

How Tight are Malthusian Constraints?

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The Historical Relevance of Malthus

“But the logic of the Malthusian model matches the empirical evidence for the preindustrial world. While even long before the Industrial Revolution small elites had an opulent lifestyle, the average person in 1800 was no better off than his or her ancestors of the Paleolithic or Neolithic.” - Clark (2007)

“During the Malthusian Epoch....technologically superior countries eventually developed denser populations, but their standards of living did not reflect the degree of their technological advancements.” - Galor (2011)

Malthusian Pressures?

“To the extent that it left such improvements to be realized in the future, eighteenth-century European farming left more room to continue growth before encountering Malthusian constraints than was present in east Asia.” - Pomeranz (2000)

“Clearly the shortage of many resources grew more severe [in China] ... A major cause of these shortages was of course the continuing growth of the population under conditions of relative technological standstill.” - Elvin (1973)

“In both regions population grew over the period 1680 to 1850: In China from an estimated 150 million to an estimated 400-410 million and in Western Europe from some seventy-five million to some 170 million and in both there were Malthusian pressures.” - Vries (2013)

The Continued Relevance of Malthus

“[Africa’s] countries remain mired in a Malthusian crisis of high mortality, high fertility, and rapid population growth (with an accompanying state of chronic extreme poverty)” - Conley, McCord, and Sachs (2007)

“The Malthusian channel by which a high level of population reduces income per capita is still relevant in poor developing countries that have large rural populations dependent on agriculture, as well as in countries that are heavily reliant on mineral or energy exports.” - Weil and Wilde (2009)

“Our general aim is to build a theory of economy-environment interactions capable of addressing one of the main future challenges ... how to sustain innovation-driven income per capita growth in a habitat that has a finite carrying capacity of people.” - Peretto and Valente (2015)

Malthusian?

What do Malthusian constraints (or pressures, or limits, or crises, or bounds) refer to?

- The use of land/resources in production?
- High population density?
- Low population density?
- High fertility rates?
- Low mortality rates?
- The level of living standards?
- The biological subsistence level of living standards?

Definition and Estimation

Here, the Malthusian constraint is the *elasticity of output with respect to agricultural land*. This answers:

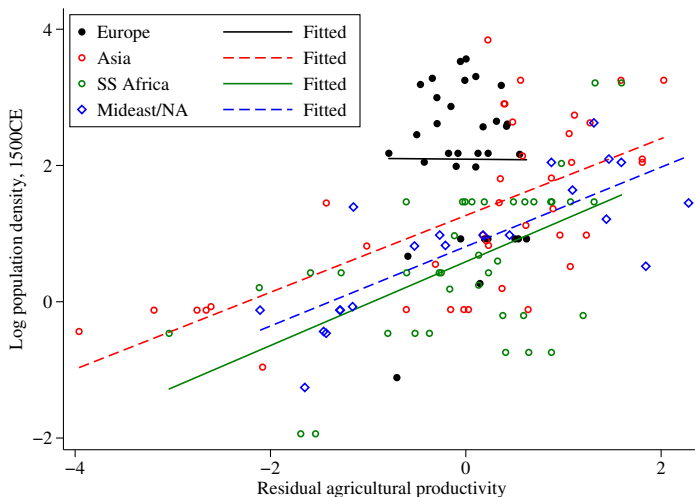
- How important is land to production?
- How does the average product of labor change with respect to labor?
- How sensitive is economy (real wage, agric. labor share) to a *change* in population?

Estimate constraint from the relationship of density and inherent agricultural productivity (TFP)

- If density is sensitive to productivity \Rightarrow *loose* Malthusian constraint
- If density is insensitive to productivity \Rightarrow *tight* Malthusian constraint

Toy Model

Density and Productivity, by Region, 1500CE



Data from Ashraf and Galor (2010). Residual plot using their controls except continent FE.

In this paper

Estimate the Malthusian constraint:

- Use relationship of *rural* density and agro-climatic agricultural TFP
- Estimates come from *within*-province variation
- Population data from HYDE
- Agro-climatic TFP built from Galor and Özak data on caloric suitability

We find:

- Constraints range from 0.090 to 0.308 by region
- Variation is related to agro-climatic conditions
- Temperate, cold, regular rain \Rightarrow tight Malthusian constraints
- Equatorial, hot, seasonal rain \Rightarrow loose Malthusian constraints

Implications

Pattern of results:

- No strict relationship with development
- Rich places today do not (did not?) have looser Malthusian constraints

But..

- Constraint determines how sensitive real wages and agric. labor share are to shocks
- Tight constraint \Rightarrow very sensitive (and v.v.)

So the following differ with constraints:

- Population shocks (epidemics, mortality transition)
- Population dynamics (time to reach s.s.)
- Technology shocks (new crops/better crops/inputs)

Some Related Literature:

- **Geography and development:** Olsson and Hibbs (2005); Ashraf and Galor (2011); Nunn and Qian (2011); Nunn and Puga (2012); Michalopoulos (2012); Alesina, Giuliano, Nunn (2013); Cook (2014a,b); Fenske (2014); Alsan (2015); Ashraf and Michalopoulos (2015); Dalgaard, Knudsen, Selaya (2015); Galor and Özak (2016); Litina (2016); Andersen, Dalgaard, Selaya (2016); Frankema and Papaioannou (2017)
- **Malthusian and UGT models:** Galor (2011); Galor and Weil (2000); Galor and Moav (2002); Hansen and Prescott (2002); Doepke (2004); Cervellati and Sunde (2005); Lägerlof (2006); Crafts and Mills (2009); Strulik and Weisdorf (2008), Voigtländer and Voth (2013a,b)
- **Agriculture and development:** Gollin, Parente, Rogerson (2007); Restuccia, Yang, Zhu (2008); Weil and Wilde (2009), Gollin (2010)
- **Motamed, Florax, Masters** (2014): Pattern/date of urbanization at grid-cell level based on agro-climatic conditions
- **Henderson, Squires, Storeygard, Weil** (2016): Spatial organization of economic activity, relationship to geographic conditions

A Model of Density and Productivity

Region I contains a set of districts, each denoted by i , with aggregate agricultural production

$$Y_i = A_i X_i^\beta \left(K_{Ai}^\alpha L_{Ai}^{1-\alpha} \right)^{1-\beta} \quad (1)$$

