ECON 7343 - Homework 3

Due Friday, Sep. 22nd

1. Consider a two-country version of the Lucas human capital model. Production in both countries is $y_i = k_i^{\alpha} (u_i h_i)^{1-\alpha}$ and capital accumulation is $\dot{k}_i = s y_i - (n+\delta) k_i$. So far all of this is like the model presented in class. For human capital accumulation, we add in a new assumption. Let h_l be the level of human capital per capita in the country that has a higher level of human capital (”the leader”), and h_f be the level in a country that has less human capital (”the follower”). We assume that human capital in the leader country works just like Lucas' model: $\dot{h}_l = \phi (1-u_l) h_l$. Human capital in the following country is produced by two methods. First there is production through basic accumulation, as before. But also, there is a spillover of human capital from the leader country. The amount of the spillover depends on the difference in their human capital stocks. So $\dot{h}_f = \phi (1-u_f) h_f + \beta (h_f - h_l)$ and $\beta > 0$. Assume the two countries have the same savings rate, but $u_1 < u_2$. Notice that this does not necessarily identify the leader and follower. A) Describe the steady state of the model and solve for each country's growth in steady state. B) Which country is the leader and which is the follower? C) Solve for the relative level of human capital per person in the two countries in steady state. How do the parameters ϕ and β affect the ratio h_1/h_2? D) Solve for the relative consumption in the two countries in steady state. How (and why) do ϕ and β affect the relative consumption ratio c_1/c_2?

2. The production function for the economy is $Y = K^{\alpha} R^{\beta} (EL)^{1-\alpha-\beta}$. K is capital, R is a non-renewable resource, E is technology, and L is labor. They accumulate/grow according to the following equations

$$\frac{\Delta K_{t+1}}{K_t} = s_K \frac{Y_t}{K_t} - \delta \qquad (1)$$

$$\frac{\Delta E_{t+1}}{E_t} = g \qquad (2)$$

$$\frac{\Delta L_{t+1}}{L_t} = n \qquad (3)$$

$$\frac{\Delta R_{t+1}}{R_t} = -s_R \qquad (4)$$

$$(5)$$

where the last equation is telling you that the non-renewable resource is declining at the rate s_R. The rates s, δ, g, n, and s_R are exogenous. You know that $\delta > s_R$.

   (A) What is the steady state growth rate of capital per efficiency unit (K/EL) and resources per efficiency unit (R/EL)?

   (B) What is the steady state ratio of capital to output (K/Y)?

   (C) What is the steady state growth rate of output *per capita*?

   (D) Under what conditions on s_R will the growth rate of output per capita be positive?

   (E) Assume that the economy has s_R such that growth in output per capita is positive. At time T, there is a drop in s_R. Draw a diagram showing how output per capita changes over time following this change in the depletion rate of resources.

3. Consider a Solow model with positive rates of population growth, depreciation, and technological change. Imagine a country is in steady state, and suddenly its rate of technological change increases. Describe how output per efficiency unit evolves over time. Describe how output per person evolves over time. If you have trouble with the math, draw the graphs.

4. Two countries are described by the Solow model with $y = k^{1/2}$. In both, $n + \delta = 0.1$. In country A, $s = 0.1$ while in country B, savings are a function of the capital stock, $s = 0.2\left(\frac{1}{1+k}\right)$. A) Show that the two countries have the same steady state, B) Solve for the growth rate of income per person. If both countries start with the same stock of capital per person, which country will grow faster? Will this country always grow faster?

5. Consider a Solow model with positive rates of population growth, depreciation, and technological change. The economy starts in steady state, and has Cobb-Douglas production of $\tilde{y} = \tilde{k}^{\alpha}$. There is a negative shock to E, efficiency. Describe what happens immediately to capital per worker and output per worker. Plot the time path of capital per worker and output per worker following the negative shock to E. Now, consider the same negative shock, but in an economy with a production function of $\tilde{y} = \tilde{k}^{\beta}$, where $\beta > \alpha$. Does it take more or less time for the β economy to reach steady state after the negative shock?