Latex Assignment17

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Ex 12.3.2

1. Let $A = \begin{pmatrix} 2 & 4 \\ 3 & 2 \end{pmatrix}, B = \begin{pmatrix} 1 & 3 \\ -2 & 5 \end{pmatrix}, C = \begin{pmatrix} -2 & 5 \\ 3 & 4 \end{pmatrix}$. Find each of the following:
   (i) $A + B$
   (ii) $A - B$
   (iii) $3A - C$
   (iv) AB
   (v) BA

2. Compute the following:
   (i) $\begin{pmatrix} a & b \\ -b & a \end{pmatrix} + \begin{pmatrix} a & b \\ -b & a \end{pmatrix}$
   (ii) $\begin{pmatrix} a^2 + b^2 & b^2 + c^2 \\ a^2 + c^2 & a^2 + b^2 \end{pmatrix} + \begin{pmatrix} 2ab & 2ac \\ -2ac & -2ab \end{pmatrix}$
   (iii) $\begin{pmatrix} -1 & 4 & -6 \\ 8 & 5 & 16 \\ 2 & 8 & 5 \end{pmatrix} + \begin{pmatrix} 12 & 7 & 6 \\ 8 & 0 & 5 \\ 3 & 2 & 4 \end{pmatrix}$
   (iv) $\begin{pmatrix} \cos^2 x & \sin^2 x \\ \sin^2 x & \cos^2 x \end{pmatrix} + \begin{pmatrix} \sin^2 x & \cos^2 x \\ \cos^2 x & \sin^2 x \end{pmatrix}$

3. Compute the following products:
   (i) $\begin{pmatrix} a & b \\ b & -a \end{pmatrix} \begin{pmatrix} a & -b \\ b & a \end{pmatrix}$
   (ii) $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \begin{pmatrix} 2 & 3 & 4 \end{pmatrix}$
   (iii) $\begin{pmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{pmatrix} \begin{pmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 5 & 0 & 5 \end{pmatrix}$

(iv) $\begin{pmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{pmatrix}$

(v) $\begin{pmatrix} 3 & -1 & 3 \\ 1 & 0 & 2 \end{pmatrix}\begin{pmatrix} 2 & -3 \\ 1 & 0 \\ 3 & 1 \end{pmatrix}$

4. If $A = \begin{pmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{pmatrix}$ and $C = \begin{pmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{pmatrix}$, then compute $(A + B)$ and $(B + C)$. Also, verify that $A + (B - C) = (A + B) - C$.

5. If $A = \begin{pmatrix} \frac{2}{3} & 1 & \frac{5}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{4}{3} \\ \frac{7}{3} & 2 & \frac{2}{3} \end{pmatrix}$ and $B = \begin{pmatrix} \frac{2}{5} & \frac{3}{5} & 1 \\ \frac{1}{5} & \frac{2}{5} & \frac{4}{5} \\ \frac{7}{5} & \frac{6}{5} & \frac{2}{5} \end{pmatrix}$, then compute $3A - 5B$.

6. Simplify $\cos\theta\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} + \begin{pmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{pmatrix}$.

7. Find X and Y, if:

(i) $X + Y = \begin{pmatrix} 7 & 0 \\ 2 & 5 \end{pmatrix}$ and $X - Y = \begin{pmatrix} 3 & 0 \\ 0 & 4 \end{pmatrix}$.

(ii) $2X + 3Y = \begin{pmatrix} 2 & 3 \\ 4 & 0 \end{pmatrix}$ and $3X + 2Y = \begin{pmatrix} 2 & -2 \\ -1 & 5 \end{pmatrix}$

8. Find X, if $Y = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$ and $2X + Y = \begin{pmatrix} 1 & 0 \\ -3 & 2 \end{pmatrix}$.

9. Find x and y, if $2\begin{pmatrix} 1 & 3 \\ 0 & x \end{pmatrix} + \begin{pmatrix} y & 0 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 5 & 6 \\ 1 & 8 \end{pmatrix}$.

