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From: Sarah Moran

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Introduction and Background

For the past several years, the Delaware Valley Regional Planning Commission (DVRPC) has coordinated with a working group of stakeholders representing Southeastern Pennsylvania to look for opportunities to make bicycle facility improvements on state roads in Pennsylvania as part of the resurfacing process. State routes in Pennsylvania include most collector and arterial roadways with relatively high speeds, making it difficult to accommodate comfortable biking. Due to the current lack of bicycle infrastructure in the suburbs, there is a plethora of places throughout the Philadelphia metro region that would benefit from improvements, making prioritization difficult. The impetus for this project came from the working group's desire to identify and rank critical places which would provide the maximum local and regional connectivity benefit to the bicycle network if improvements were made.

Level of Traffic Stress (LTS) is a road classification technique based on the comfort of bicyclists in the traffic stream (1). One commonly used LTS framework ranges from LTS 1 to LTS 4, classifying road segments that would be comfortable for any bicyclist to segments that only the fearless would brave on a bicycle, respectively. Over the past several years, a variety of papers have been written about different methods for assigning LTS values to road segments and how these LTS values have been used to inform planning and infrastructure decisions through connectivity analysis. Since demand modeling for bicycling is absent from many regional travel demand models (including DVRPC's), a measure of the potential use of a bicycle facility is difficult to obtain. Instead of focusing on demand, bicycle network connectivity is often used as an accessibility metric. Much of the connectivity analysis in prior research compared existing conditions to a "bike-plan buildout" scenario, or similar, to compare project merits.

This project, instead, provides an example of a different type of LTS and network connectivity application. While the finished tool will be able to compare existing conditions and potential future scenarios, it was developed for broader use. The primary purpose of this analysis was to inform changes in road design that can occur alongside resurfacing work. For example, if 50 road segments are scheduled to be resurfaced in a given paving season, which would have meaningful impacts on low-stress bike connectivity and would be worth investing in design? Even projects that might seem inconsequential on the surface, such as adding bike lanes to a short stretch of roadway where no other bike facilities exist nearby, can have a meaningful local impact if they provide connections between low-stress residential streets, and contribute to building a low-stress regional network over time.

Rather than examine already proposed projects, this analysis identifies the potential impact on network connectivity for individual road segments that are not already considered comfortable for bicycling. The result will be a ranking of road segments that, if improved to a lower LTS score, would provide the greatest benefit to low-stress bicycle connectivity. Based on the ranked results, planners can more efficiently prioritize areas for further investigation and analysis.

Literature Review

Assigning LTS

One of the most commonly used LTS schemas was developed by the Mineta Transportation Institute (1). Their research proposed a method “for classifying road segments by one of four levels of traffic stress,” based on the four types of cyclists identified by Roger Geller at the Portland Office of Transportation (2), as shown in Table 1. The Mineta study and related 2013 TRB paper (3) relied on a variety of road attributes such as the presence of a bike lane, the presence and location of on-street parking, the number of travel lanes per direction, and the speed limit to determine the level of stress on a road segment. It also examined the stress at intersections based on the presence of a signal, a right turn lane, and the speed limit and number of lanes on the street being crossed. While this analysis provides a robust, data driven prediction of cyclist comfort on a road segment, it requires a large amount of data including attributes that are difficult to obtain at a regional scale.

Table 1: LTS Categories and Rider Types

LTS	Comfortable Enough For (Cyclist Type)	Characteristics
1	Most People	<ul style="list-style-type: none">• Relaxing• Suitable for children
2	Interested, but Concerned	<ul style="list-style-type: none">• Suitable for most adults• Presenting little traffic stress
3	Enthused and Confident	<ul style="list-style-type: none">• Moderate traffic stress• Comfortable for those already riding bikes in American cities
4	Strong and Fearless	<ul style="list-style-type: none">• High traffic stress• Multilane, fast moving traffic

Source: (1)(2)

Montgomery County, Maryland published bicycle planning guidance which included design guidelines for two types of cyclists: interested but concerned cyclists and confident cyclists (4). Their guidance provided graphs for determining the recommended type of bicycle facility based on roadway attributes, including vehicular volume and speed limit. While traffic counts are available for many roads throughout the DVRPC region, it would be time and cost prohibitive to collect data on vehicle volume on every single road in the region. To fill the gaps in the volume data in the region, this project would have had to rely on estimated volumes from DVRPC’s regional travel demand model. While possible, using model-estimated volumes for this project would have introduced an additional level of complexity and potential for error.

In 2016, Lowry, Furth, and Hadden-Loh presented their Marginal Rate of Substitution (MRS) based method for LTS assignment (5). Their method assigned LTS using a few relatively easy to acquire road attributes. The initial stress level was a function of the number of lanes and the speed limit. Then the stress level was

reduced based on stress reduction factors developed for five levels of bicycle accommodation from the least protected bike route to the most protected, or comfortable, protected bike lane, as shown in Table 2. “The stress factors are marginal rates of substitution with respect to distance cycling on a multi-use trail. For example, for a 6-lane road with a 35mph speed limit and no bicycle accommodation, the stress factor of 140% implies that a cyclist would prefer traveling up to 140% farther on a multi-use trail” (5). Given the simplicity of the data requirements and the straightforward method of assigning LTS, this method was adapted for use in the DVRPC region.