- K_{Ai} is all other inputs (e.g. capital)
- L_{Ai} is agricultural labor in district i (not a single sector)
- Assume β and α are identical *within* region (but not nec. *across* regions)
- Elasticity of Y_i/L_{Ai} w.r.t. L_{Ai} is $\alpha + \beta(1 - \alpha)$ (or β is cap/labor held constant)

Mobile Factors

The wage and return to capital in each district are given by

$$\begin{aligned}w &= \phi_L \frac{Y_i}{L_i} \\r &= \phi_K \frac{Y_i}{K_i}\end{aligned}\tag{2}$$

- ϕ_L and ϕ_K are shares of output
- Shares need not equal elasticities
- Shares are identical *within* region (but not nec. *across* regions)
- Capital and labor are mobile *within* region (but not nec. *across* regions)

Solving for Labor Allocations

Given mobility of labor and capital within region,

$$\frac{K_i}{L_{Ai}} = \frac{w}{r} \frac{\phi_K}{\phi_L}. \quad (3)$$

Adding up condition for agricultural labor within region

$$\sum_{i \in I} L_{Ai} = L_A. \quad (4)$$

Solve for allocation of labor (relative to land) to district i

$$\frac{L_{Ai}}{X_i} = A_i^{1/\beta} \frac{L_A}{\sum_{j \in I} A_j^{1/\beta} X_j}. \quad (5)$$

Agricultural Labor Allocation

Take logs of L_{Ai}/X_i expression

$$\ln L_{Ai}/X_i = \frac{1}{\beta} \ln A_i + \ln \Gamma, \quad (6)$$

where

$$\Gamma = \frac{L_A}{\sum_{j \in I} A_j^{1/\beta} X_j}. \quad (7)$$

is a *region-specific* term.

- β can be estimated from elasticity of L_{Ai}/X_i w.r.t. A_i .
- $1/\beta$ small (tight), ag. workers spread evenly w/in region
- $1/\beta$ large (loose), ag. workers concentrated on high A_i

Agricultural Labor Allocation

Take logs L_{Ai}/X_i expression

$$\ln L_{Ai}/X_i = \frac{1}{\beta} \ln A_i + \ln \Gamma, \quad (8)$$

where

$$\Gamma = \frac{L_A}{\sum_{j \in I} A_j^{1/\beta} X_j}. \quad (9)$$

is a *region-specific* term.

- Γ is constant for all districts w/in region
- Ag labor relative to total labor (L_A/L) does not enter
- Expression is not unique to heavily agricultural regions (or eras)

More

Using as a Specification

Re-arranging the prior expression and adding some notation:

$$\ln A_{isc} = \alpha + \beta \ln L_{Aisc}/X_{isc} + \gamma_{sc} + \delta' \mathbf{Z}_{isc} + \epsilon_{isc}. \quad (10)$$

- District i , region/state/province s , country c
- γ_{sc} , region/country FE, pick up Γ term
- \mathbf{Z}_{isc} are additional controls
- ϵ_{isc} is error term

Using as a Specification

We moved productivity, A_{isc} and agric. density L_{Aisc}/X_{isc} to opposite sides:

$$\ln A_{isc} = \alpha + \beta \ln L_{Aisc}/X_{isc} + \gamma_{sc} + \delta' \mathbf{Z}_{isc} + \epsilon_{isc}. \quad (11)$$

- Not a causal statement
- Estimating $1/\beta \Rightarrow$ SE's on β explode

Agricultural Density Data

L_{Aisc} comes from HYDE 3.1 database (Goldewijk et al, 2011)

- Population counts for 5 degree grid-cells built from administrative data
- We aggregate data back to administrative level (e.g. districts)
- Rural population data (not agricultural)
- Main samples based on year 2000

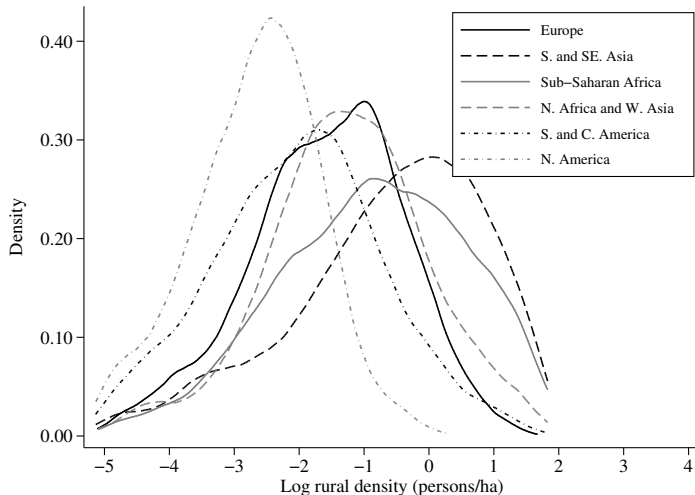
X_{isc} calculated as area of a given district

- Overstates size of agricultural land

L_{Aisc}/X_{isc} data

- Trim above 99th and below 1st percentiles
- Drop if fewer than 100 total rural residents
- 29,030 total districts

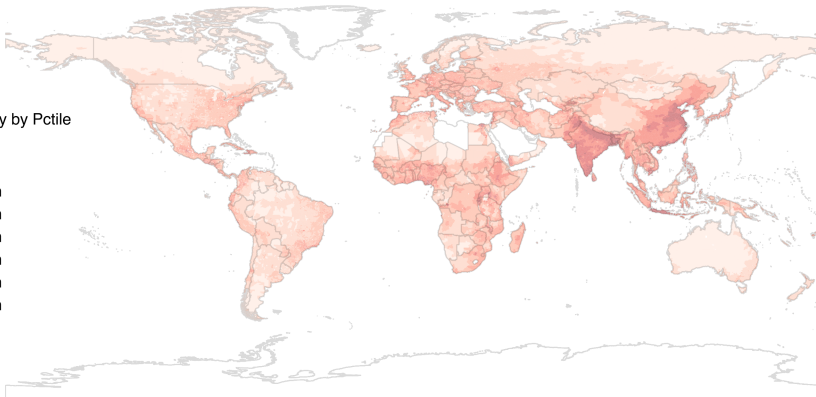
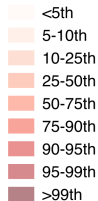
Agricultural Density Data



