A calculator was used for some portions of this assignment

1. a. $F(x) = \begin{cases} 0, & x \le 0 \\ 1-e^{-8x}, & x \ge 0 \end{cases}$

$F\left(\frac{12 \text{ min}}{60 \frac{\text{min}}{\text{hr}}}\right) \approx 1-e^{-8\cdot\frac{12}{60}} \approx 0.7981$

b. $\frac{d}{dx}\left[1-e^{-8x}\right]\bigg|_{x=x} = 8e^{-8x}$

$f(x) = \begin{cases} 0, & x \le 0 \\ 8e^{-8x}, & x \ge 0 \end{cases}$

$\int_{-\infty}^{12/60} f(x)\,dx = \int_{-\infty}^{0} 0\,dx + \int_0^{12/60} 8e^{-8x}\,dx = -e^{-8x}\Big|_0^{12/60} = -e^{-8\cdot\frac{12}{60}} - (-e^{-8\cdot 0})$

$= 1-e^{-8\cdot\frac{12}{60}}$

$= 0.7981$

2. $X \in [2, 5]$ $\quad f(x) = 2(1+x)/27$

$= \frac{2}{27} + \frac{2x}{27}$ $\quad F = \frac{x^2}{27} + \frac{2}{27}x$

a. $P(X<4) \approx \int_2^4 f(x)\,dx = \frac{16}{27}$

b. $P(3 \le X < 4) \approx \int_3^4 f(x)\,dx = \frac{1}{3}$

c. $\int 2(1+x)/27\,dx = \frac{x^2}{27} + \frac{2}{27}x = F(x)$

$P = F(4) - F(3) \approx \frac{8}{9} - \frac{5}{9} = \frac{1}{3}$

d. $P(X>4 \mid X>3) = P(X>3) \approx \int_3^5 f(x)\,dx = \frac{20}{27}$

3. a. $1 = \int_0^1 k\sqrt{x}\,dx = 1$

$1 = k\int_0^1 \sqrt{x}\,dx = k\left[\frac{2}{3}x^{3/2}\right]_0^1$

$1 = k\cdot\frac{2}{3} \Rightarrow k = 3/2$

b. $F(x) = \cancel{\frac{3}{2}\cdot\frac{2}{3}}\,x^{3/2} = x^{3/2}$

$P(0.3 < X < 0.6) \approx F(0.6) - F(0.3) \approx 0.300$

4. $f(x) = \begin{cases} \frac{1}{2000} e^{-x/2000} & , x \geq 0 \\ 0 & , x \leq 0 \end{cases}$

a. $F(x) = \begin{cases} \int \frac{1}{2000} e^{-x/2000} dx \\ 0 \end{cases} = \begin{cases} -e^{-x/2000} & , x \geq 0 \\ 0 & , x \leq 0 \end{cases}$

b. $\int_{1000}^{\infty} \frac{1}{2000} e^{-x/2000} dx = e^{-1/2} - e^{-\infty} \approx e^{-1/2} \approx 0.60653$

b. $\int_{0}^{2000} f(x)\, dx = -\frac{1}{e} + 1 \approx 0.63212$

5. $f(y) = \begin{cases} cy^2(1-y)^4 & , 0 \leq y \leq 1 \\ 0 & , \text{else} \end{cases}$

a. $1 = c\int_0^1 y^2(1-y)^4\, dy$

$1 = c/105 \Rightarrow c = 105$

b. $E(y) = \int_{-\infty}^{\infty} y \cdot f(y)\, dy = \int_0^1 y \cdot f(y)\, dy = \frac{1}{280}$

c. $f(x) = \begin{cases} \frac{1}{4} e^{-x/4} & , x \geq 0 \\ 0 & , \text{else} \end{cases}$

a. $E(x) = \int_{-\infty}^{\infty} x \cdot f(x)\, dx = \cancel{\int_{-\infty} x \cdot 0\, dx} + \int_0^{\infty} x \cdot \frac{1}{4} e^{-x/4} = -(x+4)e^{-x/4} \Big|_0^{\infty}$

$= \lim_{x \to \infty} -(x+4)e^{-x/4} + 4 = 4$

$Var(x) = E(x^2) - E(x)^2$

$= \int_{-\infty}^{\infty} x^2 f(x)\, dx - 16 = \int_0^{\infty} x^2 \cdot \frac{1}{4} e^{-x/4} dx - 16 = -(x^2+8x+32) \cdot e^{-x/4} \Big|_0^{\infty}$

$= \cancel{\lim_{x \to \infty} -(x^2+8x+32) \cdot e^{-x/4}} + 32 - 16 = 16$

7. $P = \frac{1}{10-7} = 1/3$

a. $\frac{1}{3} \cdot (8.8 - 7) = 0.6$

b. $\frac{1}{3}(9.5 - 7.4) = 0.7$

c. $\frac{1}{3}(10 - 8.5) = 0.5$

8. $f(x) = \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}}$ $g(x) = \frac{f((x-18)/2.5)}{2.5}$

a. $\int_{-\infty}^{15} g(x)\,dx \approx 0.115$
$= P(x<15)$

d. $P(17<x<21) = \int_{17}^{21} g(x)\,dx = 0.540$

b. $F(x) = \int_{-\infty}^{x} f(x)\,dx$

$F(x) = 0.2236 \Rightarrow x \approx 16.09977$

c. $P(x \geq k) = 1 - P(x<k) = 0.1814$

$F(x) = 0.8186 \Rightarrow x \approx 20.2751$

d.

9. $g(x) = \frac{f((x-200)/15)}{15}$

a. $P(x>224) = \int_{224}^{\infty} g(x)\,dx \approx 0.0548$

b. $\int_{191}^{209} g(x)\,dx \approx 0.4515$

c. $1000 \cdot \int_{230}^{\infty} g(x)\,dx \approx 22.750$

d. $0.25 = \int_{-\infty}^{x} g(x)\,dx \Rightarrow x \approx 189.883$

10. $g(x) = \frac{f((x-24)/3.8)}{3.8}$

a. $\int_{30}^{\infty} g(x)\,dx = 0.0572 = P(a)$

b. $1 - \int_{0}^{15} g(x)\,dx = 0.9912$ ~~negative [illegible]~~

c. $8{:}50 - 8{:}35 = 15$
$9{:}00 - 8{:}35 = 25$ $\therefore \int_{15}^{25} g(x)\,dx = 0.40515$

d. $0.15 = \int^{x} g(x)\,dx \Rightarrow x \approx 20.0516$

e. $P(H) = 3P(a)P(a)(1-P(a)) \approx 0.009254$