Table 2: LTS in Terms of MRS (from Lowry, Furth, & Hadden-Loh)

Roadway		Roadway Stress w/out Bicycle Accommodation	Stress Reduction from Bicycle Accommodations				Protected Bike Lane 75%
			Bike Route 5%	Sharrows 10%	Bike Lane 50%	Buffered Bike Lane 65%	
Number of Lanes	Speed Limit						
2 lanes (residential)	Up to 25 mph	10%	10%	9%	5%	4%	3%
2 lanes (residential)	30 mph	15%	14%	14%	8%	5%	4%
2-3 lanes	Up to 25 mph	20%	19%	18%	10%	7%	5%
4-5 lanes	Up to 25 mph	35%	33%	32%	18%	12%	9%
2-3 lanes	30 mph	40%	38%	36%	20%	14%	10%
6+ lanes	Up to 25 mph	67%	64%	60%	34%	23%	17%
4-5 lanes	30 mph	70%	67%	63%	35%	25%	18%
6+ lanes	30 mph	80%	76%	72%	40%	28%	20%
2-3 lanes	35+ mph	100%	95%	90%	50%	35%	25%
4-5 lanes	35+ mph	120%	114%	108%	60%	42%	30%
6+ lanes	35+ mph	140%	133%	126%	70%	49%	35%

Level of Traffic Stress Limits			
LTS 1 Limit:	10%	LTS 2 Limit:	30%
		LTS 3 Limit:	60%
		LTS 4 Limit:	no MRS limit

Source: (5)

Connectivity Analysis

Early research on LTS networks suggested that bicyclists not only seek the most direct route but also prefer not to deviate more than 2 blocks out of their way to use a known bicycle facility (6). Later research compared the “low-stress route,” one comprised of only low-stress road segments, to the most direct route. If the low-stress route between an origin and destination was more than 25 percent longer than the most direct route, the origin and destination would not be considered “connected” by a low-stress route (3).

The Lowry et al. paper proposed using “the percent of residents that can reach a majority of important destinations via low-stress bikeways” as a metric to quantify accessibility (5). The important destinations included a variety of places that residents would need or want to access on a regular basis. Shortest paths were calculated from residential parcels in their study area to key destinations. The paper introduced a modified version of the classic centrality formula to determine each road segment’s “relative importance to the network,” specifically, how many paths utilized certain road segments (5). Origins and destinations were weighted by population and employment, respectively, to ensure that the links connecting the most people to the most destinations resulted in a high overall importance to the bicycle network. Due to the regional scale of this project, obtaining the level of detail required for each destination was a difficult prospect. Additionally, in places where destinations were dense, it would require the calculation of numerous shortest paths to essentially the same location, which would increase processing time. Therefore, this method was modified for application to the DVRPC region.

Data Preparation

DVRPC's regional travel demand model was selected as the base road and trail network for this analysis. The modeled network, based on Open Street Map, contains critical attributes for link and turn LTS assignment such as number of lanes, bicycle facilities, and speed. The network is updated on a regular basis and is consistent, in terms of attribute completion and accuracy, across the region. Since routine accuracy checks and updates are completed with traffic forecasting in mind, attributes contributing to LTS assignment were scrutinized further.

To verify the number of lanes attribute, the modeled network was visually compared to Pennsylvania Department of Transportation's Road Management System lane count data. Results from the QA/QC revealed the modeled network to be very accurate, with only a few minor changes needed. These changes were also verified using 2015 aerial imagery.

Off-road bicycle facilities in the modeled network were checked using DVRPC's Circuit Trail GIS layer. This layer is maintained by staff and is rectified using aerial imagery on a frequent basis. On-road facilities were verified using data and feedback from the working group.

The speed attribute relied on the model's effective speed. Effective speed is based on a road segment's functional classification and area type (urban, suburban, rural, etc.). It is meant to represent free flow speed. One reason effective speed was chosen over speed limit is the lack of consistent speed limit data for the entire study area. Additionally, many drivers travel at speeds above the speed limit when traffic is moving freely, which impacts how comfortable a cyclist would feel on that roadway. It was hoped that effective speed would capture this effect. In some areas, the effective speed is lower than the posted speed, while in others, it is higher.

The effective speed was compared to speed limit data provided by PennDOT. Approximately 21 percent of the road segments in the study area overlapped with the PennDOT data and could be compared. Based on steering committee feedback and the average differences between the two datasets, speed limit was used in place of effective speed where the modeled speed was lower than the speed limit. Since this analysis focused on cyclist comfort, it was important to err on the side of caution and use the higher speed and therefore a higher LTS. The segments where the effective speed was higher than the PennDOT speed limit were examined closely and were ultimately not modified.

Methodology

The methodology for this analysis relied on scripts that pushed and pulled information to and from a geospatial database. Scripts were written to prepare the road network by assigning LTS. Shortest paths were calculated between millions of origin and destination pairs to identify the road segments that enabled low-stress connections between the most places throughout the region. This section describes this process in detail.