Agricultural Density Data

Legend

Rural density by Pctile



Agricultural Productivity Data

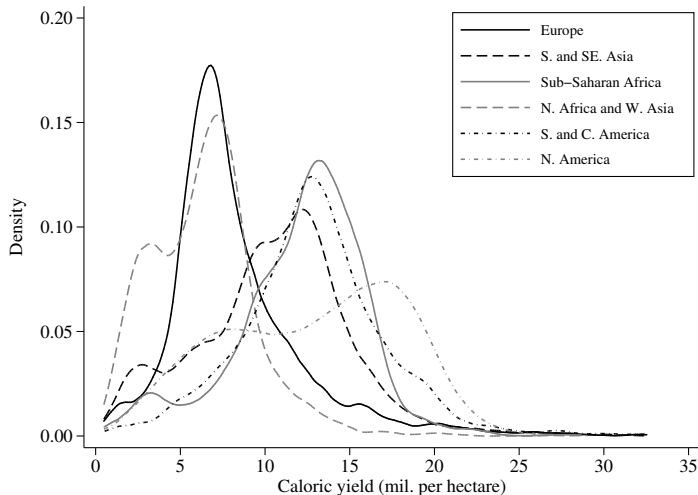
A_{isc} is built similar to Galor and Özak (2016) caloric suitability index

- Data from GAEZ on agro-climatic possible yield (in raw tons) for each crop
- Combine with nutritional information by crop (total calories per raw ton)
- For each grid cell, determine max calories across crops
- Total max calories across grid cells in district, divide by total area
- As in Galor and Özak, holds technology assumptions constant
- Trim above 99th and below 1st percentile

Summary

Crops

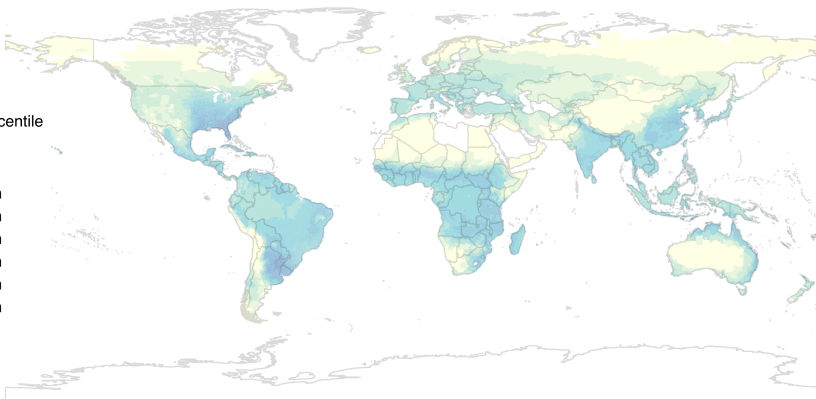
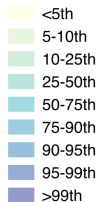
Agricultural Productivity Data



Agricultural Productivity Data

Legend

Yield by percentile



Several data questions being worked on:

- Validating HYDE population data against administrative data
- Comparing rural population to agricultural population
- Rules for assigning grid-cells to districts
- Finer level population data for select countries?
- Using alternative set of crops for max calories
- District shapes for missing countries (e.g. Libya, Saudi Arabia)

Henderson et al (2016) on spatial distribution of economic activity

- Urban activity correlated with (caused by?) high agricultural productivity (in some places)
- Low rural density because of urban activity
- $\text{Corr}(\epsilon_{isc}, \ln L_{isc}/X_{isc}) < 0$

Include two controls at the district level in \mathbf{Z}_{isc} for urban/economic activity:

- Night lights density: follows Henderson et al (2016) using Global Radiance Calibrated data
- Urban percent of population: from HYDE

Spatial Errors and Hypothesis Testing

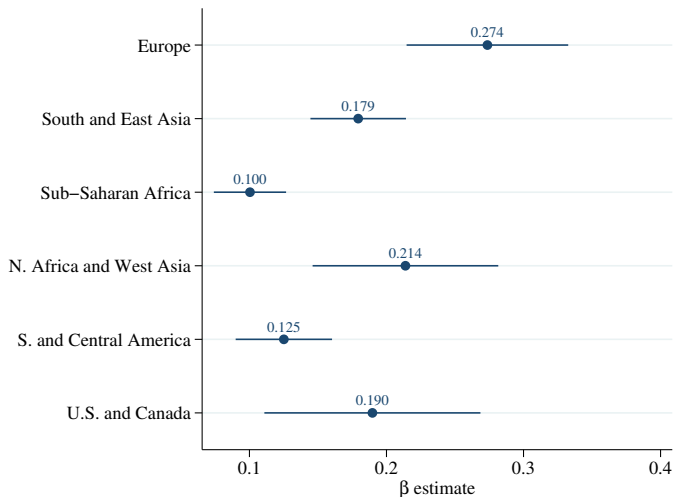
Assume ϵ_{isc} has spatial auto-correlation. Use Conley s.e. (500km window).

Two hypothesis tests:

- Is the land constraint binding?
 - $H_0 : \beta = 0$ vs. two-sided alt
- Is the land constraint the same in two samples (e.g. Europe and Sub-Saharan Africa)?
 - $H_0 : \beta = \beta^{Ref}$ vs. two-sided alt
 - β^{Ref} from ad hoc “reference” sample
 - Implemented with interaction regression combining given and reference sample

Interaction

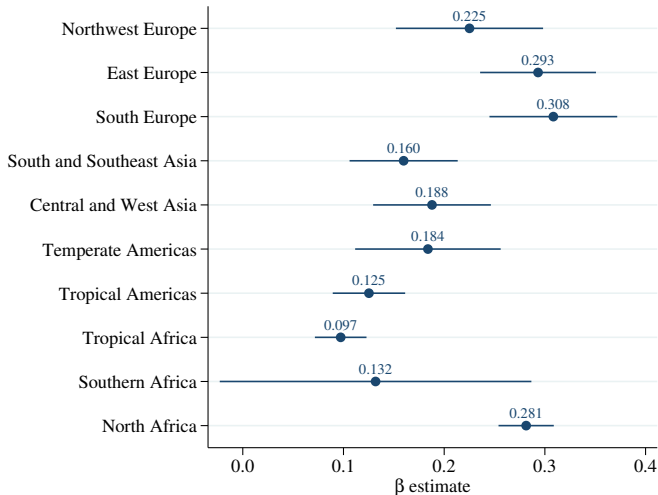
Results by Major Region



Results by Major Region

Dependent Variable: Log caloric yield (A_{isc})						
	Region:					
	Europe (1)	East & South Asia (2)	Sub- Saharan Africa (3)	North Africa & West Asia (4)	South & Central America (5)	U.S. and Canada (6)
Log rural density	0.274 (0.030)	0.179 (0.018)	0.100 (0.013)	0.214 (0.035)	0.125 (0.018)	0.190 (0.040)
p-value $\beta = 0$	0.000	0.000	0.000	0.000	0.000	0.000
p-value $\beta = \beta^{Eur}$.	0.007	0.000	0.190	0.000	0.095
Countries	34	24	43	18	25	2
Observations	7514	6761	3210	2762	9131	2782
Adjusted R-square	0.27	0.23	0.23	0.24	0.16	0.24

