0.1 Individual utility

Let individual utility be described by the following

$$V_j = \omega_c \ln c_j + \omega_h \ln h_j + \omega_q \ln Q_j + \omega_d \ln(1/CDR_j). \tag{1}$$

where c_j is consumption of non-housing goods and services, and h_j is consumption of housing services. The preference weights ω_c and ω_h fulfill, without loss of generality, $\omega_c + \omega_h = 1$.

The amenity value Q_j and the crude death rate CDR_j are taken as given by an individual in location j. They choose, however, the amounts of c_j and h_j to purchase. The budget constraint facing an individual in location j is

$$w_j = c_j + r_j h_j \tag{2}$$

where r_j is the rental price of housing (relative to consumption goods), and w_j is the wage. Optimizing over consumption and housing, we get that $c_j = \omega_c w_j$ and $h_j = \omega_h w_j / r_j$. This results in utility of

$$V_j = \ln w_j - \omega_h \ln r_j + \omega_q \ln Q_j + \omega_d \ln(1/CDR_j) + \Omega, \tag{3}$$

where $\Omega = \omega_c \ln \omega_c + \omega_h \ln \omega_h$.

For our purposes, we care about the growth rate of utility in each location, which is given by

$$\hat{V}_j = \hat{w}_j - \omega_h \hat{r}_j + \omega_q \hat{Q}_j - \omega_d C\hat{D}R_j. \tag{4}$$

We can evaluate each of these separate terms to find the effect of population growth on utility growth.

Wages: To determine the wage in a given location, we assume a production function of the form

$$Y_j = A_j K_j^{\alpha_j} X_j^{\beta_j} N_j^{1-\alpha_j-\beta_j} \tag{5}$$

where A_j is productivity, K_j is capital, and X_j is land. Note that the shares α_j and β_j are unique to a given location. Assuming that labor earns its marginal product, the wage in location j is given by

$$w_j = 1 - \alpha_j - \beta_j A_j K_j^{\alpha_j} X_j^{\beta_j} N_j^{-\alpha_j-\beta_j}. \tag{6}$$

We can allow explicitly for agglomeration effects by letting productivity be a function of the labor force, as in

$$A_j = B_j N_j^{\gamma_j} \tag{7}$$

where B_j is the inherent productivity in location j, and $\gamma_j > 0$ captures the agglomeration effect in location j.

With this specification for productivity, and the expression for the wage, the growth rate of the wage is given by

$$\hat{w}_j = \hat{a}_j^w + (\gamma_j - \alpha_j - \beta_j)\hat{N}_j, \tag{8}$$

where $\hat{a}_j^w = \hat{B}_j + \alpha_j \hat{K}_j$ is the growth of productivity and capital that is independent of the growth rate in the labor force. The effect of growth in the labor force may be positive or negative, depending on how big the agglomeration effects (γ_j) are relative to the importance of capital and land in production ($\alpha_j + \beta_j$).

In terms of our baseline model, we can write the above as

$$\hat{w}_j = \hat{a}_j^w + \epsilon_j^w \hat{N}_j, \tag{9}$$

where $\epsilon_j^w = \gamma_j - \alpha_j - \beta_j$ is the elasticity of the wage with respect to population size.

Housing: The second term in utility growth involves the rental price of housing. Given some limitations on housing due to land constraints and/or regulations, then it should be the case that rents are rising with N_j. We could write this in a reduced form as

$$\hat{r}_j = \hat{a}_j^r \epsilon_j^h \hat{N}_j \tag{10}$$

where ϵ_j^h is the elasticity of rents (housing price) with respect to population size. The value $\hat{a}_j^r$ is exogenous growth in rental rates in location j for any reason unrelated to population size.

Amenities: The third term in utility growth involves amenties. Assuming some effect of population size on the quantity (or implicit value) of amenities available, then we could write, again in reduced form,

$$\hat{Q}_j = \hat{a}_j^q + \epsilon_j^q \hat{N}_j. \tag{11}$$

Here, $\hat{a}_j^q$ is exogenous growth in the value of amenities, and ϵ_j^g is the elasticity of amenities with respect to population size.

Crude death rate: As stated in the main paper, we are taking the growth of crude death rates to be exogenous, and unrelated to population size.

