Public Capital and Economic Development

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Abstract

This paper presents new evidence on the sources of cross-country income differences by investigating the role of public capital. Public capital represents, on average, a little more than one-fourth of the total capital stock. The share of public capital in total capital is higher in poor countries, and cross-country differences in private capital stocks are larger than previously thought. Building off of a growth model with public capital, and explicitly incorporating the non-rival and congestion features of public capital, I extend the standard developing accounting framework. When public capital is a public good and complement to other factors, I find that factors of production account for almost twice as much of cross-country differences in output per worker relative to the standard exercise in which private and public capital are treated as perfect substitutes. Under congestion, the contribution of factors of production is smaller, but still larger than in the standard exercise. My findings have implications for economic policy involving the provision of public capital.

Key words: Income differences, Public Good, Public Capital, Private capital, Development Accounting.

JEL Classifications: O11 · O41 · H41.

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1 Introduction

What role does public capital play in explaining differences in per capita income across countries? The standard development accounting framework for assessing cross-country per capita income differences typically combines public and private capital assuming they are perfect substitutes. This ignores the fact that a great deal of public capital is non-rival. In other words, it ignores the public-ness of public capital. The goal of this paper is to measure public capital stocks for a wide range of countries, and then develop and implement a development accounting framework that explicitly includes for the non-rival aspects of public capital.

Public capital, such as transportation infrastructure, public sector research and development (R&D), national security, and the rule of law, tend to raise the productivity of private capital and it is an essential feature of developed economies. Most public capital is either non-excludable and non-rival, which makes government involvement in the provision of it essential. Hence, national and international policymakers and institutions have devoted resources and energy to promoting and pursuing public capital projects as a part of the modernization process.[1]

The first contribution of the paper is providing new evidence on the stock of public and private capital across countries. Using a wide variety of data sources, I estimate the stock of public capital for 90 countries (including rich, middle-income, and poor countries). Averaging over time and across countries, public capital represents 27% of the total capital stock of a country. There is a great deal of variation across countries from 9% (Australia) to 67% (Eritrea). I also find that the share of public capital is negatively correlated with output per worker. The flip side of these facts is that the cross-country variation in private capital stocks is larger than previously reported. For example, the 90-10 ratio (rich over poor) of the capital-to-labor is 94 for total capital; I find that for private capital the ratio is 145. On the other hand, for public capital, the ratio is 31.

The second contribution of the paper is to formulate a framework for including non-rival public capital in a development accounting exercise. My framework draws from a growth model with public capital. In particular, I draw from the model of Glomm and Ravikumar (1999), which in turn, draws from Barro (1990). In the framework, governments exogenously provide non-rival public capital, which is imperfectly substitutable for private capital. The latter is chosen by profit-maximizing firms. Output is produced by public capital, private capital, and labor. This output is used for consumption, private investment, and to build public capital. The government finances its public capital investment with taxation. A key feature of the production function is that output is constant returns to scale in private factors. This implies that competitive equilibrium profits are zero, and that the economy exhibits aggregate increasing returns. The way the aggregate increasing returns show up is that output per

[1]For example international organizations, like the World Economic Forum (2004) argues that there is a large global demand for infrastructure that is not satisfied (http://reports.weforum.org/strategic-infrastructure-2014/introduction-the-operations-and-maintenance-om-imperative/the-global-infrastructure-gap/#view/fn-4). The World Bank Group provides financial support to developing and developed countries (http://www.worldbank.org/en/topic/financialsector/brief/infrastructure-finance).

worker is a function of the aggregate stock of public capital. This is in contrast to private capital, in which only the per worker amount is relevant for output per worker. The framework is extended to include congestion of public capital, as well.

I then calibrate the production function with public capital. The most important parameter to calibrate is the elasticity of output with respect to public capital (λ). My methodology explicitly takes into account the aggregate increasing returns, which is essential, because the elasticity on public capital also captures the extent of the increasing returns. Specifically, using the theoretical framework, as well as the income accounting framework of Cooley and Prescott (1995), I develop a mapping from the elasticity to the public capital income share, and from the public capital income share to public capital income. The framework also tells me how to measure public capital income. I calibrate the elasticity of output with respect to both public and private capital using US national income and products accounts (NIPA) data. The elasticity with respect to public capital (λ) is 0.09, and the elasticity with respect to private capital (α) is 0.32. To my knowledge, this is the first estimate based on a theoretical framework and national accounts data.

With these parameters, and with measures of public and private capital, and gross domestic product (GDP) per worker for the sample of countries mentioned above, I conduct a development accounting exercise. I use the Caselli (2005) measure that computes the ratio of the cross-country variance of the logs of output owing to differences in capital stocks to the cross-country variance of the log of output. In the standard exercise that treats public and private capital as perfect substitutes, this ratio is 25%.[2] In other words, factors of production can account for 25% of the observed variation in output per worker. However, in my framework with public capital, the ratio is 40%. Thus, factors of production account for a substantially larger fraction of differences in output per worker. The main reason is that in poor countries there tends to be a significantly smaller aggregate amount of public capital than in rich countries. Moreover, the aggregate differences more than offset the fact that poor countries have a greater share of public capital (to total capital) than do rich countries. Similar qualitative results are obtained when considering an alternative statistic that normalizes private capital with output (instead of labor) (as in Klenow and Rodríguez-Clare, 1997).

Of course, much public capital is non-rival only to a limit. Highways are non-rival, for example, until traffic is sufficiently large that speeds slow down. Thus, my benchmark case can be considered a natural upper bound. Consequently, I extend the analysis to include congestion. Congestion is introduced in the theoretical framework in the following way. I consider the case in which the degree of non-rivalry of public capital depends on the number of workers in a country. Specifically, the stock of public capital in a country is divided by a power function of its number of workers. A special case of this congestion technology is constant aggregate returns to scale. This extension of the analysis requires the re-calibration of the elasticity of output with respect to public capital. This modification implies a substantial

[2] The value of 25% for this statistic is smaller than the one found in some part of the literature. For example, in Caselli (2005) it is around 40%. The reason is the different sample used. The ratio of output per worker between rich and poor countries is 60 in my sample, whereas in Caselli (2005) it is 34.

decrease of the calibrated value of the elasticity and, as a result, differences in factors of production account for less of the observed cross-country income differences than in the case of public capital as a pure public good. When I repeat the development accounting exercise for an extreme case of congestion, I find that the contribution of factors of production is now 30%, 10 percentage points below my benchmark estimate. Clearly, the presence of congestion blunts the non-rival effects of public capital.

It is useful to compare my development accounting exercises with and without congestion to those in which there are constant returns to scale in public capital, private capital, and labor. I re-calibrate the parameters with this framework and find that the elasticity on public capital is quite similar to that of my benchmark framework, 0.08. Note though that the elasticity applies to public capital per worker, not aggregate public capital. With the re-calibration, I conduct the development accounting exercise and find that the ratio of variances of output per worker is 33%. This is roughly in the middle between my benchmark results and the standard development accounting results.

Even with my benchmark results, the conclusion that total factor productivity (TFP) differences across countries account for the majority of the per capita output differences remains. The contribution of my paper is that in a world in which public capital continues to be an important part of output, and of countries' development strategies, it is important to appropriately quantify the essential features of public capital - its non-rivalness and its congestion effects in any development accounting exercise. In addition to its methodological contributions of incorporating public capital into development accounting, this paper provides the range by which cross-country differences in public capital account for income differences across countries.

Related Literature Starting with the pioneering work of Barro (1990) there is a large literature that includes productive public capital into one-sector growth models. Important papers are Jones et al. (1993), Futagami et al. (1993), Glomm and Ravikumar (1994, 1999), Fisher and Turnovsky (1998), and more recently; Baier and Glomm (2001), Rioja (2001), Chatterjee (2007) and Agénor (2010). Their emphasis was mostly on the effect of government investment policy in productive public capital on long-run growth. Unlike these authors, I focus on differences in levels across countries –that is, in cross-country differences in output per worker– and thus my work relates more to the macroeconomics literature on economic development. An exception to this strand of the literature is the work of Chakraborty and Lahiri (2007), who also focus on differences in levels. They introduce productive public capital into a neoclassical production function with constant returns to scale. They do not focus on the non-rivalness and congestion of public capital, and public capital is not directly measured as I do here. In addition, they take from previous literature a higher value for elasticity of output with respect public capital (0.17). These authors find that differences in factors of production can explain a much higher fraction of the observed income dispersion than in my exercise.

My work closely relates to the work that is more empirical and takes the theoretical framework as a benchmark to focus on the determinants of cross-country income differences using the development accounting approach. Along this line, several papers have contributed to establishing a consensus that TFP differences are more important than differences in factors of production in accounting for cross-country income differences. (See, for example, Klenow and Rodríguez-Clare, 1997; Hsieh and Klenow, 2010; Prescott, 1997; Hall and Jones, 1999; Caselli, 2005). Caselli (2005), for example, uses a standard development accounting exercise with a Cobb-Douglas production function, leading to the conclusion that factors of production explain less than 40% of the observed differences in income across countries.[3] However, the separation of capital into public and private is not considered in his analysis.

As noted, the values of the elasticity of output with respect to public capital is crucial for the quantitative answers. Following the work of Aschauer (1989), there is a large literature on the estimation of this parameter (see Glomm and Ravikumar, 1997, for a review). Aschauer (1989) estimates the elasticity with respect to public capital stock to be 0.39, one of the highest in the literature. Using a different approach and Australian data, Otto and Voss (1998) estimate a value of this parameter of 0.06. Similar small values are also reported in Holtz-Eakin (1994). According to my findings the elasticity is 0.09 when it is a pure public good and it is reduced when public capital is subject to congestion. To estimate the elasticity I use a value of the return rate of public capital investments of 9.7% which coincides with the one reported in Otto and Voss (1998) for Australia and in Allen and Arkolakis (2014) for the US highway system.

My work is not the first to provide estimates capital stocks for both types of capital. Other authors have done so with a smaller sample of countries. Examples are the estimates provided by the by Kamps (2004), Arestoff and Hurlin (2006). The International Monetary Fund (IMF) in a contemporaneous work provide estimates for larger sample of countries that closely coincide with mine. Finally, my public capital stocks estimates can be compared to alternatives measures of public capital. Canning (1998) constructs an index using data on kilometers of paved roads, railways, power capacity and telephone lines to represent the stock for a sample of countries. For a comparable sample of countries, his measure of public capital is highly correlated with my measure. Piketty and Zucman (2014) use data from censuses of wealth to estimate private and public wealth for 9 developed countries. In my estimates, both the ratio of public capital to income and the ratio of public capital to private capital are highly correlated with the figures reported by these authors.

A recent literature strand on trade is related to this paper. Agglomeration effects (and congestion) are important parts of the frameworks used in this literature and thus they are connected with the literature on urban economics. The trade literature in this area focuses on the effect of the transportation network, sometimes considering specific infrastructure projects, on output. Examples of this work are Donaldson (2017), Cosar and Demir (2016), and Allen and Arkolakis (2014). In urban economics literature, increasing returns are essential to un-

[3]The value of this statistic in my sample is 25%. As explained later, the reason is that the ratio of output per worker between rich and poor countries is 60 in my sample, whereas in Caselli (2005) it is 34.

derstand the formation of cities which are the result of the trade-off between agglomeration economies and the cost of urban congestion (see Duranton and Puga, 2004, for an excellent review). The available cross-country data do not allow estimating different categories of public capital (e.g., roads) to compare the different approaches, but the results presented in my paper can be used to connect the findings of these authors to the development literature. For example, in the case of Allen and Arkolakis (2014), the authors estimate a rate of return of investments in the US highway system of 9%, which is similar my estimate for the public capital stock. In my case, the analysis is similar but considering the provision of public capital by the government at the aggregate level.

The paper proceeds as follows. In Section 2 I present a model that rationalizes my benchmark specification of the production function that separates capital into private and public, which is then used in my empirical analysis. In Section 3, I present my estimates of capital stocks for my sample of countries and uncover new facts regarding the share of each type of capital across countries. In Section 4 I present the calibration of the production function. The development accounting for my benchmark technological specification is presented in Section 5. Section 6 extends my analysis for the case of congested public capital. I conclude in Section 7.

2 Public Capital in Production: A Framework

In this Section I develop the theoretical framework that incorporates productive public capital in the production of output of a closed economy. The model is a special case of the model developed in Glomm and Ravikumar (1999) in which public capital enters as an external input and has public goods properties. I use the equilibrium of the model as the basis for my development accounting exercise. I use the production function, and other features of the model to guide the calibration of its parameters. The model is briefly described below.

Technology The technology to produce output requires the use of three inputs: private capital K^p, public capital K^g and labor services. Public capital is external to the firm and is provided by the government. Output at time t, Y_t, is given by:

$$Y_t = A\left(K_t^g\right)^{\lambda}\left(K_t^p\right)^{\alpha}\left(L_t\right)^{1-\alpha}, \tag{1}$$

where A denotes a TFP parameter, α represents the elasticity of output with respect to private capital and λ denotes the elasticity of output with respect to public capital.