Assigning LTS

Road segment LTS assignments relied on the MRS based stress values presented in Lowry et al. (5). Based on the number of lanes, effective modeled speed, and the presence and type of a bicycle facility,

LTS values were assigned to road segments using a script. The following upper limits were used to distinguish between the four levels of traffic stress:

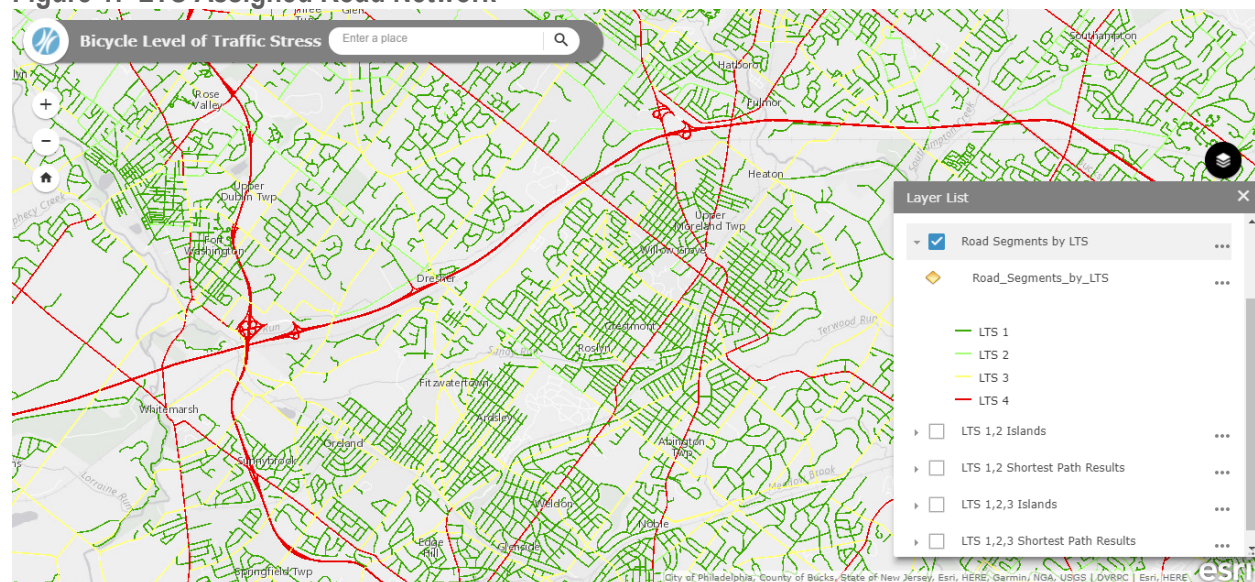
- LTS 1: 10%
- LTS 2: 30%
- LTS 3: 60%
- LTS 4: no limit

Once LTS values were assigned to road segments, they were used to assign LTS values to turning movements. Lowry et al. considered traffic signal, functional priority, number of lanes, and speed limit of the cross street when assigning LTS to turns (5). Traffic signal and functional priority data was not consistently available at the regional scale. Therefore, this analysis relied on a modified turn LTS assignment method, somewhat based on the “weakest link” logic presented in the Mineta report (1). Each turn direction was treated differently:

- Right turns were assigned the LTS of the road segment to which the turn was being made
- Left turns were assigned the LTS of the road segment to which the turn was being made, multiplied by two
 - Left turns are typically more difficult, and therefore more stressful, than right turns
 - Doubling the LTS value for left turns ensures this stress is reflected in the turn cost
- Straight movements were assigned the maximum LTS value of all the links at the intersection
 - This way, the impact of crossing a high-stress street was accounted for
 - Crossing a comfortable street while on a stressful street did not significantly lower the cost of the route
- U-turns were excluded

The LTS assigned road network, as shown in Figure 1, was based on DVRPC’s regional travel demand model network supplemented with the attributes necessary for connectivity analysis. Symbolizing the road segments by LTS values provided a picture of existing cyclist comfort throughout the study area. Since turns occur at the intersection of road segments, they were not visible in the network.

Figure 1: LTS Assigned Road Network



After LTS values were assigned to the road network, the following base input data was loaded into a database:

- LTS assigned road network
- Road network nodes
- Census block centroids

Census block centroids functioned as origins and destinations for connectivity analysis. A script identified the closest node to each block centroid to serve as the network start and end points of each shortest path calculation. A list of combinations of these closest nodes was compiled as inputs to the shortest path searches.

Next, the network data was prepared for shortest path searches. First, a script created a subset of road segments to be included in the “tolerable network.” For the first analysis, “tolerable” links are those considered to have an acceptable LTS for most bicycle riders—in this case LTS 1 and 2 roads. Since the shortest paths in this analysis were intended to be able to be used by everyone, the more stressful LTS 3 and 4 roads were excluded from the network to ensure that shortest paths would not traverse them. In the second analysis, the tolerable network included LTS 1, 2, and 3 roads.

