```
restart;
n := 10;
```

$$n := 10 \quad (1)$$

```
Delta := 1 / n;
```

$$\Delta := \frac{1}{10} \quad (2)$$

```
# Построим кубический сплайн
cubic_spline := proc(f, x)
  local i;
  local p := seq(i * Delta, i = 1 ..n);
  local lin_eqs := [c[0] = 0, c[n] = 0, a[0] = f(0)];
  for i from 1 to n do:
    lin_eqs := [
      op(lin_eqs),
      a[i] = f(p[i]),
      b[i] = (a[i] − a[i − 1]) / Delta − (2 * c[i − 1] + c[i]) / 3 * Delta,
      d[i] = (c[i] − c[i − 1]) / 3 / Delta
    ];
    if i ≠ n then:
      lin_eqs := [
        op(lin_eqs),
        (c[i − 1] + 4 * c[i] + c[i + 1]) * Delta = 3 * (a[i + 1] − 2 * a[i] + a[i − 1]) / Delta
      ];
    end if
  end do:
  assign(fsolve(lin_eqs));

  return piecewise(
    seq(`if`(i :: odd, x ≤ (i + 1) / 2 * Delta, a[i/2] + b[i/2] * (x−p[i/2]) + c[i/2] * (x−p[i
    /2])^2 + d[i/2] * (x−p[i/2])^3), i = 1 .. 2 * n)
  );
end proc:

# Рассмотрим функцию из статьи https://ceur−ws.org/Vol−1638/Paper63.pdf
```

$u(x) := \cos\left(\frac{\mathrm{Pi} \cdot x}{2}\right) + \exp\left(-\frac{x}{\mathrm{epsilon}}\right);$

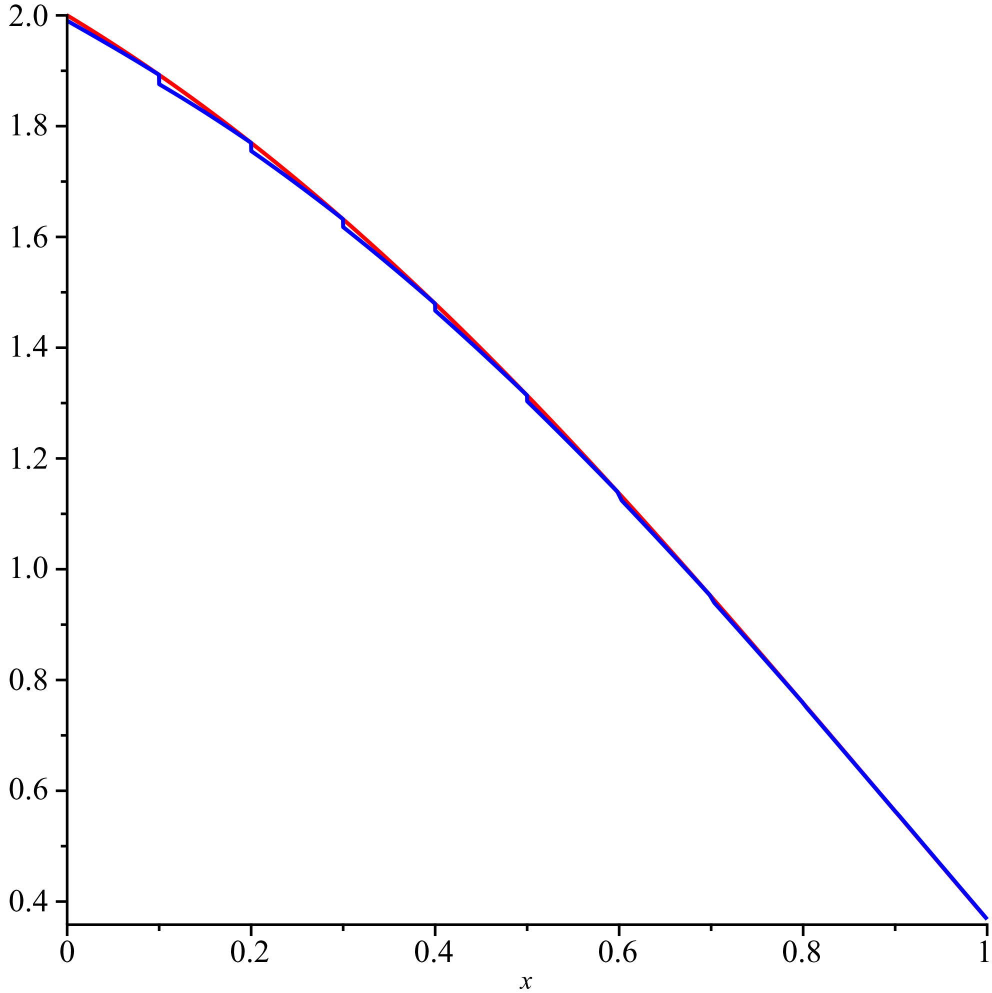
$$u := x \mapsto \cos\left(\frac{x \cdot \pi}{2}\right) + e^{-\frac{x}{\epsilon}} \quad (3)$$

```
# Попробуем пронаблюдать, что при меньших eps приближенеи получается хуже
```