In per-worker terms we have:

$$y_t = A\left(K_t^g\right)^{\lambda}\left(k_t^p\right)^{\alpha}. \tag{2}$$

This is the baseline technology that I use in my development accounting exercise. Note that output per worker depends on private capital per worker and on the aggregate stock of public capital. Examples of such public capital could be roads, harbors, and so on offered by the public sector. In this specification public capital is completely non-rival and thus cross-country differences in the aggregate levels of public capital will affect output per worker. However, its effect will be determined by the value of the parameter λ, a central aspect of my development accounting results. Similar to a strand of the literature in endogenous growth (see Romer, 1986; Lucas, 1988, among others), there may be gains to combining countries. There has been suggestive empirical evidence suggesting the existence of aggregate increasing returns (see Hall, 1990; Basu and Fernald, 1997). That said, I also allow for public capital to be subject to congestion. Allowing for congestion significantly reduces the effect of increasing returns.[4]

Firms The technology exhibits constant returns to scale with respect to the private inputs and thus, without loss of generality, I consider just one competitive firm. The firm operates the technology and chooses the amount of private capital and labor to rent taking as given the amount of public capital and prices r_t and w_t. Formally, the firms solve the following static problem:

$$\max_{K_t^p, L_t} A\left(K_t^g\right)^{\lambda}\left(K_t^p\right)^{\alpha}\left(L_t\right)^{1-\alpha} - r_t K_t^p - w_t L_t$$

$$s.to \quad K_t^p, L_t \geq 0, \quad given \quad K_t^g, r_t, w_t.$$

Households The economy is populated by a large number of identical and infinitely lived households that derive utility from consuming a single good. The number of households is N and there is no population growth. The household maximizes

$$\sum_{t=0}^{\infty} \beta^t u(c_t), \tag{3}$$

where $0 < \beta < 1$, c_t is the consumption of the representative household at time t and u is the utility function, which is increasing, strictly concave and twice continuously differentiable with $\lim_{c\to 0} u'(c) = \infty$. The representative household has one unit of time that is supplied inelastically to the labor market and is endowed with an initial capital stock of private capital $k_0^p > 0$. Every period the household rents capital at the rate r_t and earns wages, w_t. The household income is taxed at rate τ_t. Capital depreciates at rate δ_p. Every period the household decides how much to consume and the capital stock for the next period. Formally, the household problem is to choose $\{c_t, k_{t+1}^p\}_{t=0}^{\infty}$ to maximize (3) given k_0^p, the tax rate τ_t and prices subject to the private capital accumulation equation,

$$k_{t+1}^p = (1-\delta_p)k_t^p + i_t^p \quad t = 0, 1, ... \tag{4}$$

[4] As presented in Section 6, the case of constant returns to scale in the aggregate (i.e. where the technology is $Y = A\left(K^g\right)^{\lambda}\left(K^p\right)^{\alpha}\left(L\right)^{1-\alpha-\lambda}$) could be interpreted as a special case of congested public capital.

where i_t^p denotes the investment done by the household. The household budget constraint is given by:

$$\sum_{t=0}^{\infty} p_t(c_t + i_t) = \sum_{t=0}^{\infty}[p_t(1-\tau_t)(w_t + r_t k_t^p) + \pi_t], \tag{5}$$

where p_t is the price of the consumption good and π_t is the share of profits at time t.

Government Every period, the government makes public capital investments, I^g and public capital obeys the following law of motion

$$K_{t+1}^g = (1-\delta_g)K_t^g + I_t^g \quad t = 0, 1, ... \tag{6}$$

where $\delta \in (0,1)$ is the depreciation rate. The initial stock of public capital is K_0^g. The government collects taxes and the proceedings are used to finance the investments in public capital:

$$I_t^g = \tau_t(w_t N + r_t K_t^p) \tag{7}$$

The model provides a framework for the role of each type of agents in the economy and it is going to be followed when using the data to calibrate the parameters of the production function. In Section I of the Appendix I provide the definition of the equilibrium and discuss some of its properties together with the conditions for the existence of a uniqueness and the optimal government policy provided in Glomm and Ravikumar (1999).

3 Measuring Factors of Production

I start my empirical analysis by measuring the quantity of both public and private capital stocks for 90 countries using investment data until 2010. I present the methodology for measuring the public and private capital stocks, and then I present the results. I also present results for measures of human capital and for public capital when part of the investments are wasted.

Physical Capital As is standard in the literature (see, for example, Hall and Jones, 1999; Caselli, 2005), I use the perpetual inventory method to compute aggregate capital stocks. In this way, the *private* capital accumulation equation is

$$K^p{}_{it} = p_{it} I^p{}_{it} + (1-\delta_p) K^p{}_{it-1}, \tag{8}$$

where $K^p{}_{it}$ is aggregate private capital stock of country i in period t, $I^p{}_{it}$ is aggregate private investment in country i in period t, δ_p is the depreciation rate of private capital and p_{it} represents the price of investment goods. I use a measure of the relative price of investment goods in international dollars to assign the values for p_{it} as reported in the PWT. Changes in the relative price of investment goods reflect the technological improvements in investment goods sector as interpreted in Greenwood et al. (1997). Indeed, the cross-country average of the relative price change in the period is -3.3% per year. Cross-country differences in the relative price of investment goods will predict differences in the investment rates and in the capital-to-output ratios across countries. Thus, they could reflect distortions in the accumulation process as interpreted in Restuccia and Urrutia (2001).

I follow the same procedure to measure aggregate *public* capital stocks:

$$K^g{}_{it} = p_{it} I^g{}_{it} + (1 - \delta_g) K^g{}_{it-1}, \tag{9}$$

where $K^g{}_{it}$ is aggregate public capital stock of country i in period t, $I^g{}_{it}$ is aggregate public investment in country i in period t and δ_g is the depreciation rate of public capital. Note that I assume that the prices of investment goods are the same for both types of capital.[5]

The perpetual inventory method requires to obtain an initial level for each type of capital stock. As it is standard in the literature (see Caselli, 2005, for a discussion), I use the framework of the Solow growth model and assume that countries are in their balanced growth path. Therefore, for private capital we have that

$$K^p{}_{i0} = \frac{I^p{}_{i0}}{[(1 + \Upsilon)(1 + n_i) - (1 - \delta_p)]}, \tag{10}$$

where Υ is the rate of technological progress common for all countries and n_i is the population growth rate of country i.

Similarly, for public capital the expression for the initial stock is given by

$$K^g{}_{i0} = \frac{I^g{}_{i0}}{[(1 + \Upsilon)(1 + n_i) - (1 - \delta_g)]}. \tag{11}$$

Note that these equations give us the needed initial capital stocks provided we have the investment (the first data point in the time series) and population growth rates data for each country and can approximate the growth rate of technological progress. In Section B of the Appendix I analyze the robustness of the results to the use of this approach to measure the initial capital stocks.

I now have all the ingredients needed to compute the stocks. I use cross-country private investment and total investment data from the World Bank Development Indicators (WDI (2016)) and Africa Development Indicators (ADI, 2016) databases as well as the OECD.Stat

[5]It could be that they are different. However, there are data limitations. The relative price of investment goods could be different for the two types of capital (public and private), but unfortunately there are no comparable (cross-country) data to perform such a detailed analysis. One reason might be that the composition of the capital stock may differ, for example, public capital in one country could have more structures than equipment compared with another country. However, to my knowledge there are no data that can be used to perform such a detailed decomposition for every country in the sample, although the depreciation rates of the two different types of capital are very similar (calibrated for the US as shown below). In Section H of the Appendix I discuss this assumption as well as the consideration of country specific depreciation rates.

database from the first period for which data are available for each country to 2010. By using these data, I calculate public investment time-series data for each country and year. All data are converted into a common basket of goods, or as it is also called, into international dollars, by using price data from the Penn World Tables (PWT Heston et al., 2012). The sample includes the 90 countries listed in Section A of the Appendix, which also includes more detailed information about the data.

To calculate initial capital stocks, from the PWT I obtain population data for each country that are then used to compute average population growth rates. In addition, I use the data on the real US GDP per worker (also from the PWT) to compute its average growth rate in the period 1950-2003 (1.8%) and use it as my approximation for the world growth rate of technological progress; that is, $\Upsilon = 0.018$.

Finally, I approximate the depreciation rates δ_p and δ_g to the implicit average scrapping rate of the US capital stock. I calculate scrapping rates for private and public capital stocks for each period from 1929-2010 by using NIPA accounts data. Specifically, for each type of capital and for each year I divide the amount of capital depreciated provided in NIPA by the net capital stock figures (also from NIPA). Then I compute the average in the period. I obtain a depreciation rate of 4.6% for both types of capital. I assume that this rate is the same for all countries and is not time varying.

Cross-country differences in the depreciation rate can arise due to the different composition of the stock. Changes in the depreciation rate would change the relative weight of past versus recent investments in the perpetual inventory method. In the development literature it typically assumed that the depreciation rate is the same across countries. The reason is that the development accounting results are not sensitive to this assumption. [6] As for the depreciation rate of public capital, Arestoff and Hurlin (2006) use data on the depreciation rates for different types of assets in the United States and the weight of some assets in Latin American countries, they provide estimates of depreciation rates for public capital in developing countries. The estimated values range from 2.52% to 2.77%. Thus, there is not much variation across these developing countries and, on average, they are in the ballpark of the one I obtained for the United States when I do not include defense capital, i.e. 3%. In Section H of the Appendix I provide a more elaborate discussion of this assumption.

Results I have estimates for 90 countries -see Table A.1 in Section A of the Appendix- for the year 2010. Three facts are salient. First, public capital is sizable: It accounts for an average (across countries) of 27% of the total capital stock. Second, it greatly varies across countries: It ranges from 9% (Australia) to 67% (Eritrea), of the total stock of physical capital. Third, and more importantly, it is negatively correlated with output per worker: the correlation coefficient is −0.65. As clearly shown in Figure 1, the fraction of public (private) capital is substantially lower (higher) in rich countries.

[6] In Caselli (2005) it is assumed to be 6% and then the author conducts a sensitivity analysis to show that the changes in his results are minimal.

In Table 1, I compare capital stocks across countries by presenting figures of ratios of capital stocks for the average top 10th and bottom 10th percentiles of countries in the income distribution of the sample.[7] There are large cross-country differences in the stock of capital. The total capital stock of a rich country is 199 times larger than the capital stock of a poor country. In per worker terms, it is 94 times larger. When separating the stock between private and public, I find the differences are larger for the case of private capital. The private capital stock of a rich country is 284 times larger than the stock of a poor country. In per worker terms, the average worker of a rich country has 145 times higher capital stock of the average worker of a poor country. As for public capital, the average rich country has 83 times the public capital stock of a poor country. In per worker terms, a rich country has 31 times the public capital stock of a poor country.

Figures 2 and 3 show investment rates in international dollars. Investment rates of private capital are positively correlated with output per worker, and investment rates of public capital are negatively correlated with output per worker. These differences in investment rates across countries between the two types of physical capital are driving the differences in capital stocks.[8] Private capital investment rates are much higher in rich countries compared with poor countries. Conversely, public capital investment rates are much higher in poor countries.

I now examine how my measures of capital stocks vary with output per worker. My study allows analysis of cross-country differences in capital stocks by type of capital. Figure 4 shows total capital-to-output ratios for the 90 countries in the sample. As is clear in the figure, there are large differences in capital-to-output ratios across countries, and capital-to-output ratios are positively correlated with output per worker. The mean capital-to-output ratio in the sample is 2.9 and varies greatly across countries, with very low capital stocks like Tajikistan (0.55) and countries with very high capital stocks like Japan (5.25). Figure 5 shows private capital-to-output ratios and output per worker. Similar to the case of total capital, private capital stocks are positively correlated with output per worker. However, the case of public capital is different as shown in Figure 6: Public capital-to-output ratios are negatively correlated with output per worker. Poor countries have a very different composition of capital stock. They have less public capital than rich countries but its share in total capital is substantially higher than rich countries.

Explaining such cross-country disparities in capital-to-output ratios has been an important part of recent research in economic development (see Restuccia and Urrutia, 2001; Hsieh and Klenow, 2007). We can interpret the differences in capital-to-output ratios as evidence of the relative distortion in capital accumulation between rich and poor countries. The separation of capital into public and private stocks allows us to exclusively focus our analysis on the private sector, and this result strongly suggests that the private sector accumulation process

[7]Section A of the Appendix provides detailed tables with the estimates obtained for each country, but for the sake of simplicity and following the development literature, I focus here on the comparison of the tails of the distribution.

[8]The sensitivity of the results to the assumptions regarding the calculation of initial capital stocks is discussed in Section B of the appendix.

could be more distorted than what has been previously thought. Additionally, these facts uncover cross-country differences in public capital policies reflected in the very different compositions of the physical capital stocks between poor and rich countries. Putting these results in perspective and taking advantage of the availability of data across time, the United States can be used as a benchmark by observing the evolution of the US public and private capital stocks. The share of public capital in total capital stock (without including defense capital) is on average 15% in the period 1929-2010 and does not appear to be related to the evolution of US gross domestic product (GDP) per worker. These results challenge the views that consider the lack of public capital as a barrier to economic development.

My work is not the first to provide estimates of capital stocks for both types of capital. Other authors have done so with a smaller sample of countries. Examples are works by Kamps (2004), who estimates public capital stocks for 22 Organisation for Co-operation and Development (OECD) countries, and Arestoff and Hurlin (2006), who estimate public capital stocks for 26 developing countries. More recently in an effort that is contemporaneous to this paper and building on Kamps (2004) the International Monetary Fund started to provide its own estimates on public and private capital stocks for a set of countries.[9] The IMF arrived at estimates that are highly correlated to my measures; the correlation coefficient is 0.99 for both types of capital. Other authors have followed a methodology that differs from the perpetual inventory method. This is the case in Canning (1998), who used data on kilometers of paved roads, railways, power capacity and telephone lines to measure the stock for a sample of countries. For example, for a comparable sample of countries, my measure of public capital stocks is highly correlated to the index of roads for the year 2000 (the correlation coefficient is 0.72).[10] Finally, my capital stocks estimates can be compared to the ones obtained by Piketty and Zucman (2014) who use data from censuses of wealth to estimate private and public wealth for 9 developed countries. I compare my figures with the ones reported in by these authors. Both the ratio of public capital to income and the ratio of public capital to private capital are highly correlated: the correlation coefficients are 0.71 and 0.65, respectively. As for the the private capital to income ratio, the correlation coefficient is 0.25 and their figures are on average higher than mine. The reason is the valuation of both real state and corporate capital is different as they take their market value instead of using past investment as I do when using the perpetual inventory method. The work of Pritchett (2000) also relates to the estimation of capital stocks, this is discussed in what follows.

