Confidence intervals

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1 Assumptions

We have a collection of random variables $X_1, X_2, ..., X_n$. To build a confidence interval we need to do some assumptions (the same as in the CLT):

i $X_1, X_2, ..., X_n$ are equally distributed.

ii $X_1, X_2, ..., X_n$ are independent.

iii The standard deviation is finite.

2 For the mean

- If we know the variance or the standard deviation, then directly from the CLT:

$$(\overline{X} - z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}, \overline{X} + z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}})$$

  where σ is the standard deviation of each variable and $z_{\frac{\alpha}{2}}$ is the value that the $N(0,1)$ takes for the probability $\frac{\alpha}{2}$.

- With unknown variance:

$$(\overline{X} - t_{\frac{\alpha}{2}}\frac{s}{\sqrt{n}}, \overline{X} + t_{\frac{\alpha}{2}}\frac{s}{\sqrt{n}})$$

  Where s is the quasi deviation:

$$s = \sqrt{\frac{Var(X)n}{n-1}}$$

  and t is the $t - student$ distribution with $k = n - 1$ degrees of freedom.

3 For the variance

We have the next confidence interval:

$$(\frac{(n-1)s^2}{\chi^2_{\alpha/2}}, \frac{(n-1)s^2}{\chi^2_{1-\alpha/2}})$$

Where $\chi^2_{\alpha/2}$ and $\chi^2_{1-\alpha/2}$ are the quantiles for the distribution Chi-squared with n-1 degrees of freedom.

4 For the proportion

$$(p - z_{\frac{\alpha}{2}} \times \sqrt{\frac{p \times q}{n}}, p + z_{\frac{\alpha}{2}} \times \sqrt{\frac{p \times q}{n}})$$

Where p is the probability of success and q is $q = 1 - p$ and $z_{\frac{\alpha}{2}}$ is the value that the $N(0,1)$ takes for the probability $\frac{\alpha}{2}$