SMB

→ $A^* = A^T$ if A is a real matrix.

→ Gates :

- X = not
- CNOT = XOR
- Tiffoli = CCNOT = AND (3 Qbit) (1 + 2 = 3)

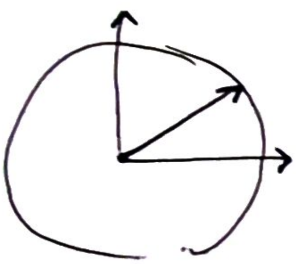
• control
⊕ target

CX - controlled x

→ NOT + Tiffoli = Universal

→ Gates:

- Hadamard = H-gate = probabilistic operator.
- Z → inverts the phase : CZ - controlled z



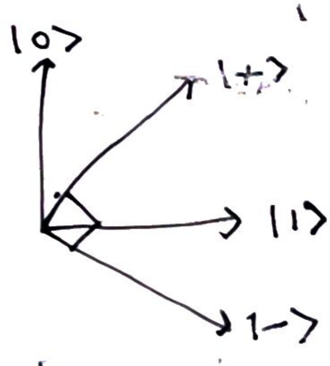
Amplitude magnitude : 0.75 (complex)
Phase : 45°

Classic

[ ] 2^1
[ ][ ] 2^2
⋮

$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$



$|00\rangle = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ $|01\rangle = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ $|10\rangle = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ $|11\rangle = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ 2 Qubit basis.

One qubit Basis

$|+\rangle = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \cdot \sqrt{\frac{1}{2}}$

$= \overline{|0\rangle + |1\rangle}$

$|-\rangle = \sqrt{\frac{1}{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$= \overline{|0\rangle - |1\rangle}$

$H = \sqrt{1/2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ → Gates have to be unitary or hermitian adjoint

→ Unitary ⇒ $U^*U = UU^* = UU^T = I$

→ Hermitian adjoint = conjugate transpose
$= U^{+}U = UU^{+} = I$

→ Real analog of unitary matrix is orthogonal mat.

→ Product states vs entangled states

→ 'phase kickback' when both target and control have superposn.

QC

→ A bit : 0 or 1

→ Qubits representation on state space:

$|0\rangle = \begin{bmatrix}1\\0\end{bmatrix}$ $\qquad |1\rangle = \begin{bmatrix}0\\1\end{bmatrix}$

$|01\rangle = \begin{bmatrix}0\\1\\0\\0\end{bmatrix}$ $\qquad |11\rangle = \begin{bmatrix}0\\0\\0\\1\end{bmatrix}$

To be $\boxed{\text{Complex space}}$

$|\alpha|^2+|\beta|^2=1$

constraint

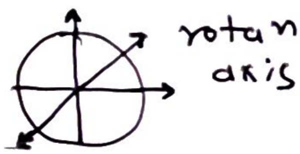
→ Superposition = Linear combination

→ $|0\rangle$ and $|1\rangle$ are orthonormal vectors

* $NOT(\alpha|0\rangle+\beta|1\rangle) = \alpha|1\rangle+\beta|0\rangle$

↑

x-gate = exchange

X = 

$X = \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$

Thinking of left-to-right as passage of time vs a wire

→ gates are functions. fn application as matrix multiplican

$|\psi\rangle$: denoting arbitrary quantum state

* Hadamard

$$H(\alpha|0\rangle + \beta|1\rangle) = \alpha\underbrace{\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right)}_{}^{H|0\rangle} + \beta\left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)$$

$H|0\rangle$ ↓ $\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right)$; $H|1\rangle$ ↓ $\left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right)$

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$|\psi\rangle \text{ ---}\boxed{H}\text{---}\boxed{X}\text{--- } = XH|\psi\rangle$$

→ q-Gates are unitary matrices (e.g. $U = X$ or $U = H$)

$$U^{\dagger}U \overset{[INT]}{=} I \quad ; \quad U^{\dagger} = (U^{T})^{*}$$

Adjoint or dagger operation or Hermitian conjugate / conjugate

complex transpose

→ X, Y, Z · Pauli matrices

* $Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$ * $Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

* ordinary rotaⁿ of 2-d plane by θ:

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Intuition

[INT] → Unitary matrices preserve the length of their inputs. They are simply rotations and reflections.

→ $|\psi\rangle^{\dagger} = \langle\psi|$ → $\| |\psi\rangle \|^2 = \langle\psi||\psi\rangle.$
$= \langle\psi|\psi\rangle$

→ $(M|\psi\rangle)^{\dagger} = \langle\psi|M^{\dagger}$

→ For any matrix M, $\|M|\psi\rangle\|^2 = \langle\psi|M^{\dagger}M|\psi\rangle$

→ $|e_j\rangle$: unit vector with a 1 in the j^{th} component

$|\psi\rangle = |e_0\rangle + |e_1\rangle$; $|e_0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $|e_1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

↑ one qubit

$\langle e_j|M|e_k\rangle$ = jk^{th} element of M.

→ possible combinations of a two-qubit system:

$|\psi\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$

* CNOT : $|x, y\rangle \xrightarrow{CNOT} \underbrace{|x, y \oplus x\rangle}$

classical equivalent

$\oplus$ = Add. modulo 2

$|\psi\rangle \xrightarrow{CNOT} \alpha|00\rangle + \beta|01\rangle + \gamma|11\rangle + \delta|10\rangle$

$(CX)(H)|00\rangle = \dfrac{|00\rangle + |11\rangle}{\sqrt{2}}$

↑ entangled state

$|0\rangle|1\rangle = |01\rangle$; $|0\rangle|0\rangle = |00\rangle$; ...

→ $(\alpha|0\rangle + \beta|1\rangle)(\gamma|0\rangle + \delta|1\rangle)$

$= \alpha\gamma|00\rangle + \alpha\delta|01\rangle + \beta\gamma|10\rangle + \beta\delta|11\rangle$

two single-qubit states, when combined

→ 2^n computational basis for an n-qubit system

→ Measurements in computational basis, collapsing probabilities.

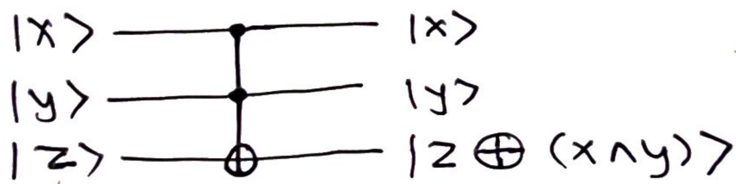
→ Qutrits with three basis: $|0\rangle, |1\rangle, |2\rangle$

→ $e^{i\theta} I$: global phase factor [A gate]

→ $e^{i\theta} = \cos\theta + i\sin\theta$

** A unitary gate with two input qubits always gives two qubits as output.

* Toffoli



if $|z\rangle = |0\rangle$. Toffoli ≃ classical AND

if $|z\rangle = |1\rangle$: its NAND

→ "Programmable matter"

→ Shor's quantum factoring algo.

→ John Preskill on simulating q-field theory

[3 → 2 / 3 → 1 / 2 → 1] side note