

Production with Pareto distributions

Due Feb 5th

You should write down a model of production that has the following features:

1. There is a final goods producer who aggregates intermediate goods from different firms, and there is a continuum of firms of mass M . The elasticity of substitution across those firms is σ .
2. The firms produce output using a standard production function of $y(i) = z(i)x(i)$, where $z(i)$ is the productivity of firm i , and $x(i)$ is the amount of input they use.
3. Firms must pay a fixed cost in terms of labor to operate, f . So their profits are $\pi(i) = p(i)z(i)x(i) - wx(i) - f$, where w is the wage rate of inputs, and $p(i)$ is the price they can charge for their output. Firms are monopolists, so they take into account their effect on the price when they make their decisions.
4. There is free entry into producing goods. So the mass of firms M is dictated by a zero profit condition for firms.
5. The productivity of a firm is known to that firm before they decide to operate. The productivity is drawn from a Pareto distribution with $G(z) = 1 - \left(\frac{z_{min}}{z}\right)^\alpha$.

Given this, you should solve your model for the following

1. The cutoff value of productivity, call it \bar{z} , above which a firm will decide to operate
2. An expression for aggregate productivity (in terms of the parameters of the Pareto and the cutoff \bar{z})
3. An expression for aggregate output
4. An expression for the amount that any given firm j produces, $y(j)$.

Using this, you should discuss how aggregate productivity relates to the size of the Pareto parameter α , and what that means about the relationship of inequality in firm sizes and the size of aggregate productivity.

You should also discuss how changing the size of the fixed costs f changes the measure of aggregate productivity and output.

This should all be written up in Latex. I won't accept hand-written copies.