```
12=2.3.7
3 # gcd(a, 42) = 1 then 168/a6-1
                                                166-2.2.2.3.7
    Because the self a & 42 is I we know that a cannot divide any of 42's
   factors.
                         Zta 3ta 7ta
        42 + 2.3.7
    similancers congruence for each of the factory of 168.
        1661 a 6-I
                                  - a must be odd as z far
                                - Allodd jumbers can be represented to 1 1 1 1-1
       4-1=0 (mod 148)
                                (4k+1)(4k+1) = 8(2k2+K)+1 E 8n+1
                                (4K-1)(4K-1) = 6(2K2-K)+1 E 8m+1
        ac = 1 (med 168)
                               this any odd ummber squared is of firm Ex+1
                              therefore a2 = 1 (uned 8)
     == 1 anod 7)
                                        (a2)3 = a6 = 13 = 1 (med 8)
    a = 1 (med 7)
    (True by ferment's
     Theorem &s
       a = 1 (mod 3)
     Beenge 3ta, a # 0 cmod 37 and they either
                                 a = 2 cmod 3)
        a = 1 (med 3)
                                  Z' = 1 cmod 3)
      16 = 1 (mpd 3)
                                  (4 = 1 (mod 3)
        1=1 cmod3)
                                  1= 1 and 3) V
   Because all 3 parts of 9h similarions congrue are quaranteed to held, In original might also hold and so might the 1 68 divide
```

4. gcd (a, 133) = gcd (b, 133) = 1 tun 133 138-618 - because 133 = 7.9 gcd(a,7)=ged(b,7)=gcd(a,9)=gcd(b,9)=1 : 617 019 - so we can apply fermaty Little theorem giving ~ = 6 = 1 (mod 9) + a = 6 = 1 (mod 7) the popesition can be rewritten as a similarrow congruence. 133 | 918 - 618 18-618 = 0 (mod 133) a18 = 618 (mod 133)

a'8=6'8 (mod 7) 218 = 618 emod 9) (a) = (64) (mod 7) (a) = (b) (mad 9)

congruencies from formats theorem... substituting for

(1) = (1) (mod 7) (1) = (1) cmod 7)

1=1 (med 7) 1=1 cmod 7)

. as both words of the smultoning comprise hold todaying so must the proposition of

6. 13 = a (mod 3.7.13) + a & I The proposition can be rewritten of a summaning confinence a = a const B) a13 = a (mod 3) a's = a (mod +) I fame by words , F FLT we can certy the conquerer · a" = a" (mod 7) for all possible values f (a2) = a2 (mod 7) as 13 joinne a. (exhaustion) 12+ c= 2 7 = c (mod 7) a mod 3 E & 0,1,23 y 9=0 (mod 3) SJ 9=1 (mod 3) = 1=2.00 = time by corollary of , 1'= 1 (mod 3/ 2 3 = [ = 1 = 10 = 0.5] FLT & 7 & prime 013 = 0 (mod 3) 0=0 (med3) 1=1 (mod3) 3192=2 (max - ) Because all 3 parts of the simultaneous congruence have seen for all a the proposition myst solgo, hold, 10+3 10=205 72. 300 mod 10 =? 3=1 (mod 2) 34=1 (mod 5) 3100 = 1100 = 1 (mod2) (34) = 125 = 1 (mod 5) 3100 = 1 (mod 10) Because there is a smultimory solution @ 1 7b. 3= a cmod 10) Drop can be rewritten of smarteneway congressed a = a (mod 5) as Ex cmod Z) proof by exhauxtion time by corollary of FLT 03 = 0 (mod 2) [3 = 1 (mod 2) as 5 is ovime 0=0 (mod2)

suggests pup.

Escarge the representative simplement congruency holds for all at I we come

confirm the proposition.

6. 15 = a (mod 3.7.13) + a & I The proposition can be rewriter of a summaning congruence a = a comp 1 13) 13 = a (mod 3) " = a (mod 7) · a = a (mod 7) we can certy the conquere I fore by worlding (q2) = q2 (mod 7) for all possible values of as 13 is orine a. (Khugtian) let c = 2 % c (mod 7) a mod 3 E & 0,1,23 time by corollary if a= 1 (mod 3) = a = 2 (mod 3) y a = 0 (mod 3) FLT & 7 & prime 1=1(mod 3) 213 = 2 (mod 3) 013 = 0 (mod 3) 1=1 (mod 3) 8192 = 2 (mod 30). 0=0 (med3) 20=1= 2 (mod 3) Because all 3 parts of the Simultaneous congruence have been prooven to hold for all a the proposition my solgo, hold, 10+3 10=205 7. 3 " mod 10 = ? 3 = 1 (mod 5) 3 = 1 (mod 2) 3100 = 1 (mod 107) (34) = 125 = 1 (mod 5) 3100 = 1100 = 1 (mod 2) 7b. 3= a cm.d 10) Because there is a small troubly solution (a) I Peop can be rewritten of sountaining congruence 1 = a (mod 5) as = a cmod Z) true by corollary of FLT proof by exhaustion 05 = 0 (mod 2) 15 = 1 (mod 2) 0= 0 (mod 2) 1= 1 (mod 2) as 5 is prime susperty pup. tocase he representative similaring congruency holds for all at I we will

confirm the proposition.

4. pisprine 1,6 EZ pta,6

a. if I = b (medp) then x = b (mod p)

a corollary of FLT states that at = a (mode) when as a rise and act,

PEa (mod P)

6 = 6 and e)

a = b (mod P)

a = b (mod e) \*

b. if a = b (midp) thun a = b (mod )

ch ---

9. See attached sayemath cole

4. pisprine a, b EZ pta, b a. if a = b (modp) then a = b (modp) a corollary of FLT states that at = a consol to when o's prival and the resulting this are can simplify.

$$a^P \equiv a \pmod{P}$$
  $b^T \equiv b \pmod{P}$ 

a = b (mod P)