# Calculus 2. Practice 2

**1)** Sketch the region of integration and evaluate the integral $\int_0^1 \int_0^{y^2} 3y^3 e^{xy}\, dx\, dy$.

**2)** Sketch the region of integration, determine the convenient order of integration, and evaluate the integral.

**2a)** $\int_0^2 \int_0^{4-x^2} \frac{xe^{2y}}{4-y}\, dy\, dx$ **2b)** $\int_0^8 \int_{\sqrt[3]{x}}^2 \frac{dy\, dx}{y^4+1}$

**3)** Change the cartesian integral into an equivalent polar integral, then evaluate the integral.

**3a)** $\int_0^2 \int_0^{\sqrt{4-y^2}} (x^2+y^2)\, dx\, dy$ **3b)** $\int_0^1 \int_0^{\sqrt{1-x^2}} e^{-(x^2+y^2)}\, dy\, dx$

**4)** Evaluate the integrals

**4a)** $\int_0^1 \int_0^{3-3x} \int_0^{3-3x-y} dz\, dy\, dx$ **4b)** $\int_0^{\pi} \int_0^{\ln(\sin y)} \int_{-\infty}^{z} e^x\, dx\, dz\, dy.$

**5)** Evaluate the integral

$$\int_0^4 \int_0^1 \int_{2y}^2 \frac{4\cos(x^2)}{2\sqrt{z}}\, dx\, dy\, dz$$

by changing the order of integration in an appropriate way.

**6)** Convert **3a)** into cylindrical coordinates, convert **3b)** into spherical coordinates. Then evaluate the new integrals.

**6a)** $\int_0^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{-(x^2+y^2)}^{(x^2+y^2)} 21xy^2\, dz\, dy\, dx$ **6b)** $\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^1 dz\, dy\, dx.$

**7)** Integrate $f$ over the given curve.

**7a)** $f(x,y) = \frac{x+y^2}{\sqrt{1+x^2}}$, $C$: $y = \frac{x^2}{2}$ from $(1, 1/2)$ to $(0,0)$.

**7b)** $f(x,y) = x+y$, $C$: $x^2+y^2 = 4$ in the first quadrant from $(2,0)$ to $(0,2)$.

**7c)** $f(x,y,z) = x\sqrt{y} - 3z^2$, $\mathbf{r}(t) = \cos(2t)\mathbf{i} + \sin(2t)\mathbf{j} + 5t\mathbf{k}$, $0 \le t \le 2\pi$.

**8)** Find the work done by force $\mathbf{F}$ on a particle that moves along the given path.

**8a)** $\mathbf{F} = (3x^2 - 3x)\mathbf{i} + 3z\mathbf{j} + \mathbf{k}$, over the straight-line path $\mathbf{r}(t) = t\mathbf{i} + t\mathbf{j} + t\mathbf{k}$, $0 \le t \le 1$.

**8b)** $\mathbf{F} = (y+z)\mathbf{i} + (z+x)\mathbf{j} + (x+y)\mathbf{k}$, over the curved path $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t^4\mathbf{k}$, $0 \le t \le 1$.

**9)** Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ for the vector field $\mathbf{F} = y\mathbf{i} - x\mathbf{j}$ counterclockwise along the unit circle $x^2+y^2 = 1$ from $(1,0)$ to $(0,1)$.

**10)** Given the fields $\mathbf{F}_1 = (y\sin z)\mathbf{i} + (x\sin z)\mathbf{j} + (xy\cos z)\mathbf{k}$ and $\mathbf{F}_2 = y\mathbf{i} + (x+z)\mathbf{j} - y\mathbf{k}$ determine if they are conservative or not. In affirmative case, find the potential function.

**11)** Show that the differential form in the integral is exact, then evaluate the integral (i.e. show that the vector field $\mathbf{F} = < 2xy, x^2 - z^2, -2yz >$ is conservative and the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ can be evaluated independently on the choice of the path C from A=[0,0,0] to B=[1,2,3])

$$\int_{(0,0,0)}^{(1,2,3)} 2xy\, dx + (x^2 - z^2)\, dy - 2yz\, dz.$$

**12)** Show that the work done by the force $\mathbf{F} = (x^2+y)\mathbf{i} + (y^2+x)\mathbf{j} + ze^z\mathbf{k}$ over a path from $(1,0,0)$ to $(1,0,1)$ is independent on the path, find the work.

**13)** Apply Green's theorem to evaluate the integral $\oint_C 3y\, dx + 2x\, dy$, where $C$ is the boundary of $0 \le x \le \pi$, $0 \le y \le \sin x$.

**14)** Use Green's theorem to find the work done by $\mathbf{F} = 2xy^3\mathbf{i} + 4x^2y^2\mathbf{j}$ in moving a particle once counterclockwise around the boundary of the "triangular" region in the first quadrant enclosed by the $x$-axis, the line $x = 1$, and the curve $y = x^3$.