Calculus 2. Practice 1

1) Find and sketch the domains of the given functions

1a) $f(x, y) = \ln(xy - 1)$ **1b)** $f(x, y) = xy\sqrt{x^2 + y}$

2) Evaluate the following limits (with or without the use of polar coordinates)

2a) $\lim_{(x,y,z)\to(0,0,0)} \frac{xy+yz+xz}{x^2+y^2+z^2}$ **2b)** $\lim_{(x,y,z)\to(0,0,0)} \frac{x^2y^2z^2}{x^2+y^2+z^2}$

2c) $\lim_{(x,y)\to(0,0)} \frac{\sin(x^2+y^2)}{x^2+y^2}$ **2d)** $\lim_{(x,y,z)\to(0,0,0)} (x^2 + y^2)^{x^2y^2}$

4) **4a)** Given the function $f(x, y)e^{xy^2}$. Find f_x, $f_y(1, 1)$. Verify that $f_{xy} = f_{yx}$. Evaluate f_{xxy}.

4b) Given the function $f(x, y, z) = x^5 + yz^2 + \sin(xy) + \cos(zx)$. Evaluate f_x, f_{yx}, f_{zz}.

7) Find the linearization $l(x, y, z)$ of $f(x, y, z) = e^{xy^2} + x^4yz$ at $(1, 1, 1)$.

8) Use the chain rule to find the indicated (partial) derivatives.

8a) $\frac{dw}{dt}$ where $w = \frac{x}{y} + \frac{y}{z}$, $x = \sqrt{t}$, $y = \cos(2t)$, $z = e^{-3t}$

8b) $\frac{\partial z}{\partial s}$, $\frac{\partial z}{\partial t}$, where $z = xe^y + ye^{-x}$, $x = s^2t$, $y = st^2$

9) Verify the assumption of the Implicit Function Theorem to prove that the equation $xe^y + \sin(x, y) + y - \ln 2 = 0$ defines y as a differentiable function of x around $(0, \ln 2)$. Use implicit differentiation to find $\frac{dy}{dx}$ at the given point.

10) Find the gradient of f, evaluate the gradient at the given point P and find the rate of change of f at P in the direction of the given vector $\mathbf{u}$.

10a) $f(x, y) = e^x \sin y$, $P(1, \frac{\pi}{4})$, $\mathbf{u} = (-1, 2)$

10b) $f(x, y, z) = xy + yz^2 + xz^3$, $P(2, 0, 3)$, $\mathbf{u} = (-2, -1, 2)$.

11) Find the directional derivative of the given function at the given point in the direction of the given vector.

11a) $f(x, y) = e^x \cos y$, $P(1, \frac{\pi}{6})$, $\mathbf{u} = \mathbf{i} - \mathbf{j}$

11b) $f(x, y, z) = z^3 - x^2y$, $P(1, 6, 2)$, $\mathbf{u} = (3, 4, 12)$.

12) Find the directions in which the functions increase and decrease most rapidly at P. Find the derivatives of the functions in these directions.

12a) $f(x, y) = x^2y + e^{xy} \sin y$, $P(1, 0)$

12b) $f(x, y, z) = xe^y + z^2$, $P(1, \ln 2, \frac{1}{2})$.

13) Find all local maxima, local minima, and saddle points of the following functions:

1a) $f(x, y) = 2x^2 + 3xy + 4y^2 - 5x + 2y$ **1b)** $f(x, y) = x^3 - y^3 - 2xy + 6$