1. 701 is prime
2. 101, 103, 107, 109, 113, 127, 131, 137, 139, 149
3. Assuming that $\sqrt{p}$ is rational we could find two relatively prime integers

$$\sqrt{p} = \frac{a}{b} \qquad a,b \in \mathbb{Z}^+ \qquad \gcd(a,b)=1$$

$$p = \frac{a^2}{b^2}$$

$$a^2 = pb^2 \quad \therefore \quad \gcd(a^2, b^2) = p \quad !!!$$

By removing the radical we find that $\gcd(a^2, b^2)$ must be p which is nonsensical as because $a+b$ are relatively prime the FTA tells us that they and as a result their squares must have a unique prime factorization and thus the $\gcd(a^2, b^2)$ should be 1.

4. Section 3.2 of the textbook explains how the largest prime factor p for a composite integer a must satisfy $p \leq \sqrt{a}$

Because the largest number in the set is 9999 and $\sqrt{9999}$ is 99 with some remainder, the largest prime < 100 is 97 which is the value from the question.

5. ~~TODO~~
6. Consider $n! - 1$

FTA tells us that it must either be prime or a product of primes

in the case that it is prime it trivally satisfies the inequality when replacing p.

$$n < n! - 1 < n!$$

in the case that $n!-1$ is not prime it must have a prime divisor p (By FTA)

$$p \mid (n!-1)$$

if p were $\leq n$ it would also divide $n!$ which would be nonsensical as we proved in the previous homework assignment that the $\gcd(n, n-1) = 1$

thus the prime factor p must satisfy the inequality

$$p \mid (n!-1)$$

$$\Downarrow$$

$$n < p < n! - 1 < n$$

$$n < p < n!$$

7. $7-5=5-3$ (2) $n=1-3$ - index start at zero

$59-53=53-47$ (6) $n=14-16$

$163-157=157-151$ (6) $n=35-37$

$179-173=173-167$ (6) $n=38-40$

$223-211=211-199$ (12) $n=45-47$

8. numerator: $(1 \cdot p_2 \cdot \ldots \cdot p_n) + (p_3 \cdot \ldots \cdot p_n) + \ldots + p_n$

denominator: $p_1 \cdot \ldots \cdot p_n$

yikes