$u[1](x) := \cos\left(\frac{\mathrm{Pi} \cdot x}{2}\right) + \exp(-x);$

$$u_1 := x \mapsto \cos\left(\frac{x \cdot \pi}{2}\right) + e^{-x} \quad (4)$$

```
max_deviation := max(seq(abs(u[1](i) − cubic_spline(u[1], i)), i = 0 ..1, Delta))  = 0.009663625
plot([u[1](x), cubic_spline(u[1], x)], x = 0 ..1, color = [red, blue]);
```

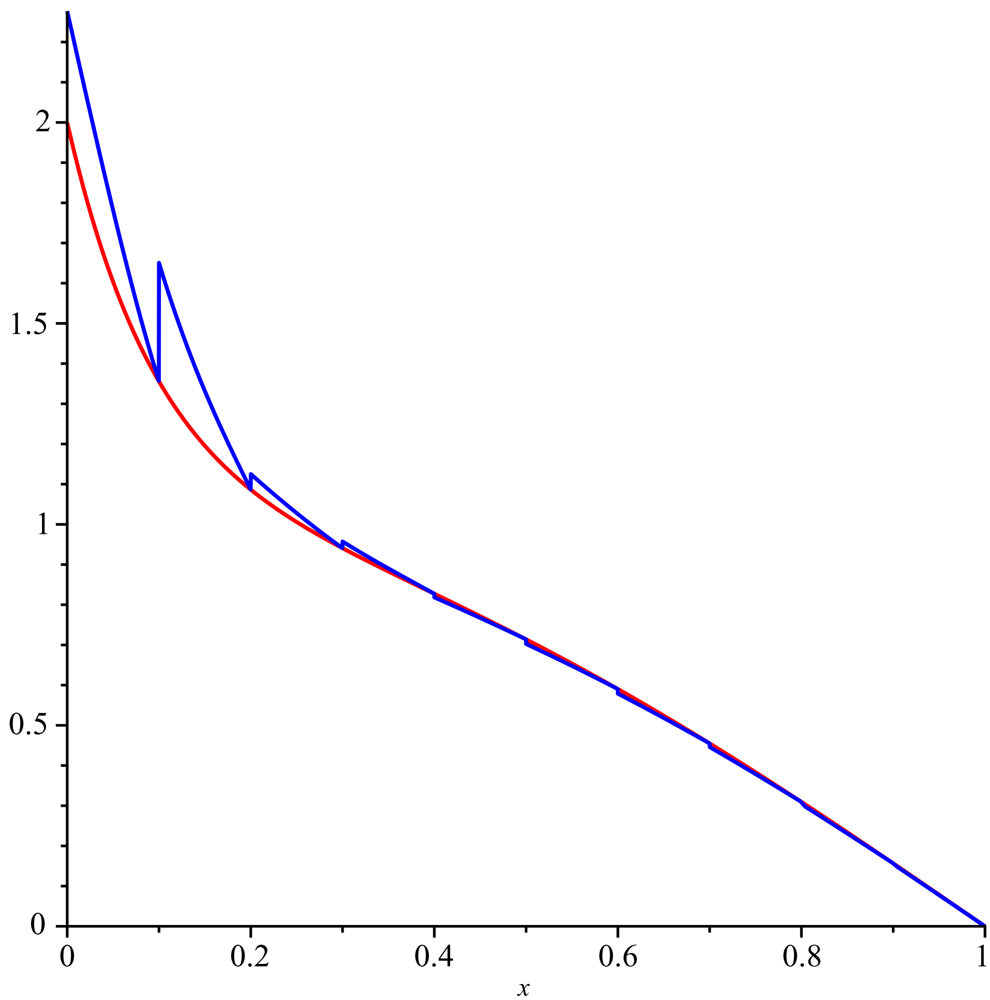


$u[2](x) := \cos\left(\frac{\mathrm{Pi}\cdot x}{2}\right) + \exp(-10\cdot x);$