Results by Sub-Region



Results by Sub-Region

Dependent Variable in both panels: Log caloric yield (A_{isc})

Panel A

Sub-Region:

	North & Western Europe (1)	Eastern Europe (2)	Southern Europe (3)	Excl. China	
				South & Southeast Asia (4)	Central & West Asia (5)
Log rural density	0.225 (0.037)	0.293 (0.029)	0.308 (0.032)	0.160 (0.027)	0.188 (0.030)
p-value $\beta = 0$	0.000	0.000	0.000	0.000	0.000
p-value $\beta = \beta^{NWEur}$.	0.140	0.061	0.157	0.435
Countries	16	9	9	13	18
Observations	1628	4772	1114	3921	2762
Adjusted R-square	0.13	0.28	0.20	0.15	0.15

Results by Sub-Region

Dependent Variable in both panels: Log caloric yield (A_{isc})

Panel B

Sub-Region:

	Temperate Americas	Tropical Americas	Tropical Africa	South Africa	North Africa
Log rural density	0.184 (0.037)	0.125 (0.018)	0.097 (0.013)	0.132 (0.079)	0.281 (0.014)
p-value $\beta = 0$	0.000	0.000	0.000	0.095	0.000
p-value $\beta = \beta^{NWEur}$	0.431	0.016	0.001	0.285	0.158
Countries	5	22	39	4	5
Observations	3183	8730	3032	178	1147
Adjusted R-square	0.14	0.09	0.10	0.12	0.20

Results for China, Japan, Korea

Dependent Variable: Log caloric yield (A_{isc})					
	All China (1)	Temperate China (2)	Sub-Trop China (3)	Japan (4)	N. & S. Korea (5)
Residuals	0.416 (0.088)	0.522 (0.063)	0.118 (0.023)	0.178 (0.009)	0.214 (0.061)
p-value $\beta = 0$	0.000	0.000	0.000	0.000	0.001
p-value $\beta = \beta^{Temp}$			0.000		
Observations	266	130	136	1039	311
Adjusted R-square	0.59	0.72	0.14	0.18	0.27

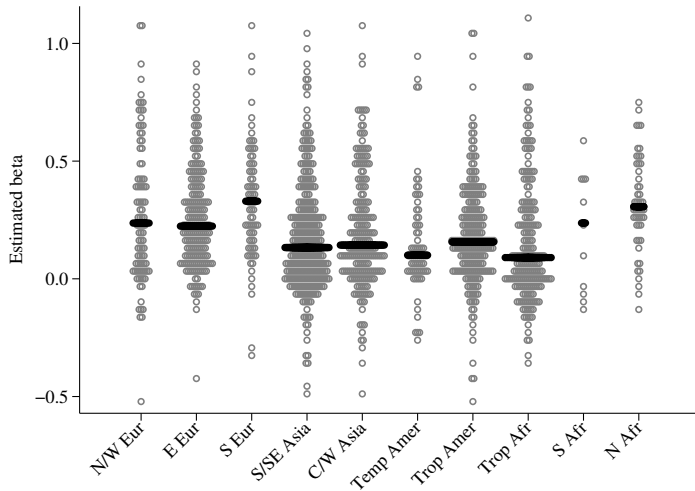
Results by Province

Regions and sub-regions assume β constant within region/sub-region.

Instead, estimate β individually for each province

- Only provinces with 6 or more districts (1,340 provinces)
- ... so really big SE on any individual estimate
- Look at pattern of β 's for each sub-region

Results by Province



Results by Province

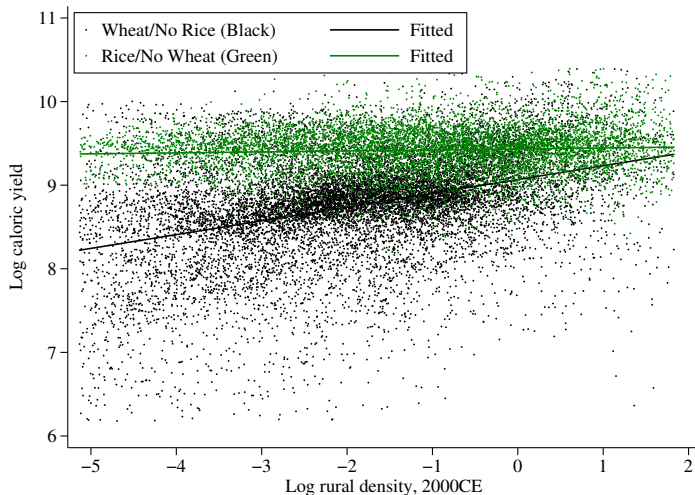
Sub-region	Prov.	Mean	SD	Percentiles:				
				10th	25th	50th	75th	90th
Northwest Europe	96	0.25	0.40	-0.09	0.04	0.23	0.46	0.73
Eastern Europe	192	0.26	0.23	0.04	0.10	0.22	0.40	0.56
Southern Europe	73	0.29	0.38	0.07	0.16	0.33	0.48	0.60
South and S. East Asia	290	0.19	0.27	-0.05	0.02	0.14	0.31	0.55
Central and West. Asia	211	0.22	0.32	-0.06	0.03	0.14	0.35	0.55
Temperate Americas	70	0.15	0.23	-0.05	0.02	0.10	0.24	0.39
Tropical Americas	224	0.17	0.29	-0.07	0.04	0.15	0.31	0.44
Tropical Africa	209	0.18	0.53	-0.11	-0.01	0.09	0.29	0.58
Southern Africa	12	0.07	0.49	-0.11	-0.09	0.17	0.41	0.43
Northern Africa	53	0.30	0.36	0.03	0.22	0.31	0.48	0.64