Wasteful Public Capital Investments Differences in the relative price of investment goods across countries could reflect distortions in the capital accumulation process. These distortions could lead to wastefulness of investments in the public capital stock. In other words, a portion of government investment goods may be lost because of inefficiencies, corruption, and so on. Pritchett (2000) accounts for wasteful public capital investment and shows that the

[9] http://www.imf.org/external/np/fad/publicinvestment/#5

[10] The data for 2000 used to make the comparison were obtained from an updated database provided by the authors at https://www.hsph.harvard.edu/david-canning/data-sets/.

actual investment effort is not necessarily what the data represent and varies across countries. Related to this paper is the work by Hulten (1996), who distinguishes between public capital stock used effectively or ineffectively. Chakraborty and Lahiri (2007) use a similar idea: A portion of the public capital investments is lost because agents in charge of carrying out public investment projects do not have the incentives to do their best. These loses in public capital investments could also be thought as a way to incorporate cross-country differences in the quality of their institutions. More transparent, better designed systems that efficiently allocate public investments are plausibly part of what drives income differences.

One way to incorporate these additional distortions in the accumulation of public capital is by adding an additional parameter to the perpetual inventory method equation. Thus, for country i

$$K_{it}^{g} = \gamma_i p_{it} I_{it}^{g} + (1-\delta_g) K_{it-1}^{g},$$

where K_{it}^{g} is the aggregate public capital stock of country i in period t, I_{it}^{g} is the public capital investment of country i in period t, δ_g is the depreciation rate, and γ_i is a parameter that represents the effectiveness of public investment to build public capital; that is, the portion of the public investment that actually contributes to building the stock of public capital. For a developed country this parameter may be close to 1 and, for a developing country, depending on the severity of the distortion, it would be less than 1.

According to the estimation results in Pritchett (2000), half or more of government investment spending has not created equivalent capital. In other words, at least half of the total government expenditures in investment goods is lost and does not actually contribute to building the stock of public capital. Olken (2007) provides an alternative approach to estimating the losses of infrastructure projects due to corruption or inefficiencies. In this study the author designs and conducts a randomized, controlled field experiment in 608 Indonesian villages building a village road as part of a nationwide infrastructure project. This study is useful as it provides a direct measure of corruption in road projects by using a team of engineers and surveyors to estimate the cost of the roads. The engineers' estimates are then compared with the actual expenditures and the difference is the estimate of the missing expenditures. Missing expenditures are estimated to range from 24% to 27% on average.

Although the results in Olken (2007) are for Indonesia, they have the advantage that they are a direct measure well suited to the method I use to measure capital stock. Thus, I assume $\gamma_{rich} = 1$ and $\gamma_{poor} = 0.7$, which means that 70% of the total amount of investment in public capital in poor countries actually contributes to build the stock (i.e. 30% is wasted). Private capital and human capital stocks are the same as before. The public capital-to-output ratio in the rich countries is 0.5 as before (there are no losses in public investment). However, for the case of poor countries the public capital-to-output ratio is now 0.7, lower than before owing to the extra distortion in the accumulation of this type of capital. As a result, rich and poor countries appear more similar in terms of this statistic using this alternative measure. In addition, due to the changes in the measurement in the public capital stock the total capital-to-output ratio in poor countries is lower (2 versus 2.2) and thus the ratio of total capital-to-output ratios between rich and poor countries is now 1.8 (previously it was 1.6). Therefore, cross-country differences in the capital stocks are now larger and thus will increase the role of physical capital in accounting for development, as explored in Section 5.

Human Capital To appropriately compare my results with the literature on development accounting, I construct human capital stocks in the simplest possible way following Caselli (2005) and Hall and Jones (1999). Human capital per worker in country i, denoted as h_i, is an increasing function of the average years of schooling S_i per worker in that country:

$$h_i = e^{\phi_{S_i} S_i}, \tag{12}$$

where ϕ_{S_i} is a coefficient that depends on the value of S_i and represents the returns on years of schooling. To compute human capital stocks I use data on the average years of schooling in the population older than 25 years of age in country i and provided by Barro and Lee (2013) for 2010. From Psacharopoulos (1994) I take the following estimates of ϕ_S:

- 0.13 for $S \leq 4$,
- 0.10 for $4 < S \leq 8$, and
- 0.07 for $8 < S$.

These values for the rates of returns on schooling are from the Mincerian wage regressions estimated in Psacharopoulos (1994) for a large sample of countries. Specifically, the rate of return for the first 4 years of schooling is the average for sub-Saharan Africa; between 4 and 8 years is the average for the world as a whole; and for more than 8 years of schooling is the average for the OECD countries. The Table in Section A of the Appendix contains the results. As in the previous literature I find differences in the stock of human capital across countries: The ratio of the stock of human capital between rich (top 10%) and poor (bottom 10%) countries is 2.

4 Calibration of the Production Function

An important part of my development accounting exercise is determining the parameter values of the production function. The standard development accounting exercises do not separate capital into private and public and rely on national accounts data for developed countries to calibrate the share of capital in output. In my framework, I need to calibrate an elasticity for each type of capital. The challenge is that how public capital enters into the production function, as well as how its is financed, affects the calibration procedure. Moreover, the procedures followed in the national accounts create an additional challenge as the income from public capital is not reported, and the measurement of the income from private capital is not trivial to compute. Similar to Cooley and Prescott (1995), my strategy follows the theoretical framework from Section 2, which guides the use of the national account data (NIPA tables) to obtain values for the parameters.[11] Due to data availability, I calibrate the production function for the United States and, as common in the literature following the work of Gollin (2002), I assume the parameters are constant across countries.[12]

The assumed technology renders constant shares of each of the inputs used. However, due to aggregate increasing returns, the sum of the shares do not add up to 1. Thus, different from the case of aggregate constant returns to scale (where all the output is exhausted), the elasticities of output with respect to the inputs are not equal to their income shares. To see this, first note that the marginal products of each input are given by $MPK^p = \alpha \frac{Y}{K^p}$ (marginal product of private capital), $MPK^g = \lambda \frac{Y}{K^g}$ (marginal product of public capital), and $MPL = (1-\alpha)\frac{Y}{L}$ (marginal product of labor). Thus, the sum of all the shares of inputs is given by

$$MPK^g \frac{K^g}{Y} + MPK^p \frac{K^p}{Y} + MPL \frac{L}{Y} = 1 + \lambda, \tag{13}$$

where the term $1+\lambda$ represents the returns to scale. Therefore, when using NIPA data to calibrate the technology parameters we need to adjust the data correspondingly. Define the share of public and private capital, θ_g and θ_p, respectively as

$$\theta_g = \frac{MPK^g K^g}{MPK^g K^g + MPK^p K^p + MPL L}, \tag{14}$$

and

$$\theta_p = \frac{MPK^p K^p}{MPK^g K^g + MPK^p K^p + MPL L}. \tag{15}$$

Substituting the expressions for the marginal products and using (13) we have that

$$\theta_g = \frac{\lambda}{\lambda + 1}, \tag{16}$$

and

$$\theta_p = \frac{\alpha}{\lambda + 1}. \tag{17}$$

Thus, given θ_g and θ_p we can solve for α and λ.[13]

To measure θ_g and θ_p we turn to use the national accounts data. In the national accounts we have that

$$\theta_p = \frac{Inc_p}{Total\ income}, \tag{18}$$

and

$$\theta_g = \frac{Inc_g}{Total\ income}, \tag{19}$$

where Inc_p and Inc_g are the income from private and public capital, respectively. To be

[11] The NIPA data are obtained from $https://www.bea.gov/national/$.

[12] Notice that I follow the literature in assuming a unitary elasticity of substitution between the two types of capital. In Section E of the appendix I develop the case of CES between public and private capital.

[13] The same idea is found in Hall (1990) and is applicable to a technology with only one type of capital and labor that also exhibits increasing returns to scale.

consistent with the model, total income is defined as

$$Total\ income = GNP + Inc_g - T_y + T_g, \tag{20}$$

where GNP is the gross national product as defined in the national accounts, T_y are all the taxes on production, and T_g are the taxes to exclusively finance investments in public capital.[14] In this case, public capital income is added to GNP provided that this income is not measured and thus not included in GNP. In addition, the theoretical framework assumes all the existing taxation in production is exclusively used to fund investment in public capital and thus we need to subtract all the taxes on production reported by NIPA (a large fraction of the taxes collected are used to fund other types of government expenditures) and add the amount of taxes that exclusively fund investment in public capital done by the government.

The income approach as defined by NIPA contains all the taxes on production and imports. In addition, the line corresponding to corporate profits in the income approach includes the total amount of corporate income taxes paid by corporations. Thus, to be consistent, I approximate T_y as

$$T_y = Taxes\ on\ production\ and\ imports + Taxes\ on\ coporate\ profits - Custom\ duties. \tag{21}$$

T_g is not reported in NIPA data but it does report the total amount of government expenses on investment in public capital. Thus, by assuming the government runs a balanced budget (as in the model) I can use that amount as a measure of T_g.

Specifically, I take the $Taxes\ on\ production\ and\ imports$ from line 1 of Table 3.5, $Taxes\ on\ coporate\ profits$ from line 16 on Table 1.10 and $Custom\ duties$ from line 14 of Table 3.5. In addition, to obtain T_g, I take the total amount of government expenses on investment in public capital from line 39 of Table 3.1.

I now turn to measure Inc_p. To calculate Inc_p I use the income data from the corporate sector reported in NIPA data from 1929 to 2010 and I follow the procedure outlined in Cooley and Prescott (1995) to assign the proprietor income to capital. I provide the details in Section C of the Appendix.

To measure Inc_g I need to add all the public capital income. It is given by the sum of depreciation (DEP_g) and the net returns from the stock of public capital (Net returns), as follows:

$$Inc_g = DEP_g + Net\ returns. \tag{22}$$

Depreciation is reported in NIPA and it corresponds to the annual allowance for using up public capital. The net returns are not reported and I measure them by multiplying a rate of

[14]The model counterpart of T_g is $\tau(wL + rK)$. I also use the model assumption that the government runs a balanced budget.

return on public capital, r_{pub}, by the value of the net stock of public capital (Net stock), i.e.,

$$Net\, returns = r_{pub}{}^{*}(Net\ stock). \tag{23}$$

From line 1 in NIPA Tables 7.1A and 7.1B, I obtain data for the amount of depreciation of the US government (federal, state, and local) fixed assets (DEP_G in equation (22)), and from line 1 in Tables 7.1A and 7.1B I have the value of the net stock of US government fixed assets (Net stock) from 1929 to 2010. The stock includes federal, state and local assets. The types of assets are equipment, buildings, highways and streets, military facilities, sewer and water facilities, conservation and development, research and development and software.[15]

For r_{pub} I take the rate of return on private capital investment calculated as follows:[16] The return rate (net) for private capital (r), is given by

$$r = \frac{Inc_p - DEP_p}{K}, \tag{24}$$

where K is the net stock of private capital obtained from line 1 in NIPA Table 6.1 for each year for the period considered. Thus, using the obtained values of Inc_p, I calculate r for each year; the mean is 9.7%.

Once I compute Inc_g and Inc_p, I use equations (18), (19) and (20) together with (16) and (17) to obtain average values of the parameters in the period 1929-2010. I obtain $\alpha = 0.32$ and $\lambda = 0.09$.

Note that when the capital stock is separated into private and public capital the elasticity of output with respect to public capital is much smaller than that of private capital and more importantly, smaller than that of some of the references in the literature. For example, in Aschauer (1989), with the same specification of technology as mine, employs econometric methods to estimate $\lambda = 0.39$.[17] I can use his estimate in conjunction with my framework to back out the implicit rate of return of public investment. The calculation is simple as one only needs to iterate backward and use the system of equations to solve for the rate of return for each year and then compute the average rate of return. For example, to obtain the value estimated in Aschauer (1989) one would need to assume a rate of return to public capital of 58%; which is 6 times higher than the one I estimate, as well as the one reported by Otto and Voss (1998) and in Allen and Arkolakis (2014).

As mentioned earlier, the standard approach in the literature is to ignore the separation

[15] The assets of public enterprises are not considered. In Section G of the appendix I consider the case when military or national defense assets are not included.

[16] Although my estimates are consistent with the estimates reported in Otto and Voss (1998) for Australia and in Allen and Arkolakis (2014) for highways systems, one can argue that there are excess returns on public capital investment. An example of a higher rate of return in the literature is the one estimated in Fernald (1999). Assuming excess returns on this type of investment would imply a departure from the proposed theoretical framework. Nevertheless, to analyze the scope of such a modification in the empirical results, in Section F of the Appendix I provide estimates and results for the development accounting by using the return rate estimated in Fernald (1999).

[17] Aschauer (1989) fails to reject the hypothesis of aggregate constant returns. However, Aschauer (1990), rejects the same hypothesis and thus finds evidence supporting the existence of aggregate increasing returns.

of capital between private and public and simply take it as an aggregate. In this case, the contribution of aggregate capital is 0.31.[18]

5 Development Accounting

The results of the development accounting exercise are presented here. As is common in the literature, the approach consists of assuming that each country is represented by a closed economy in its steady-state equilibrium. In my case, I introduce public capital in the technology following the theoretical approach presented in Section 2. Equipped with measures of inputs for each country and values for the parameters of the production function, I ask how much of the observed cross-county income dispersion is due to differences in factors of production. For this purpose, I follow the literature and define a statistic that uses the production function to decompose the observed variances of income in logs. First, I include the measure of human capital and transform (2) to write it as:

$$y_i = A_i \tilde{y}_i, \tag{25}$$

where $\tilde{y}_i = (K^g)^{\lambda}(k^p)^{\alpha}h^{1-\alpha}$; that is, the output per worker implied by the model for country i when only factors of production are taken into account.[19]

An alternative approach follows Klenow and Rodríguez-Clare (1997) and define output per worker as

$$y_i = (A_i)^{\frac{1}{1-\alpha}} \hat{y}_i \tag{26}$$

where $\hat{y}_i = (K^g)^{\frac{\lambda}{(1-\alpha)}}(\frac{k^p}{y_i})^{\frac{\alpha}{1-\alpha}}h$.