$$u_2 := x \mapsto \cos\left(\frac{x\cdot\pi}{2}\right) + e^{-10\cdot x} \tag{5}$$

$max\_deviation := \max(seq(\mathrm{abs}(u[2](i) - cubic\_spline(u[2], i)), i = 0..1, \mathrm{Delta})) \quad = 0.276855265$

plot([u[2](x), *cubic_spline*(*u*[2], *x*)], x = 0..1, color = [red, blue])



$$u_3 := x \mapsto \cos\left(\frac{x \cdot \pi}{2}\right) + e^{-100 \cdot x} \quad \textbf{(6)}$$

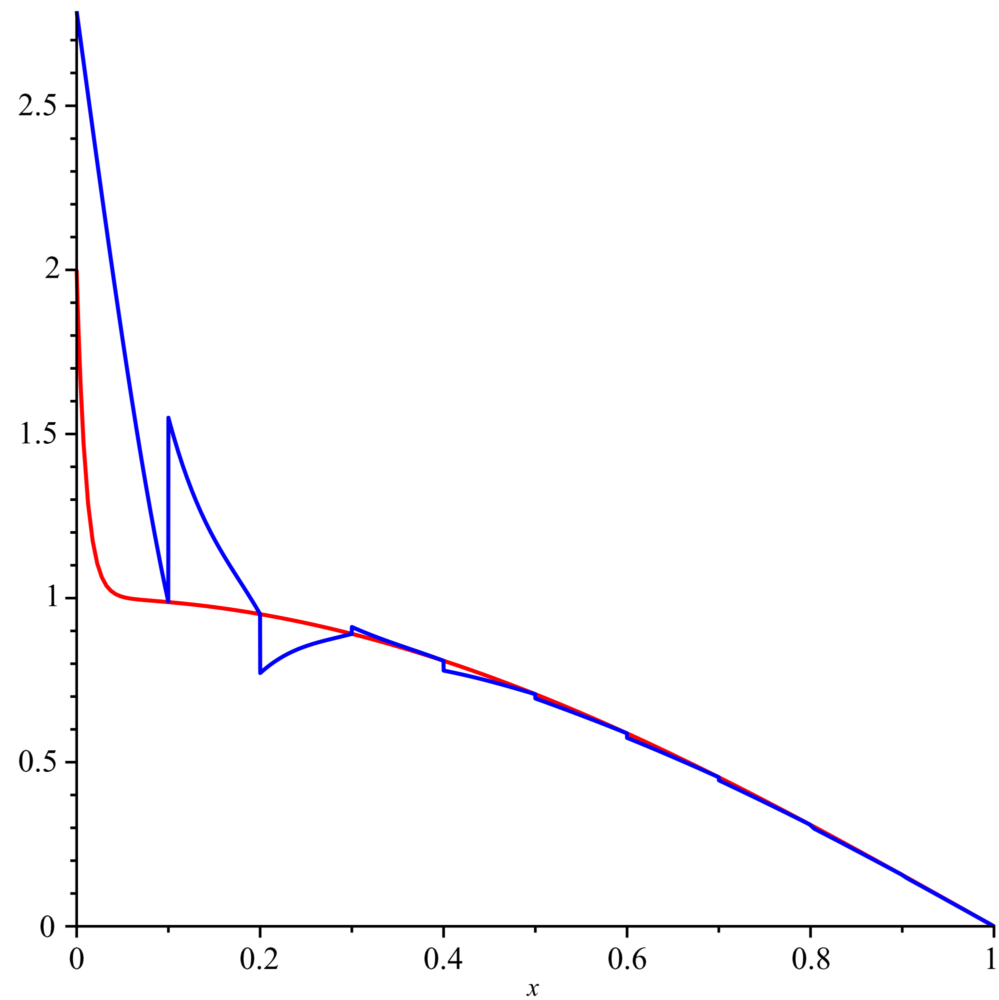
Error, invalid input: eval expects 1 or 2 arguments, but received 0

