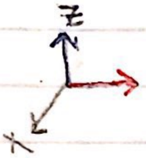
CS411 - Assignment 0A

DV Tate Testa
2019-8-26

A. $A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \ B = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}, \ C = \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}$

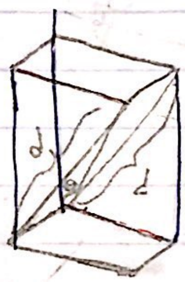
1. $2\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} - \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} - \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \\ 0 \end{bmatrix}$

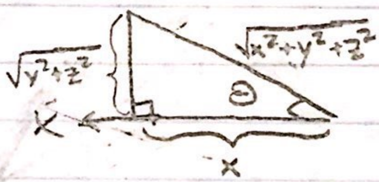
2. 

$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$

$\left\| \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \right\| = \sqrt{1^2 + \underbrace{2^2}_{4} + \underbrace{3^2}_{9}} = \sqrt{14}$

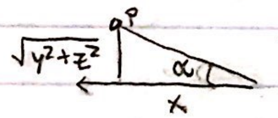
Angle from x-axis



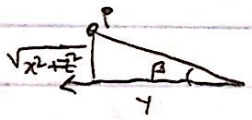


$\theta = \tan^{-1}\left(\frac{\sqrt{y^2+z^2}}{x}\right) = \tan^{-1}\left[\frac{\sqrt{13}}{1}\right] \approx 1.300 \text{ radians} \approx 74.499°$

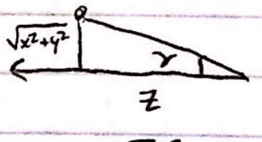
3. $\hat{A} = \left\langle \frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}} \right\rangle$

4. 

$\alpha = \tan^{-1}\left(\frac{\sqrt{y^2+z^2}}{x}\right) = \tan^{-1}(\sqrt{13}) \approx 1.300 \text{ radians}$



$\beta = \tan^{-1}\left(\frac{\sqrt{x^2+z^2}}{y}\right) = \tan^{-1}\left(\frac{\sqrt{10}}{2}\right) \approx 1.007 \text{ radians}$



$\gamma = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right) = \tan^{-1}\left(\frac{\sqrt{5}}{3}\right) \approx 0.641 \text{ radians}$

5. $A \cdot B = 4 + 10 + 18 = 32$

$B \cdot A = 4 + 10 + 18 = 32$

6. $A \cdot B = 32$ $\quad \|A\| = \sqrt{1^2+2^2+3^2} = \sqrt{14}$ $\quad \|B\| = \sqrt{4^2+5^2+6^2} = \sqrt{16+25+36} = \sqrt{77}$

$\vec{A} \cdot \vec{B} = \|\vec{A}\| \|\vec{B}\| \cos\theta$

$\theta = \cos^{-1}\left(\frac{\vec{A} \cdot \vec{B}}{\|\vec{A}\| \|\vec{B}\|}\right) = \cos^{-1}\left(\frac{32}{\sqrt{14}\sqrt{77}}\right) = 0.226 \text{ radians}$

7. $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix} = \begin{vmatrix} 2 & 3 \\ 5 & 6 \end{vmatrix} \hat{i} - \begin{vmatrix} 1 & 3 \\ 4 & 6 \end{vmatrix} \hat{j} + \begin{vmatrix} 1 & 2 \\ 4 & 5 \end{vmatrix} \hat{k} = \langle -3, 6, -3 \rangle$

9.