Reduced form welfare growth: Combining the information about the different components of utility growth, we can write

$$\hat{V}_j = G_j - \epsilon_j \hat{N}_j - \omega_d C\hat{D}R_j \tag{12}$$

where

$$G_j = \hat{a}_j^w - \omega_h \hat{a}_j^r + \omega_q \hat{a}_j^q \tag{13}$$

is the exogenous growth in wages, rents, and amenities, where each are weighted as in the utility function. Similarly, the combined elasticity term, ϵ_j is

$$\begin{aligned} \epsilon_j &= \epsilon_j^w - \omega_h \epsilon_j^h + \omega_q \epsilon_j^q && (14) \\ &= \gamma_j - \alpha_j - \beta_j - \omega_h \epsilon_j^h + \omega_q \epsilon_j^q && (15) \end{aligned}$$

where the second line shows the explicit role of agglomeration effects (γ_j) and the production function parameters (α_j and β_j) on the elasticity. WHile the value of γ_j is expected to be positive, note that the rest of these terms would act to make ϵ_j negative. The values of α_j and β_j indicate how quickly wages decline with the number of workers. The relationship of rents to population size is presumably positive, so the term $\omega_h \epsilon_j^h$ is positive, and this is subtracted

0.2 Parameter values

The expression in (15) shows us what explicit types of information we need to quantify the aggregate elasticity of welfare with respect to population size. For each in turn, we can evaluate them based on outside literature. Ideally, we would be able to provide a separate estimate of the parameter by location, although as we will describe below, this will not always be feasible.

Agglomeration effects: The value of γ_j has been estimated XXXXX, and appears to have a maximum value of γ

Factor shares: The values of α_j and β_j (which capture the importance of capital and land in production in location j, respectively), can be estimated from factor share information. Individually, α_j and β_j would be equal to the factor share of capital and land, assuming factor markets are competitive. Alternatively, their sum, $\alpha_j + \beta_j$, could be recovered if we know the *labor* share in a location.

The major issue here is that factor shares are not reported by location, but typically only by sector. So in order to leverage factor share information, we first have to take some position on what sectors belong to which locations. Of course, many of these sectors operate in all locations (i.e. retail) so there is not a one-to-one mapping. Rather, we think of the locations as having different weights on which sectors are relatively important.

In the rural location, the agriculture is obviously going to be the dominant sector, with smaller weights on manufacturing and services activities. For the formal sector, we believe that the factor shares will be driven by sectors involving heavy manufacturing (i.e. chemicals or equipment), wholesale trade, information, and services like finance. For the informal sector, the factor shares will depend mainly on sectors like light manufacturing (i.e. textiles), retail and/or commerce, and personal services.

Valentinyi and Herrendorf (2008) provide estimates of the combined land and capital shares, $\alpha_j + \beta_j$, for several broad sub-sectors of the U.S. economy. In agriculture, this share is 0.54, manufacturing consumption has a share of 0.40, equipment investment a share of 0.34, construction a share of 0.21, and services a share of 0.34.

Young (2010) looks at labor shares at a more disaggregate level for the United States, and over a longer period of time, but does not make the same adjustments for proprietors income that Valentinyia nd Herrendorf do. With that caveat in mind, his evidence indicates that sectors we consider to be more heavily represented in informal locations have lower values of $\alpha_j + \beta_j$. Specifically, for general services the labor share is 0.689, indicating that the $\alpha + \beta$ here is about .311. In contrast, for finance the labor share is 0.444, for $\alpha + \beta$

Rents: For the effect of renta

Amenities: For amenities, we need to establish both the utility weight ω_q, as well as their elasticity with respect to population.

1 Historical outcomes

Table 1: **HISTORICAL OUTCOMES USING THE CALIBRATED MODEL, 1800-1940**

	1800:			1900:			1940:		
Scenario	Urb. Rate	Inf. Rate	Urb. Size	Urb. Rate	Inf. Rate	Urb. Size	Urb. Rate	Inf. Rate	Urb. Size
1. *Observed data:*	12.5	50.0	1.0	40.0	30.0	7.0	50.0	20.0	12.9
2. Calibrated model, exog. fert.	12.5	50.0	1.0	41.1	31.3	14.8	56.3	18.7	29.9
3. Calibrated model, exog. fert., fast UMT	12.5	50.0	1.0	43.4	37.3	21.2	57.8	24.1	43.6
4. Calibrated model, exog. fert., fast UMT, large UMT	12.5	50.0	1.0	50.3	48.5	31.7	63.7	36.7	72.8
5. Calibrated model, endog. fert.	12.5	50.0	1.0	38.9	22.7	8.8	55.3	13.0	18.8
6. Calibrated model, endog. fert., fast UMT	12.5	50.0	1.0	42.0	32.7	16.5	57.0	20.7	34.9
7. Calibrated model, endog. fert., fast UMT, large UMT	12.5	50.0	1.0	51.7	51.7	37.9	65.9	43.2	104.2