The Elasticity of Aggregate Output with Respect to Labor and Capital

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Research questions

- What are the elasticities of GDP with respect to capital and labor?
- Have those elasticities changed over time?
- Are those elasticities similar across countries?

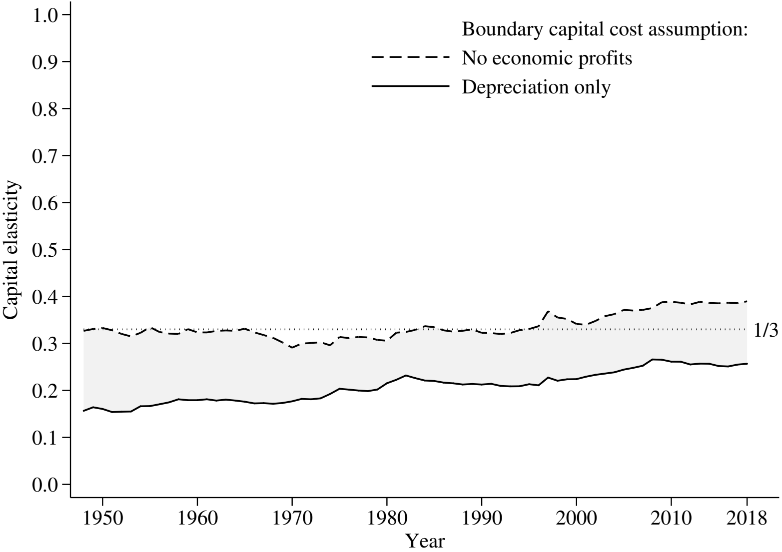
Summary

This paper answers those questions:

- Calculate elasticities for US 1948-2018, OECD 1995-2015
- Applies the methodology of Baqaee and Farhi (2017, 2018)
- Industry-specific cost structure, market power, and input-output relationships
- Creates bounds for those elasticities given issues with measuring capital costs

Preview of U.S. results

Robert Solow was kind-of, sort-of right?



Relevance and contribution

The answer informs us on:

- Consequences of aggregate shocks
- Convergence speed, transition dynamics
- Distribution of GDP to labor, capital, profits
- Validity of decades of macro papers calibrated using $\alpha = 1/3$

Relevance and contribution

Literature on labor's share of GDP: Gollin (2002); Young and Zuleta (2013a,b); Elsby, Hobijn, and Sahin (2013); Karabarbounis and Neiman (2014); Gomme and Rupert (2014); Rognlie (2015); Barkai (2017); Smith, Yagan, Zidar, Zwick (2017); Karabarbounis and Neiman (2018); Koh, Santaeulalia-Llopis, Zheng (2018)

Differences and similarities:

- Elasticities don't equal shares if markups > 1
- Elasticities could provide part of explanation for labor share decline
- Elasticity calculation explicitly at industry level vs. aggregate
- Same data and imputation problems

Theoretical setting

Borrowed completely from Baqaee and Farhi (2017, 2018)

Each industry i has constant-returns cost function. Industry i has costs as follows:

$$COST_i = COST_{iM} + COST_{iK} + COST_{iL} \tag{1}$$

The first term is total intermediate costs from J total industries:

$$COST_{iM} = \sum_{j=1}^{J} COST_{ij} \tag{2}$$

Theoretical setting

Cost shares for intermediates defined as

$$\lambda_{ij} = \frac{COST_{ij}}{COST_i} \tag{3}$$

and for factors of production as

$$\lambda_{iK} = \frac{COST_{iK}}{COST_i} \tag{4}$$

$$\lambda_{iL} = \frac{COST_{iL}}{COST_i}. \tag{5}$$

Theoretical setting

Build the matrix of cost shares

$$\Lambda = \begin{bmatrix} \lambda_{11} & \lambda_{12} & \cdots & \lambda_{1J} & \lambda_{1K} & \lambda_{1L} \\ \lambda_{21} & \lambda_{22} & \cdots & \lambda_{2J} & \lambda_{2K} & \lambda_{2L} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \lambda_{J1} & \lambda_{J2} & \cdots & \lambda_{JJ} & \lambda_{JK} & \lambda_{JL} \\ 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 \end{bmatrix} \tag{6}$$

where labor and capital are treated as “industries” that provide an input to other industries.