10. Solve the equation for x, y, z and t, if $2\begin{pmatrix} x & y \\ z & t \end{pmatrix} + 3\begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix} = 3\begin{pmatrix} 3 & 5 \\ 4 & 6 \end{pmatrix}$.

11. If $x\begin{pmatrix} 2 \\ 3 \end{pmatrix} + y\begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 10 \\ 5 \end{pmatrix}$, find the values of x and y.

12. Given $3\begin{pmatrix} x & y \\ z & w \end{pmatrix} = \begin{pmatrix} x & 6 \\ -1 & 2w \end{pmatrix} + \begin{pmatrix} 4 & x + y \\ z + w & 3 \end{pmatrix}$, find the values of x, y, z and w.

13. If $F(x) = \begin{bmatrix} cosx & -sinx & 0 \\ sinx & cosx & 0 \\ 0 & 0 & 1 \end{bmatrix}$, show that $F(x) + F(y) = F(x + y)$.

14. Show that:

(i) $\begin{pmatrix} 5 & -1 \\ 6 & 7 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix} \neq \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 5 & -1 \\ 6 & 7 \end{pmatrix}$.

(ii) $\begin{pmatrix}1 & 2 & 3\\0 & 1 & 0\\1 & 1 & 0\end{pmatrix}\begin{pmatrix}-1 & 1 & 0\\0 & -1 & 1\\2 & 3 & 4\end{pmatrix} \neq \begin{pmatrix}-1 & 1 & 0\\0 & -1 & 1\\2 & 3 & 4\end{pmatrix}\begin{pmatrix}1 & 2 & 3\\0 & 1 & 0\\1 & 1 & 0\end{pmatrix}$

15. Find $A^2 - 5A + 6I$, if $A = \begin{pmatrix}2 & 0 & 1\\2 & 1 & 3\\1 & -1 & 0\end{pmatrix}$.

16. If $A = \begin{pmatrix}1 & 0 & 2\\0 & 2 & 1\\2 & 0 & 3\end{pmatrix}$, prove that $A^3 - 6A^2 + 7A + 2I = 0$.

17. If $A = \begin{pmatrix}3 & -2\\4 & -2\end{pmatrix}$ and $I = \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$, find k so that $A^2 = kA - 2I$.

18. If $A = \begin{pmatrix}0 & -\tan\frac{\alpha}{2}\\ \tan\frac{\alpha}{2} & 0\end{pmatrix}$ and I is the identity matrix of order 2, show that $I + A = (I - A)\begin{pmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{pmatrix}$.

19. A trust fund has ₹ 30000 that must be invested in two different types of bonds. The first bomd pays 5% interest per year, and the secon bond pays 7% interest per year. Using matrix multiplication, detemine how to divide ₹ 30000 among the 2 types of bonds. If the trust fund must obtain an annual total interest of:

    (a) ₹ 1800
    (b) ₹ 2000

20. The bookshop of a particular school has 10 dozen chemistry books, 8 dozen phyaics books, 10 dozen economics books. Their selling prices are ₹ 80, ₹ 60 and ₹ 40 each respectively. Find the total amount the bookshop will receive from selling all the books using matrix algebra.
    Assume X, Y, Z, W and P are matrices of order $2 \times n, 3 \times k, 2 \times p, n \times 3$ and $p \times k$, respectively. Choose the correct answer in 21 and 22.

21. The restriction on n, k and p so that $PY + WY$ will be defined are:

    (a) $k = 3, p = n$
    (b) k is arbitrary, $p = 2$.
    (c) p is arbitrary, $k = 3$
    (d) $k = 2, p = 3$

22. If $n = p$, then order of the matrix $7X - 52$ is:

    (a) $p \times 2$
    (b) $2 \times n$
    (c) $n \times 3$
    (d) $p \times n$