Results by Crop

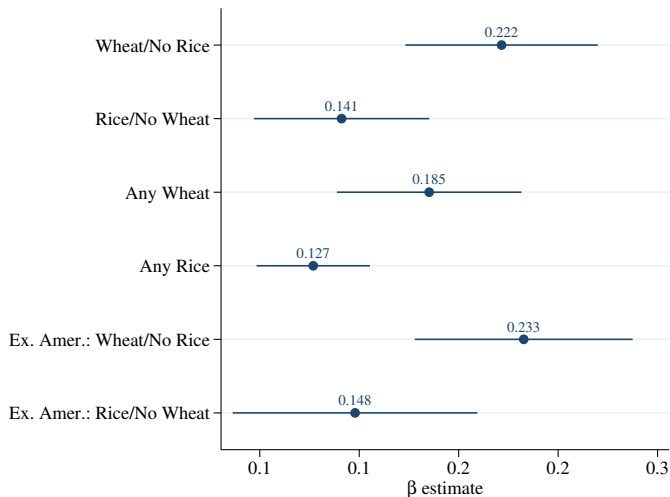
Region and sub-region results appear correlated with agro-climatic zones:

- Run samples defined by agro-climatic zones
- Zones based on GAEZ suitability indices for each crop (0 to 100)
- Index is based purely on climate and soil characteristics
- Define samples using 0 vs > 0 suitability
- Not estimating a crop-specific production function

Results by Crop



Results by Crop



Results by Crop

Dependent Variable in all panels: Log caloric yield (A_{isc})

Panel A: Wheat and rice

Inclusion by crop suitability:

	Entire world:				Ex. Americas:	
	Wheat>0 Rice=0 (1)	Wheat=0 Rice>0 (2)	Wheat>0 (3)	Rice>0 (4)	Wheat>0 Rice=0 (5)	Wheat=0 Rice>0 (6)
Log rural density	0.222 (0.025)	0.141 (0.022)	0.185 (0.024)	0.127 (0.015)	0.233 (0.028)	0.148 (0.031)
p-value $\beta = 0$	0.000	0.000	0.000	0.000	0.000	0.000
Countries	106	74	135	132	86	52
Observations	12627	7796	24431	19600	10185	4439
Adjusted R-square	0.18	0.14	0.15	0.12	0.21	0.14

Results by Crop

Dependent Variable in all panels: Log caloric yield (A_{isc})

Panel B: Tropical crops

Inclusion is wheat suitability = 0, but:

	Cassava>0	Cowpea>0	Maize>0	Pearl Millet>0	Sweet Potato>0	Yams>0
Log rural density	0.143 (0.022)	0.146 (0.021)	0.146 (0.021)	0.158 (0.021)	0.147 (0.021)	0.143 (0.021)
p-value $\beta = 0$	0.000	0.000	0.000	0.000	0.000	0.000
Countries	74	80	78	72	77	78
Observations	8052	8312	8377	6590	8354	8269
Adjusted R-square	0.14	0.13	0.13	0.14	0.13	0.13

Results by Crop

Dependent Variable in all panels: Log caloric yield (A_{isc})

Panel C: Temperate crops

Inclusion is rice suitability = 0, but:

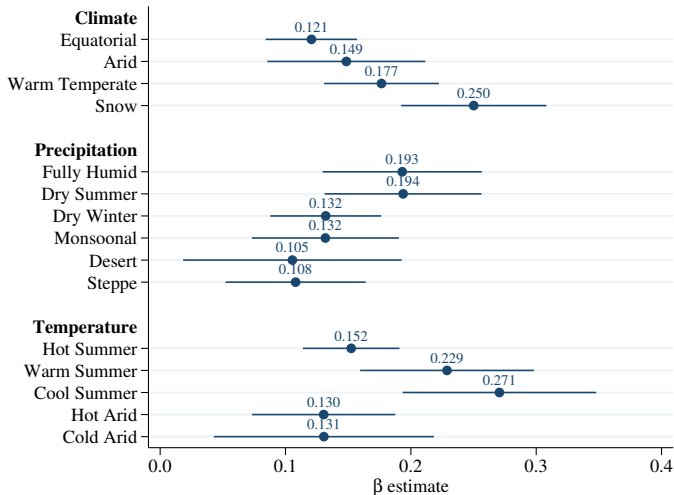
	Barley>0	Buck-wheat>0	Oats>0	Flax>0	Rye>0	White Potato>0
Log rural density	0.222 (0.025)	0.222 (0.026)	0.229 (0.025)	0.222 (0.026)	0.229 (0.025)	0.222 (0.024)
p-value $\beta = 0$	0.000	0.000	0.000	0.000	0.000	0.000
Countries	106	76	72	74	72	105
Observations	12627	11162	11089	11035	11106	12494
Adjusted R-square	0.18	0.19	0.20	0.20	0.20	0.19

Results by Climate Zone

Crop suitability based in part on climate conditions.

- Create samples based on Köppen-Geiger zones
- Three layers: Climate, Precipitation, Temperature
- Each layer has multiple types (e.g. Climate is Equatorial, Arid, ...)
- Create samples where districts have >50% of land in a given type
- E.g. create sample where all districts are >50% “dry winters”

Results by Climate Zone



Results by Climate Zone

Dependent Variable in all panels: Log caloric yield (A_{isc})

Panel A: Climate Zones

	Equatorial (1)	Arid (2)	Temperate (3)	Snow (4)
Log rural density	0.121 (0.019)	0.149 (0.032)	0.177 (0.023)	0.250 (0.030)
p-value $\beta = 0$	0.000	0.000	0.000	0.000
Countries	81	57	94	43
Observations	10752	2675	13019	6058
Adjusted R-square	0.12	0.07	0.16	0.25

Results by Precipitation Zone

Panel B: Precipitation Zones						
	Fully Humid (1)	Dry Summer (2)	Dry Winter (3)	Monsoons (4)	Desert (5)	Steppe (6)
Log rural density	0.193 (0.032)	0.194 (0.032)	0.132 (0.023)	0.132 (0.030)	0.105 (0.044)	0.108 (0.028)
p-value $\beta = 0$	0.000	0.000	0.000	0.000	0.018	0.000
Countries	99	46	75	43	34	55
Observations	16371	3067	8683	1729	390	2270
Adjusted R-square	0.18	0.19	0.12	0.13	0.03	0.05

Results by Temperature Zone

Panel C: Temperature Zones					
	Hot Summer (1)	Warm Summer (2)	Cool Summer (3)	Hot Arid (4)	Cold Arid (5)
Log rural density	0.152 (0.020)	0.229 (0.035)	0.271 (0.039)	0.130 (0.029)	0.131 (0.045)
p-value $\beta = 0$	0.000	0.000	0.000	0.000	0.004
Countries	61	86	26	45	26
Observations	8749	9751	487	1594	1065
Adjusted R-square	0.15	0.22	0.12	0.05	0.07

Explanations?