Theoretical setting

Value-added shares of GDP, $VA = \sum_{j=1}^{J} va_j$,

$$\gamma_j = \frac{va_j}{VA}. \tag{7}$$

Collect in a vector,

$$\Gamma' = \begin{bmatrix} \gamma_1 & \gamma_2 & \cdots & \gamma_J & 0 & 0 \end{bmatrix} \tag{8}$$

Theoretical setting

Calculate “cost-based” Domar weights - value-added weights times the Leontief inverse.

$$E = \Gamma'(I - \Lambda)^{-1} \tag{9}$$

The structure of E is as follows,

$$E = \begin{bmatrix} \epsilon_1 & \epsilon_2 & \cdots & \epsilon_J & \epsilon_K & \epsilon_L \end{bmatrix} \tag{10}$$

Theoretical setting

Baqaee and Farhi prove that the entries ϵ_K and ϵ_L are the elasticity of aggregate output (GDP) with respect to the aggregate stock of K and L.

- Domar weights capture elasticity of GDP w.r.t. productivity in industry i, capturing downstream and upstream effects
- For labor and capital “industries”, productivity increase is expansion of their supply.
- Only downstream effects of labor and capital, as they purchase no inputs.
- “Cost-based” Domar weights deals with arbitrary markups of prices over costs in industries.

Theoretical setting

This nests your favorite methods for estimating ϵ_K and ϵ_L.

Solow (1957)

- No I/O structure: $\lambda_{ij} = 0$ for any two industries
- Zero profits: $\lambda_{iK} = RK_i/VA_i$ and $\lambda_{iL} = wL_i/VA_i$
- Result is that $\epsilon_K = RK/VA$ and $\epsilon_L = wL/VA$

Theoretical setting

Hall (1988, 1990)

- No I/O structure: $\lambda_{ij} = 0$ for any two industries
- Non-zero profits: $\lambda_{iK} = RK_i/(RK_i + wL_i)$ and $\lambda_{iL} = wL_i/(RK_i + wL_i)$
- Result is that $\epsilon_K = RK/(RK + wL)$ and $\epsilon_L = wL/(RK + wL)$
- Aside on markup (μ) and shares of GDP (similar for labor):

$$\epsilon_K = \frac{Y}{RK + wL}\frac{RK}{Y} = \mu\frac{RK}{Y} \tag{11}$$

Theoretical setting

Hulten (1978)

- I/O structure: $\lambda_{ij} \neq 0$ for any two industries
- Zero profits: $\lambda_{iK} = RK_i/VA_i$ and $\lambda_{iL} = wL_i/VA_i$
- Result is that $\epsilon_K = RK/VA$ and $\epsilon_L = wL/VA$
- Envelope result. No distortions so I/O structure is irrelevant

Implementation

Big issues in plugging data into the Baqaee and Farhi structure given national accounts. Simplifying

$$GDP = COMP + TAX + PROP + ROS \tag{12}$$

Cannot cleanly extract labor or capital costs for any industry

1. Proprietors income ($PROP$) contains labor costs, capital costs, economic profits
2. Residual operating surplus (ROS) contains capital costs and economic profits

Problem measuring costs ....

Implementation

....and the industry definitions are not consistent over time.

Series	I/O table	National accounts	Capital Stock
1947-62	NAICS 2012 (47 ind)	SIC 1972	BEA/NAICS 2012
1963-86	NAICS 2012 (65 ind)	SIC 1972	BEA/NAICS 2012
1987-96	NAICS 2012 (65 ind)	SIC 1987	BEA/NAICS 2012
1997-18	NAICS 2012 (71 ind)	NAICS 2012	BEA/NAICS 2012

All data is from the BEA. Used BEA crosswalks and own assumptions to map all data into NAICS 2012 coding that matched the I/O tables.