Overall cost fields were added to the road segments and turns for use as the path search impedance. For road segments, the overall cost was a function of the link length and assigned LTS value:

$$\text{Cost} = \text{Link Length} \times (1 + \text{Link LTS})$$

The turn cost was a function of the assigned turn LTS, the direction of the intersection movement, and the base turn distance of 30ft (0.005 miles), as used in the Lowey et al. paper (5).

$$\begin{aligned}\text{Straight Cost} &= 0.005 \times (1 + \text{Turn LTS}) \\ \text{Right Turn Cost} &= 0.005 \times (1 + 1 + \text{Turn LTS}) \\ \text{Left Turn Cost} &= 0.005 \times (1 + 2 + \text{Turn LTS})\end{aligned}$$

Therefore, turns were accounted for but had a minimal impact on route selection since the overall cost of turning was small. In the future, additional research could be done to determine the sensitivity of the shortest path search results to turn cost.

Then, the script created a table to hold a subset of the nodes that correspond with the tolerable link subset. Turns were imported from the model and also subsetting to correspond with the tolerable links and nodes.

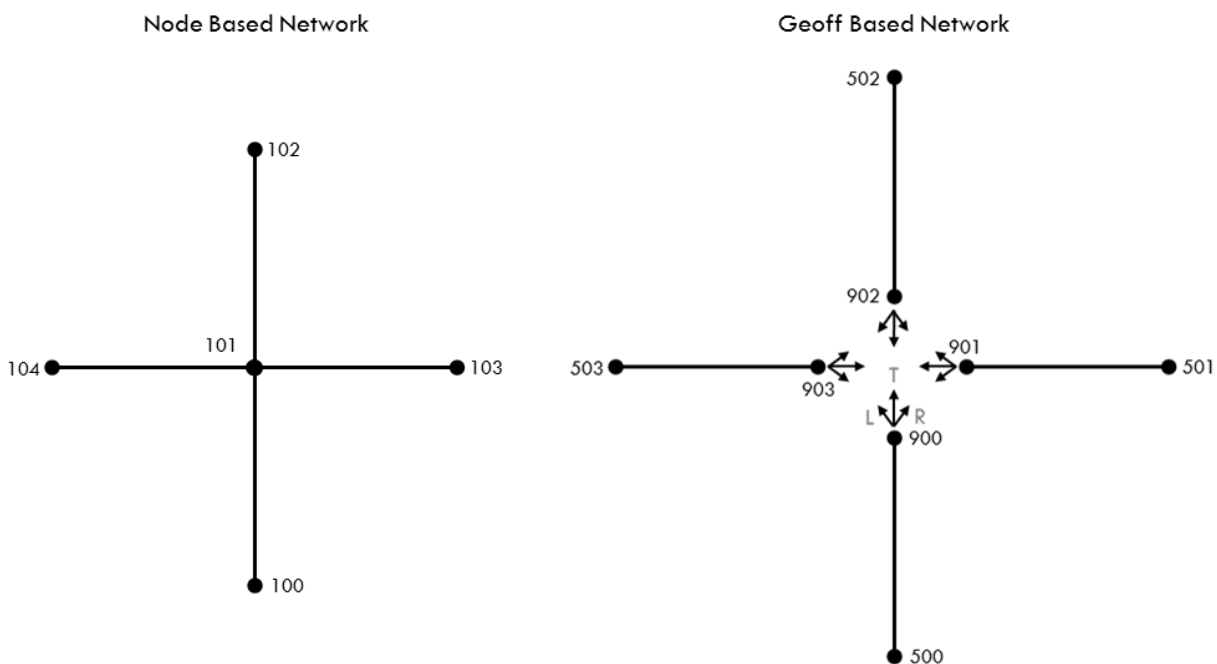
Creating the Routable Network

The next critical step was to combine turns and links in the same table to create a complete routable network. The process was to explode every node in the network into repeated nodes at the same location; one for each approach of the intersection. These exploded nodes were termed “geoffs.” In an effort to simplify discussions about how to create and use them, rather than referring to them as “turn nodes,” an easily confused combination of other component names, geoff will be used to describe the exploded nodes going forward.

Looking at the “Node Based Network” diagram in Figure 2, nodes are numbered, starting with 100. Links are identified using a “from-node” and a “to-node.” For example, the upward direction of the bottom link is identified by from-node 100 and to-node 101. A right turn starting at node 100 and ending at node 103 would be identified using the following node sequence: from-node 100, via-node 101, to-node 103.

The “Geoff Based Network” diagram in Figure 2 represents the same intersection, this time based on geoffs. When combining the links and turns into a master network table, all nodes were converted to sets of geoffs. Instead of being represented by a different number and combination of node identifiers (from and to for links; from, via, and to for turns), the creation of geoffs allowed for each network component to be identified by a from-geoff and a to-geoff. This made it possible to combine what were previously links and turns into a single, routable, master network table with both types of components. For example, the bottom link is still identified using a from and to location; from-geoff = 500, to-geoff = 900. Once at 900, to make a right turn, one would travel along “R,” from geoff 900 to geoff 901. Then one could continue from geoff 901 to geoff 501. Upon reaching 501, there would be another turning movement before connecting to the next link.