I then apply logarithms and then the variance operator to equations (25) and (26) to obtain:

$$var\left[log(y)\right] = var\left[log(A)\right] + var\left[log(\tilde{y})\right] + 2\ cov\left[log(A), log(\tilde{y})\right], \tag{27}$$

and

$$var\left[log(y)\right] = var\left[log(A)\right] + var\left[log(\hat{y})\right] + 2\ cov\left[log(A), log(\hat{y})\right], \tag{28}$$

Following Caselli (2005), I assume that $var\left[log(A)\right] = cov\left[log(A), log(\hat{y})\right] = cov\left[log(A), log(\tilde{y})\right] = 0$, and define the statistics $varlog_{\tilde{y}}$ and $varlog_{\hat{y}}$ as

$$varlog_{\tilde{y}} = \frac{var\left[log(\tilde{y})\right]}{var\left[log(y)\right]} \tag{29}$$

[18]In this case we just consider total capital and all the adjustments related to taxes are ignored. To compute α we just need to sum the income from capital reported in NIPA (i.e. $I_p + DEP_g$) and divide it by GNP.

[19]The role of human capital in development is not the focus of this paper. For the sake of comparison with the development accounting literature I exogenously include human capital stocks in the simplest possible way following Caselli (2005) and Hall and Jones (1999). However, abstracting from human capital does not alter the conclusions of the paper.

and

$$varlog_{\hat{y}} = \frac{var\left[log(\hat{y})\right]}{var\left[log(y)\right]}.^{20} \tag{30}$$

where y is the output per worker observed in the data. Thus, this statistic indicates how much of the observed cross-country variation in income per worker is accounted for by variation in factors of production.[21]In the appendix, I provide the development accounting results for other statistics used in the literature.

To implement the exercise, I use cross-country data on real GDP per worker from the PWT. I also use the measures of capital stocks obtained in Section 3 and the calibrated values for the elasticities of each type of capital obtained in Section 4.

Table 2 show the main results for $varlog_{\tilde{y}}$ and $varlog_{\hat{y}}$. I present results for the case in which public capital and private capital are simply summed together (first row) and for my benchmark specification. It also includes the case with no waste in public investment in poor countries (second row) and when there are wasteful public capital investments (third row).

First, in the standard case (physical capital is not decomposed into public and private), the fraction of the observed income dispersion accounted for by factors of production is 25% in the case of $varlog_{\tilde{y}}$ and 9% in the case of $varlog_{\hat{y}}$. [22]

Differences in factors of production in my benchmark specification, account for a larger share of the observed income dispersion. $varlog_{\tilde{y}}$ is now 39% when there are no losses in public investment (wasteless) and 40% when there are losses in public capital investment in poor countries (wasteful). These are substantial increases when compared to the standar case.

When using $\hat{y}$, the separation of capital into public and private increases $varlog_{\hat{y}}$ to 22% (wasteless) and to 23% (wasteful). The importance of factors of production more than doubles, although it still accounts less than 1/4 of the total income dispersion. Thus, the separation of capital into public and private stocks increases the fraction of differences in income across countries that can be accounted for by differences in capital stocks. In the case of $\hat{y}$ it more than doubles.

My framework assumes public capital is a pure public good, and thus it is the aggregate stock of public capital (not divided by the number of workers) that affects output per worker. Although the share of public capital in total capital is smaller in rich countries, they still have substantially higher public capital stocks than poor countries, as shown in Table 1. There

[20]There is no consensus in the literature regarding the specification that is better to conduct the development accounting exercise. For some authors $\tilde{y}$ is cleaner as it is invariant to differences in A, which is clearly not the case for $\hat{y}$. When computing *varlog* we implicitly make the assumption that A is the same across countries and thus $\tilde{y}$ seems more appropriate (see Caselli, 2005; Klenow and Rodríguez-Clare, 1997, for a discussion).

[21]In Caselli (2005) this statistic is called "success" in the sense that it measures the success of the model proposed to account for the observed income differences.

[22]These values are lower than the ones reported in Caselli (2005). As expected, they depend on the sample of countries considered as well as the income figures. The main differences come from the income figures used in this paper. the ratio of output per worker between rich and poor countries is 60 in my sample and 34 in Caselli (2005)'s sample. This is clear if I restrict my sample and the sample in Caselli (2005) to include the same countries. As for the lower values of the statistics when using $\hat{y}$, that is also the case in Caselli (2005). The reason is that A is positively correlated with $\tilde{y}$ and $\tilde{y}$ does not vary as much.

is high cross-country variation in the stock of public capital (the coefficient of variation is 3.4), and although its contribution to output is lower than in the case of private capital, (λ is 0.09), it still significantly increases *varlog* by adding variation to the cross-country variation in private capital. For the case of private capital, it enters in per worker terms. The cross-country variation in the per worker stocks of private capital is slightly higher than the variation of the per worker total stocks of capital; the coefficient of variation of the former is 1.15 and 1.10 for the latter and, compared to the standard accounting, it has a similar effect since its share is similar (α is similar).

I can decompose the contribution of each type of capital stock by computing $varlog_{\tilde{y}}$ assuming that only private and human capital stocks varies across countries. In this case, $varlog_{\tilde{y}} = 0.30$, which is higher than the value of 0.25 obtained in the standard accounting but lower than the value of 0.39 of my benchmark case. This suggests that my treatment of public and private capital leads to a larger role for both factors of production.

As is clear by now, the calibrated values of λ and α are crucial in determining the contribution of public and private capital to explaining cross-country income differences. Given the variation in capital stocks, the higher the values of these parameters, the higher the fraction of observed cross-country income differences that can be accounted for by differences in factors of production, particularly public capital, across countries. My framework allows the use of values of λ provided in the literature together with my measures of the stocks to address the role of public capital in development accounting. For example, one can consider the value of this parameter estimated in Aschauer (1989), an important referent in the literature; as mentioned earlier, the estimated value is 0.39. If this is the case, given the same measures of capital stocks we can calculate the value of $varlog_{\tilde{y}}$ is now 89% and $varlog_{\hat{y}}$ is now 98%, which means we can explain almost all the observed variation of income across countries. As a consequence, we would be close to solving the development problem and the role of cross-country differences in TFP (A in my specifications) in explaining cross-country income differences is minimal. However, my results point to a different conclusion. Although the role of differences in factors or production in explaining cross-country income differences is higher, TFP differences still play a substantial role.

Thus, the value of λ plays a crucial role and as noted, the implicit rate of return of public capital investment associated with $\lambda = 0.39$ is 6 times higher than the one I estimate for private capital investments. In addition, and no less important, is how public capital enters into the production function. In this example it is a pure public good, but even if I assume such high rates of return (and thus a high λ) the result is significantly affected if I consider the case of congestion as analyzed in the following Section.

6 Development Accounting with Congestible Public Capital

In my benchmark specification, the production function exhibits aggregate increasing returns and public capital is a pure public good. This is a departure from some strand of the previous literature on the topic that considers aggregate constant returns (see Barro, 1990; Chakraborty and Lahiri, 2007, among others). Under such constant returns, output per worker would be

$$y = A\left(k^{g}\right)^{\lambda}\left(k^{p}\right)^{\alpha}. \tag{31}$$

Note that the main difference with respect to my specification (given by equation (2)) is that public capital now enters in per-worker terms instead of as an aggregate. Both the calibration and the development accounting can be done using this specification and would render qualitative similar results.Specifically, we can add my measures of human capital and recalibrate the elasticities to compute $varlog_{\tilde{y}}$. In this case the factor shares are equal to the elasticities and by following the procedure outlined in Section 4 I get $\lambda = 0.08$ and $\alpha = 0.30$. When using these values we have that $varlog_{\tilde{y}} = 0.33$ which is in between my benchmark result and the standard development accounting exercise. Although, public capital is in per worker terms, there still large differences across countries: as shown in Table 1 public capital per worker in the average rich country is 30 times larger than in poor countries.

Perhaps a more useful interpretation is that the inclusion of public capital in this form is associated with the more general idea of introducing congestion in the use of the public good. That is, public capital is a special input in the sense that it has public good features, but also could be subject to congestion in its use. The services from public capital goods decrease as more agents use them and thus, the effect of the size of a country in output per worker is reduced. [23] Allowing for congestion, public capital is not a pure public good and potentially we can allow for different degrees of non-rivalry. The model presented in Section 2 can be extended to the case of congestion.[24].

One possible way to specify congestion is by assuming that

$$\widehat{K}^{g} = \frac{K^{g}}{L^{\theta}}. \tag{32}$$

Above, we associate the degree of congestion by considering the number of workers in a country with θ controlling the degree of non-rivalry. Note that when $\theta = 1$ we are in the case of constant aggregate returns specified in (31).[25]

Following the same concept as in Section 5, we redefine the production technology as follows:

$$Y_i = A_i\left(\widehat{K}^{g}\right)^{\lambda}\left(K_i^{p}\right)^{\alpha}\left(h_i L_i\right)^{1-\alpha}. \tag{33}$$

[23]Fernald (1999) provides empirical evidence about the importance of congestion in the case of the US road system.

[24]In Glomm and Ravikumar (1999) the authors provide the proofs for the existence and uniqueness of the competitive equilibrium in which public capital

[25]In reality, we lack a theory that guides the specification of the congestion in the context of this model. For example, we can think of a more generic specification of congestion like the one proposed in Glomm and Ravikumar (1994), where the stock public capital is given by $\widehat{K}^{g} = \frac{K^{g}}{(K^{p})^{1-\epsilon}L^{\epsilon}}$, where K^{g} and K^{p} are aggregate stocks of public and private capital, respectively, and L is aggregate labor.

Therefore, as before there are constant returns to scale in private inputs and increasing returns to scale at the aggregate level. However, the scale effects are diminished by the presence of the term L^{θ}.

In per-worker terms we have

$$y_i = A_i \left(\widehat{K}^g\right)^{\lambda} \left(k_i^p\right)^{\alpha} h_i^{1-\alpha}. \tag{34}$$

It is important to understand that congestion affects the calibration of λ. The reason is that the public capital is now rival, and thus the value of the flow of services (the income from public capital) obtained from it is reduced. I follow the same procedure discussed in Section 4 to obtain public and private capital income as well as total income. The only difference is that public capital income, Inc_g, is now divided by L^{θ}. With direct way to calibrate θ, I perform the calculation for different values of this parameter. This modification affects the value obtained for α (recall that the income from public capital is part of total income), and more importantly it affects the value of λ because now the income from public capital is deflated by the number of workers (to the power of θ).

Table 3 presents the results. First, it is important to notice the change in the value of the elasticity. It shows the values for different degrees of non-rivalry, specifically for $\theta = \{0, 0.05, 0.10, 0.20\}$. As expected, as θ increases, λ is reduced: from 0.09 (its value in the benchmark case) to virtually 0. Although it is not shown in the table, the values are almost the same for $\theta > 0.2$; that is, the contribution of public capital to output is close to zero for high degrees of rivalry of the public good.

I now perform the development accounting exercise by using the measures of inputs already estimated and the new values of the parameters. Table 3 shows the values of $varlog_{\tilde{y}}$ and $varlog_{\hat{y}}$. As we introduce rivalry in the use of public capital; the contribution of cross-country differences in factors of production decreases. For example, for $\theta = 0.05$, $varlog_{\tilde{y}} = 33\%$ and, for $\theta \geq 0.1$ $varlog_{\tilde{y}} = 30\%$. Compared to the standard accounting, the inclusion of public capital still increases the explanatory power of factors of production, but it is lower than the case of public capital as pure public good. Although cross-country differences in public capital are less important with higher degree of rivalry, the cross-country dispersion of private capital is higher than the dispersion of total capital and that raises its contribution.

7 Concluding Remarks

This paper addresses the role of public capital in economic development. It uses a model and develops a method to incorporate the non-rivalness and congestion effects of public capital into development accounting. The standard development accounting framework for assessing cross-country income differences typically combines public and private capital assuming they are perfect substitutes and thus ignores these special features of public capital.

Quantitatively, the paper offers new insights on the sources of cross-country income differences. First, it provides new facts on public and private capital stocks across countries. Although rich countries have more public capital than poor countries, the share of public capital in the total capital stock of countries is negatively correlated with output per worker. In addition, cross-country differences in private capital stocks are larger than previously thought. Furthermore, it offers new estimates of the elasticities of output with respect to each type of capital by using a new method to calibrate these parameters based on national accounts data. Both the measures of capital stocks and the values of the elasticities are crucial in determining the role of public capital in development accounting. I find that when public capital is a pure public good, differences in factors of production account for almost twice as much of the observed variation in income per worker relative to the standard development accounting. The more congested public capital is, the lower its elasticity, thus, under congestion, differences in the stocks of capital play a smaller role in accounting for cross-country income differences.

A country's stock of public capital is directly affected by economic policy and by the quality of governmental institutions. My development accounting results shed new light on the effects of policies on economic development as the inclusion of public capital diminishes the role of TFP differences in accounting for cross-country income differences. However, TFP differences still play a major role. In addition, the fact that poor countries have relatively more public capital may provide information about the importance of the interplay between economic policy and the potential distortions that affect the investment decisions made by the private sector, a topic that I hope encourages future research.

This paper addresses many of the difficulties associated with the inclusion of public capital, mainly those related to the public good features of public capital. It also uncover new challenges. Regarding the measure of the stock, future effort should be devoted to collecting cross-country data on the different types of goods that compose the stock of public capital. Although this paper provides new measures of the elasticity of output with respect to public capital, it does so using US national accounts. A more detailed cross-country analysis would require the same quality of data for developing countries. On the theoretical side, much effort has been devoted to understanding agglomeration externalities at the city level. But, there has been less research on the returns arising from the production side in the presence of inputs with public goods features. This will surely contribute to a better understanding of the returns of public capital investment.