8. $\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix} = \begin{vmatrix} 2 & 3 \\ 5 & 6 \end{vmatrix}\hat{i} - \begin{vmatrix} 1 & 3 \\ 4 & 6 \end{vmatrix}\hat{j} + \begin{vmatrix} 1 & 2 \\ 4 & 5 \end{vmatrix}\hat{k} = \langle -3, 6, -3 \rangle$

$\vec{B}\times\vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 5 & 6 \\ 1 & 2 & 3 \end{vmatrix} = \begin{vmatrix} 5 & 6 \\ 2 & 3 \end{vmatrix}\hat{i} - \begin{vmatrix} 4 & 6 \\ 1 & 3 \end{vmatrix}\hat{j} + \begin{vmatrix} 4 & 5 \\ 1 & 2 \end{vmatrix}\hat{k} = \langle 3, -6, 3 \rangle$

9. See 7

10. $x\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + y\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} + z\begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix} = 0$

$\begin{vmatrix} 1 & 4 & -1 \\ 2 & 5 & 1 \\ 3 & 6 & 3 \end{vmatrix} = 0$

$\therefore$ linearly dependant

$x + 4y - z = 0 \Rightarrow z = x + 4y$

$2x + 5y + z = 0 \Rightarrow 2x + 5y + x + 4y = 0$

$3x + 6y + 3z = 0 \qquad 3x + 9y = 0$

$-x = -3y$

$z = -3y + 4y$

$z = y$

$-9y + 6y + 3y = 0$

$x = 0$

$y = 0$?

$z = 0$

11. $[1, 2, 3]\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = [4 + 10 + 18] = [32]$

$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}[4, 5, 6] = \begin{bmatrix} 4 & 5 & 6 \\ 8 & 10 & 12 \\ 12 & 15 & 18 \end{bmatrix}$

CS 411 - Assignment 0B

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B. $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ -1 & 1 & 3 \end{bmatrix}$

1. $$2\begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4 & 6 \\ 8 & -4 & 6 \\ 0 & 10 & -2 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 5 \\ 6 & -5 & 10 \\ -3 & 12 & -3 \end{bmatrix}$$

2. $$AB = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} (1\cdot1+2\cdot2+3\cdot3) & (1\cdot2+2\cdot1+3\cdot-2) & (1\cdot1+2\cdot-4+3\cdot1) \\ (4\cdot1+-2\cdot2+3\cdot3) & (4\cdot2+-2\cdot1+3\cdot-2) & (4\cdot1+-2\cdot-4+3\cdot1) \\ (0+5\cdot2+-1\cdot3) & (0\cdot2+5\cdot1+-1\cdot-2) & (0\cdot1+5\cdot-4+-1\cdot1) \end{bmatrix}$$

$$= \begin{bmatrix} 14 & -2 & -4 \\ 9 & 0 & 15 \\ 7 & 7 & -21 \end{bmatrix}$$

$$BA = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} (1\cdot1+2\cdot4+1\cdot0) & (1\cdot2+2\cdot-2+1\cdot5) & (1\cdot3+2\cdot3+1\cdot-1) \\ (2\cdot1+1\cdot4+-4\cdot0) & (2\cdot2+1\cdot-2+-4\cdot5) & (2\cdot3+1\cdot3+-4\cdot-1) \\ (3\cdot1+-2\cdot4+1\cdot0) & (3\cdot2+-2\cdot-2+1\cdot5) & (3\cdot3+-2\cdot3+1\cdot-1) \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 3 & 8 \\ 6 & -18 & 13 \\ -5 & 15 & 2 \end{bmatrix}$$

3. $$(AB)^T = \left(\begin{bmatrix} 14 & -2 & -4 \\ 9 & 0 & 15 \\ 7 & 7 & -21 \end{bmatrix}\right)^T = \begin{bmatrix} 14 & 9 & 7 \\ -2 & 0 & 7 \\ -4 & 15 & -21 \end{bmatrix}$$

$$A^T B^T = \begin{bmatrix} 1 & 4 & 0 \\ 2 & -2 & 5 \\ 3 & 3 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & -2 \\ 1 & -4 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 6 & -5 \\ 3 & -18 & 15 \\ 8 & 13 & 2 \end{bmatrix}$$

48 + 8

4. $$\begin{vmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{vmatrix} = 1\begin{vmatrix} -2 & 3 \\ 5 & -1 \end{vmatrix} - 2\begin{vmatrix} 4 & 3 \\ 0 & -1 \end{vmatrix} + 3\begin{vmatrix} 4 & -2 \\ 0 & 5 \end{vmatrix} = -12 + 8 + 60 = 55 \leftarrow |A|$$

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ -1 & 1 & 3 \end{vmatrix} = 1\begin{vmatrix} 5 & 6 \\ 1 & 3 \end{vmatrix} - 2\begin{vmatrix} 4 & 6 \\ -1 & 3 \end{vmatrix} + 3\begin{vmatrix} 4 & 5 \\ -1 & 1 \end{vmatrix} = 9 + 36 + 27 = 12 \leftarrow |C|$$

5. ~~A?: <1,2,3> · <4,-2,3> = 4-4+9 = 9 ≠ 0~~ NOTA!