$u[3](x) := \cos\left(\frac{\mathrm{Pi} \cdot x}{2}\right) + \exp(-10 \cdot x)$;

$$u_3 := x \mapsto \cos\left(\frac{x \cdot \pi}{2}\right) + e^{-10 \cdot x} \quad \textbf{(7)}$$

max_deviation := max(*seq*(abs(*u*[3](*i*) − *cubic_spline*(*u*[3], *i*)) , *i* = 0 ..1, Delta)) = 0.788242127

plot([u[3](x), *cubic_spline*(*u*[3], *x*)] , x = 0 ..1, color = [red, blue])



 # Построим квадратичный B-сплайн

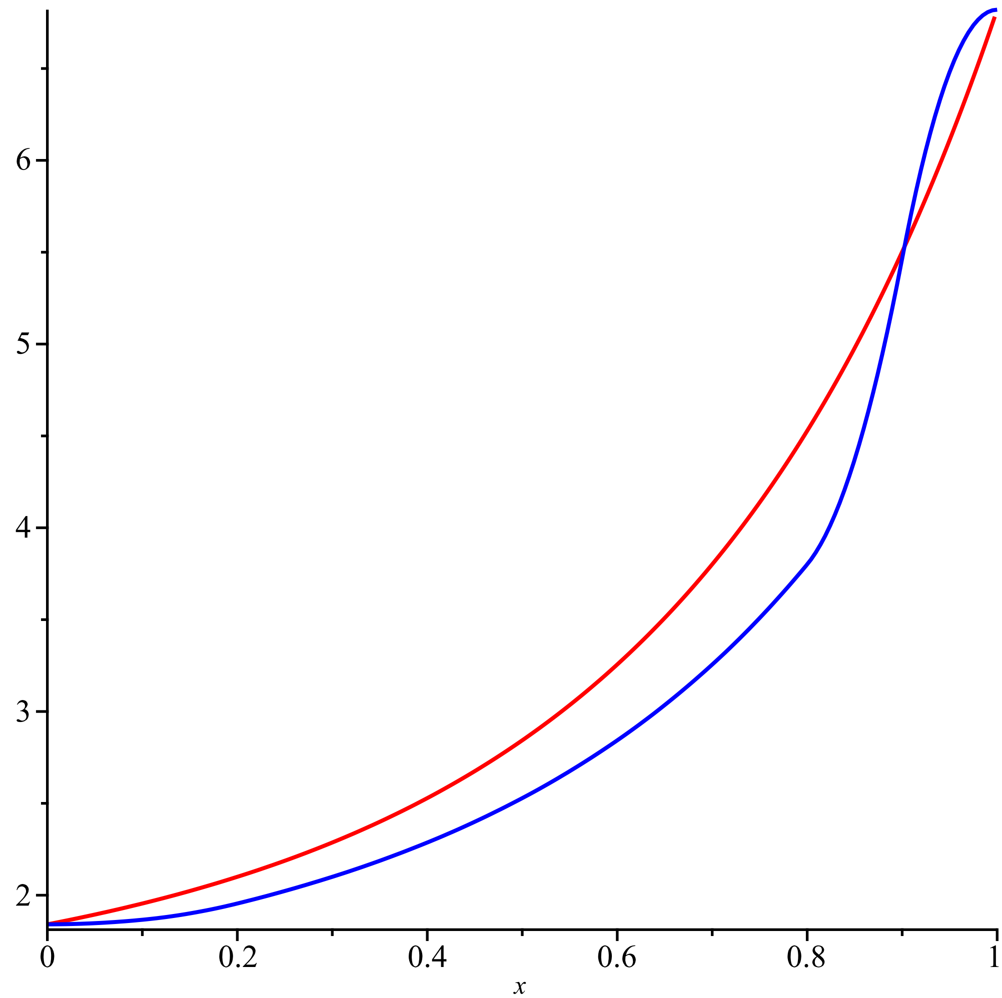
```
b_spline := proc(f, x)
   local i;
   local B;
   local eps := 10^(-9)
   local p := [-2 * eps, -eps, seq(i * Delta, i = 0 ..n), 1 + eps, 1 + 2 * eps];
   local b := [f(0), f(0), seq(1/2 * (-f(p[i]) + 4 * f((p[i] + p[i + 1])/2) - f(p[i + 1])), i
     = 3 ..n), f(1), f(1)];
   B[0] := (j, x) → if p[j] ≤ x < p[j + 1] then 1 else 0 end if;
   B[1] := (j, x) → (x - p[j]) / (p[j + 1] - p[j]) * B[0](j, x) + (p[j + 2] - x) / (p[j + 2]
     - p[j + 1]) * B[0](j + 1, x);
   B[2] := (j, x) → (x - p[j]) / (p[j + 2] - p[j]) * B[1](j, x) + (p[j + 3] - x) / (p[j + 3]
     - p[j + 1]) * B[1](j + 1, x);
   return add(b[i] * B[2](i, x), i = 1 ..n + 2);
end proc:
```

