# 2D Inverted Pendulum

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Equations of Motion

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## 1 Terms

- Subscripts
  - **b**: body
  - **m**: motor
  - **g**: gravity, IE a force or moment due strictly to gravity
  - **f**: friction, frictional forces or moments
  - **o**: origin point of the system

## 2 Equations Of Motion

$$\sum M = I\alpha \tag{1}$$

$$-M_{bg} + M_{bf} - M_{mg} + T_m - M_{mf} = I\alpha_b \tag{2}$$

$$M_{bg} = m_b g * l_b sin(\theta_b), M_{mg} = m_m g * l_m sin(\theta_b) \tag{3}$$

$$M_{bf} = C_b\dot{\theta}_b, M_{mf} = C_b\dot{\theta}_b \tag{4}$$

$$I = I_{bo} + I_{mo} \tag{5}$$

$$I_{mo} = I_m + m_w l^2 \tag{6}$$

$$-m_b g * l_b sin(\theta_b) + C_b\dot{\theta}_b - m_m g * l_m sin(\theta_b) + T_m - C_b\dot{\theta}_b = (I_{bo} + I_{mo})\alpha_b \tag{7}$$

$$\ddot{\theta}_b = \frac{m_b g * l_b sin(\theta_b) - C_b\dot{\theta}_b + m_m g * l_m sin(\theta_b) - T_m + C_b\dot{\theta}_b}{I_{bo} + I_m + m_m l^2} \tag{8}$$

$$\ddot{\theta}_b = \frac{g sin(\theta_b)(m_b l_b + m_m l_m) - C_b\dot{\theta}_b - T_m + C_b\dot{\theta}_b}{I_{bo} + I_m + m_m l^2} \tag{9}$$