ECON 7343 - Homework 1

Due Friday, Sep. 8th

1. Let the intensive production function be $y_t = k_t^{\alpha}$. Using the Solow model with a constant savings rate of s, solve for the value of k^*. Given this steady state value, solve for the steady state value of output per capita, y^*, and the steady state value of consumption per capita, c^*.

2. Show that if $F(K_t, N_t)$ has the properties that

$$F_K > 0,\ F_{KK} \leq 0 \ \text{ and } \ F_N > 0,\ F_{NN} \leq 0. \tag{1}$$

   then it must be the case that $f'(k_t) > 0$ and $f''(k_t) \leq 0$.

3. Another common production function used in macro is the following:

$$Y = (AK^{(\sigma-1)/\sigma} + BL^{(\sigma-1)/\sigma})^{\sigma/(\sigma-1)} \tag{2}$$

   Using this production function A) what are the marginal products of capital and labor? B) What is the MRTS between capital and labor? C) What is the elasticity of substitution between capital and labor? D) What does the production function look like as σ goes to 1? As σ goes to infinity? As σ goes to 0? [To get an idea of where these lead, consider what the isoquants look like in each case]

4. The production function is $y = k^{1/2}$. Suppose that capital per worker is 400. The savings rate is 50%. The depreciation rate is 5%. There is no population growth or technical change. A) Is the country at its steady state of output per capita, above its steady state, or below its steady state? B) Now suppose there is a large immigration into the country. The population quadruples. The new immigrants do not bring any new capital with them. Following the immigration is output per capita above, below, or at the steady state? C) What is the growth rate of income per capita immediately after the immigration?

5. There are two countries, A and B, each described by the Solow model. In both, the production function is $k^{1/2}$, and the depreciation rate is 0.02. There is no technological change. In A, the savings rates i 0.2, and population growth is zero. In country B the savings rate is 0.4 and population growth is 0.02. Both countries are observed to have income per capita of 5 at time zero. Draw a graph with time on the horizontal axis and income per capita on the vertical axis showing how the level of income per capita in the two countries will evolve over time.

6. Consider a Solow model in which the growth rate of population is endogenous. Specifically

$$n = \begin{cases} n_h & \text{if } y < \bar{y} \\ \\ n_l & \text{if } y \geq \bar{y} \end{cases} \tag{3}$$

   where $n_h > n_l$. Production is $y = k^{\alpha}$. Capital depreciates at δ and there is no technological change. The savings rate is exogenous. A) Draw graphs showing the different possible configurations of the

steady state values of capital per person (i.e. single steady states, multiple steady states) and label each steady state as stable or unstable. B) For what values of the savings rate with the model display multiple steady states? For what values will there be a single steady state?

7. Consider a country with a production function of $y = k^{\alpha}$. Population grows at the rate n and capital depreciates at the rate δ. There is no technological change. Consumption is each to a constant fraction of output, denoted $\bar{c}$. In addition, every period a payment in the amount of p per capita must be made to the foreign power which provides protection for this country. All output that is not consumed or paid to the foreign power is invested. A) Write down the differential equation governing the evolution of the per capita stock of capital in this country. B) Draw the Solow diagram for this country. Is there more than one equilibrium level of the capital stock and of output? Is so, identify all the equilibria. Indicate the dynamics on the diagram - that is, show to which equilibria an economy will move given it's initial stock of capital per person.

8. An economy is described by the Solow model with $y = k^{1/2}$, and some values of n and δ. Currently the savings rate is 0.6, and the country is at its steady state. Two course of action are proposed. One is to lower savings to 0.5, and the other is to lower savings to 0.4. Graphs the paths of consumption per capita over time following the introduction of each policy, and show how these consumption levels compare to consumption in the old steady state. Explain.

9. There are two countries described by the Solow model. They both have the same savings rates, population growth rates, and depreciation rates. There is no technological change. The production function in both is $y = k^{\alpha}$. In country 1 there is a proportional tax rate of τ on income. Savings is done from post-tax income. In country 2 there is constant per capita tax of Ψ per person. Savings is done out of post-tax income. The size of the per person tax Ψ is set so that taxes per person are equal to those collected in steady state by the country with the proportional tax. A) Solve for Ψ. B) Describe the dynamics of the two economies. Show that there are different possible configurations of steady states in country 2 relative to country 1. C) Show how the different configurations of steady states in 2 relative to 1 depend on the parameters. Derive the cases in which each possible configuration will occur.

10. The economy is described by the Solow model. There is no technological change, so $\Delta k_{t+1} = s_t y_t - (n+\delta)k$. Output is $y_t = k_t^{\alpha}$. Savings are non-standard. The savings rate increases with income per capita. Specifically, $s_t = \overline{s} k_t^{\beta}$ where $\overline{s}$ is a baseline savings rate and $0 < \beta < 1$. Additionally, assume that $\alpha + \beta < 1$.

    (A) What is the steady state value of capital per person, k^{ss}?

    (B) What is the Golden rule level of $\overline{s}$ that maximizes consumption in the steady state?

    (C) Under what conditions is the Golden rule level of $\overline{s}$ greater than the Golden rule savings rate in the standard Solow model?