Evidence suggests:

- Tight constraints: temperate/snow, fully humid/dry summer, warm/cool summers
- Loose constraints: equatorial/arid, dry winter/monsoon, hot summer/arid

Why looser constraints in “tropics”?

- Positive(?): Multiple cropping, longer growing periods, more sun, more rain during growing periods \Rightarrow land area isn't binding?
- Negative(?): Soil leaching, lack of frost \Rightarrow land is less useful?

Robustness

“Robust” meaning

- Absolute and relative size of β estimates across samples are similar
- Hypothesis tests return similar results

Robustness checks:

- Use province level data (with country FE) [Results](#)
- Use rural density from 1900 from HYDE [Results](#)
- Use untrimmed samples of rural density and/or agricultural productivity
- Use districts with fewer than 100 rural residents
- Clustered standard errors (at province level)

Measurement Error

Measurement error in rural density data

- Creates attenuation bias
- Is measurement error more pronounced in some regions (e.g. SE Asia) and driving results?
- True variance of $\ln L_{Aisc}/X_{isc}$ would have to be one-third of measured variance
- Implies one-third of districts have rural density mis-stated by factor of > 2 or < 0.5

Measurement Error

Measurement error of land area, X_{isc} , specifically

- We are using total land area, not agricultural area, so X_{isc} always overstated
- Systematic mismeasurement of districts within a province not a problem
- Variation in systematic mismeasurement across provinces not a problem (FE)
- Problem is *variation in variation of mismeasurement* across provinces
- Some provinces have more geographic variation across districts?
- ... provinces with low β estimates do not have higher variance of geographic characteristics

Mobility of Workers?

Mobility of workers across districts within a province?

- If workers immobile, may create similar densities across districts
- Estimated β would be falsely high?
- Would have to be that frictions more prevalent in high- β places (e.g. Europe or N. Africa)

Mobility of Workers?

Mobility of workers across districts within a province?

- If workers immobile, may create similar densities across districts
- Estimated β would be falsely high?
- Would have to be that frictions more prevalent in high- β places (e.g. Europe or N. Africa)
- Or, workers immobile, but demographic behavior varies widely by district?
- Estimated β would be falsely low?
- Would have to be that demography is more variable in low- β places (e.g. Sub-Saharan Africa)

Elasticity of Substitution?

What if land and labor do not have elasticity of subs. equal to one?

- Elasticity of output w.r.t. land depends on rural density L_A/X
- With EOS more than one, higher density, lower elasticity
- Do results fit this?
 - South/SE Asia, some SS Afr are high density, low β
 - ..but C/S America, other SS Afr are low density, low β
 - ..but N America lowest density, not highest β

Density?

Back to the model

If each person consumes c_A in agric. goods, with L people, then:

$$c_A L = \sum_{i \in I} A_i X_i^\beta \left(K_{Ai}^\alpha L_{Ai}^{1-\alpha} \right)^{1-\beta}, \quad (12)$$

Assume that capital and labor are mobile between districts within region I , but also between agric. and non-agric. Can solve for

$$\frac{L_A}{L} = \left(\frac{c_A L^\beta}{(K/L)^{\alpha(1-\beta)} \left(\sum_{i \in I} A_i^{1/\beta} X_i \right)^\beta} \right)^{1/(1-\beta)} \quad (13)$$

where K/L is aggregate capital/labor ratio.

Real Wage

With labor mobile between agric. and non-agric., then

$$p_N w_N = p_A w_A \quad (14)$$

where p_j is nominal price of good j , and w_j is wage in terms of output in j .
Combine with $w_A = \phi_L Y_A / L_A$ definition from before to get

$$\frac{p_N w_N}{p_A} = \frac{\phi_L C_A}{L_A / L}. \quad (15)$$

This is a “grain wage”. Non-agricultural nominal wage deflated by the price of agricultural goods.

Elasticities

Holding c_A constant,

With respect to population shocks

- $\frac{\partial L_A/L}{\partial L} \frac{L}{L_A/L} = \frac{\beta}{1-\beta}$
- $\frac{\partial p_N w_N / p_A}{\partial L} \frac{L}{p_N w_N / p_A} = -\frac{\beta}{1-\beta}$

With respect to productivity shocks in agric. (equal across all districts)

- $\frac{\partial L_A/L}{\partial A} \frac{A}{L_A/L} = \frac{1}{1-\beta}$
- $\frac{\partial p_N w_N / p_A}{\partial A} \frac{A}{p_N w_N / p_A} = -\frac{1}{1-\beta}$

Calibration

Base Model

Implications

Three settings where the Malthusian constraint might matter

- **Effect of Black Death:** Large effects on European development (Voigtländer and Voth, 2013a,b) due to tight constraint? Similar epidemics in Asia w/o major changes due to loose constraint?
- **Involution:** Higher densities and output, but not living standards, in response to productivity (Geertz, 1963; Huang, 1990) due to loose constraint?
- **Response to agric. technology/inputs:** Necessary increase to match rich countries in TFP/inputs is larger with loose constraint (Eberhardt and Vollrath, 2016a,b)

Conclusion

- Define the Malthusian constraint as the elasticity of output w.r.t. land
- Estimate constraint from variation in rural density within provinces
- Constraint is “tight” (0.20-0.30) in temperate areas (N. China, Europe, US/Canada, S. Africa)
- Constraint is “loose” (0.10-0.15) in tropical areas (S. China, SE Asia, C. Africa, S/C America)
- Constraint dictates the sensitivity of L_A/L and real wage to population and productivity

A Toy Model

Single sector production with productivity, A , fixed factor, X , and labor, L .

$$Y = AX^{\beta}L^{1-\beta} \quad (16)$$

Average product of labor is:

$$\frac{Y}{L} = A \left(\frac{X}{L} \right)^{\beta}. \quad (17)$$

Elasticity of average product with respect to L depends on β .

