Homework assignment 3

Compile

Use `make all` to compile the program to a MPI binary. The serial reference implementation is also included but not relevant to the assginment

Binary Usage

```
mpirun -n <slots> ./gauss_mpi <N> [random seed]
- N: Dimension of the matricies
- random seed: seed for populating the matricies
- slots: number of threads to use
```

Additionally the makefile includes targets for the tests required in the assignment

Run Tests

To get an idea of the performance characteristics you can use `make test_assgined` (1,2,4,8,12,16) or `make test_1_22` (1-22). Otherwise you can use the produced binary directly as described above.

Notice on rewrite

Because the number of changes involved for this assignment were relatively signficant (changing structures and such)

Optimizations

- The provided serial algorithm performs the elimination operation on the entire row despite elements not in the upper triangular matrix being known to result in zeros or being used in the back-substitution phase. Instead I simply left those values to what they were initialized so that we don't have to waste resources on them
- Minor changes to improve cache efficiency
- I normalized the diagonal pivots to 1 before sending the row to the other ranks such as to reduce the amount of expensive divisions. It's possible that like in hw2 this didn't provide a meaningful improvement in performance however I feel that the threads would have to synchronize over this anyway so there would be a delay regardless unlike with the shared memory progra
  - This part was optimized by first finding multiplying the entire row by the reciporocal of the pivot as this is more efficient than division

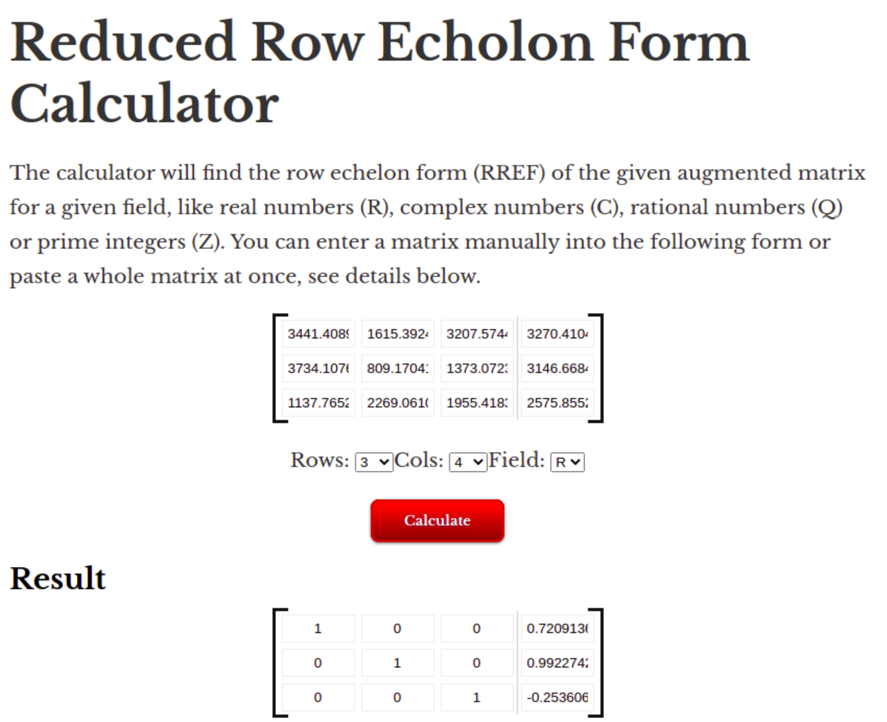
Algorithm description

- The algorithm is similar to the provided serial algorithm
- for each diagonal element - e

- use threads to apply elementary row operations on the rows below it that would eliminate the values in the column below e
- for each row r, below e's row in it's own thread
  * apply subtract a multiple of e's row from r such that it would eliminate the element in the column below e in r
  * don't change the elements in or left of e's column
- synchronize all threads

Verification of Correctness

Although the algorithm very closely follows what I was taught in linear algebra last semester, I did verify the results. - Consistent results were confirmed by running with the same seed several times with various different core counts - Notably difficult was finding faulty behavior when the work was not evenly divisible by the number of ranks - Results were checked via an online RREF calculator



```
--------------------------------------------------------------
matrix(3):
        3441.40894, 1615.39246, 3207.57446 | 3270.41040
        3734.10767,  809.17041, 1373.07239 | 3146.66846
        1137.76526, 2269.06104, 1955.41833 | 2575.85522

matrix(3):
           1.00000,    0.46940,    0.93205 |    0.95031
        3734.10767,    1.00000,    2.23323 |    0.42591
        1137.76526, 1734.99561,    1.00000 |   -0.25361

vector: [ 0.72,  0.99, -0.25]
gausian elimination time: 0.000004 seconds
total time: 0.000005 seconds

[tate@archbook hw3]$
```

Algorithm Performance

As shown in the below image, the parallelism priveds a noticable improvement in performance, however there are dimishing returns, giving the algorithm sublinear scaling.

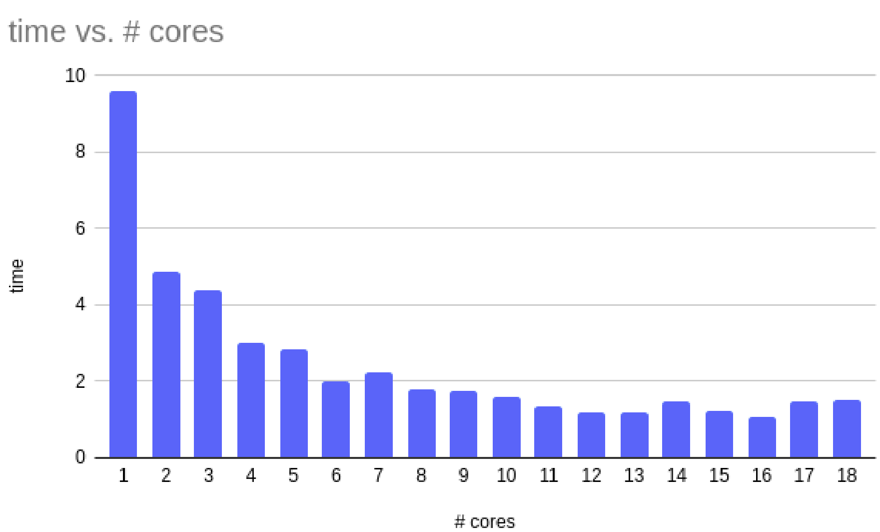


Figure 1: image

slots	time (s)
1	9.601567
2	4.850437
3	4.389371
4	2.987821
5	2.828795
6	1.980991
7	2.222434
8	1.793964
9	1.771634
10	1.609408
11	1.341887
12	1.18755
13	1.175471
14	1.488021
15	1.223894
16	1.069273
17	1.457165
18	1.494384
19	1.723393
20	1.561171
21	1.267043
22	1.188439