 # Проверим сплайны на некоторых функциях

$f(x) := \sin(\cos(x)) + \exp(x^2 + \sin(x));$

$$f := x \mapsto \sin(\cos(x)) + e^{x^2 + \sin(x)} \qquad \textbf{(8)}$$

plot([f(x), 'b_spline(f, x)'], x = 0 ..1, color = [red, blue])



$max\_deviation := \max(seq(\mathrm{abs}(f(i) - b\_spline(f, i)), i = 0 ..1, \mathrm{Delta}))$

$$max\_deviation := \frac{5\sin\left(\cos\left(\frac{4}{5}\right)\right)}{4} + \frac{5\,e^{\frac{16}{25}+\sin\left(\frac{4}{5}\right)}}{4} + \frac{\sin\left(\cos\left(\frac{3}{5}\right)\right)}{4} + \frac{e^{\frac{9}{25}+\sin\left(\frac{3}{5}\right)}}{4} - \sin\left(\cos\left(\frac{13}{20}\right)\right) - e^{\frac{169}{400}+\sin\left(\frac{13}{20}\right)} + \frac{\sin\left(\cos\left(\frac{7}{10}\right)\right)}{2} + \frac{e^{\frac{49}{100}+\sin\left(\frac{7}{10}\right)}}{2} - \sin\left(\cos\left(\frac{3}{4}\right)\right) - e^{\frac{9}{16}+\sin\left(\frac{3}{4}\right)} \tag{9}$$