Implementation

Absence of precise cost information for capital and labor. Strategy:

- I/O table reports actual costs of intermediates, no problem
- Allocate proprietors income to calculate labor costs
- Construct different series of ϵ_K and ϵ_L based on capital cost assumptions
- Try to *bound* the elasticities based on theory/data
- Undertake variations on assumptions, do they stay in bounds?

Implementation

Labor costs: Allocate a portion of proprietors income to labor. General principle:

$$COST_{iLt} = COMP_{it} + PROP_{it}\left(\frac{COMP_{it}}{VA_{it} - PROP_{it}}\right). \tag{13}$$

This follows Gomme and Rupert (2004).

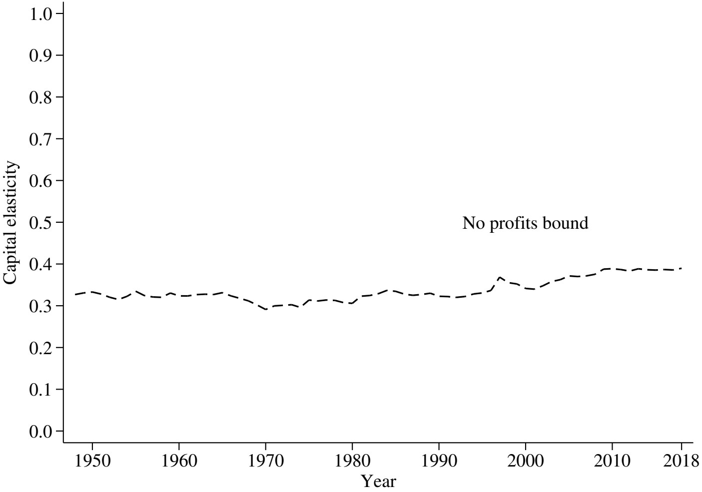
Implementation

Capital costs: Set the *upper bound* for capital costs by assuming there are zero profits.

$$COST_{iKt}^{NoProf} = VA_i - COST_{iLt}. \tag{14}$$

Gives an *upper bound* for ϵ_K. Note this will be the *lower bound* for ϵ_L.

Implementation



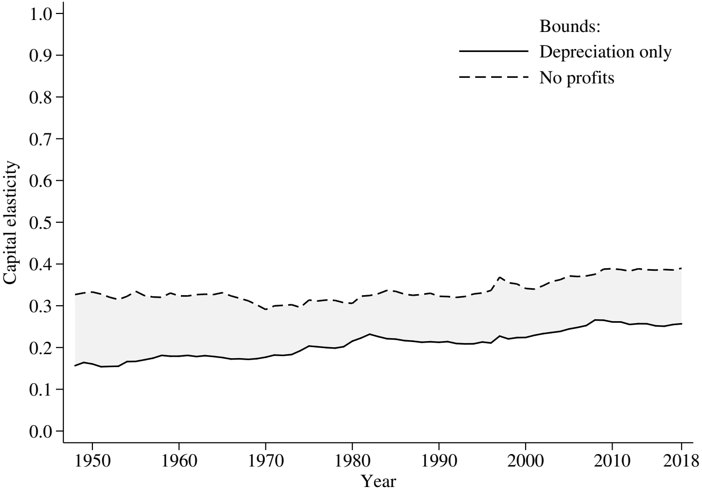
Implementation

Capital costs: Set the *lower bound* for capital costs by using the cost of depreciation ($DEPR_{it}$), which is reported by industry. Assumes zero financing costs of existing capital stock.

$$COST^{Depr}_{iKt} = DEPR_{it}. \tag{15}$$

Gives an *lower bound* for ϵ_K. Note this will be the *upper bound* for ϵ_L.