B?: $\langle 1,2,1\rangle \cdot \langle 2,1,-4\rangle = 2+2-4 \overset{\checkmark}{=} 0$ $\quad \langle 2,1,-4\rangle \cdot \langle 3,-2,1\rangle = 6-2-4 \overset{\checkmark}{=} 0$

$\langle 1,2,1\rangle \cdot \langle 3,-2,1\rangle = 3-4+1 \overset{\checkmark}{=} 0$

B's rows form orthogonal set

6. $A^{-1} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix}^{-1}$

$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{vmatrix} = 55$ (see #4)

$$A^{-1} = \begin{bmatrix} \begin{vmatrix} -2 & 3 \\ 5 & -1 \end{vmatrix} & -\begin{vmatrix} 4 & 3 \\ 0 & -1 \end{vmatrix} & \begin{vmatrix} 4 & -2 \\ 0 & 5 \end{vmatrix} \\ -\begin{vmatrix} 2 & 3 \\ 5 & -1 \end{vmatrix} & \begin{vmatrix} 1 & 3 \\ 0 & -1 \end{vmatrix} & -\begin{vmatrix} 1 & 2 \\ 0 & 5 \end{vmatrix} \\ \begin{vmatrix} 2 & 3 \\ -2 & 3 \end{vmatrix} & -\begin{vmatrix} 1 & 3 \\ 4 & 3 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 4 & -2 \end{vmatrix} \end{bmatrix}^T \left(\frac{1}{|A|}\right)$$

$$\begin{bmatrix} -13 & 4 & 20 \\ 17 & -1 & -5 \\ 12 & 9 & -10 \end{bmatrix}^T \left(\frac{1}{|A|}\right)$$

$$\begin{bmatrix} -13/55 & 17/55 & 12/55 \\ 4/55 & -1/55 & 9/55 \\ 4/11 & -1/11 & -2/11 \end{bmatrix}$$

$B^{-1} = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix}^{-1}$

$|B| = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{vmatrix} = 1\begin{vmatrix} 1 & -4 \\ -2 & 1 \end{vmatrix} - 2\begin{vmatrix} 2 & -4 \\ 3 & 1 \end{vmatrix} + 1\begin{vmatrix} 2 & 1 \\ 3 & -2 \end{vmatrix} = -7 + 20 - 7 = -42$

$$B^{-1} = \begin{bmatrix} \begin{vmatrix} 1 & -4 \\ -2 & 1 \end{vmatrix} & -\begin{vmatrix} 2 & -4 \\ 3 & 1 \end{vmatrix} & \begin{vmatrix} 2 & 1 \\ 3 & -2 \end{vmatrix} \\ -\begin{vmatrix} 2 & 1 \\ -2 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 3 & -2 \end{vmatrix} \\ \begin{vmatrix} 2 & 1 \\ 1 & -4 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 2 & -4 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} \end{bmatrix}^T \left(\frac{1}{|B|}\right)$$

$$= \begin{bmatrix} 1/6 & 2/21 & 3/14 \\ 1/3 & 1/21 & -1/7 \\ 1/6 & -4/21 & 1/14 \end{bmatrix}$$

CS 411 Assignment OC

C. $f(x) = x^2 + 3$

$g(x, y) = x^2 + y^2$

1. $f'(x) = 2x$

$f''(x) = 2$

2. $\frac{\partial g}{\partial x} = 2x$ $\qquad$ $\frac{\partial g}{\partial y} = 2y$