

# Nested production structures

Due Jan 29th

For this assignment, you need to write down a model that has the following features.

1. There are  $J$  top level industries (i.e. agriculture or services)
2. Within each top level industry  $j$ , there are  $K_j$  sub-industries (i.e. textiles or footwear under manufacturing)
3. Within each sub-industry  $jk$ , there are  $N_{jk}$  firms, who are all monopolistic competitors with one another.

The preferences of individuals are simple. They consume a top level final good, and they prefer to consume more of it. The top level final good is provided by a (competitive) final goods firm that combines the output of the  $J$  top-level industries in a CES production function with an elasticity of substitution of  $\gamma$ . There is also a preference/production weight of  $\phi_j$  on each industry.

Each separate industry  $j$  has a (competitive) final goods firm that combines the output of the  $K_j$  sub-industries below it in a CES production function with an elasticity of substitution  $\theta_j$ , and then sells this final good to consumers. Each sub-industry has a (competitive) final goods firm that combines the output of the  $N_{jk}$  firms below it in a CES production function with an elasticity of substitution  $\sigma_{jk}$ , and then sells this final good to the industry level final goods firm.

Each firm  $i$  in sub-industry  $jk$  has a production function of  $Y_{ijk} = A_{ijk}X_{ijk}$ , where  $A_{ijk}$  is their productivity and  $X_{ijk}$  is the amount of input that they use. Each of these firms is a monopolist in producing their specific good. There is a total stock of  $X$  of the input, and it is mobile across firms, sub-industries, and industries. All firms are price-takers in the market for the input, which earns a wage  $w$ .

The consumers are the providers of the input, and  $Xw$  is their total income, which they spend on the various industry goods.

You should provide a solution for the following items

1. The input share of any firm  $ijk$  in their sub-industry  $jk$ , so  $X_{ijk}/X_{jk}$ .
2. The input share of any sub-industry in their industry, so  $X_{jk}/X_j$ .
3. The input share of any industry in total inputs, so  $X_j/X$ .
4. The input share of any firm in total inputs, so  $X_{ijk}/X$ .
5. An expression for aggregate output, meaning the production of the top level final good aggregator that sells to consumers

Given those solutions, you should provide a discussion of the following:

1. Under what conditions will an increase in firm level productivity,  $A_{ijk}$ , would result in a larger expenditure share and input share for that firm.

2. Under what conditions will an increase in the average productivity of a sub-industry, call it  $\bar{A}_{jk}$ , result in a larger expenditure share and input share for each firm in that sub-industry?
3. Under what conditions will an increase in the average productivity of an industry, call it  $\bar{A}_j$ , result in a larger expenditure share and input share for each sub-industry in that industry?

Your answer must be written in Latex. I won't accept hand-written versions. If you are not familiar with Latex, now is a great time to learn it. You can get a sample document [here](#) which contains basic instructions. Otherwise, introductions to Latex are available on Google, or upper level graduate students can offer some assistance.