Докажете равенството: $\underbrace{\arcsin\frac{1}{2}\sqrt{2-\sqrt{2-2x}}}_{\beta}=\frac{\pi}{8}+\frac{1}{4}\underbrace{\arcsin x}_{\alpha},\ |x|\leq 1$

$\alpha,\beta\in[-\frac{\pi}{2},\frac{\pi}{2}]$

$$\begin{aligned}
\sin\beta &= \frac{1}{2}\sqrt{2-\sqrt{2-2x}}\\
&= \frac{1}{2}\sqrt{2-\sqrt{2-2\underbrace{\sin\alpha}_{=2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}}}\\
&= \frac{1}{2}\sqrt{2-\sqrt{2}\sqrt{\underbrace{1}_{=\sin^2\frac{\alpha}{2}+\cos^2\frac{\alpha}{2}}-2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}}\\
&= \frac{\sqrt{2}}{2}\sqrt{1-\frac{1}{\sqrt{2}}\sqrt{\left(\cos\frac{\alpha}{2}-\sin\frac{\alpha}{2}\right)^2}}\\
&= \frac{\sqrt{2}}{2}\sqrt{1+\underbrace{\frac{1}{\sqrt{2}}}_{=\cos\frac{\pi}{4}}\sin\frac{\alpha}{2}-\underbrace{\frac{1}{\sqrt{2}}}_{=\sin\frac{\pi}{4}}\cos\frac{\alpha}{2}}\\
&= \frac{\sqrt{2}}{2}\sqrt{1+\underbrace{\sin\frac{\alpha}{2}\cos\frac{\pi}{4}-\cos\frac{\alpha}{2}\sin\frac{\pi}{4}}_{=\sin\left(\frac{\alpha}{2}-\frac{\pi}{4}\right)}=}
\end{aligned}$$

Тук използвахме, че$(\sin\alpha\cos\beta+\cos\alpha\sin\beta=\sin(\alpha+\beta))$

$$=\frac{\sqrt{2}}{2}\sqrt{\underbrace{1}_{=\sin^2\left(\frac{\alpha}{4}-\frac{\pi}{8}\right)+\cos^2\left(\frac{\alpha}{4}-\frac{\pi}{8}\right)}+\underbrace{\sin\left(\frac{\alpha}{2}-\frac{\pi}{4}\right)}_{=2\sin\left(\frac{\alpha}{4}-\frac{\pi}{8}\right)\cos\left(\frac{\alpha}{4}-\frac{\pi}{8}\right)}}$$

$$=\frac{\sqrt{2}}{2}\sqrt{\left(\sin\left(\frac{\alpha}{4}-\frac{\pi}{8}\right)+\cos\left(\frac{\alpha}{4}-\frac{\pi}{8}\right)\right)^2}$$

$$=\frac{\sqrt{2}}{2}\sin\left(\frac{\alpha}{4}-\frac{\pi}{8}\right)+\frac{\sqrt{2}}{2}\cos\left(\frac{\alpha}{4}-\frac{\pi}{8}\right)$$

$$=\sin\left(\frac{\alpha}{4}-\frac{\pi}{8}+\frac{\pi}{4}\right)=\sin\left(\frac{\alpha}{4}+\frac{\pi}{8}\right)$$