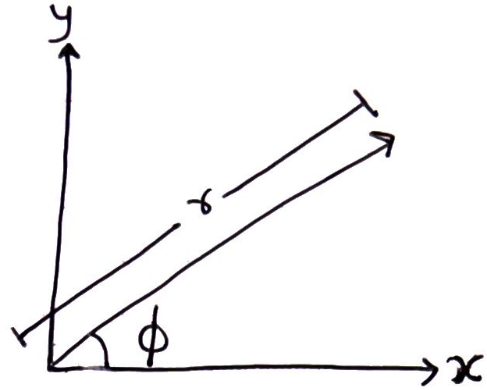
Week 4

* complex numbers:

$$re^{i\phi} = r(\cos\phi + i\sin\phi)$$

$$re^{i\phi} = r\,\mathrm{cis}\,\phi = r\angle\phi$$

$$r = |z| = \sqrt{x^2+y^2}$$



* New set of basis:

$$|+i\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$$

$$|-i\rangle = \frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle)$$

} eigenstates of σ_y [y-measure.]

→ z-measurement : σ_z : $|0\rangle, |1\rangle$

→ x-measurement : σ_x : $|+\rangle, |-\rangle$

* A state collapses to what we measure: The Born rule

* Bloch sphere: Block vector $\vec{r} = \begin{pmatrix} \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\theta \end{pmatrix}$

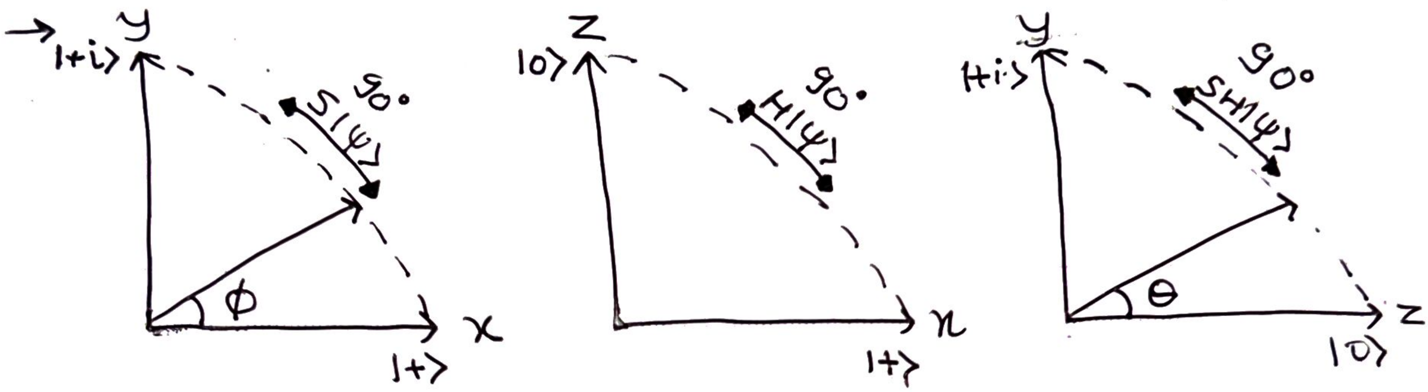
→ z-measure. is a projection of $|\psi\rangle$ on the z-axis ... the same for x, y axes.

$\theta \in [0, \pi]$ where $|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$

→ θ is the angle on Bloch sphere, but $\frac{\theta}{2}$ is the angle corresponding to Hilbert space.

→ y-axis on the Block sphere correspond to the y-measure. orthogonal states ~~and similary for x,z axes:~~ Pauli gates are a $180°$ rotatn in the Bloch Sphere, so $90°$ in Hilbert Space over the corresponding axis.

→



[Block sphere in 2D]

→ x-gate: bit-flip, z-gate: phase-flip, y-gate: both-flip

⇛ Tensor products are used to describe multipartite states,

e.g. $|a\rangle \otimes |b\rangle = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \otimes \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} a_1 b_1 \\ a_1 b_2 \\ a_2 b_1 \\ a_2 b_2 \end{pmatrix} = |ab\rangle$

is a bipartite state

* Entangled states create two sets of basis states. Described by ϕ^-, ϕ^+, ψ^-, ψ^+ in Week 2 These are also known as Bell states.

→ If (CX)(H) is applied to a 2-qubit state before measurement, it is known as bell measure.
[We use (H)(CX) to get an entangled state.]