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Figures

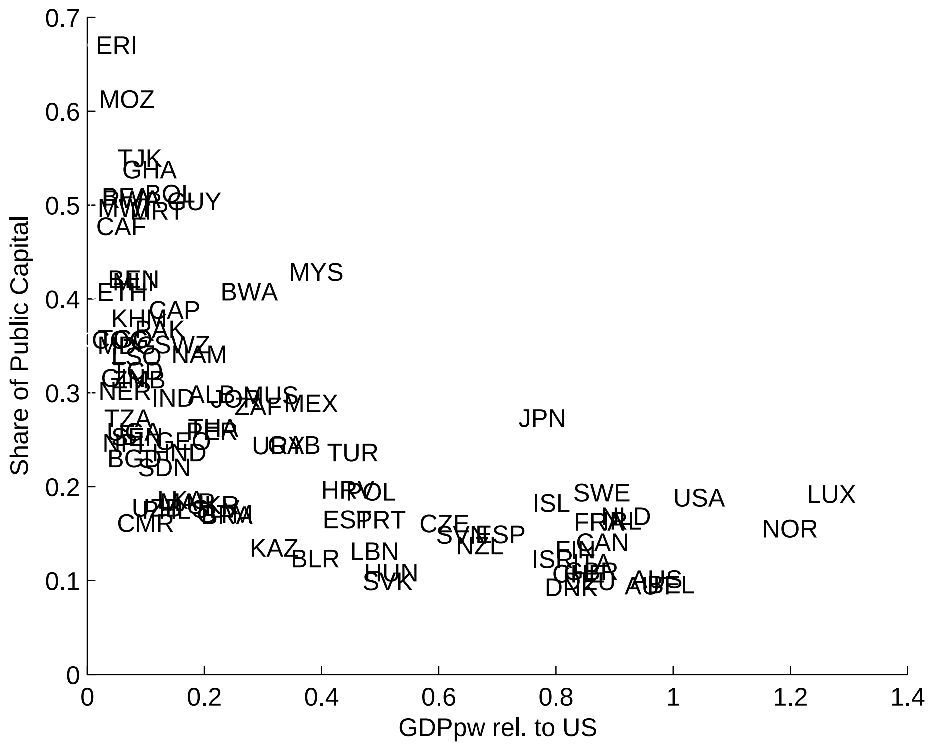

Figure 1: The figure shows the gross domestic product per worker (GDP) per worker of each of the countries in the sample relative to US GDP (GDPpw rel. to US, x-axis) and the share of public capital stock in the stock of total capital for the same countries (y-axis). See Section A in the Appendix for country abbreviations.

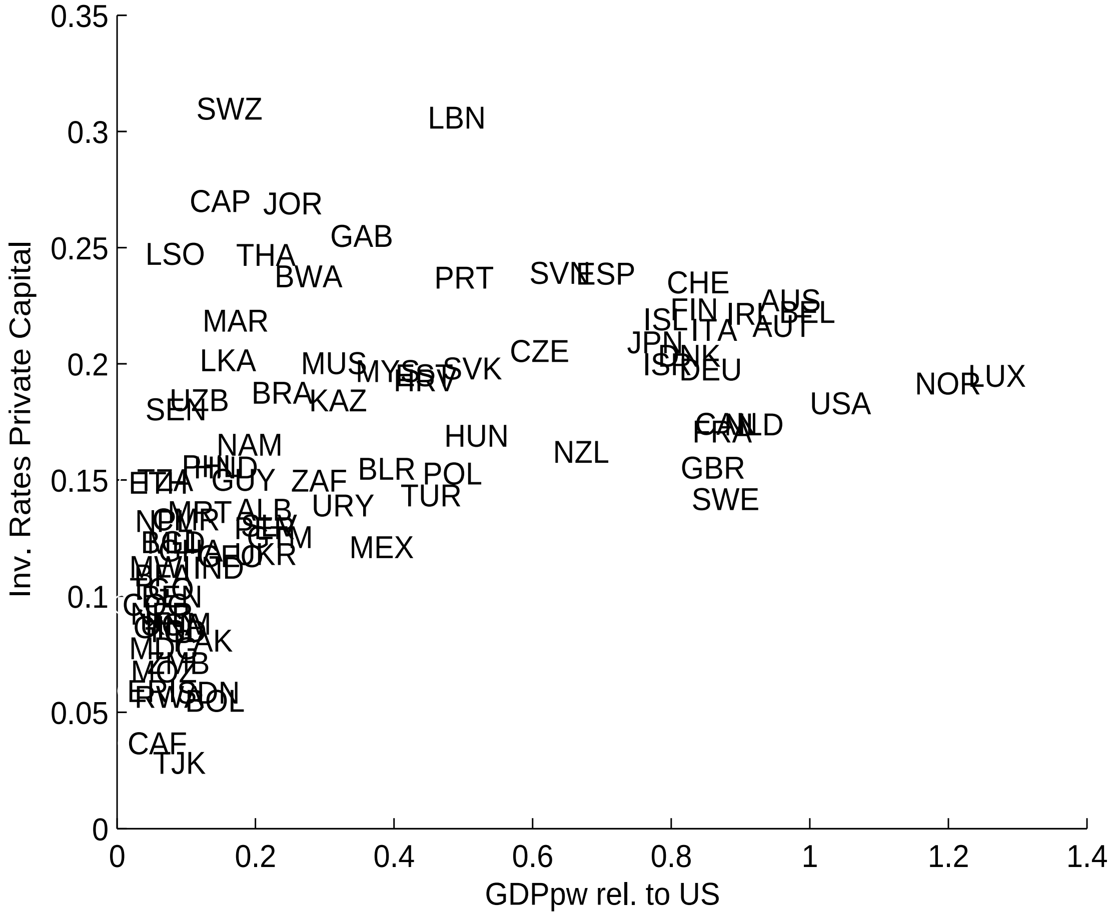

Figure 2: The figure shows the gross domestic product (GDP) per worker of each of the countries in the sample relative to US GDP (GDPpw rel. to US; x-axis) and the ratio of investment in private capital to GDP for the same countries (y-axis). See Section A in the Appendix for country abbreviations.

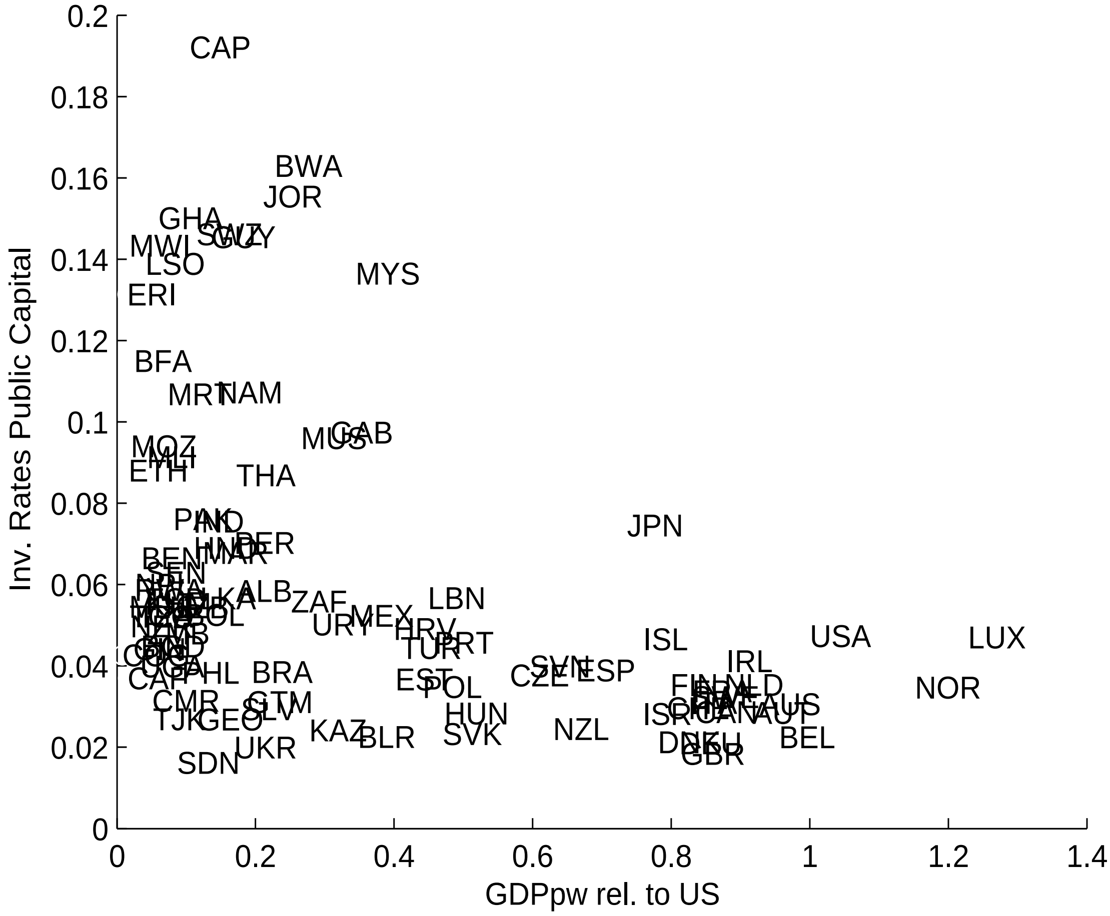

Figure 3: The figure shows the gross domestic product (GDP) per worker of each of the countries in the sample relative to US GDP (GDPpw rel. to US; x-axis) and the ratio of investment (Inv.) in public capital to GDP for the same countries (y-axis). See Section A in the Appendix for country abbreviations.

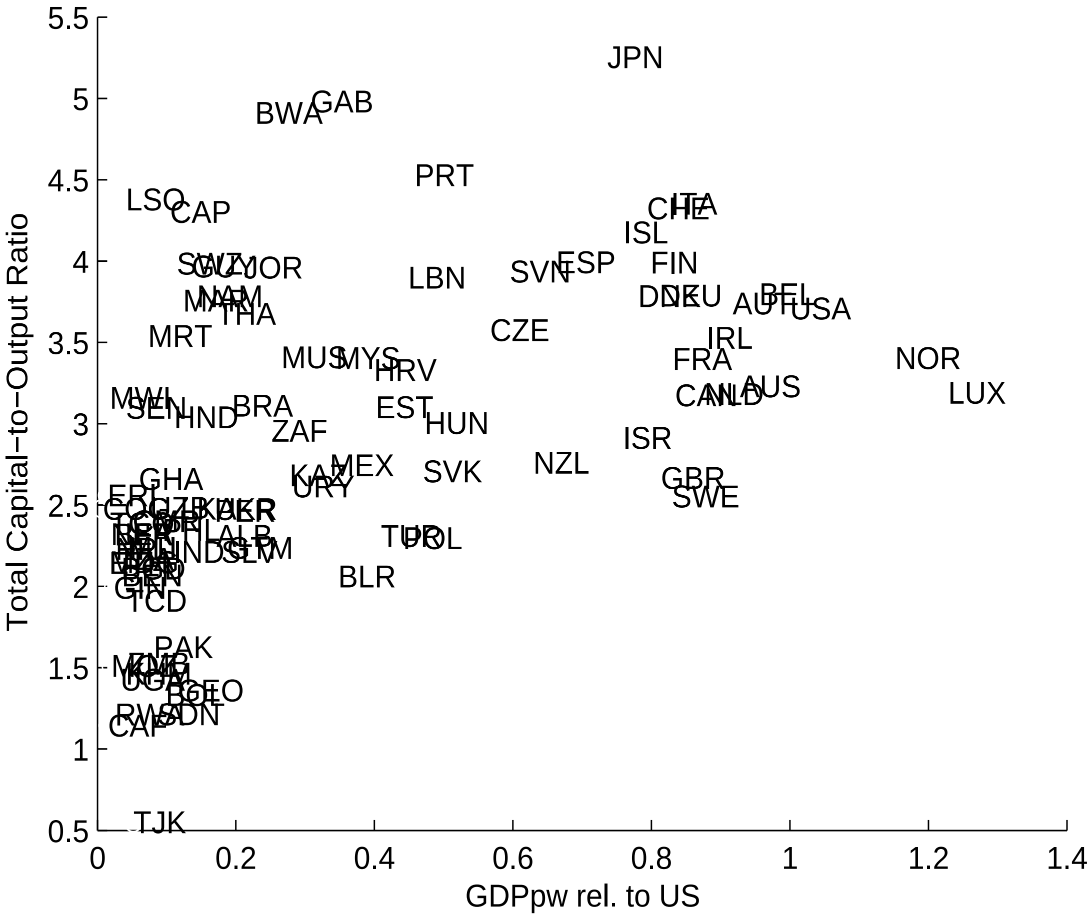

Figure 4: The figure shows the gross domestic product (GDP) per worker of each of the countries in the sample relative to US GDP (GDPpw rel. to US; x-axis) and the ratio of total capital stock to GDP for the same countries (y-axis). See Section A in the Appendix for country abbreviations.