Implementation



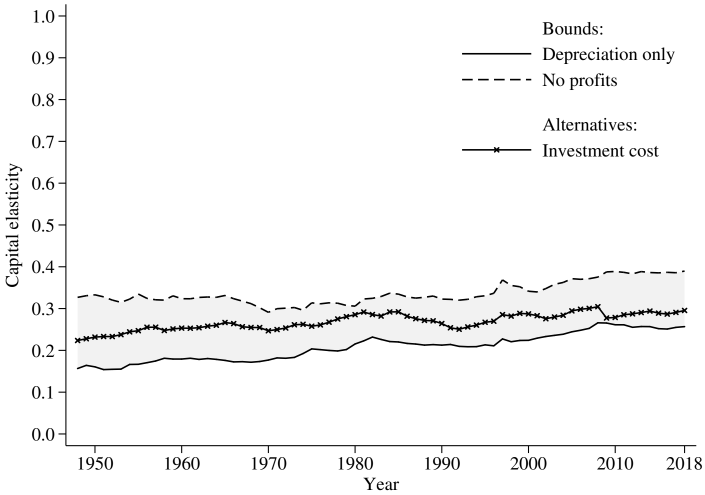
Implementation

Capital costs: Total investment (INV_{it}) is reported by industry. Combines replacement of depreciation and purchase of new capital goods. In Golden rule world $INV = RK$.

$$COST^{Inv}_{iKt} = INV_{it}. \tag{16}$$

Calculate ϵ_K and ϵ_L.

Implementation



Implementation

Capital costs: Calculate the user cost of capital by industry. Three types of capital (structure, equipment, IP).

$$COST_{iKt}^{User} = \sum_{j \in st,eq,ip} K_{ijt} R_{ijt}. \tag{17}$$

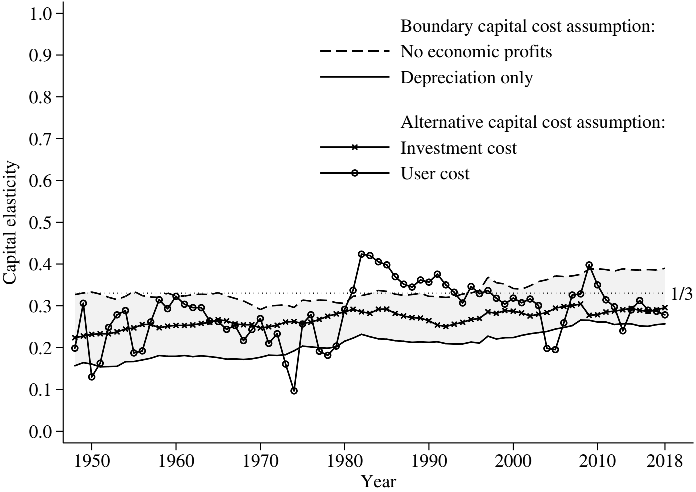
where

$$R_{ijt} = (Int_{it} - E[\pi_{ijt}] + \delta_{ijt}) \frac{1 - z_{jt}\tau_t}{1 - \tau_t} \tag{18}$$

is the rental rate of each type.

- Int_{it}: nominal interest rate facing industry i
- $E[\pi_{ijt}]$: expected inflation of capital type j for industry i
- δ_{ijt}: depreciation of capital type j for industry i (BEA)
- z_{jt}: depreciation allowance for capital type j in tax code (BEA)
- τ_t: effective corporate tax rate (BEA)