Figure 2: Node Based Network vs. Geoff Based Network

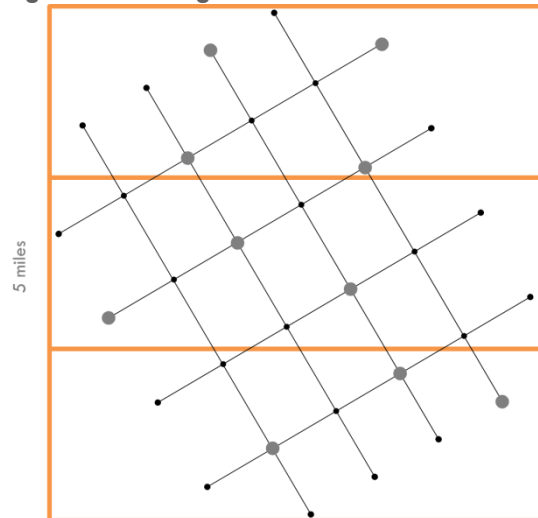


Island Analysis and Moving Frame

Testing revealed that the large routable network comprised of links and turns resulted in unacceptably long processing times. Island analysis and moving frames were introduced to divide the network dataset into more manageable subnetworks. The routable network was analyzed as a graph and divided into connected components, or groups of link “islands,” to streamline analysis. A field was added to the master network table to identify the island to which each road and turn belonged. A subnetwork table was created for each individual island. Since an origin on one island would not be connected to a destination on another island, it was possible to calculate shortest paths on one island at a time, drastically reducing the number of records being searched for each calculation.

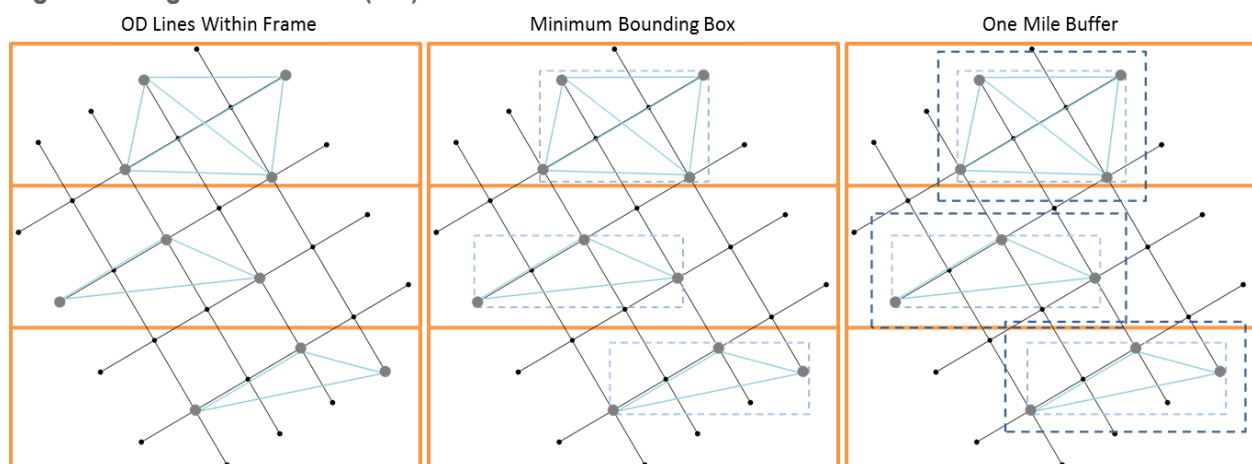
The moving frame concept was introduced to further divide large routable islands into even smaller subnetworks for more efficient calculations. The moving frame started with a minimum bounding box around the island. The box was then divided into 5 mile vertical frames, starting at the top as shown in Figure 3. If the island's bounding box was not evenly divided by 5, the remainder was given its own frame.