$\xrightarrow{\text{at 5 digits}}$

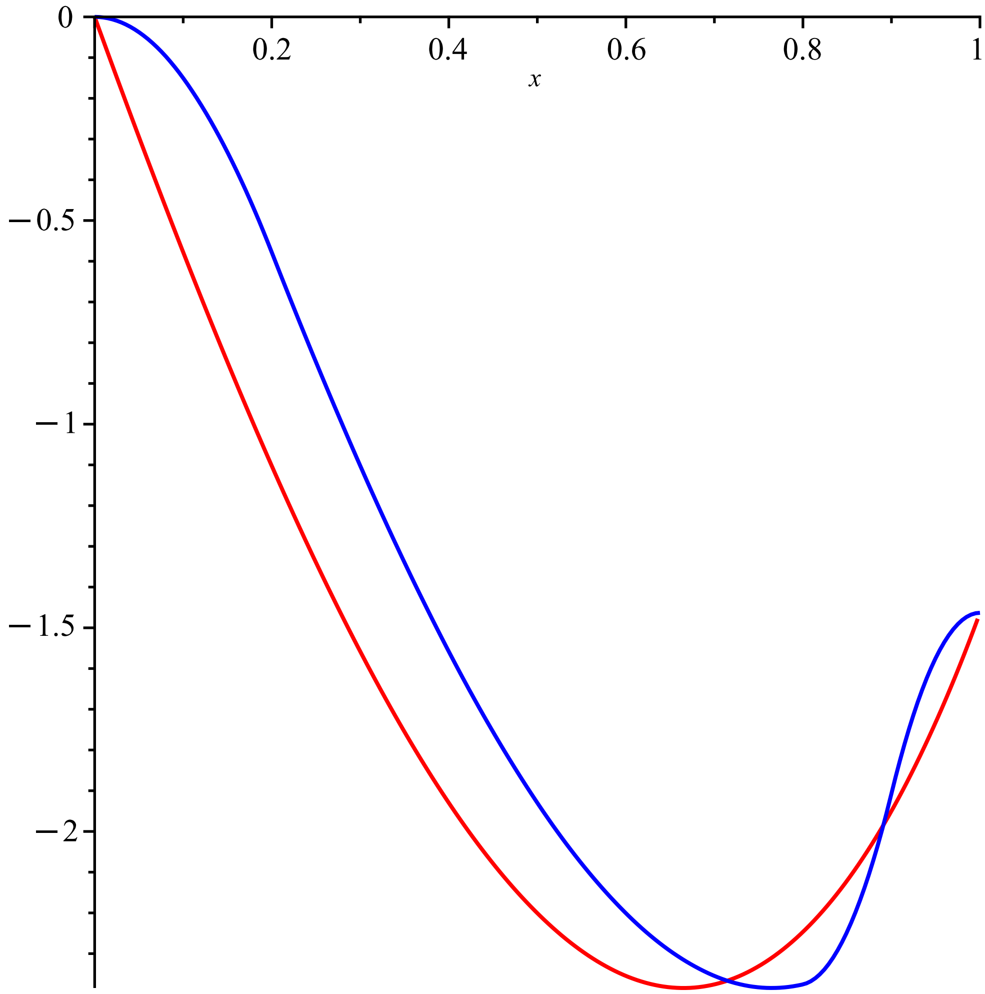
$$0.7268 \tag{10}$$

Error, invalid input: evalf[5] expects 1 argument, but received 0

$f(x) := x^2 + \frac{\tan(x)\cdot x}{x+1} - 6\,x\cdot\cos(x);$

$$f := x \mapsto x^2 + \frac{\tan(x)\cdot x}{x+1} - 6\cdot x\cdot\cos(x) \tag{11}$$

plot([f(x), 'b_spline(f, x)'], x = 0 ..1, color = [red, blue])



$max\_deviation := \max(seq(\mathrm{abs}(f(i) - b\_spline(f, i)), i = 0..1, \mathrm{Delta}))$

$$max\_deviation := -\frac{3}{100} - \frac{5\tan\left(\frac{1}{5}\right)}{24} + \frac{3\cos\left(\frac{1}{5}\right)}{2} + \frac{\tan\left(\frac{1}{20}\right)}{21} - \frac{3\cos\left(\frac{1}{20}\right)}{10} - \frac{\tan\left(\frac{1}{10}\right)}{22} + \frac{3\cos\left(\frac{1}{10}\right)}{10} + \frac{3\tan\left(\frac{3}{20}\right)}{23} - \frac{9\cos\left(\frac{3}{20}\right)}{10} \tag{12}$$