Implementation



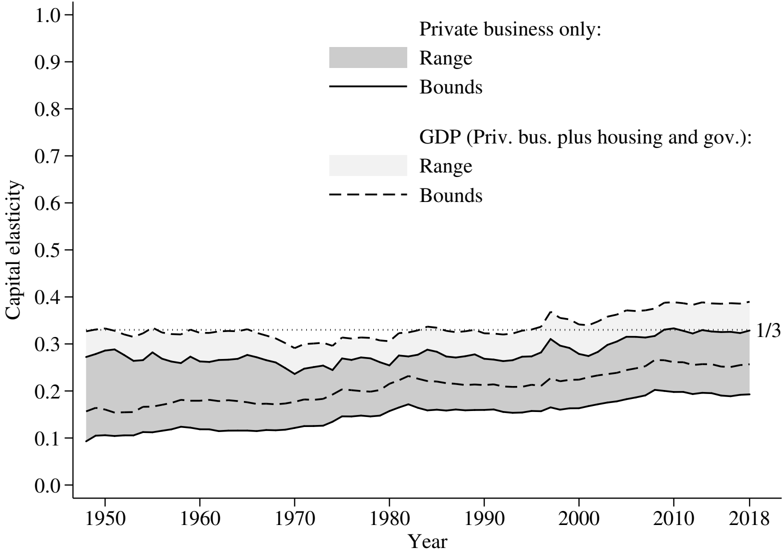
Variations

Private sector business only:

- Exclude government (cost shares not far from average)
- Exclude housing (relatively high capital and low labor cost)

Lower implied capital elasticity (and higher labor elasticity)

Variations

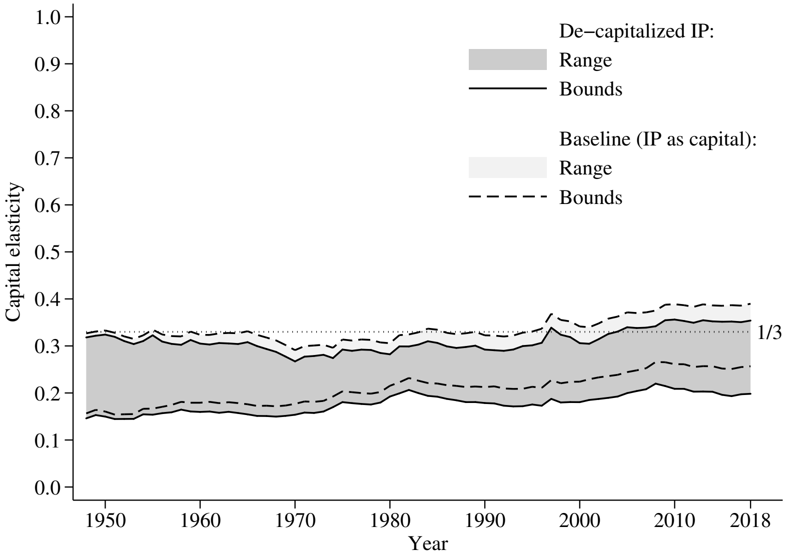

Private business only:
Range
Bounds
GDP (Priv. bus. plus housing and gov.):
Range
Bounds
Capital elasticity
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
1/3
1950
1960
1970
1980
1990
2000
2010
2018
Year

Variations

Intellectual property?

- Elasticity rises over time
- But may be because data on IP from pre-1990 is scarce?
- Koh, Santaeulalia-Llopis, Zheng (2018): aggregate labor share falls due to IP accounting

Variations



De−capitalized IP:
Range
Bounds
Baseline (IP as capital):
Range
Bounds
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
Capital elasticity
1/3
1950
1960
1970
1980
1990
2000
2010
2018
Year

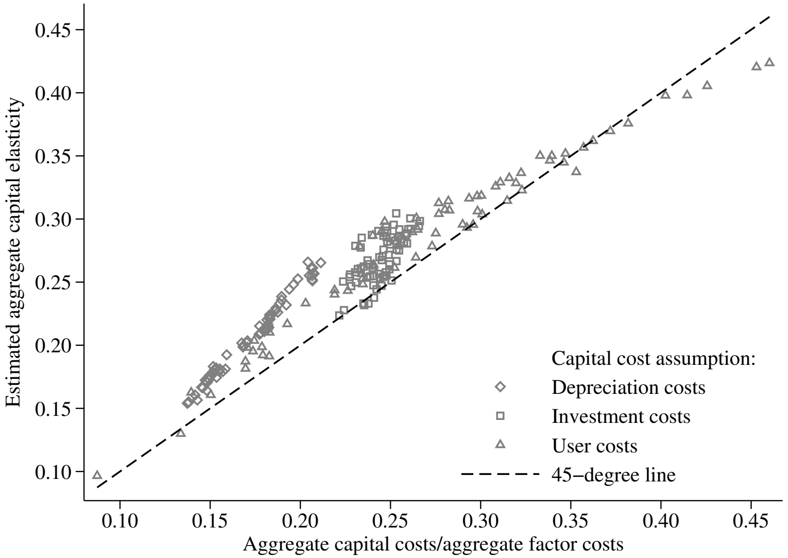
Aggregate cost shares

Define the following aggregate cost share for capital:

