

1. 3 Balls ; 3 bowls ;  $Y = \#$  empty bowls

2 empty bowls : 1 way  $\begin{smallmatrix} \bullet & \bullet & \bullet \\ \cup & \cup & \cup \end{smallmatrix}$

1 empty bowl : 6 ways  $\begin{smallmatrix} \bullet & \bullet & \bullet \\ \cup & \cup & \cup \end{smallmatrix}$  x 3 permutations

0 empty bowls : 1 way  $\begin{smallmatrix} \bullet & \bullet & \bullet \\ \cup & \cup & \cup \end{smallmatrix}$

$$P(H)P(T)$$

$$P(H)P(H)$$

$$Y=2 : \frac{1}{8} = 0.125$$

$$Y=1 : \frac{6}{8} = 0.75$$

$$Y=0 : \frac{1}{8} = 0.125$$

2. 7 TVs : 2 defective 5 good  
Choose 3 ;  $X = \#$  defective

$$0 \text{ defective} : 5 \cdot 4 \cdot 3 = 60$$

$$P(X=0) : \frac{60}{110} = 0.54$$

$$1 \text{ defective} : 5 \cdot 4 \cdot 2 = 40$$

$$P(X=1) : \frac{40}{110} = 0.36$$

$$2 \text{ defective} : 5 \cdot 2 \cdot 1 = 10$$

$$P(X=2) : \frac{10}{110} = 0.09$$

$$3 \text{ defective} : 0$$

$$P(X=3) : \frac{0}{110} = 0$$

3.  $W = \#$  heads -  $\#$  tails in 3 coin tosses

$$a. \text{ s.t. } HHH \Rightarrow W=3$$

$$HHT \Rightarrow W=1$$

$$HTH \Rightarrow W=1$$

$$HTT \Rightarrow W=-1$$

$$THT \Rightarrow W=-1$$

$$THT \Rightarrow W=-1$$

$$TTH \Rightarrow W=-1$$

$$TTT \Rightarrow W=-3$$

$$W \in \{3, 1, -1, -3\}$$

$$b. P(W=3) = \frac{1}{8} = 0.125$$

$$P(W=1) = \frac{3}{8} = 0.375$$

$$P(W=-1) = \frac{3}{8} = 0.375$$

$$P(W=-3) = \frac{1}{8} = 0.125$$

$$c. E(W) = \frac{1}{8}(3) + \frac{3}{8}(1) + \frac{3}{8}(-1) + \frac{1}{8}(-3) = 0$$

$$\text{Var}(W) = \frac{1}{8}(3)^2 + \frac{3}{8}(1)^2 + \frac{3}{8}(-1)^2 + \frac{1}{8}(-3)^2 = 3$$

4.

$X$	-3	0	9
$P$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$

$$a. E(X) = \frac{1}{6}(-3) + \frac{1}{2}(0) + \frac{1}{3}(9) = 5.5$$

$$E(X^2) = \frac{1}{6}(-3)^2 + \frac{1}{2}(0)^2 + \frac{1}{3}(9)^2 = 46.5$$

$$b. \text{Var}(X) = \frac{1}{6}(-3-5.5)^2 + \frac{1}{2}(0-5.5)^2 + \frac{1}{3}(9-5.5)^2 = 16.25$$

$$c. E(X^3) = \frac{1}{6}(-3-5.5)^3 + \frac{1}{2}(0-5.5)^3 + \frac{1}{3}(9-5.5)^3 = 242$$

5.  $X$ : green die  $Y$ : red die

$$a. \mu = \frac{1}{6 \cdot 6} \sum_{x=1}^6 \sum_{y=1}^6 2x - y = 3.5$$

$$b. \text{Var}(2X - Y) = \frac{1}{6 \cdot 6} \sum_{x=1}^6 \sum_{y=1}^6 (2x - y - \mu)^2 = \frac{525}{36} = 14.58\bar{3}$$

$$c. \mu = \frac{1}{6 \cdot 6} \sum_{x=1}^6 \sum_{y=1}^6 x + 3y - 5 = \frac{324}{36} = 9$$

$$s^2 = \frac{1}{6 \cdot 6} \sum_{x=1}^6 \sum_{y=1}^6 (x + 3y - 5 - 9)^2 = \frac{1050}{36} = 29.1\bar{6}$$

6. 50% companies  $P(A) = 0.5$

$$a. P(X \in [2, 5]) = \frac{15 + 20 + 15 + 4}{49} = 0.875$$

$$b. P(X < 3) = \frac{1 + 6 + 15}{49} = 0.34375$$

7.  $P(A) = 0.6$   $X = \# \text{ contracting}$

$$a. P(X=0) = (0.6)^5 = 0.07776$$

$$b. P(X < 2) = 1 + 0.2592 = 0.3369$$

$$c. P(X > 3) = 0.0512 + 0.0768 = 0.128$$

$$P(X=0) = \binom{6}{0} \left(\frac{1}{2}\right)^6 = \frac{1}{64}$$

$$P(X=1) = \binom{6}{1} \left(\frac{1}{2}\right)^6 = \frac{6}{64}$$

$$P(X=2) = \binom{6}{2} \left(\frac{1}{2}\right)^6 = \frac{15}{64}$$

$$P(X=3) = \binom{6}{3} \left(\frac{1}{2}\right)^6 = \frac{20}{64}$$

$$P(X=4) = \binom{6}{4} \left(\frac{1}{2}\right)^6 = \frac{15}{64}$$

$$P(X=5) = \binom{6}{5} \left(\frac{1}{2}\right)^6 = \frac{6}{64}$$

$$P(X=6) = \binom{6}{6} \left(\frac{1}{2}\right)^6 = \frac{1}{64}$$

$$P(X=0) = \binom{6}{0} (0.4)^5 = 0.07776$$

$$P(X=1) = \binom{6}{1} (0.4)^4 (0.4)^1 = 0.2592$$

$$P(X=2) = \binom{6}{2} (0.4)^3 (0.4)^2 = 0.3456$$

$$P(X=3) = \binom{6}{3} (0.4)^2 (0.4)^3 = 0.2304$$

$$P(X=4) = \binom{6}{4} (0.4)^1 (0.4)^4 = 0.0768$$

$$P(X=5) = \binom{6}{5} (0.4)^0 (0.4)^5 = 0.0512$$

8.  $P(A) = 0.7$   $X = \# \text{ trials}$   $\text{pos}$

$$a. P(X=3) = (1-0.7)^2 (0.7) = 0.063$$

$$b. \sum_{i=0}^2 (1-0.7)^i (0.7) = 0.973$$

9.  $P(A) = 0.1$

$$a. P(X=2) = (0.9)(0.1) = 0.09$$

$$b. P(X=3, 0.3) = \binom{4}{2} (0.9)^2 (0.1)^2 (0.1) = 0.00486$$

10.  $P(B) = 0.8$

$$a. P(X=6, 0.4) = \binom{3}{3} (0.8)^3 (0.2)^3 (0.8) = 0.164$$

$$b. P(X=7, 0.4) = (0.2)^2 (0.4) = 0.032$$