A Toy Model

Assume population process works such that average product is always c , density is

$$\frac{L}{X} = \left(\frac{A}{c} \right)^{1/\beta} . \quad (18)$$

- Density does not tell us whether land constraint is tight or loose
- But β tells us how sensitive density is to productivity

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Interaction Regression

Combine a given sample with the reference sample (denoted by *Ref*). Run the following regression with interaction terms

$$\ln A_{isc} = \beta \ln L_{Aisc}/X_{isc} + (\beta^{Ref} - \beta) \ln L_{Aisc}/X_{isc} \times I(Ref) + \gamma_{sc} + \delta' \mathbf{Z}_{isc} + (\delta^{Ref} - \delta)' \mathbf{Z}_{isc} \times I(Ref) + \epsilon_{isc}. \quad (19)$$

where $I(Ref)$ is an indicator for the reference region. Our hypothesis test is $H_0 : \beta^{Ref} - \beta = 0$, the coefficient on the interaction term for rural density.

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Results by Major Region, 1900 CE

Dependent Variable: Log caloric yield (A_{isc})						
	Region:					
	Europe (1)	East & South Asia (2)	Sub- Saharan Africa (3)	North Africa & West Asia (4)	South & Central America (5)	U.S. and Canada (6)
Log rural density	0.413 (0.046)	0.254 (0.054)	0.061 (0.032)	0.269 (0.035)	0.070 (0.029)	0.316 (0.064)
p-value $\beta = 0$	0.000	0.000	0.054	0.000	0.016	0.000
p-value $\beta = \beta^{Eur}$.	0.025	0.000	0.013	0.000	0.217
Countries	34	24	43	18	25	2
Observations	7514	6761	3210	2762	9131	2782
Adjusted R-square	0.43	0.34	0.31	0.37	0.24	0.40

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Results by Sub-Region, 1900 CE

Dependent Variable in both panels: Log caloric yield (A_{isc})

Panel A

Sub-Region:

	North & Western Europe (1)	Eastern Europe (2)	Southern Europe (3)	Excl. China	
				South & Southeast Asia (4)	Central & West Asia (5)
Log rural density	0.363 (0.055)	0.414 (0.027)	0.291 (0.025)	0.090 (0.035)	0.264 (0.062)
p-value $\beta = 0$	0.000	0.000	0.000	0.010	0.000
p-value $\beta = \beta^{NWEur}$.	0.247	0.323	0.000	0.232
Countries	16	9	9	13	18
Observations	1628	4772	1114	3921	2762
Adjusted R-square	0.35	0.45	0.31	0.24	0.26

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Results by Sub-Region, 1900 CE

Dependent Variable in both panels: Log caloric yield (A_{isc})

Panel B

Sub-Region:

	Temperate Americas	Tropical Americas	Tropical Africa	South Africa	North Africa
Log rural density	0.200 (0.059)	0.080 (0.028)	0.049 (0.033)	0.257 (0.091)	0.296 (0.041)
p-value $\beta = 0$	0.001	0.004	0.130	0.005	0.000
p-value $\beta = \beta^{NWEur}$	0.042	0.000	0.000	0.316	0.322
Countries	5	22	39	4	5
Observations	3183	8730	3032	178	1147
Adjusted R-square	0.22	0.12	0.16	0.34	0.30

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Results by Major Region, 2000 CE, Provinces

Dependent Variable: Log caloric yield (A_{isc})						
	Region:					
	Europe (1)	East & South Asia (2)	Sub- Saharan Africa (3)	North Africa & West Asia (4)	South & Central America (5)	U.S. and Canada (6)
Log rural density	0.385 (0.101)	0.267 (0.092)	0.124 (0.044)	0.382 (0.112)	0.027 (0.068)	0.111 (0.280)
p-value $\beta = 0$	0.000	0.004	0.005	0.001	0.697	0.693
p-value $\beta = \beta^{Eur}$.	0.389	0.018	0.983	0.003	0.358
Countries	34	23	43	18	23	2
Observations	507	570	525	282	355	47
Adjusted R-square	0.16	0.19	0.13	0.19	0.08	0.13

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Results by Sub-Region, 2000 CE, Provinces

Dependent Variable in both panels: Log caloric yield (A_{isc})

Panel A

Sub-Region:

	North & Western Europe (1)	Eastern Europe (2)	Southern Europe (3)	Excl. China	
				South & Southeast Asia (4)	Central & West Asia (5)
Log rural density	0.662 (0.114)	0.343 (0.074)	0.169 (0.102)	0.044 (0.016)	0.384 (0.069)
p-value $\beta = 0$	0.000	0.000	0.099	0.007	0.000
p-value $\beta = \beta^{NWEur}$.	0.006	0.004	0.000	0.037
Countries	16	9	9	13	18
Observations	166	206	135	370	303
Adjusted R-square	0.20	0.24	0.15	0.17	0.24

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Results by Sub-Region, 2000 CE, Provinces

Dependent Variable in both panels: Log caloric yield (A_{isc})

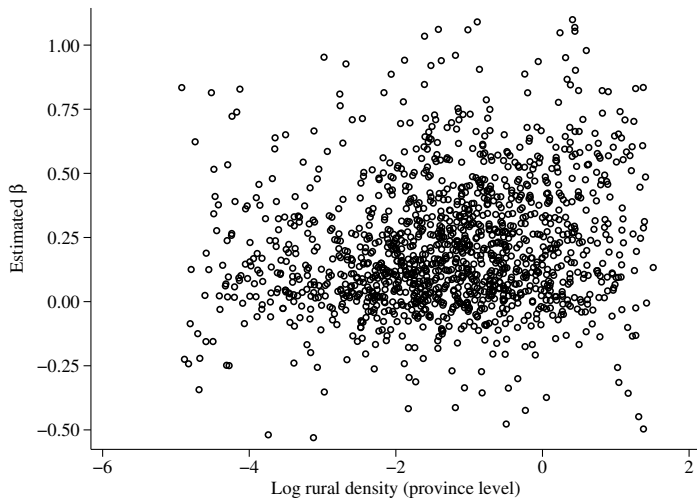
Panel B

Sub-Region:

