1. a. Goldbach Conjecture: any even integer can be represented as the sum of two other integers being either prime or 1

b. f(x) is integer polynomial...

~~[illegible] for a polynomial~~

f(x) = p

the polynomial must be a constant function so that its output does not vary based on input

c. ~~-321 / 100~~

$321 = 3 \cdot 100 + 21$

$-321 = -3 \cdot 100 - 21 \leftarrow -21$

$7 \cdot 2 = 14$

2. $\binom{30}{4}^{100}$

$\binom{30}{4} = \frac{30!}{(30-4)!\,4!} = \frac{30!}{26!} = \frac{30 \cdot 29 \cdot 28 \cdot 27}{4 \cdot 3 \cdot 2 \cdot 1} = \frac{5 \cdot 7 \cdot 29 \cdot 27}{1}$

$= 5 \cdot 7 \cdot 29 \cdot 3 \cdot 3 \cdot 3$

it divides 5, 7, 29, 3

3. $\gcd(9k+4, 2k+1) = 1$

9+4 13 3

5. $x \equiv 57^{100} \pmod{62}$

$x \equiv 57^{38} \pmod{62}$

$57^2 = 1249 \equiv 9 \pmod{62}$

$57^4 \equiv 9^2 \equiv 81 \equiv 19 \pmod{62}$

$57^8 \equiv 19^2 \equiv 361 \equiv 51 \pmod{62}$

$57^{16} \equiv 51^2 \equiv 2601 \equiv 59 \pmod{62}$

$57^{32} \equiv 59^2 \equiv 9 \pmod{62}$

$57^{64} \equiv 9^2 \equiv 19 \pmod{62}$

$57^{100} \equiv 57^{64} 57^{32} 57^4 \equiv 19 \cdot 9 \cdot 19 \equiv 361 \cdot 9 \equiv 3249 \equiv 25 \pmod{62}$

$x \equiv 25 \pmod{62}$

$x = 25$ is smallest on $\mathbb{Z}^+$

100 = 64 + 32 + 4

38 = 32 + 4 + 2

$62 \overline{)1249}$ = 20 r 9

$62 \overline{)361}$ = 5 r 51

$62 \overline{)3249}$ = 52 r 25

$57 \times 57 = 3249$, $19 \times 19 = 361$, $51 \times 51 = 2601$

$62 = 2 \cdot 31$

6. a. $35x + 14y = 93$

No solutions as $\gcd(35,14) \nmid 93$

$7 \nmid 93$

$\gcd(35,14)$

$35 = 2 \cdot 14 + 7$

$14 = 2 \cdot 7$ ← $\gcd(35,14)$

b. $35x + 14y = 91$

$7 \mid 91$ ✓

$x_0 = 3 \quad y_0 = -2$

$x = x_0 + \left(\frac{b}{d}\right)t = 3 + 2t$

$y = y_0 - \left(\frac{a}{d}\right)t = -2 - 5t$

$x \cdot 5 \cdot 7 + 2 \cdot 7 y = 13 \cdot 7$

$7 \overline{)91}$ = 13, $\therefore 7 \mid 91$

$x =$

8. $x \equiv 1 \pmod{3}$
$x \equiv 2 \pmod{5}$
$x \equiv 3 \pmod{7}$

3, 5 and 7 are all prime & they mutually relatively prime allowing us to apply the chinese remainder theorem

$n = 3 \cdot 5 \cdot 7 = 105$

$N_1 = \frac{n}{3} = 35$ $\quad N_2 = \frac{n}{5} = 21$ $\quad N_3 = \frac{n}{7} = 15$

$35x \equiv 1 \pmod{3}$ $\quad 21x \equiv 1 \pmod{5}$ $\quad 15x_3 \equiv 1 \pmod{7}$

$x_1 = 2$ $\quad x_2 = 1$ $\quad x_3 = 1$

$x \equiv 1 \cdot 35 \cdot 2 + 2 \cdot 21 \cdot 1 + 3 \cdot 15 \cdot 1 \pmod{105}$

$\equiv 70 + 42 + 45 \pmod{105}$

$\equiv 157 \equiv 52 \pmod{105}$

$X = 52 + 105t$

9. $34x \equiv 4 \pmod{98}$

$34x \equiv 4 \pmod{98}$

$34x - 4 = 98k$

$34x - 98k = 4$

$x_0 = 1 \quad k = 1$

$x = 1 + \left(\frac{34}{2}\right)t$

gcd(a,n) | b ?
gcd(98,34)
$98 = 2 \cdot 34 + 30$
$34 = 1 \cdot 30 + 4$
$30 = 7 \cdot 4 + 2$
$4 = 2 \cdot 2$
gcd

$2 | 2$ ✓

2 solutions

$4 = 34 - 30$
$= 34 - 98 - 2 \cdot 34$
$= 98 - 34$