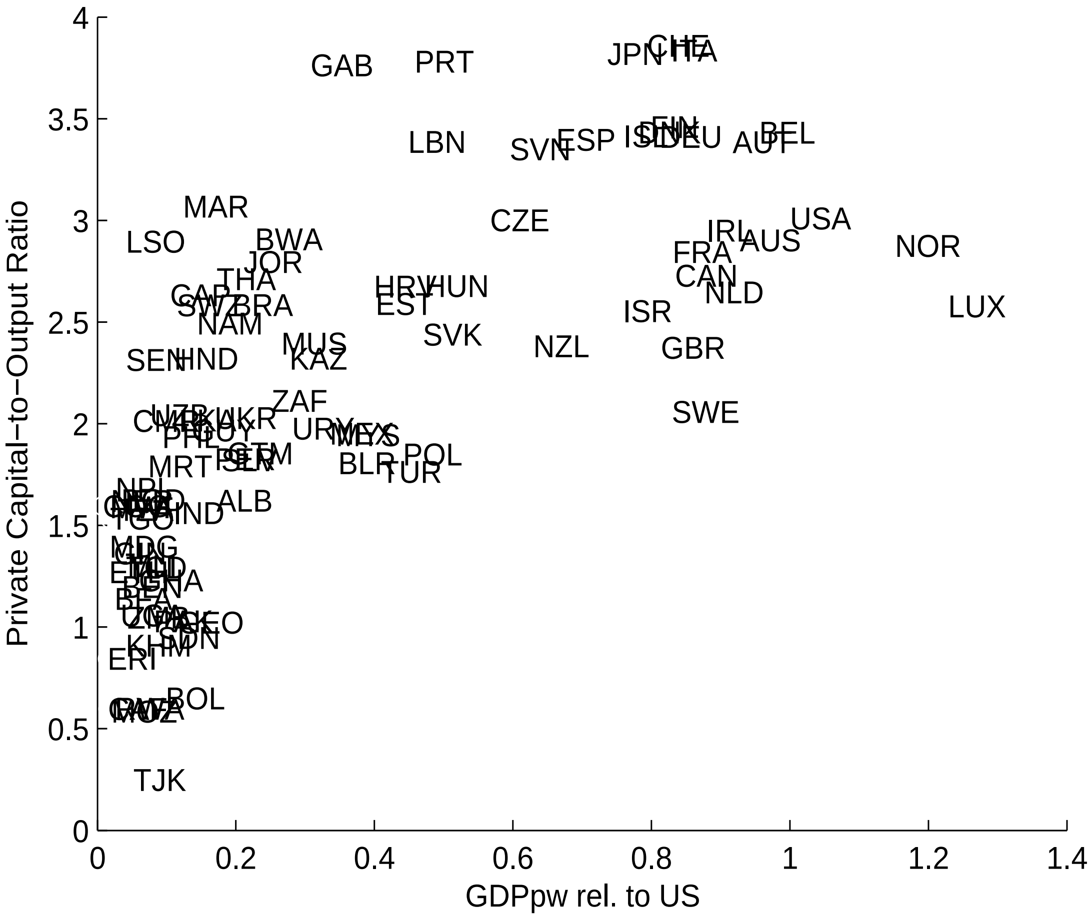

Figure 5: The figure shows the gross domestic product (GDP) per worker of each of the countries in the sample relative to US GDP (GDPpw rel. to US; x-axis) and the ratio of private capital stock to GDP for the same countries (y-axis). See Section A in the Appendix for country abbreviations.

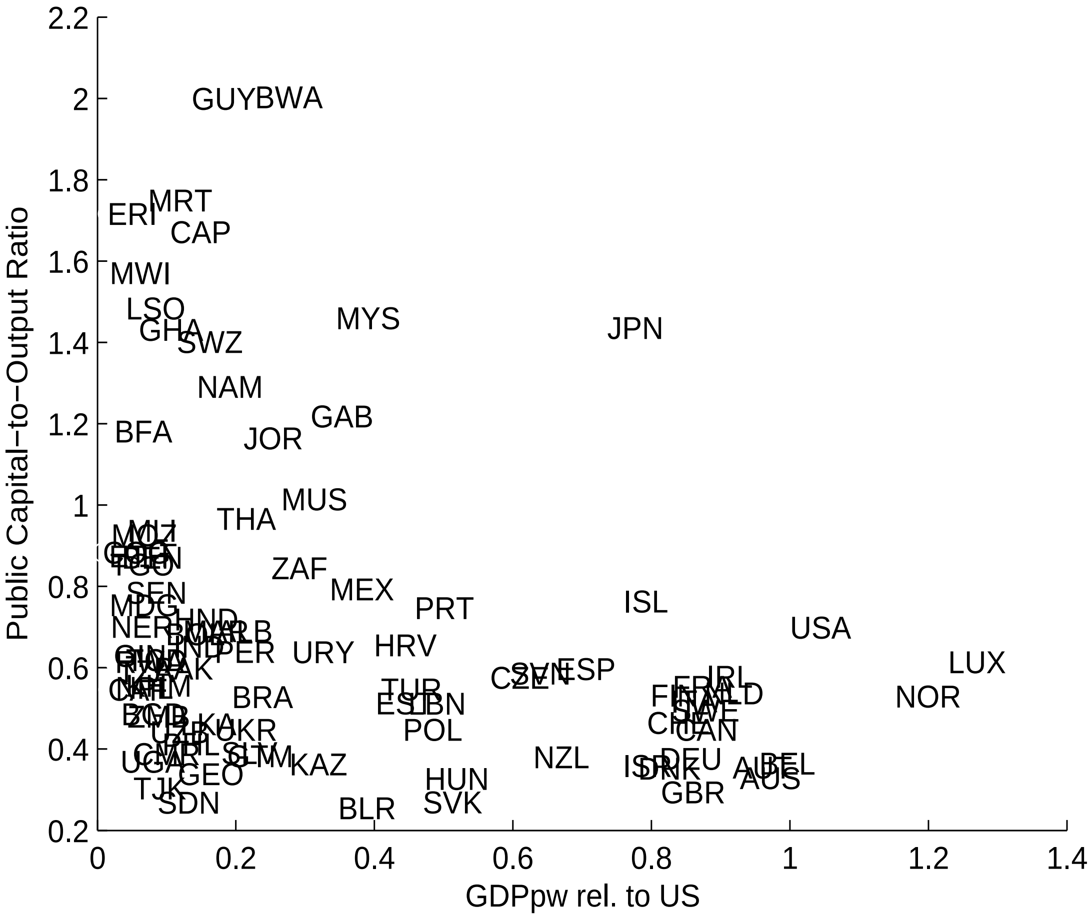

Figure 6: The figure shows the gross domestic product (GDP) per worker of each of the countries in the sample relative to US GDP (GDPpw rel. to US; x-axis) and the ratio of public capital stock to GDP for the same countries (y-axis). See Section A in the Appendix for country abbreviations.

Tables

Table 1: Capital Stocks Ratios

Capital-Labor Ratios	Rich countries (top 10%)	Poor countries (bottom 10%)	*Rich/Poor*
K^{pub}/L	$40,215.4$	$1,309.2$	30.7
K^{priv}/L	$235,689.5$	$1,629.8$	144.6
K^{tot}/L	$275,904.9$	$2,938.9$	93.9
Capital-to-output	Rich countries (top 10%)	Poor countries (bottom 10%)	*Rich/Poor*
K^{pub}/Y	0.5	1.0	0.5
K^{priv}/Y	2.9	1.2	2.4
K^{tot}/Y	3.4	2.2	1.6
Capital Ratios			*Rich/Poor*
K^{pub}			83.2
K^{priv}			283.5
K^{tot}			199.0

Note: This table presents the public, private and total capital stocks, in per worker terms and, divided by output; for the average country in the top 10% (Rich) and bottom 10% (Poor) of the distribution of gross domestic product per worker.

Table 2: Development Accounting

(1)	(2)	(3)	(4)
		$varlog_{\tilde{y}}$ (%)	$varlog_{\hat{y}}$ (%)
No public capital		25	9
With public capital	Wasteless	39	22
	Wasteful	40	23

Note: This table presents the statistics that represent the fraction of the observed cross-country income differences that can be accounted for by differences in factor of production. As defined in Section 5 the statistic $varlog$ computes the cross-country variance of logs of the output predicted by the model over the variance of logs of the observed output. It presents the values for this statistic both in the case of specifying output per worker using capital-to-labor ratios ($varlog_{\tilde{y}}$) and using capital-to-output ratios ($varlog_{\hat{y}}$). It separates the cases that include public capital and the case in which the capital stock is not separated into private and public as in the standard development accounting exercise. In addition, as described in Section 3, when public capital is included, it presents the case in which poor countries waste part of the investment in public capital (Wasteful) and the case when they do not (Wasteless).

Table 3: Development Accounting with Congested Public Capital

(1)	(2)	(3)	(4)	(5)
	Parameter Values		$varlog_{\tilde{y}}$ (%)	$varlog_{\hat{y}}$ (%)
No public capital			25	9
With public capital	$\theta = 0.00, \alpha = 0.32, \lambda = 0.09$		39	22
	$\theta = 0.05, \alpha = 0.32, \lambda = 0.04$		33	16
	$\theta = 0.10, \alpha = 0.32, \lambda = 0.01$		30	13
	$\theta = 0.20, \alpha = 0.32, \lambda = 0.00$		30	12

Note: This table presents the statistic that represents the fraction of the observed cross-country income differences that can be accounted for by differences in factor of production. As defined in section 5 the statistic *varlog* computes the cross-country variance of logs of the output predicted by the model over the variance of logs of the observed output. It presents the values for this statistic both in the case of specifying output per worker using capital-to-labor ratios ($varlog_{\tilde{y}}$) and using capital-to-output ratios ($varlog_{\hat{y}}$). It separates the cases that include public capital and the case in which the capital stock is not separated into private and public as in the standard accounting exercise. The second column presents the values of the calibrated parameters in each case in which public capital is considered. As explained in Section 6 α represents the elasticity of output with respect to private capital, λ the elasticity of output with respect to public capital and θ governs the degree of non-rivalry of public capital. The higher the value of θ the more rival public capital is.

Appendix

A Data

I obtain cross-country time-series data on investment from three sources. For some countries I use data on total and private sector Gross Fixed Capital Formation (GFCF) from the WDI (2016) and ADI (2016) (series codes are NE.GDI.FPRV.ZS and NE.GDI.TOT.ZS). For the most developed countries I use the data on public sector GFCF from the OECD.Stat Extract online database (series codes P51 and GP51P). Having either the public or the private investment allows me to distinguish them from the total investment. The first data point varies with countries (the earliest data points is in 1960). I drop countries for which I do not have data before 1995. The final sample includes 90 countries listed in Table A.1.

From the PWT (Heston et al., 2012) I use the variables $cgdp$, ppp, POP, pi, $xrat$, ci, ki, $rgdpl$, POP, $rgdpch$ and $rgdpwok$.

I first use the share of investment in GDP constant and current prices, ki and ci, respectively; and GDP in constant and current prices ($rgdp$ and $cgdp$, respectively) to construct a deflator to use to obtain investment data for types of capital in constant international dollars (i.e. a common international basket of goods). I define it as $ppp_{def} = (\frac{ki}{100} \times rgdpl)/(\frac{ci}{100} * cgdp)$. I then adjust the investment series of every country by the cross-country differences in the price of investment goods, which is my approximation to reflect the distortion in the accumulation of capital as explained in the main text. It is defined as $pi_{ppp} = pi \times \frac{xrat}{100}$ where pi are prices of investment goods, and $XRAT$ are purchase power parity exchange rates.

In addition, to calculate population growth rates for each country, I compute the average growth rate of the population as reported in the variable POP. In addition, my measure of output per worker is $rgdpwok$, which is also used together with $rgdpch$ (real GDP per capita using the chain rule) and POP to obtain the number of workers in each country.

The following table contains the values for output per worker and stocks of capital used to construct the figures and to perform the development accounting exercise.

Table A.1: Capital Stocks and Output per Worker

Code	Country	y	L	k^p/L	k^g/L	h
ALB	Albania	14172.86	1394570.14	22982.34	9782.90	2.92
AUS	Australia	76408.97	10942302.29	221740.53	25126.72	3.26
AUT	Austria	75536.54	4195950.64	255738.11	26796.93	2.85
BGD	Bangladesh	2818.80	75933183.12	4578.88	1369.99	2.07
BLR	Belarus	28623.24	4534802.20	51705.99	7303.31	3.25
BEL	Belgium	78703.36	4709127.96	270124.82	28704.95	3.07
BEN	Benin	2879.05	3701815.49	3446.68	2506.97	1.78

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Code	Country	y	L	K^p/L	K^g/L	h
BOL	Bolivia	8111.30	4591323.00	5272.90	5536.29	2.60
BWA	Botswana	18730.03	1048277.10	54469.28	37521.79	2.85
BRA	Brazil	15974.63	104795033.30	41276.10	8439.25	2.53
BFA	Burkina Faso	2029.91	7440546.41	2312.26	2397.16	1.19
KHM	Cambodia	3355.56	8149795.23	3049.60	1865.38	1.84
CMR	Cameroon	4173.19	8082136.79	8405.17	1618.46	2.12
CAN	Canada	68692.18	18235068.06	187373.78	30760.32	3.43
CAP	Cape Verde	8648.99	230341.51	22762.60	14453.09	1.60
CAF	Central African Republic	1254.94	2273093.11	750.52	685.59	1.66
TCD	Chad	3377.54	4153771.71	4370.00	2091.01	1.22
COG	Congo, Dem. Rep.	628.03	26754591.82	1001.42	554.85	1.51
HRV	Croatia	32876.37	2002782.00	87947.08	21554.26	3.20
CZE	Czech Republic	46700.11	5110889.94	140158.42	26869.80	3.55
DNK	Denmark	64299.53	2891194.92	220736.89	22647.64	3.20
SLV	El Salvador	14723.08	2535670.18	26816.15	5754.35	2.50
ERI	Eritrea	1190.07	2862245.69	1004.33	2042.46	1.58
EST	Estonia	33081.21	663988.95	85674.95	16951.48	3.38
ETH	Ethiopia	1384.03	43269977.61	1758.89	1209.47	1.38
FIN	Finland	65769.22	2635862.76	227548.92	34975.79	2.90
FRA	France	68386.06	29643526.60	194574.47	37841.78	3.07
GAB	Gabon	25340.41	603448.26	95345.54	30868.71	2.63
GEO	Georgia	9550.50	2457162.98	9763.24	3232.60	3.38
DEU	Germany	66826.37	41648011.67	227982.64	25149.81	3.45
GHA	Ghana	4927.93	10343978.12	6058.82	7052.30	2.31
GTM	Guatemala	15423.99	5351317.28	28601.28	5906.18	1.81
GIN	Guinea	1921.46	4232318.64	2618.49	1208.93	1.24
GUY	Guyana	11205.38	299087.07	22049.77	22403.89	2.70
HND	Honduras	9099.45	3142916.02	21122.32	6545.19	2.13
HUN	Hungary	38902.16	4252739.48	104182.32	12714.45	3.33
ISL	Iceland	62589.28	175763.70	213645.21	47755.00	3.15
IND	India	9010.21	452737123.30	14062.38	5878.58	2.14
IRL	Ireland	72446.14	2225546.67	213748.37	41899.31	3.37
ISR	Israel	62451.93	3065681.89	159558.25	22390.20	3.43
ITA	Italy	68233.30	25264808.08	261711.00	35340.29	2.86
JPN	Japan	60642.57	65756581.73	231608.60	87045.01	3.27
JOR	Jordan	17377.03	1645532.69	48583.88	20228.51	2.85
KAZ	Kazakhstan	22851.09	8324032.37	53026.60	8286.09	3.21
LBN	Lebanon	36942.18	1418174.43	125096.12	18913.96	2.53