Figure 3: Moving Frame Network Division



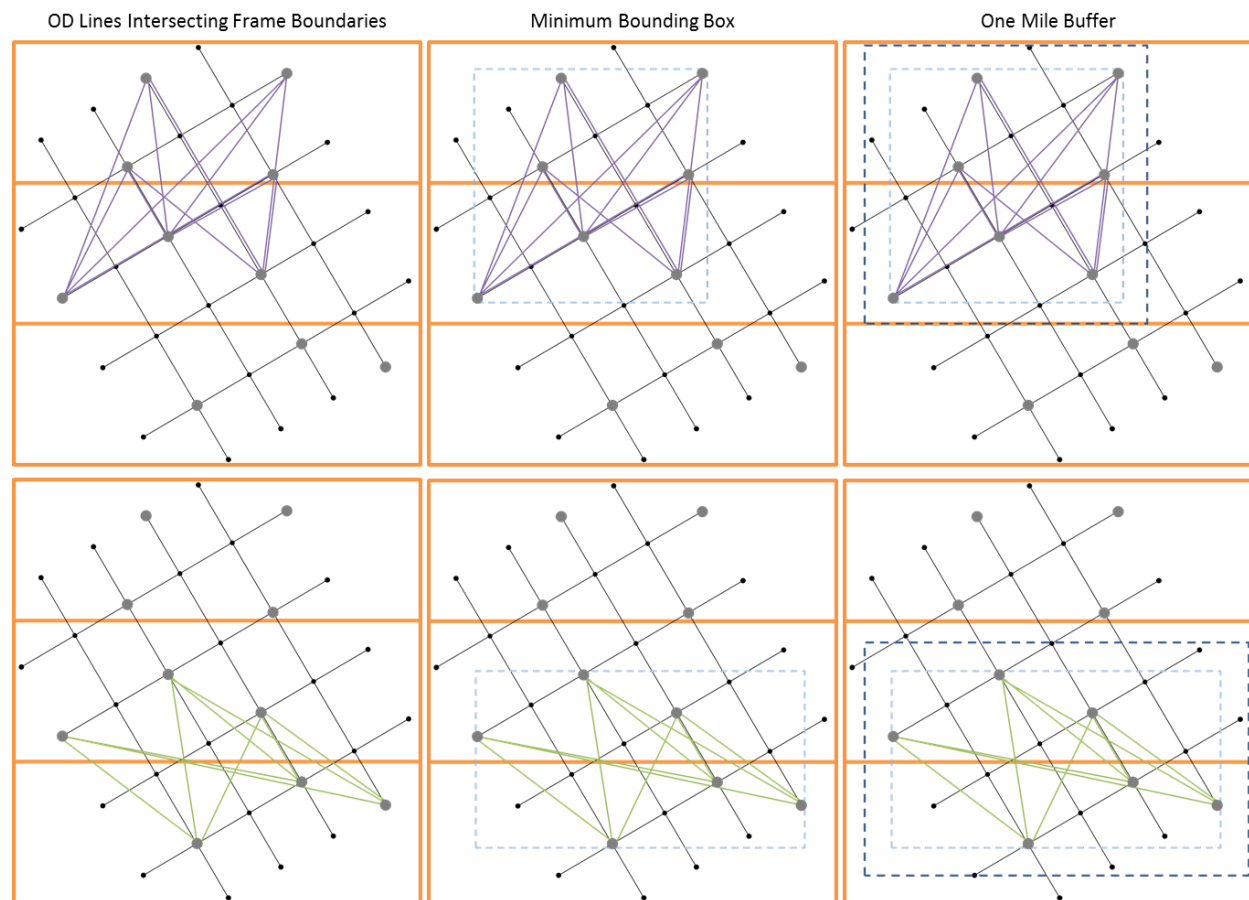
In the example in Figure 3, the larger gray geoffs—which appear similar to nodes—represent those that were identified as closest to imaginary block centroids and will therefore serve as the origins and destinations for the example. Once the boundaries of the frames were determined, straight lines were drawn between each origin and destination pair. First, the lines that were completely contained within frame boundaries were selected, as shown by the blue lines in Figure 4. A minimum bounding box was drawn around the group of lines in each frame. Then, a buffer of one mile was added before clipping the network, to allow for shortest paths that are required to travel slightly out of the way to connect the origin and destination. Analysis was done on a full network to help determine an adequate buffer size while maintaining the benefit of smaller network chunks for shortest path calculations.

Figure 4: Origin-Destination (OD) Pairs Connected Within a Frame



Then, the lines that intersected frame boundaries were selected, as shown by the purple and green lines in Figure 5. Similar to the lines within the frames, a minimum bounding box was drawn around the intersecting lines and an additional one-mile buffer was added before clipping the network. The clipped networks were saved into temporary tables to be used in shortest path calculations for the island in question. The origin and destination pairs for the lines in the selection were also saved into a corresponding temporary table to ensure that only shortest paths for the pairs connected in each frame were calculated for the correct corresponding subnetwork.

Figure 5: Origin-Destination (OD) Pairs Connected Across Frames



Shortest Paths

Shortest paths between each origin-destination (OD) pair were calculated using the directed Dijkstra algorithm (7). Inputs included the subnetwork, or island on which the OD pair was located, the geoff ID of the origin and destination, and the weight, or calculated cost of each network component. For each OD pair, if a path was found, the algorithm returned the length of the shortest path and a list of components (links and turns) used to connect the origin and destination. The components of each path were added to an output table for post processing. Finally, the components were tallied to determine the number of times each unique component served to connect a shortest path.

Discussion

According to Roger Geller's analysis of the four types of cyclists, supported by national surveys conducted by Portland State University (8), the slight majority of people fall into the "interested but concerned group" (2). This means that a lot of potential cyclists would ride their bicycles more if they had more comfortable places to ride. It was important to target this group when prioritizing infrastructure improvements. The first step in identifying and ranking the critical places which would most benefit the bicycle network was to assign LTS values to road segments and turns. LTS was used along with distance to evaluate the overall cost of a route between two census blocks. High stress roads (LTS 3 and 4), considered uncomfortable for most cyclists, were eliminated to create a picture of the existing tolerable road network. As mentioned earlier, the tolerable network (LTS 1 and 2) served as the basis for the first round of shortest path calculations between all census blocks in the study area.

