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- · Class Holl no.: 26
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Tutorial -4

Q. For each of the following recurrences, give an expression for the runtime T(n) if the recurrence can be solved with the Master Theorem. Otherwise, indicate that the Master Theorem does not apply:

1) $T(n) = 3T(n/2) + n^2$

According to Master's Theorem,

If T(n) = aT(n/b) + f(n), $a \ge 1$, b > 1and $c = log_b a$, then if:

• $f(n) < n^c \Rightarrow T(n) = \Theta(n^c)$

• $f(n) = n^c \Rightarrow T(n) = O(n^c log n)$

• $f(n) > n^c \Rightarrow T(n) = \Theta(f(n)) - T_o - (n)T$

Here, a=3, b=2 $c = log_2 3 = 1.58$ $g(n) = n^2$, $n^6 = n^{4.58}$

": f(n) > n" $\Rightarrow [T(n) = O(n^2)]$

2) $T(n) = 4T(n/2) + n^2$

 \rightarrow Mere, a=4, b=2 $c=log_24=2$

 $f(n) = n^2, \quad n^c = n^2$

°°° $f(n) = n^c$ ⇒ $T(n) = \theta(n^2 \log n)$

3) $T(n) = T(n/2) + 2^n$

 \rightarrow Here, $b(n) = 2^n$ which is not a polynomial.

10 = (4) + 4-24 ~

1 h < 0 = 1 (m) = 0 (m h

then It

 \Rightarrow T(n) = O(n)

... Master's Theorem Joes not apply.

4)
$$T(n) = 2^n T(n/2) + n^n$$

-> Hore, a=2" which is not a constant.

... Master's Theorem does not apply.

5)
$$T(n) = 16T(n/4) + n$$

→ Mere,
$$a = 16$$
, $b = 4$
 $c = \log_4 16 = 2$

$$f(n) = n , n^c = n^2$$

"
$$f(n) < n^c \Rightarrow T(n) = O(n^2)$$

6)
$$T(n) = 2T(n/2) + n \log n$$

-> According to extended Master's Theorem,

If
$$T(n) = aT(n/b) + O(n^k log^p n)$$
; $a \ge 1, b > 1, k \ge 0$.
then if:

 $p \in R$

•
$$a > b^h \Rightarrow T(n) = \theta(n^{\log_b a})$$

•
$$a = b^h$$
, then if:

$$\sqrt{p} > -1 \Rightarrow T(n) = O(n^{\log_p a} \log_p^{p+1})$$

$$\checkmark p < -1 \Rightarrow T(n) = O(n \log_{b} a)$$

$$\checkmark p \ge 0 \Rightarrow T(n) = \Theta(n^k \log p_n)$$

$$\sqrt{p} < 0 \Rightarrow T(n) = \theta(nk)$$

Here,
$$a = 2$$
, $b = 2$, $h = 1$, $p = 1$

Now,
$$b^{k} = 2^{4} = 2 = a$$

and $b > -1$

$$\Rightarrow T(n) = O(n^{\log_2 2} \log^{1+1} n)$$

$$\Rightarrow \left[T(n) = O\left(n\log^2 n\right)\right]$$

7)
$$T(n) = 2T(n/2) + n/\log n$$

3) Using extended Master's Theorem,
 $T(n) = a.T(n/b) + a(b)$

$$T(n) = aT(n/b) + \theta(n^{k}log^{k}n)$$

Here, $a = 2$, $b = 2$, $k = 1$, $p = -1$

Now,
$$b^{k} = 2^{1} = 2 = a$$

and $b = -1$

$$\Rightarrow T(n) = O(n^{\log_2 2} \log \log n)$$

$$\Rightarrow T(n) = \Theta(n \log \log n)$$

8)
$$T(n) = 2T(n/4) + n^{0.51}$$

$$\rightarrow$$
 Move, $a = 2$, $b = 4$
 $c = log_4 2 = 0.5$

$$f(n) = n^{0.51}, n^{c} = n^{0.5}$$

"
$$f(n) > n^c \Rightarrow \left[T(n) = O(n^{0.51})\right]$$

a)
$$T(n) = 0.5 T(n/2) + 1/n$$

E=1, ==5, 0-0;

し = モロを外する

M = 0, $M_{\rm c} = M_{\rm c}$

10)
$$T(n) = 16T(n/4) + n!$$

$$\rightarrow$$
 Here, $a=16$, $b=4$

$$c=\log_4 16=2$$

$$f(n) = n!$$
, $n^c = n^2$

":
$$f(n) > n^c \Rightarrow T(n) = O(n!)$$

11)
$$T(n) = 4T(n/2) + \log n$$
 $\rightarrow (laing extended Maister's Theorem)$
 $T(n) = aT(n/b) + \Theta(n^k log^k n)$

