem

Theorem (M.)

An ∞-category is presentable $\iff$ it is limit ∞-sketchable.

Corollary

The ∞-category of models of a limit ∞-sketch in a presentable ∞-category is presentable.

Future work

- **Generalization:** A ∞-category is accessible if, and only if, it is equivalent to the ∞-category of models of an ∞-sketch.

  A **sketch** is a limit sketch with a set of cocones which are sent to colimit cocones by any model.

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  $\implies$ **Model-independent** version of this presentation!

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- Accessibility, presentability, sketches, and representation theorem for ∞-**cosmoi** (Riehl and Verity 2022)

  $\implies$ **Model-independent** version of this presentation!

- Formalize this work with a proof assistant which supports synthetic ∞-categories like `rzk`.

References

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Thank you for listening!

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LIMIT SKETCHES AND PRESENTABILITY

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