$$s_{Kt}^{Cost} = \frac{\sum_{j \in J} COST_{jKt}}{\sum_{j \in J} COST_{jKt} + COST_{jLt}}. \tag{19}$$

If there were no I/O relatonships, then $\epsilon_{Kt} \to s_{Kt}^{Cost}$.

Aggregate cost shares



Decomposition

From the calculation of E,

$$\epsilon_{Kt} = \sum_{i \in J} va_{it} \ell_{iKt} \tag{20}$$

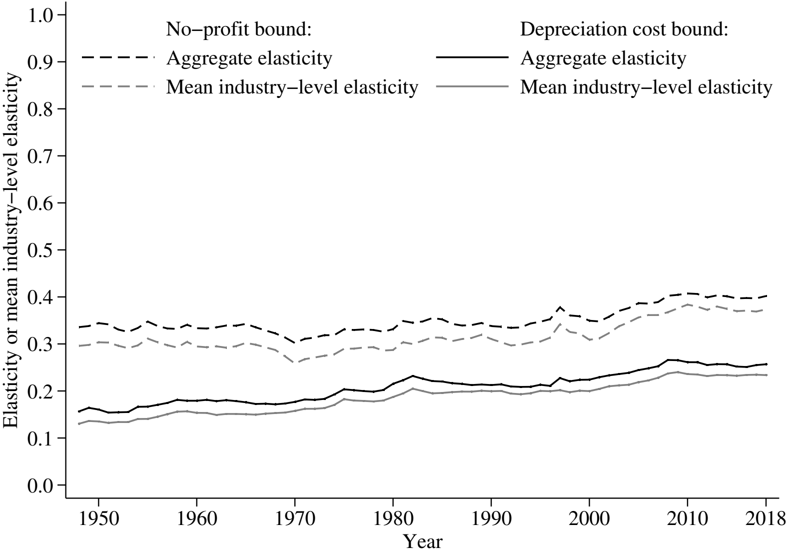
where va_{it} are value-added shares and ℓ_{iKt} are Leontief inverse entries.

Do an Olley-Pakes type decomposition of ϵ_{Kt}

$$\epsilon_{Kt} = \bar{\ell}_{Kt} + \sum_{i \in J} (va_{it} - \overline{va}_t)(\ell_{iKt} - \bar{\ell}_{Kt}), \tag{21}$$

where $\bar{\ell}_{Kt}$ is mean industry elasticity, and summation is covariance of industry elasticity and size of industry.

Decomposition



Comparison

OECD: Create similar bounds for OECD countries using data 2005-2015.

- STAN database for I/O accounts and national accounts data by industry
- STAN does not separate out proprietors income
- 2005-2015: ISIC v.4
- Less re-mapping necessary between I/O and national accounts

Comparison

Labor costs: Without proprietors income

$$COST_{iLt} = COMP_{it} + SELF_{it}\frac{COMP_{it}}{EMPL_{it}}. \tag{22}$$

- $SELF_{it}$ are self-employed and $EMPL_{it}$ are formal employees.
- Probably understates labor costs as proprietors tend to be high wage.