Here, $a = 4$, $b = 2$, $b = 0$, $b = 1$
 $Now, b^k = 2^0 = 1 < a$
 $\Rightarrow T(n) = \Theta(n^{\log_2 4})$
 $\Rightarrow \overline{T(n)} = \Theta(n^2)$

12) $T(n) = \sqrt{n} T(n/2) + \log n$
 $\rightarrow \text{Here}, a = \sqrt{n} \text{ which is not a constant.}$
 $\therefore \text{ Master's Theorem does not alphy.}$

13) $T(n) = 3T(n/2) + n$
 $\rightarrow \text{Here}, a = 3, b = 2$
 $c = \log_2 3 = 1.58$
 $f(n) = n, n^c = n^{1.58}$
 $\therefore f(n) < n^c \Rightarrow \overline{T(n)} = \Theta(n^{3.58})$

14) $T(n) = 3T(n/3) + \sqrt{n}$
 $\rightarrow \text{Here}, a = 3, b = 3$
 $c = \log_3 3 = 1$
 $f(n) = \sqrt{n}, n^c = n$
 $\therefore f(n) < n^c \Rightarrow \overline{T(n)} = \Theta(n)$

15) $T(n) = 4T(n/2) + kn$
 $\rightarrow \text{Here}, a = 4, b = 2$
 $c = \log_2 4 = 2$
 $f(n) = kn, n^c = n^2$
 $\therefore f(n) < n^c \Rightarrow \overline{T(n)} = \Theta(n^2)$

16)
$$T(n) = 3T(n/4) + n \log n$$
 \Rightarrow Using extended Master is Theorem,

 $T(n) = aT(n/b) + 0 (n^k \log^k n)$

Hove, $a = 3$, $b = 4$, $k = 1$, $p = 1$

Now, $b^k = 4^l = 4 > a$

and $b \ge 0$
 $\Rightarrow T(n) = \theta (n^l \log^2 n)$
 $\Rightarrow T(n) = \theta (n \log n)$

17) $T(n) = 3T(n/3) + n/2$
 \Rightarrow Mere, $a = 3$, $b = 3$
 $c = \log_3 3 = 1$
 $f(n) = n/2 \approx n$, $n^c = n$
 $\therefore f(n) = n^c \Rightarrow T(n/3) + n^2 \log n$

18) $T(n) = 6 T(n/3) + n^2 \log n$
 \Rightarrow Using extended Master's Theorem,

 $T(n) = a T(n/b) + \theta (n^k \log^k n)$

Here, $a = 6$, $b = 3$, $k = 2$, $p = 1$

Now, $b^k = 3^2 = 9 > a$

and $b \ge 0$
 $\Rightarrow T(n) = \theta (n^2 \log^4 n)$
 $\Rightarrow T(n) = \theta (n^2 \log^4 n)$
 $\Rightarrow T(n) = 4T(n/2) + n/\log n$

19) $T(n) = 4T(n/2) + n/\log n$
 \Rightarrow Using extended Master's Theorem,

 $T(n) = aT(n/b) + \theta (n^k \log^k n)$

Here, $a = 4$, $b = 2$, $k = 1$, $p = -1$

Now, $b^k = 2^l = 2 < a$
 $\Rightarrow T(n) = \theta (n^{log_3 4}) \Rightarrow T(n) = \theta (n^2)$

20) $T(n) = 64T(n/8) - n^2 \log n$ \rightarrow Here, $\xi(n) = -n^2 \log n$ which is negative. ... Master's Theorem does not apply. 21) $T(n) = 7T(n/3) + n^2$ Merch - 4 - 4 / West \rightarrow Here, a=7, b=3osq in $c = log_37 = 1.77$ $f(n) = n^2$, $n^c = n^{1.77}$ $:: f(n) > n^c \Rightarrow T(n) = O(n^2)$ 22) T(n) = T(n/2) + n(2-cos n)-> Hore, the Regularity condition is violated. {ab(n/b) < c f(n); c < 1} .. Master's Theorem does not apply. 2 T(n) = 6 T(n/3) + n 269n Washing Endanded Merster's Therewarns T(n) = a T(n/b) + Q(n 1 20g/n) Menco 2=6, b=3, k=2, p=7

Towns and noted Most is Therewish, $T(n) = \alpha T(n/b) + \Theta(n^{k} \log k)$ Here, $\alpha = 6$, b = 3, k = 5, k = 4Notes, $k = 3^{2} = q > \alpha$ and $k \geq 0$ $T(n) = \Theta(n^{k} \log^{k} n)$ $T(n) = O(n^{k} \log^{k} n)$

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