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Code	Country	y	L	K^p/L	K^g/L	h
LSO	Lesotho	3385.25	790864.27	9803.86	5023.34	2.06
LUX	Luxembourg	101180.47	371691.88	260823.03	62121.20	3.14
MDG	Madagascar	1434.05	10426541.89	2002.53	1081.02	1.93
MWI	Malawi	1456.29	6954303.61	2315.31	2287.08	1.85
MYS	Malaysia	28367.99	11916745.48	55149.77	41401.66	3.02
MLI	Mali	3570.02	3856654.52	4643.08	3344.51	1.30
MRT	Mauritania	6005.45	1034602.63	10756.30	10505.42	1.80
MUS	Mauritius	21871.99	601378.51	52380.69	22184.16	2.71
MEX	Mexico	27625.47	48607700.78	53925.36	21912.63	2.70
MAR	Morocco	10165.04	11270778.59	31192.31	7014.27	1.88
MOZ	Mozambique	1649.69	10616406.41	964.98	1528.09	1.29
NAM	Namibia	11820.91	866160.90	29473.71	15255.98	2.13
NPL	Nepal	2139.09	15500444.09	3596.14	1178.43	1.75
NLD	Netherlands	72177.94	8880232.72	191020.10	38756.42	3.22
NZL	New Zealand	51829.68	2279952.71	123455.58	19702.36	3.13
NER	Niger	1583.43	5234393.03	2565.58	1109.85	1.29
NOR	Norway	94862.99	2488800.60	272763.96	50219.13	3.27
PAK	Pakistan	6680.65	63405177.01	6871.62	3996.30	1.90
PER	Peru	13931.12	15407820.47	25446.13	8903.76	2.72
PHL	Philippines	7694.24	41468867.58	14889.88	3172.00	2.64
POL	Poland	36343.29	17679812.50	67228.45	16271.34	3.21
PRT	Portugal	37706.00	5632516.96	142559.47	28174.29	2.44
RWA	Rwanda	2083.36	5440633.93	1247.66	1279.70	1.77
SEN	Senegal	3392.85	5336701.53	7850.93	2661.09	1.44
SVK	Slovak Republic	38697.01	2726075.58	94382.72	10411.02	3.55
SVN	Slovenia	49037.83	1017251.90	164279.36	28725.77	3.33
ZAF	South Africa	20678.94	17842604.54	43681.63	17473.05	2.87
ESP	Spain	54538.80	23305933.19	185253.39	32541.00	2.99
LKA	Sri Lanka	9852.15	8695666.23	19873.81	4544.06	2.94
SDN	Sudan	7125.58	14110223.46	6748.99	1912.24	1.54
SWZ	Swaziland	9429.97	530182.77	24362.68	13214.99	1.90
SWE	Sweden	68296.17	4800667.82	140583.32	33822.84	3.28
CHE	Switzerland	65369.89	4662238.49	252256.87	30386.22	3.60
TJK	Tajikistan	4262.69	3099283.84	1058.56	1293.97	2.99
TZA	Tanzania	2387.09	20682191.83	3764.24	1413.28	1.91
THA	Thailand	14154.38	37796310.94	38376.21	13673.44	2.56
TGO	Togo	1502.92	3212055.98	2304.58	1283.98	1.99
TUR	Turkey	33704.55	24095164.23	59459.16	18426.52	2.33

Continued on next page

Code	Country	y	L	K^p/L	K^g/L	h
UGA	Uganda	2742.71	13416249.76	2901.47	1012.88	2.03
UKR	Ukraine	13915.41	22990637.82	28227.12	6225.12	3.17
GBR	United Kingdom	67025.06	31876990.26	159068.60	19682.46	3.42
USA	United States	82359.26	155814672.60	247873.07	57553.27	3.64
URY	Uruguay	23125.47	1672653.34	45712.36	14766.68	2.59
UZB	Uzbekistan	6240.38	11967308.81	12748.09	2757.21	2.93
ZMB	Zambia	3559.00	5738271.61	3725.76	1708.86	2.39

Note: y is gross domestic product (GDP) per worker for the year 2010 in 2005 international dollars, L is the number of workers in 2010, K^p/L is the stock of private capital per worker for the year 2010 in 2005 international dollars, K^g/L is the stock of public capital per worker for the year 2010 in 2005 international dollars and h is the measure of the stock of human capital for the year 2010. All the statistics used in the text can be calculated by using the information contained in this table.

B Initial Capital Stock

I use equations (10) and (11) to compute an initial measure of the stocks. To analyze the impact of this method of calculating initial stocks, I follow Caselli (2005) by computing the portion of the initial stock (which I call η_{K^i} for $i = p, g$) that survives the sample period, given the depreciation rate δ. In other words, what fraction of the initial stock is part of the stock in 2010? This is given by

$$\eta^j_{K^i} = \frac{(1-\delta_i)^t K^i_0}{(1-\delta_i)^t K^i_0 + \sum_{i=0}^t (1-\delta_i)^t {I^i}_{t-i}}$$

for country j and for $i = p, g$, where t is the number of periods for which data for that country are available and 0 represents the year for which the first data point on investment for that country is available. The average across countries for η_{K^p} is 0.14 and for η_{K^g} is 0.15. In addition, the values for each country are positively correlated with GDP per worker, indicating that public capital stock for rich countries may be overestimated. Most of the countries that drive up the η values are the those that were formerly part of the Soviet Union because data on investment are available only from the beginning of the 1990s. Although the importance of the initial capital stock is on average not too high, I perform the development accounting exercise for two different and smaller sub samples of countries. The first one includes those with values of $\eta < 0.25$ and the second those with values of $\eta < 0.10$. The first sample consists of 70 countries with values of $\eta_{K^p} = 0.09$ and $\eta_{K^g} = 0.11$. The second sample consists of 37 countries with values of $\eta_{K^p} = 0.04$ and $\eta_{K^g} = 0.07$. Interestingly, when these two sub samples are used both the results in terms of the capital stocks and development are qualitatively similar to the ones obtained with the original sample. For example, with the first sub sample of 70 countries, the standard accounting without public capital renders $varlog_{\tilde{y}} = 0.25$. With public capital (wasteless), $varlog_{\tilde{y}} = 0.37$ (it is 0.39 using the whole sample). Using the second sub sample of 37 countries, $varlog_{\tilde{y}} = 0.26$ without public capital, and $varlog_{\tilde{y}} = 0.40$ with public capital (wasteless).

C Measuring Income from Private Capital in the NIPA accounts

In this Section I explain how I obtain the income from private capital using NIPA accounts. I basically follow the procedure outlined in in Cooley and Prescott (1995). Using their terminology, I first define income from private capital as unambiguous income (UI) plus its ambiguous component (AI) plus depreciation (DEP_p). Thus

$$Inc_p = UI + AI + DEP_p. \tag{35}$$

The unambiguous component of private capital income is given by

$$UI = Rental\, income + Corporate\, profits\ + Net\, interest - Taxes\, on\, coporate\, profits + T_g \tag{36}$$

Note that to be consistent with the definition of output I have subtracted the amount of corporate profits taxes and I have added the amount of taxes paid to fund the government investment in public capital.

As for the ambiguous component of income from private capital, it includes proprietors income (PI) and the difference between net national product (NNP) and national income (NI). Here I follow the same strategy as in Cooley and Prescott (1995): I assign this ambiguous income according to the share of private capital in measured GNP (which I call α_M), which is defined as

$$\alpha_M = \frac{Inc_p}{GNP},; \tag{37}$$

thus,

$$Inc_p = \alpha_M GNP. \tag{38}$$

Therefore,

$$AI = \alpha_M[PI + (NNP - NI)]. \tag{39}$$

Then from (35) and (38) we have

$$UI + AI + DEP_p = \alpha_M GNP, \tag{40}$$

and substituting (39) yields

$$UI + \alpha_M(PI + NNP - NI) + DEP_p = \alpha_M GNP. \tag{41}$$

From (41) we can solve for α_M as follows:

$$\alpha_M = \frac{UI + DEP_p}{GNP - (PI + NNP - NI)}. \tag{42}$$

I calculate UI by using data on the three terms on the right-hand side of (36) obtained from NIPA Table 1.12 for each year from 1929 to 2010; specifically, lines 12, 13 and 18 are rental income, corporate profits and net interest, respectively. In addition, PI (line 9) is obtained. From NIPA Table 6.4 I obtain DEP_p (line 1) and from NIPA Table 1.7.5, NNP (line 14), NI (line 16), and GNP (line 4) for the same period. I compute α_M for each year from 1929 to 2010 (the average over this period is 0.32). By using the value obtained for α_M in each year and (38) I calculate I_p from 1929 to 2010.

D Development Accounting with Sub samples and Alternative Statistics

In the paper I have used the *varlog* statistic to measure the contribution of factors of production to explain cross-country income differences. However, in the literature on development accounting we can find alternative statistics (see Caselli, 2005, for a review). The most common are those that compare different parts of the distribution of income. The most standard are those that compare the bottom and top 10% of the income distribution, sometimes associated with comparing the average rich and poor countries. Suppose I want to calculate the output predicted by the specification of the production function for a rich country (represented by the top 10% of the distribution) and a poor country (represented by the bottom 10% of the distribution), denoted by y_{90} and y_{10}, respectively. Thus, using (2), the ratios of predicted outputs can be expressed as

$$\frac{y_{90}}{y_{10}} = \frac{A_{90}}{A_{10}}\frac{\tilde{y}_{90}}{\tilde{y}_{10}}. \tag{43}$$

The left-hand side of equations (43) -the ratio of rich-to-poor country income- is observable through data on countries GDPs per worker. We want to dichotomize this ratio of observed aggregate income into its component parts, as represented by the expressions on the right-hand side of equations (43). Now, the ratio of TFPs -that is, $\frac{A_{90}}{A_{10}}$, between rich and poor countries- is not observable and so I measure the ratios $\frac{\tilde{y}_{90}}{\tilde{y}_{10}}$ given values for the parameters and capital stocks. This allows the determination of the portion of the differences in the observed income ratios that can be explained by differences in factors of production. This is done by computing

$$9010ratio_{\tilde{y}} = \frac{\tilde{y}_{90}/\tilde{y}_{10}}{y_{90}/y_{10}}. \tag{44}$$

The same calculations can be done for the average countries in the top and bottom 25% of the income distribution as well as the average country below and above the median. I call these ratios $7525ratio_{\tilde{y}}$ and $5050ratio_{\tilde{y}}$, respectively. In Table A.2 I present the results.

As is clear in the table, when moving to compare the tails of the distribution the value of the statistic falls, which means the more heterogeneous the group of countries we compare, the less the variation in factors of production explains income differences between the groups.

Table A.2: Development Accounting

(1)	(2)	(3)	(4)
	$5050ratio_{\tilde{y}}$ (%)	$7525ratio_{\tilde{y}}$ (%)	$9010ratio_{\tilde{y}}$ (%)
No public capital	34	18	11
With public capital	43	27	18

Note: This table presents the statistics that represent the fraction of the observed cross-country income differences that can be accounted for by differences in factor of production. I use three different statistics defined above: the $9010ratio_{\tilde{y}}$, the $7525ratio_{\tilde{y}}$ and the $5050ratio_{\tilde{y}}$, that use the capital-to-labor ratios as defined in Section 5. It separates the cases that includes public capital and the one in which the capital stock is not separated into private and public as in the standard accounting exercise. In addition, as described in Section 3, when including public capital it only presents the case in which countries do not waste any portion of the investment in the public capital stock.

When using the 9010*ratio* the fractions $\frac{\tilde{y}_{90}}{\tilde{y}_{10}}$ and $\frac{\hat{y}_{90}}{\hat{y}_{10}}$ take the value of 6.7 and 2.5 in the standard accounting, respectively (in the case of wasteless investments). In the data, it is 59.8. As a result, $9010ratio_{\hat{y}} = 11\%$.[26] Due to the inclusion of public capital its value increases. The value of the fraction $\frac{\tilde{y}_{90}}{\tilde{y}_{10}}$ is 9.6. As a result, and for the same reasons explained for *varlog*, the value of $9010ratio_{\tilde{y}}$ increases 63% (it changes from 0.11 to 0.18). In the case of the 5050*ratio* and 7525*ratio*, their values also increases but to a lesser extent. Therefore, the results are qualitatively similar to the ones obtained using *varlog* as summarized in Section 5.

Another interesting aspect to explore is to perform the development accounting exercise for different samples of countries. Groups of countries can be defined either through geography or through other considerations. In Table A.3 I present $varlog_{\tilde{y}}$ for differen sub samples.

With the exception of the group of European countries, the separation of the sample does not seem to make a difference in terms of the value of *varlog* for the standard development accounting without separating capital into public and private. As for the change in *varlog* due to the inclusion of public capital, the main differences with respect to the value for all the sample comes from considering the Americas and the group of OECD countries.