Comparison

Capital costs: Do similar bounding exercise

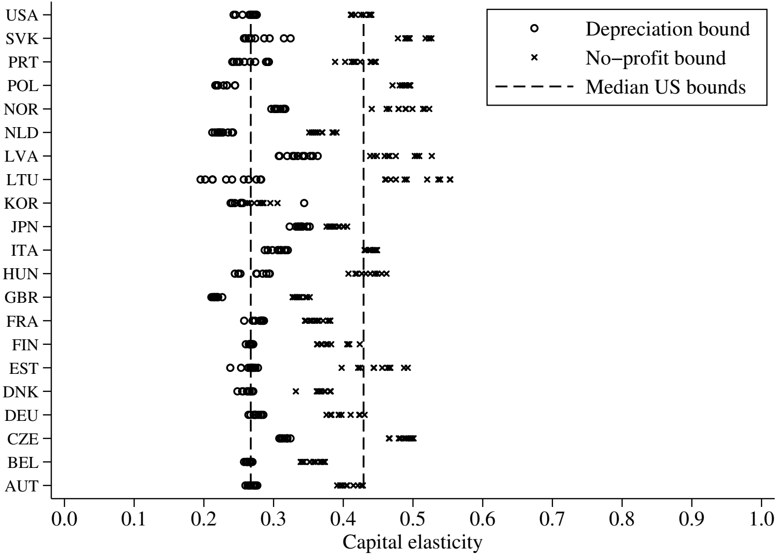
Upper bound on capital elasticity using

$$COST_{iKt}^{NoProf} = VA_i - COST_{iLt}. \tag{23}$$

Lower bound on capital elasticity using

$$COST_{iKt}^{Depr} = DEPR_{it}. \tag{24}$$

Comparison



USA
SVK
PRT
POL
NOR
NLD
LVA
LTU
KOR
JPN
ITA
HUN
GBR
FRA
FIN
EST
DNK
DEU
CZE
BEL
AUT
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Capital elasticity
Depreciation bound
No−profit bound
Median US bounds

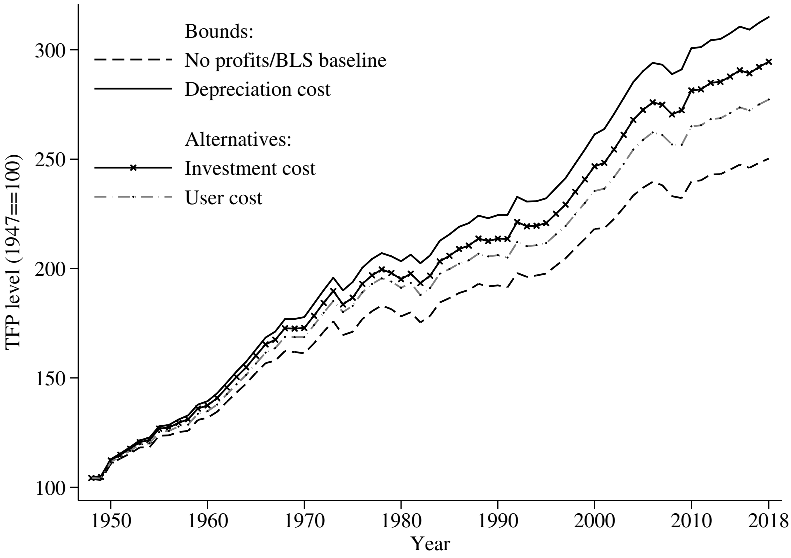
Growth Accounting

For a typical growth accounting exercise:

$$d \ln TFP_t = d \ln Y_t - \epsilon_{Kt} d \ln K_t - \epsilon_{Lt} d \ln L_t. \tag{25}$$

- The implied growth in TFP depends on the elasticities
- BLS uses the “no-profit” assumption only
- Effect is ambiguous

Growth Accounting



Conclusions

- Naive capital elasticity (1/3) is an upper bound for most of period
- ...but this bound shifted up over time
- Scope of economic activity matters: housing pulls up capital elasticity
- Structural change did not appear to drive shifts, within-industry changes
- Consistent across the OECD for short time frame

Remaining work, caveats, and questions...

- What are second-order effects for large changes in K or L?
- Are cost structures consistent with industry-level estimates?
- Can infer markups from elasticities and shares. Consistent with firm-level evidence?
- Expand OECD coverage over time