Week 8

$$Z = \begin{array}{c c} & \langle 0| \quad \langle 1| \leftarrow \text{I/P} \\ \begin{array}{c}|0\rangle \\ |1\rangle\end{array} & \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \end{array}$$

$\uparrow$ O/P

$\langle Z \rangle$

$\uparrow$

expectation of a quantum observable (for Z-gate)

- → Coherent noise
- → Incoherent noise
- → Projection noise
- → State preparation and meas. noise

✱ The Hamiltonian for X gate:

$$\hat{H} = \frac{\hbar\omega}{2} X$$

$$U(t) = \exp(-it\hat{H}/\hbar)$$

$$\theta = \omega t$$

$$R_x(\theta) = \exp\left(-\frac{i\theta}{2}\hat{X}\right)$$

$$= \cos\left(\frac{\theta}{2}\right) I - i\sin\left(\frac{\theta}{2}\right) X$$

→ Noisy X gate: (coherent noise)

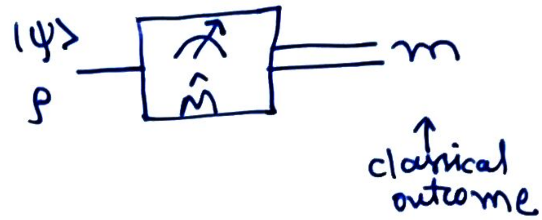
$$\tilde{X} := R_x(\pi + \varepsilon) = \exp\left(-i\frac{\pi}{2}X - i\frac{\varepsilon}{2}X\right)$$

$$= \exp\left(-i\frac{\varepsilon}{2}X\right)\exp\left(-i\frac{\pi}{2}X\right)$$

$$\therefore \quad \tilde{X} = X_{\mathcal{E}} X$$

* The standard (von Neumann) measurement:

→ $\hat{M}$ is the measurement operator/obserable.



projectors
$$\begin{cases} \hat{\pi}_0 = |0\rangle\langle 0| \\ \hat{\pi}_1 = |1\rangle\langle 1| \end{cases}$$

General form $\hat{M} = \sum_m m \, \hat{\Pi}_m$ (also known as spectral decomposition)

where $\hat{\pi}_m = |m\rangle\langle m|$

$$\hat{I} = \sum_m \hat{\Pi}_m \quad \text{(resolution)}$$

(m is a classical outcome)

e.g.
$\hat{\pi}_0 + \hat{\pi}_1 = \hat{I}$

→ For an arbitrary state $|\psi\rangle$,

$$p(m=0) = \langle\psi|0\rangle\langle 0|\psi\rangle = |\langle 0|\psi\rangle|^2$$

$$p(m=1) = \langle\psi|1\rangle\langle 1|\psi\rangle = |\langle 1|\psi\rangle|^2$$

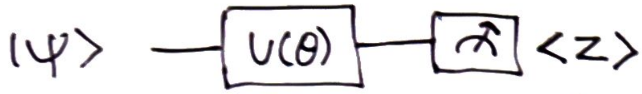
$$p(0) + p(1) = |\langle 0|\psi\rangle|^2 + |\langle 1|\psi\rangle|^2 = ||\psi\rangle|^2$$
$$= |\langle\psi|\psi\rangle|^2$$
$$= 1.$$

$$\sigma_p = \sqrt{\frac{p(1-p)}{N}}$$

s. d. of meas. fluctuations of a projection p.

★ A Quantum classifier:

→ Variational models / Parameterized circuits / Ansatz:



↑ expectation using multiple meas.

1. Data Encoding

(i) Basis encoding

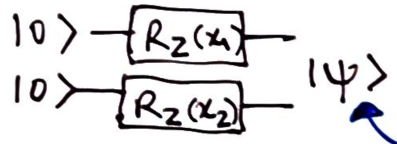
$01 \rightarrow |01\rangle$

(ii) Amplitude encoding

$\begin{bmatrix} 0.7 \\ 0.3 \end{bmatrix} \rightarrow \begin{bmatrix} 0.7 \\ 0.3 \end{bmatrix}$ (encoded using unitary rotat^ns)