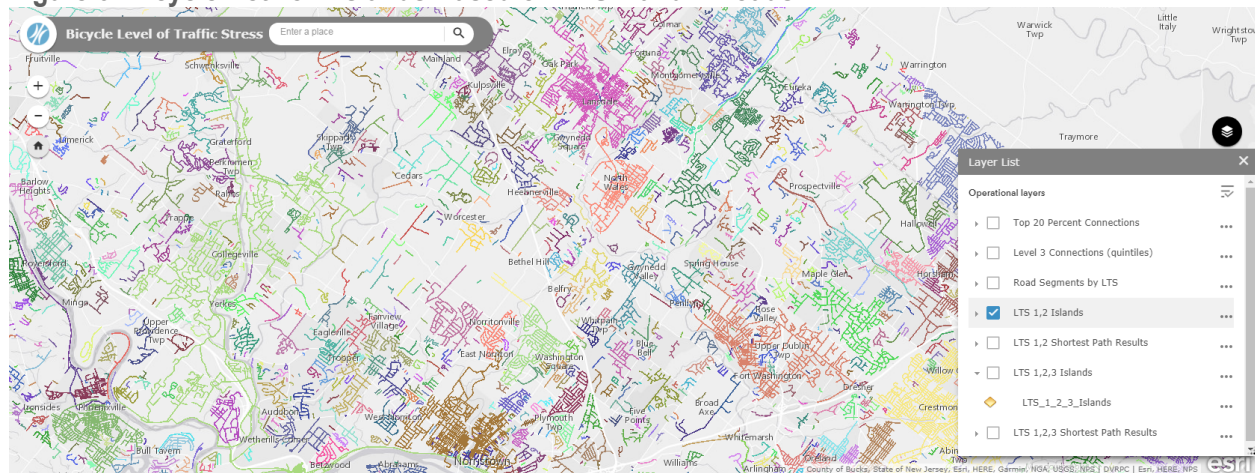
Another network was created for comparison to the existing conditions and to enable the prioritization of places that are not currently comfortable for most cyclists. This network only eliminated LTS 4 roads, leaving LTS 3 segments to be used as network options for the shortest paths connecting census blocks. The results show which LTS 3 segments, if made more comfortable for cyclists, would allow for the most connections between census blocks and, in turn, the greatest benefit the overall bicycle network.

The size of the study area and the computationally intensive analysis presented a number of challenges. Methods were iteratively modified throughout the process to overcome these issues. When preparing the origin and destination pairs for shortest path searches, steps were taken to ensure that only unique pairs of differing origin and destination nodes were calculated. In some cases, the same node was the closest to multiple block centroids. This resulted in the possibility of duplicate shortest paths being calculated between the same node pairs. Further analysis was required in post processing to account for the number of duplicates that had been removed and to add them back in to ensure the components that connected the most block centroids had the highest counts.

Due to the size of the study area, the connectivity analysis yielded extremely large tables, with the largest containing upwards of 10 billion records. Analyzing datasets this large can tax even the most powerful workstations, resulting in unacceptably slow processing times. Routinely working with large datasets throughout this project underscored the importance of implementing an optimized relational database using keys and indices.

Combining links and turns into one large routable network table quadrupled the number of records being used to calculate shortest paths, significantly slowing processing time. Link islands were identified in an effort to increase efficiency. As shown in Figure 6, islands are comprised of connected parts of the low-stress network. If a bicyclist, only comfortable on low-stress ($LTS < 3$) roads, begins a trip on a certain island, they can comfortably travel to any destination on their island. However, if this cyclist needs to reach a destination located on a different island, they will be faced with uncomfortable conditions and therefore be less likely to ride their bicycle. In terms of the implemented algorithm, no path will be found due to one of the trip ends existing on a separate island.

Figure 6: Bicycle Network Islands Based on LTS 1 and 2 Roads



Network islands, shown in Figure 6, were counted and initially visualized to compare their relative size. Although breaking the shortest path calculations into islands and running the algorithm on smaller tables drastically reduced processing time, some larger islands were considered candidates for further processing optimizations—moving frame analysis, as described in the methodology section. However, breaking up a connected island could result in some OD pairs not being connected in the network when they would be in reality. Measures were put in place to preserve these connections. Tradeoffs between processing time and maintaining completeness of the connected shortest paths were carefully balanced.

After the network islands were separated and large islands were divided into smaller frames, the subnetworks were ready for shortest path calculations. To further increase the processing speed, the shortest path searches were conducted in parallel, maximizing the utilization of available computational resources. For each island or frame, OD pairs were re-examined to ensure they were on the same island, and therefore connected. The pairs that met this qualification were added to a list of pairs that would be run through the shortest path algorithm. Initially, the processing time was so great that the connection to the database was lost; this was solved by reopening a connection to the database periodically. Additionally, if a connection could not be made for any reason, the work performed was still dumped to disk.