[26]For the sake of comparison, the fraction $\frac{\tilde{y}_{90}}{\tilde{y}_{10}}$ computed in Caselli (2005) is 7 using the standard accounting approach. What differs is the fraction in the data, which takes the value of 21 in Caselli (2005) and 60 in my sample.

Table A.3: Development Accounting in Sub samples Using $varlog_{\tilde{y}}$ (%)

(1)	(2)	(3)	(4)
Sub Sample	Obs.	No public capital	With public capital
All	90	25	39
Above median	45	24	43
Below median	45	29	45
OECD	31	27	54
Non-OECD	59	28	41
Africa	32	28	38
Europe	30	19	34
Americas	11	27	64
Asia	17	27	41

Note: This table presents the statistic $varlog_{\tilde{y}}$ that represents the fraction of the observed cross-country income differences that can be accounted for by differences in factor of production when using the capital-to-labor ratios as defined in Section 5. It separates the cases that includes public capital and the one in which the capital stock is not separated into private and public as in the standard accounting exercise. In addition, as described in Section 3, when including public capital it only presents the case in which poor countries do not waste the investment in public capital. The table shows the values of the statistic for different groups of countries: above and below the median, members and non-members of the Organisation for Economic Co-operation and Development (OECD) and then when they are grouped by continent.

E Constant Elasticity of Substitution Between Private and Public Capital

The theoretical framework presented in Section 2 proposes a technology that assumes a unitary elasticity of substitution between public and private capital stocks. A natural extension would be to consider a technology that considers a constant elasticity of substitution between these two types of capital. This extension relies on the idea that due to the nature of the service that public capital provides (affected by the degree of non-excludability and rivalry) there are public and private capital goods that can be more substitutable than others (government buildings vs. national defense capital). In reality it could be the case that due to the composition of the stock, the elasticity of substitution between the two types of capital is not one and thus, we would need to consider a more generic technology. This line of reasoning was pursued in Baier and Glomm (2001). These authors introduce a technology with constant elasticity of substitution (CES) between the two types of capital and aggregate constant returns to scale in a one-sector growth model. Based on this framework, I conduct the empirical analysis. The technology used in Baier and Glomm (2001) reads as follows:

$$Y = A\left[\gamma(K^g)^\rho + (1-\gamma)(K^p)^\rho\right]^{\frac{\alpha}{\rho}} (L_t)^{1-\alpha}, \tag{45}$$

where γ and α are share parameters and ρ is the one associated with the elasticity of substitution between private and public capital, which, in this case is $\frac{1}{1-\rho}$. When $\rho \to 1$ private and public capital become perfect substitutes and when $\rho \to -\infty$, they are perfect complements.

As is well understood, when $\rho \to 0$ we are in the special case of unitary elasticity of substitution.[27]

We need to assign values to γ, ρ and α to then use the measures of inputs and conduct the development accounting exercise as it is done in Section 5. In this way, we can define the share of each type of capital in output as

$$\theta_p = \frac{MPK^p K^p}{Y} \quad \text{and} \quad \theta_g = \frac{MPK^g K^g}{Y}, \tag{46}$$

where MPK^p and MPK^g indicate the marginal productivity of private and public capital, respectively. Thus, by using (45) we have that

$$\theta_p = \frac{\alpha(1-\gamma)(K^p)^\rho}{\gamma(K^g)^\rho + (1-\gamma)(K^p)^\rho} \tag{47}$$

and

$$\theta_g = \frac{\alpha\gamma(K^g)^\rho}{\gamma(K^g)^\rho + (1-\gamma)(K^p)^\rho}. \tag{48}$$

Thus, given values for ρ, θ_p and θ_g we have two equations to solve for α and γ. The main challenge to calibrate this production function is to obtain values for ρ, as one would need prices and quantities of both types of capital. There is no clear consensus in the literature and we can find either high elasticities of substitution (see Nadiri and Mamuneas, 1994) or low elasticities of substitution (see Berndt and Hansson, 1992). I follow Baier and Glomm (2001) and I perform the empirical analysis for different values of this parameter, specifically for $\rho = \{-0.85, -0.50, 0.25, 0.75\}$. As in Section 4, I proceed by computing the income of each type of capital to obtain θ_p and θ_g to then solve for γ and α for each value of ρ. I use my measures of inputs obtained in Section 3 and I perform the accounting exercise. The calibrated values for the parameters and the results of the accounting exercise are presented in Table A.4. As before, we assume the values of these parameters are constant across countries.

As expected, $\alpha = 0.38$ for all values of ρ, and it is also equal to the sum of the shares of both capital stocks in income. As ρ increases, γ also increases. However, the contribution of factors of production is around the same ($varlog_{\tilde{y}} = 33$) and, compared with the standard accounting without public capital, it represents an increase in the explanatory power of factors

[27] This technology can be modified to be nested to the one considered in Section 2. We would need to modify the parameterization so that in the limiting case of $\rho \to 0$, we have the specification of the production in the benchmark analysis; that is, $Y = A\left[\gamma(K^g)^\rho + (1-\gamma)(K^p)^\rho\right]^{\frac{\alpha}{\rho}} (L_t)^{1-\alpha+\alpha\gamma}$

Table A.4: Development Accounting with CES between K^g and K^p

(1)	(2)	(3)
	Parameter Values	$varlog_{\tilde{y}}$ (%)
No public capital		25
With public capital	$\rho = -0.85, \alpha = 0.38, \gamma = 0.09$	33
	$\rho = -0.50, \alpha = 0.38, \gamma = 0.13$	33
	$\rho = 0.25, \alpha = 0.38, \gamma = 0.27$	33
	$\rho = 0.75, \alpha = 0.38, \gamma = 0.42$	32

Note: This table presents the statistics that represent the fraction of the observed cross-country income differences that can be accounted for by differences in factors of production. As defined in Section 5 the statistic *varlog* computes the cross-country variance of logs of the output predicted by the model over the variance of logs of the observed output. It presents the values for these statistics when I specify output per worker using capital-to-labor ratios ($\tilde{y}$) and a technology that exhibits a CES between private and public capital. It separates the case that includes public capital and the one in which the capital stock is not separated into private and public capital as in the standard accounting exercise. In addition, it shows the values of the statistics and their change with respect to the case in which the separation between public and private capital stock is not considered. The second column presents the values of the calibrated parameters in each case in which public capital is considered. α represents the share of capital in output (private and public), γ governs the weight of each type of capital and ρ governs the elasticity of substitution between them, which is $1/(1-\rho)$.

of production of 34%.

F A Higher Rate of Return of Public Capital Investments

As discusses in Section 4, NIPA does not provide measures of the income from the public capital. The method I follow to calculate this income uses the data NIPA provides for the stock of public capital of the United States, which is multiplied by an estimate of the return rate of the private capital stock. Assuming that these returns are the same is the natural path providing the theoretical framework I propose. However, some strands of the literature have estimated rate of returns for different infrastructure projects. One important reference is the work of Fernald (1999), who using the US interstate highway system estimates a return rate of 12%. Although using a higher return rate for public capital means an important departure from my proposed theoretical framework, one could obtain new values from the income of public capital and recalibrate λ to determine the extent to which this may change the results. In this way, by assuming a rate of return for public capital investments of 12%, the value of λ is now 0.10 and the value of α is 0.32. Given my measures of the stocks of public capital, the value of $varlog$ is now 0.40 instead of 0.39.

G Defense Capital

The measures of capital stocks obtained include military or defense capital. Although national security is a public good that contributes to the production of output by firms (see Ranasinghe and Restuccia, 2018), it is possible that some countries are special in the sense that they are involved in temporary conflicts or have a stock of military capital for other reasons not closely related to the production of security. Thus, it would be helpful to separate the stocks of defense capital from the other types of public capital. Although data on cross-country military expenditures are available, these expenses are not decomposed into the different categories such that time series of expenditures in military equipment and structures can be determined. However, one can argue that the United States is part of the group of countries with a high proportion of military capital; fortunately, the data allow disaggregating it from the total stock. In the period 1929-2010 national defense capital represents 31% of total US public capital stock, reaching a maximum of 62% in 1945. This information allows me to reestimate the depreciation rates for public capital without including defense capital (see Section H in this Appendix).

Given the data restrictions just mentioned, I address the importance of defense capital by reestimating the public capital stocks for each country with the new depreciation rate and by using the available information for the United States and the procedure explained in Section 4 to recalibrate the parameters of the production function. I consider an extreme case by assuming that military capital does not have any return. In this case, $\alpha = 0.32$ and $\lambda = 0.05$. Thus (and as expected) the elasticity of output with respect to public capital is reduced. As a result, $varlog_{\tilde{y}}$ falls to 0.34. Therefore, in this extreme case the fraction of observed cross-country income differences that can be explained is still higher than in the standard accounting without public capital but lower than when the return of defense capital is included.

H Depreciation Rates

As detailed in Section 3, in my methodology to measure public capital stocks I take the average scrapping depreciation rate for US government capital to calibrate the depreciation rate of each type of capital. It is also assumed to be the same across countries. In this Section of the Appendix I consider the possibility of capital-specific depreciation rates as well as country-specific depreciation rates.

H.1 Public and Private Capital Depreciation Rates

I separate capital into two broad categories: public and private and, using US depreciation data reported by NIPA, their average depreciation rate is almost the same. As the referee points out, this finding is at odds with the notion that the depreciation rate of public capital is lower than private capital, because the stock of public capital may be composed by a higher proportion of goods with low depreciation relative to private capital. For example, a higher proportion of structures and a lesser proportion of equipment than private capital. However, I do not find differences in the composition of the stock reported by NIPA. In US data, in the period 1929-2010, the proportion of structures is on average 80% of the stock of both private and public capital. The proportion of equipment is 15% in private capital and 14% in public capital.

The proportion of structures in the stock of public capital raises if we exclude defense capital. In the same period, the proportion of structures in the US raises to 90% and of equipment falls to 4%. For these reasons, if we exclude defense capital the average depreciation rate of public capital falls to is 3%. There are good reasons to include defense capital in the stock of capital. National security is probably one of the most non-rival forms of public capital and affects the productivity of the private sector (see Ranasinghe and Restuccia, 2018, for evidence in this direction). Nevertheless, as a robustness analysis, we can apply these two different depreciation rates to the cross-country investment data to generate new estimates for capital stocks for every country. Then we can recalibrate the elasticity of output with respect to public capital assuming that defense capital has no return. In this case we obtain a value of $\lambda = 0.05$. I have proceeded in this way in the development accounting exercise and I obtain a value of $varlog_{\tilde{y}}$ of 0.34, similar to one obtained in my benchmark case.

H.2 Cross-Country Differences in Depreciation Rates

Regarding cross-country comparisons of depreciation rates, differences can arise due to the different composition of the stock across countries. Changes in the depreciation rate would change the relative weight of past versus recent investments in the perpetual inventory method. The development literature that uses the perpetual inventory method normally assumes that the depreciation rate is the same across countries. The reason is that the development accounting results are not sensitive to this assumption. In Caselli (2005) it is assumed to be 6% and then the author conducts a sensitivity analysis to show that the changes in his results are minimal.

As for the depreciation rate of public capital, Arestoff and Hurlin (2006) estimate public capital stocks for 26 developing countries. An interesting aspect of their methodology is the use of different depreciation rates. The idea is that depreciation rates in poor countries need not to be the same as the one calculated for rich countries, given the different composition of the public capital stocks observed in developing countries. For this reason, using data on the depreciation rates for different types of assets in the United States and the weight of some assets in Latin American countries, they provide estimates of depreciation rates for public capital in developing countries. The estimated values range from 2.52% to 2.77%. Thus, there is not much variation across these developing countries and, on average, they are in the ballpark of the one I obtained for the United States when I do not include defense capital, i.e. 3%. I recalculated public capital stocks for every country using a depreciation rate of 3%. I use my previous estimate of the depreciation rate of private capital of 4.6% for every country. I use these new estimates of capital stocks to perform the development accounting exercise. The value of *varlog* stays the same so my results are not sensitive to this change.

I Model Equilibrium

Following Glomm and Ravikumar (1999), we can define a competitive equilibrium for an arbitrary fiscal policy defined by $\tau \in [0,1]$ and $K^g_{t+1} > 0$ for all $t = 0, 1, ...$ For this fiscal policy, a competitive equilibrium are sequences of allocations $\{c_t, L_t, k^p_{t+1}\}^{\infty}_{t=0}$ and prices $\{p_t, r_t, w_t\}^{\infty}_{t=0}$ such that

1. $\{c_t, k^p_{t+1}\}^{\infty}_{t=0}$ solve the representative household's problem,
2. $\{L_t, Nk^p_t\}^{\infty}_{t=0}$ solve the firm's problem,
3. $C_t + K^p_{t+1} = (1-\tau_t)A\left(K^g_t\right)^{\lambda}\left(K^p_t\right)^{\alpha}(L_t)^{1-\alpha} + (1-\delta_p)K^p_t$ and
4. $L_t = N$ for $t = 0, 1, ...$

It is important to know that in Glomm and Ravikumar (1999) the authors provide the proofs for the existence and uniqueness of this competitive equilibrium, both are important features to conduct the empirical analysis. Note that, as already mentioned, households and firms take the government policy as given, and in that case the Euler equation and the transversality condition are necessary and sufficient conditions to solve for the optimal allocations in competitive equilibrium. The approach followed by the authors is to formulate an artificial planning problem to then show that given arbitrary sequences of tax rates and stocks of public capital, the solution to that problem is equivalent to the one of the competitive equilibrium. Then they show that the solution to the planning problem is unique. In addition, the authors also provide a sufficient condition for the existence of an optimal public policy. They follow the dual approach where they obtain private decision rules taking public policies as given. The government then chooses public policies to maximize an indirect utility function taking the private decision rules as given. The proofs are also applicable for a more generic case that includes the possibility of different forms of congestion of public capital. I use those theoretical results in my empirical analysis of a special case of congested public capital (see Section 6).