

1. a. Goldbach Conjecture: any even integer can be represented as the sum of two other integers being either prime or 1

b. $f(x)$ is integer polynomial

~~Any integer polynomial is a polynomial~~

$$f(x) = p$$

the polynomial must be a constant function ^{so that} its output does not vary based on input

c. ~~$(-321)100$~~

$$321 = 3 \cdot 100 + 21$$

$$-321 = -3 \cdot 100 - 21 \rightarrow (-21)$$

$$7 \cdot 2 = 14$$

2. $\binom{30}{4}^{100}$

$$\binom{30}{4} = \frac{30!}{(30-4)!4!} = \frac{30!}{26!4!} = \frac{30 \cdot 29 \cdot 28 \cdot 27}{4 \cdot 3 \cdot 2 \cdot 1} = \frac{5 \cdot 7 \cdot 29 \cdot 27}{1} = 5 \cdot 7 \cdot 29 \cdot 3 \cdot 3$$

it divides

$$\boxed{5, 7, 29, 3}$$

3. $\gcd(9k+4, 2k+1) = 1$

$$9 \cdot 4 \quad 13 \quad 3$$

$$5. x \equiv 57^{100} \pmod{62}$$

$$x \equiv 57^{38} \pmod{62}$$

$$57^2 = 1249 \equiv 9 \pmod{62}$$

$$57^4 \equiv 9^2 \equiv 81 \equiv 19 \pmod{62}$$

$$57^8 \equiv 19^2 \equiv 361 \equiv 51 \pmod{62}$$

$$57^{16} \equiv 51^2 \equiv 2601 \equiv 59 \pmod{62}$$

$$57^{32} \equiv 59^2 \equiv 9 \pmod{62}$$

$$57^{64} \equiv 9^2 \equiv 19 \pmod{62}$$

$$57^{100} \equiv 57^{64} 57^{32} 57^4 \equiv 19 \cdot 9 \cdot 19 \equiv 361 \cdot 9 \equiv 2249 \equiv 25 \pmod{62}$$

$$x \equiv 25 \pmod{62}$$

$x = 25$ is smallest on \mathbb{Z}^+

$$6. a. 35x + 14y = 93$$

NO solutions as

$$\gcd(35, 14) \nmid 93$$

$$b. 35x + 14y = 91$$

$$7 \mid 91 \checkmark$$

$$x_0 = 3, y_0 = -2$$

$$x = x_0 + \left(\frac{b}{d}\right)t = 3 + 2t$$

$$y = y_0 - \left(\frac{a}{d}\right)t = -2 - 5t$$

$$\begin{array}{r} 12 \\ 3 \overline{) 32} \\ \underline{30} \\ 2 \end{array}$$

$$100$$

$$64 + 32 + 4$$

$$38$$

$$32 + 4 + 2$$

$$\begin{array}{r} 20 \overline{) 1249} \\ \underline{1240} \\ 9 \end{array}$$

$$\begin{array}{r} 5 \overline{) 341} \\ \underline{310} \\ 31 \end{array}$$

$$\begin{array}{r} 9 \overline{) 3249} \\ \underline{361} \\ 3249 \end{array}$$

$$\begin{array}{r} 52 \overline{) 3249} \\ \underline{310} \\ 149 \\ \underline{124} \\ 25 \end{array}$$

$$\gcd(35, 14)$$

$$35 = 2 \cdot 14 + 7$$

$$14 = 2 \cdot 7$$

$$\gcd(35, 14)$$

$$\begin{array}{r} 34 \\ 57 \overline{) 1249} \\ \underline{399} \\ 2850 \\ \underline{1249} \\ 361 \end{array}$$

$$\begin{array}{r} 4 \\ 19 \overline{) 1249} \\ \underline{171} \\ 190 \\ \underline{361} \end{array}$$

$$\begin{array}{r} 62 \\ 362 \overline{) 1249} \\ \underline{186} \\ 244 \\ \underline{2310} \\ 132 \end{array}$$

$$\begin{array}{r} 51 \\ 51 \overline{) 2550} \\ \underline{2550} \\ 0 \end{array}$$

$$45 \cdot 14$$

$$\begin{array}{r} 41 \\ 62 \overline{) 2601} \\ \underline{248} \\ 121 \\ \underline{62} \\ 59 \end{array}$$

$$42 \cdot 1 \cdot 31$$

$$2 \cdot 31$$

$$62 \times$$

$$4 \cdot 1$$

$$59^2$$

$$2 \cdot 31$$

$$121$$

$$62$$

$$59$$

$$62 \cdot 3041$$

$$310 \cdot 6$$

$$381$$

$$272$$

$$9$$

$$35 \cdot 9 \cdot 70$$

$$\gcd = r = a \cdot b, \gcd(35, 14) = 7$$

$$8 \cdot 12$$

$$12 = 14 + 4$$

$$8 = 2 \cdot 4$$

$$= 2 \cdot 4$$

$$\begin{array}{r} 13 \overline{) 93} \\ \underline{7} \\ 23 \\ \underline{21} \\ 2 \end{array}$$

$$\begin{array}{r} 17 \overline{) 91} \\ \underline{7} \\ 21 \\ \underline{21} \\ 0 \end{array}$$

$$\therefore 7 \mid 91$$

$$x \cdot 57 + 2 \cdot 7y = 13 \cdot 7$$

$$3$$

$$35/7$$

$$55 \cdot 5 - 2$$

$$\begin{aligned} 8 \quad x &\equiv 1 \pmod{3} \\ x &\equiv 2 \pmod{5} \\ x &\equiv 3 \pmod{7} \end{aligned}$$

$$n = 3 \cdot 5 \cdot 7 = 105$$

$$N_1 = \frac{n}{3} = 35$$

$$35x \equiv 1 \pmod{3}$$

$$x_1 = 2$$

$$N_2 = \frac{n}{5} = 21$$

$$21x \equiv 1 \pmod{5}$$

$$x_2 = 1$$

$$N_3 = \frac{n}{7} = 15$$

$$15x_3 \equiv 1 \pmod{7}$$

$$x_3 = 1$$

$$x \equiv 1 \cdot 35 \cdot 2 + 2 \cdot 21 \cdot 1 + 3 \cdot 15 \cdot 1 \pmod{105}$$

$$\equiv 70 + 42 + 45 \pmod{105}$$

$$\equiv 157 \equiv 52 \pmod{105}$$

$$X = 52 + 105t$$

$$9. \quad 34x \equiv 4 \pmod{98}$$

$$34x \equiv 4 \pmod{98}$$

$$34x - 4 = 98k$$

$$34x - 98k = 4$$

$$x_0 = 1 \quad k = 1$$

$$x = 1 + \left(\frac{98}{34}\right)t$$

$$4 = 34 - 30$$

$$= 34 - 98 - 2 \cdot 34$$

$$= 98 - 34$$

$$\gcd(34, 98) = 2$$

$$\gcd(34, 98) = 2$$

$$18 = 2 \cdot 34 + 30$$

$$34 = 1 \cdot 30 + 4$$

$$30 = 7 \cdot 4 + 2$$

$$4 = 2 \cdot 2$$

$$2 \mid 2 \checkmark$$

$$2 \text{ solutions}$$

$$2x \equiv 1 \pmod{5}$$

$$\begin{array}{c} 2 \\ 4 \\ 6 \\ 8 \\ 10 \\ 12 \end{array}$$

$$68$$

$$70$$

$$60$$

$$65$$

$$60$$

$$60$$

$$60$$

$$105$$

$$35 \cdot 2$$

$$2$$

$$2x \equiv 1 \pmod{5}$$