$\xrightarrow{\text{at 5 digits}}$

$$0.52441 \tag{13}$$

 # Попробуем увеличить размер сетки и сравнить точность приближения

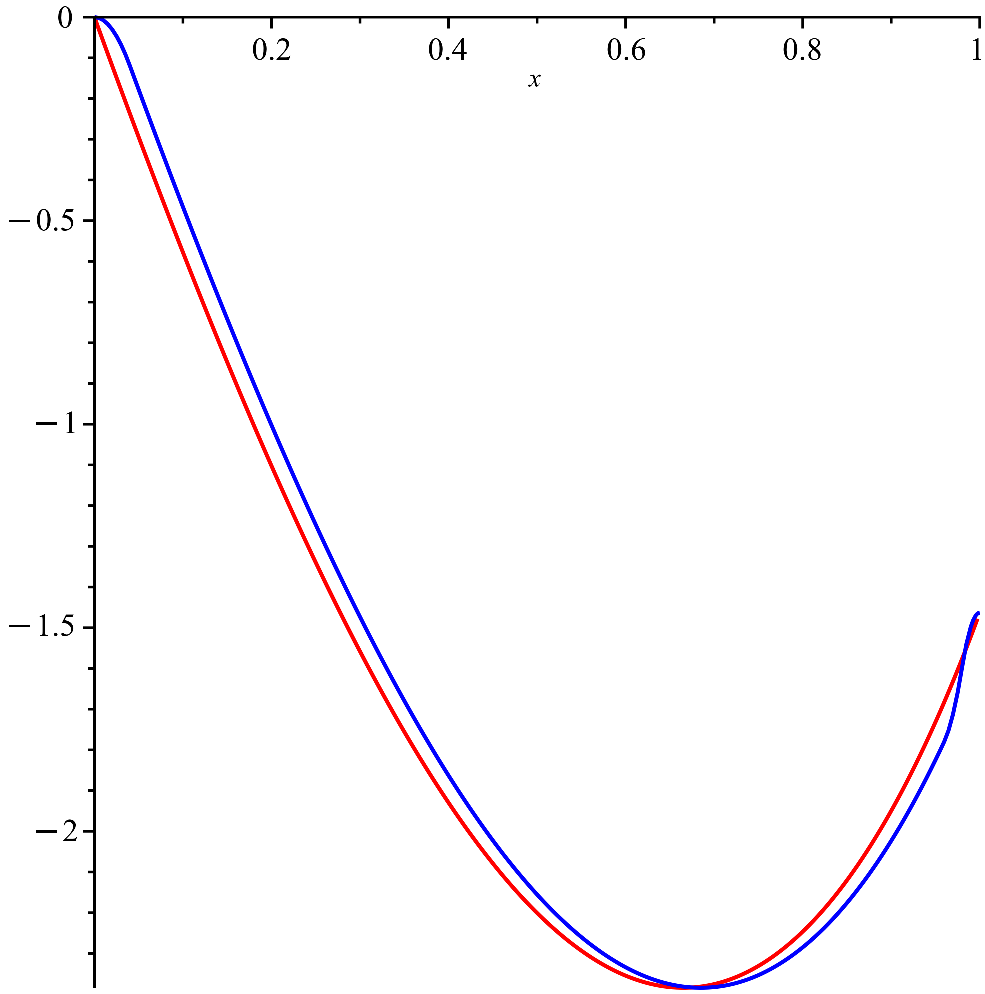
$n := 50;$

$$n := 50 \tag{14}$$

$\mathrm{Delta} := 1 / n;$

$$\Delta := \frac{1}{50} \tag{15}$$

plot([f(x), '*b_spline*(*f*, *x*)'], x = 0 ..1, color = [red, blue])



$max\_deviation := \max(seq(\mathrm{abs}(f(i) - b\_spline(f, i)), i = 0..1, \mathrm{Delta}))$

$$max\_deviation := -\frac{3}{2500} - \frac{5\tan\left(\frac{1}{25}\right)}{104} + \frac{3\cos\left(\frac{1}{25}\right)}{10} + \frac{\tan\left(\frac{1}{100}\right)}{101} - \frac{3\cos\left(\frac{1}{100}\right)}{50} - \frac{\tan\left(\frac{1}{50}\right)}{102} + \frac{3\cos\left(\frac{1}{50}\right)}{50} + \frac{3\tan\left(\frac{3}{100}\right)}{103} - \frac{9\cos\left(\frac{3}{100}\right)}{50} \quad (16)$$

$\xrightarrow{\text{at 5 digits}}$

$$0.11748 \quad (17)$$