	Temperate Americas	Tropical Americas	Tropical Africa	South Africa	North Africa
Log rural density	0.162 (0.122)	0.014 (0.070)	0.115 (0.044)	0.406 (0.224)	0.621 (0.133)
p-value $\beta = 0$	0.188	0.839	0.009	0.071	0.000
p-value $\beta = \beta^{NWEur}$	0.003	0.000	0.000	0.310	0.818
Countries	5	20	39	4	5
Observations	85	317	497	28	88
Adjusted R-square	0.14	0.08	0.13	0.21	0.28

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Relationship for β to rural density, by province



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A Toy Calibration

Set up three economies, each with

- $L = 1$
- $A = 1$
- $X = 1$

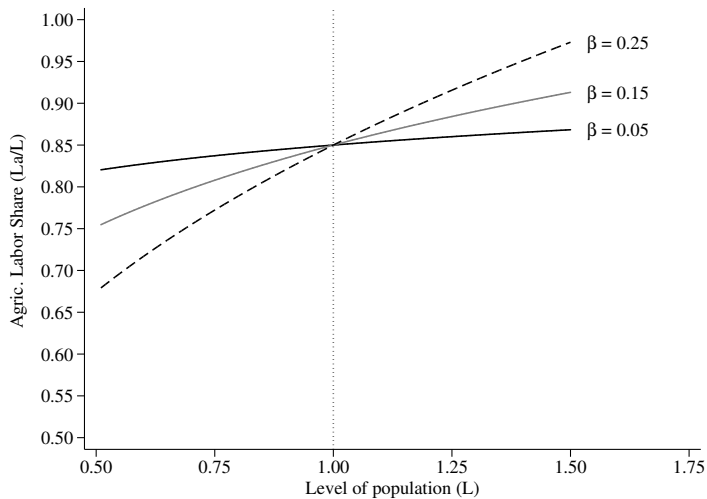
but where the value of β takes on values (0.05, 0.15, 0.25). Initial value of c_A is set so that $L_A/L = 0.85$ in each.

Look at effect of variation in L and A on

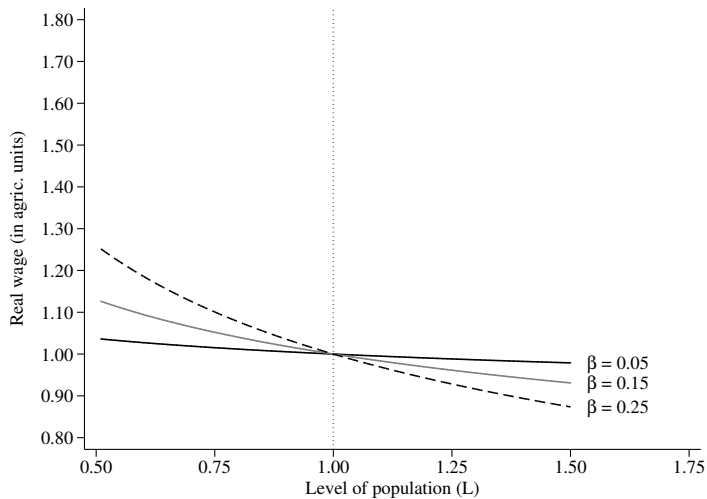
- L_A/L
- Real wage

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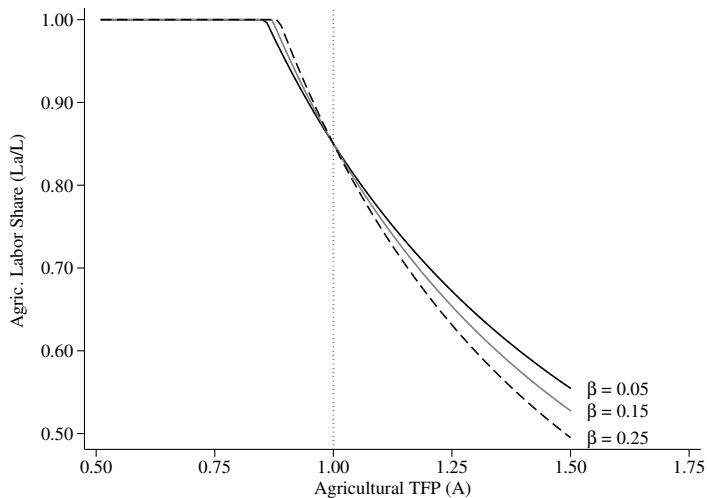
A Toy Calibration



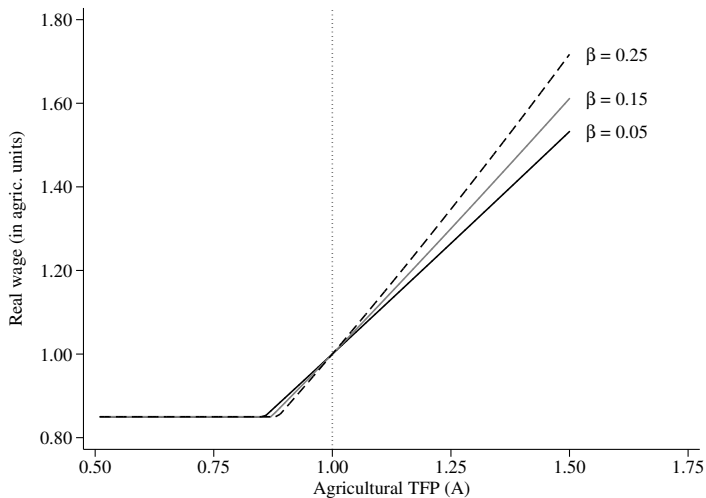
A Toy Calibration



A Toy Calibration



A Toy Calibration



Summary statistics

	Mean	SD	Percentiles:				
			10th	25th	50th	75th	90th
Rural density (persons/ha)	0.57	0.90	0.03	0.08	0.22	0.60	1.53
Caloric yield (mil cal/ha)	10.64	4.79	4.79	7.04	10.50	13.71	16.69
Urbanization rate	0.34	0.34	0.00	0.00	0.28	0.66	0.84
Log light density	1.59	0.76	1.12	1.12	1.12	1.92	2.69

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Crops used in productivity calculation

alfalfa, banana, barley, buckwheat, cassava, chickpea, cowpea, drypea, flax, foxtail millet, greengram, groundnut, indica rice, maize, oat, pearl millet, phaselous bean, pigeon pea, rye, sorghum, soybean, spring wheat, sweetpotato, rape, wet/paddy rice, wheat, winter wheat, white potato, and yams

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