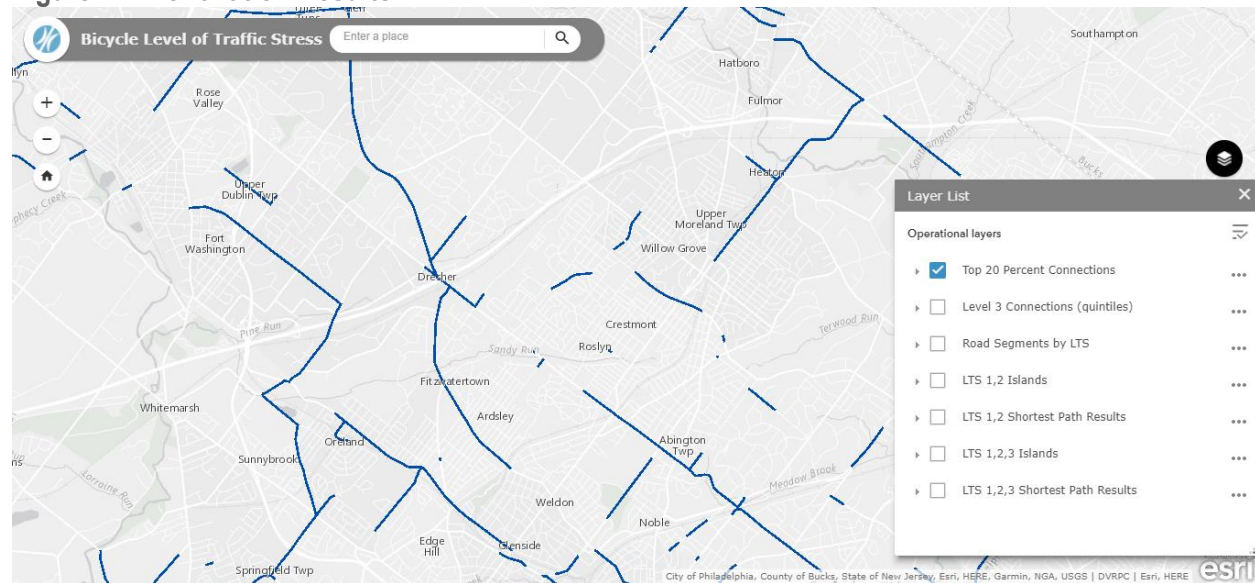
In the first few iterations of the shortest path calculations, it was estimated to take over 400 days to process the entire study area, which was unacceptable and would not meet the needs of the project. Island analysis, moving frames for large islands, and multiprocessing, added through weeks of optimization, reduced the processing time to about two weeks—still significant, but acceptable.

Results

The LTS 3 roads with the highest counts represent those that have the potential to allow the most connections between census blocks. However, being LTS 3 roads, they are not currently considered comfortable for most bicyclists. Therefore, these high count LTS 3 roads represent those where improvements to bicycle facilities, reduction in speed limit, or reduction in number of lanes to reduce the LTS would have the biggest impact on improving overall low-stress network connectivity. When examining these results, it is important to remember that the results are relative to the rest of the LTS 3 roads. Since the shortest paths were calculated on a network including all Level 3 roads, the number of connections enabled by each segment is based on every LTS 3 road being part of the low-stress network. It would be

impossible to make improvements to all LTS 3 roads at the same time. Therefore, these results serve to rank LTS 3 roads by their relative potential impact, as a resource to create a pipeline of realistic and beneficial projects. Figure 7 shows the top 20 percent of connections in Montgomery County, Pennsylvania, identifying the LTS 3 road segments that rank in the top 20 percent in terms of the number of shortest path connections enabled by that segment. This figure reveals a mix of long and short segments, each of which could have significant local benefit if made low-stress.

Figure 7: Prioritization Results



Another metric used to measure network connectivity was the total number of connected block centroids in the network. This becomes most useful when comparing before and after changes in localized analysis. While the analysis was run on the full five-county study area as a baseline for setting priorities for infrastructure investments, the process will allow for comparison between specific network improvements, processed in small sub areas, to determine the network impact of the change.

Next Steps

Currently, the results are available in a webmap and as shapefiles for download. The hope is that planners in the study area will use the webmap highlighting the LTS 3 roads that have the potential to connect the most census blocks to identify areas for further study. Site visits, community outreach, engineering, and cost estimates can help to narrow down the type of improvement possible to reduce the level of stress on the segment.

Once an improvement is proposed, pre- and post-improvement shortest path analyses on a selection of census blocks around the site can identify the number of connections the improvement would enable when implemented on its own. When visually compared, these analyses also allow planners to see the expected growth in the low-stress islands due to the proposed improvement, showing the extent of the potential network impact.

Working group members plan to use the webmap as a resource when planning for bicycle mobility and safety across the region. The tool is already being used to help identify candidates for design and striping improvements as part of the PennDOT's regularly-scheduled road resurfacing projects.

DVRPC will continue to refine and improve the analysis and tools. Analytically, future goals include the incorporation of origin and destination attributes to allow for more specific analysis, such as identifying the improvements that will help connect people to train stations or trails. Additionally, weights may be incorporated to highlight connections that are important for environmental justice or other purposes.

On the products side, another future goal is to create an interactive web-based tool which will allow users to modify the LTS network and calculate the impact of their modifications on a sub-area.

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