

Latex Assignment17

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Ex 12.3.2

1. Let $A = \begin{pmatrix} 2 & 4 \\ 3 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 3 \\ -2 & 5 \end{pmatrix}$, $C = \begin{pmatrix} -2 & 5 \\ 3 & 4 \end{pmatrix}$. Find each of the following:

(i) $A + B$

(ii) $A - B$

(iii) $3A - C$

(iv) AB

(v) BA

2. Compute the following:

(i) $\begin{pmatrix} a & b \\ -b & a \end{pmatrix} + \begin{pmatrix} a & b \\ -b & a \end{pmatrix}$

(ii) $\begin{pmatrix} a^2 + b^2 & b^2 + c^2 \\ a^2 + c^2 & a^2 + b^2 \end{pmatrix} + \begin{pmatrix} 2ab & 2ac \\ -2ac & -2ab \end{pmatrix}$

(iii) $\begin{pmatrix} -1 & 4 & -6 \\ 8 & 5 & 16 \\ 2 & 8 & 5 \end{pmatrix} + \begin{pmatrix} 12 & 7 & 6 \\ 8 & 0 & 5 \\ 3 & 2 & 4 \end{pmatrix}$

(iv) $\begin{pmatrix} \cos^2 x & \sin^2 x \\ \sin^2 x & \cos^2 x \end{pmatrix} + \begin{pmatrix} \sin^2 x & \cos^2 x \\ \cos^2 x & \sin^2 x \end{pmatrix}$

3. Compute the following products:

(i) $\begin{pmatrix} a & b \\ b & -a \end{pmatrix} \begin{pmatrix} a & -b \\ b & a \end{pmatrix}$

(ii) $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \begin{pmatrix} 2 & 3 & 4 \end{pmatrix}$

(iii) $\begin{pmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{pmatrix} \begin{pmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 5 & 0 & 5 \end{pmatrix}$

$$(iv) \begin{pmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{pmatrix}$$

$$(v) \begin{pmatrix} 3 & -1 & 3 \\ 1 & 0 & 2 \end{pmatrix} \begin{pmatrix} 2 & -3 \\ 1 & 0 \\ 3 & 1 \end{pmatrix}$$

4. If $A = \begin{pmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{pmatrix}$ and $C = \begin{pmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{pmatrix}$, then compute $(A + B)$ and $(B + C)$. Also, verify that $A + (B - C) = (A + B) - C$.

5. If $A = \begin{pmatrix} 2 & 1 & 5 \\ 1 & 2 & 3 \\ 3 & 2 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 3 & 1 \\ 1 & 2 & 4 \\ 5 & 5 & 5 \end{pmatrix}$, then compute $3A - 5B$.

6. Simplify $\cos \theta \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} + \sin \theta \begin{pmatrix} \sin \theta & -\cos \theta \\ \cos \theta & \sin \theta \end{pmatrix}$.

7. Find X and Y , if:

$$(i) X + Y = \begin{pmatrix} 7 & 0 \\ 2 & 5 \end{pmatrix} \text{ and } X - Y = \begin{pmatrix} 3 & 0 \\ 0 & 4 \end{pmatrix}.$$

$$(ii) 2X + 3Y = \begin{pmatrix} 2 & 3 \\ 4 & 0 \end{pmatrix} \text{ and } 3X + 2Y = \begin{pmatrix} 2 & -2 \\ -1 & 5 \end{pmatrix}$$

8. Find X , if $Y = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$ and $2X + Y = \begin{pmatrix} 1 & 0 \\ -3 & 2 \end{pmatrix}$.

9. Find x and y , if $2 \begin{pmatrix} 1 & 3 \\ 0 & x \end{pmatrix} + \begin{pmatrix} y & 0 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 5 & 6 \\ 1 & 8 \end{pmatrix}$.

10. Solve the equation for x, y, z and t , if $2 \begin{pmatrix} x & y \\ z & t \end{pmatrix} + 3 \begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix} = 3 \begin{pmatrix} 3 & 5 \\ 4 & 6 \end{pmatrix}$.

11. If $x \begin{pmatrix} 2 \\ 3 \end{pmatrix} + y \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 10 \\ 5 \end{pmatrix}$, find the values of x and y .

12. Given $3 \begin{pmatrix} x & y \\ z & w \end{pmatrix} = \begin{pmatrix} x & 6 \\ -1 & 2w \end{pmatrix} + \begin{pmatrix} 4 & x+y \\ z+w & 3 \end{pmatrix}$, find the values of x, y, z and w .

13. If $F(x) = \begin{pmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{pmatrix}$, show that $F(x) + F(y) = F(x + y)$.

14. Show that:

$$(i) \begin{pmatrix} 5 & -1 \\ 6 & 7 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix} \neq \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 5 & -1 \\ 6 & 7 \end{pmatrix}.$$

$$(ii) \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{pmatrix} \neq \begin{pmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix}$$

15. Find $A^2 - 5A + 6I$, if $A = \begin{pmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{pmatrix}$.
16. If $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}$, prove that $A^3 - 6A^2 + 7A + 2I = 0$.
17. If $A = \begin{pmatrix} 3 & -2 \\ 4 & -2 \end{pmatrix}$ and $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, find k so that $A^2 = kA - 2I$.
18. If $A = \begin{pmatrix} 0 & -\tan \frac{\alpha}{2} \\ \tan \frac{\alpha}{2} & 0 \end{pmatrix}$ and I is the identity matrix of order 2, show that $I + A = (I - A) \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix}$.
19. A trust fund has ₹ 30000 that must be invested in two different types of bonds. The first bond pays 5% interest per year, and the second bond pays 7% interest per year. Using matrix multiplication, determine how to divide ₹ 30000 among the 2 types of bonds. If the trust fund must obtain an annual total interest of:
- ₹ 1800
 - ₹ 2000
20. The bookshop of a particular school has 10 dozen chemistry books, 8 dozen physics books, 10 dozen economics books. Their selling prices are ₹ 80, ₹ 60 and ₹ 40 each respectively. Find the total amount the bookshop will receive from selling all the books using matrix algebra.
Assume X, Y, Z, W and P are matrices of order $2 \times n, 3 \times k, 2 \times p, n \times 3$ and $p \times k$, respectively. Choose the correct answer in 21 and 22.
21. The restriction on n, k and p so that $PY + WY$ will be defined are:
- $k = 3, p = n$
 - k is arbitrary, $p = 2$.
 - p is arbitrary, $k = 3$
 - $k = 2, p = 3$
22. If $n = p$, then order of the matrix $7X - 5Z$ is:
- $p \times 2$
 - $2 \times n$
 - $n \times 3$
 - $p \times n$