(iii) Angle encoding

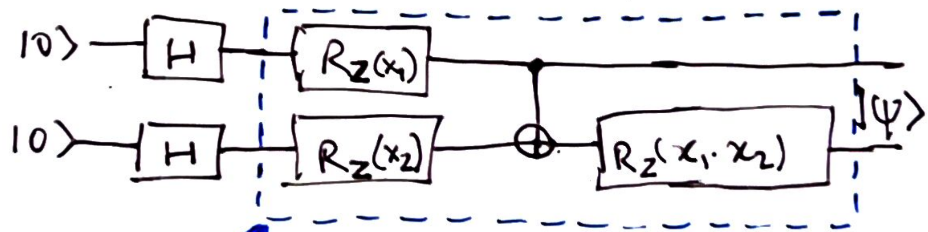
$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$:



encodes data in angle of the qubits

(iv) Higher order encoding

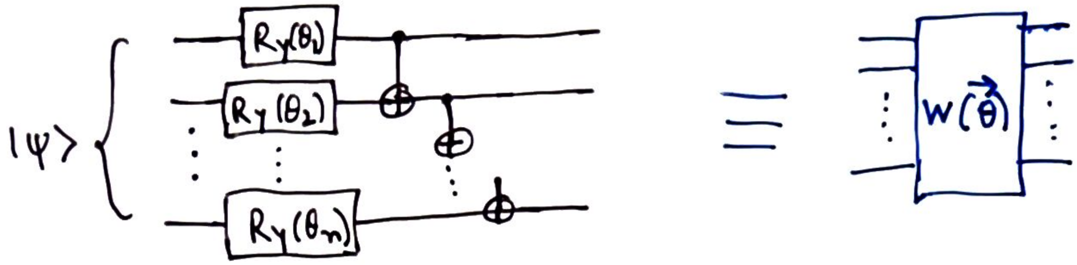
$\vec{x}$:



repeat d times : d is also called the depth of the feature

2. The model

→ Use the $|\psi\rangle$ state which has classical inputs encoded into it and apply the model $W(\vec{\theta})$ on it

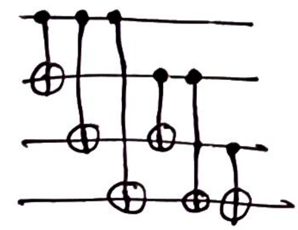


Repeated multiple times

→ Can we rotan around x, y or z axes.

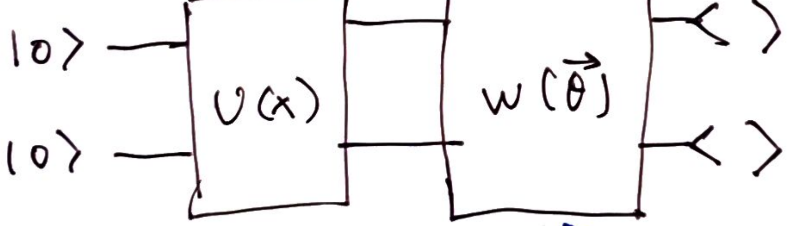
→ can we a different set of entangling controlled gates, maybe applying CNOT b/w all qubits like:

e.g:



3. Label extraction

(i) → Parity post-processing (for binary classification mapping)



e.g: even → +1
odd → −1

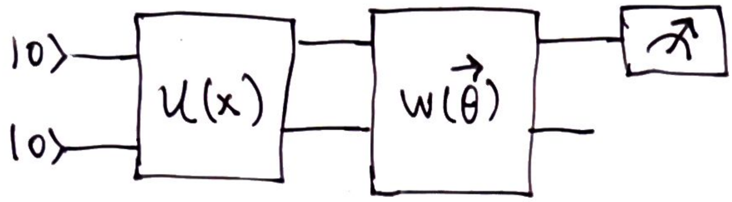
feature map

parameterized model

→ 00 → even
01 → odd
10 → odd
11 → even

$$p(\hat{y}=1) = p(00) + p(11)$$
$$p(\hat{y}=-1) = p(01) + p(10)$$
$$= 1 - p(\hat{y}=1)$$

(ii) → Measuring the first qubit



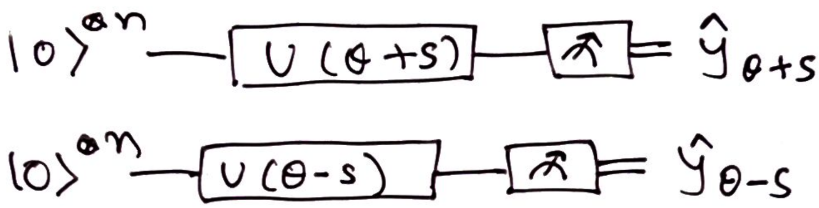
$\langle Z \rangle \rightarrow [-1, 1]$

label

if $Z < 0 \rightarrow -1$
$Z \geq 0 \rightarrow +1$

4. Optimization

(i) Parameter shift rule:



$$\text{Gradient} = \underbrace{\hat{y}_{\theta+s} - \hat{y}_{\theta-s}}_{\text{similar to finite diff. rule}}$$