$P(A|B) = \frac{P(A \cap B)}{P(B)}$

1. a. $P(\text{♂} \mid \text{secondary}) = \frac{50}{28+50} = 0.641$

b. $P(\text{not college} \mid \text{♀}) = \frac{38+28}{88} = 0.75$

$P(\text{♀}) = \frac{38+28+22}{200} = 0.44$

2. $P(A) = 0.6 \quad P(B) = 0.4 \quad P(A \cap B) = 0.15$

a. $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.15}{0.4} = 0.375$

b. $P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.15}{0.4} = 0.25$

c. $P(A \mid A \cup B) = \frac{P(A \cap (A \cup B))}{P(A \cup B)} = \frac{P(A)}{P(A)+P(B)-P(A \cap B)} = \frac{0.6}{0.4+0.6-0.15} = \frac{0.6}{0.85} = 0.706$

d. $P(A) A \cap B) = \frac{P(A \cap (A \cap B))}{P(A \cap B)} = \frac{P(A \cap B)}{P(A \cap B)} = 1.0$ this would make sense

e. $P(A \cap B \mid A \cup B) = \frac{P((A \cap B) \cap (A \cup B))}{P(A \cup B)} = \frac{P(A \cap B)}{P(A \cup B)} = \frac{0.15}{0.85} = 0.176$

3. assuming w/o replacement

a. $\frac{13-2}{52} \cdot \frac{13-3}{52} \cdot \frac{13-4}{52} = 0.00704$

b. $\frac{13-3}{52} \cdot \frac{13-4}{52} = 0.0333$

c. $\frac{13-4}{52} = 0.173$

4. $P(H) = 0.21 \quad P(W) = 0.28 \quad P(H \cap W) = 0.15$

a. $P(H) + P(W) = P(H \cup W) = 0.21 + 0.28 - 0.15 = 0.34$

b. $P(W \mid H) = \frac{P(H \cap W)}{P(H)} = \frac{0.15}{0.21} = 0.714$

c. $P(H \mid W') = \frac{P(H \cap W')}{P(W')} = \frac{0.21-0.15}{1-0.28} = 0.083$

5. $P(A) = 0.10 \quad P(B) = 0.08 \quad P(C) = 0.12$

a. $P(B \cap A') = 0.08 \cdot (1-0.10) = 0.072$

b. $P(A' \cap B' \cap C) = (1-0.10)(1-0.08)0.12 = 0.0994$

6. A: river polluted B: sample detects pollution C: fishing permitted

$P(A) = 0.30$ $P(B|A) = 0.75$ $P(B|A') = 0.20$ $P(C|A\cap B) = 0.20$

$P(C|A'\cap B) = 0.15$ $P(C|A\cap B') = 0.80$ $P(C|A'\cap B') = 0.90$

$P(B|A) = 0.25$

$P(A'\cap B) = P(B|A')(1-P(A)) = 0.20\cdot(1-0.30) = 0.14$

a. $P(A\cap B) = P(B|A)P(A) = 0.75\cdot 0.30 = 0.225$

$P(C\cap A\cap B) = P(C|A\cap B)P(A\cap B) = 0.20\cdot 0.225 = \boxed{0.045}$

b $P(B'|A) = 1 - P(B|A) = 0.25$

$P(B'\cap A) = P(B'|A)P(A) = 0.25\cdot 0.3 = 0.075$

$P(B'|A') = 1 - P(B|A') = 0.80$

$P(B'\cap A') = P(B'|A')P(A') = P(B'|A')(1-P(A)) = 0.8\cdot 0.7 = 0.56$

$P(A\cap B'\cap C) = P(C|A\cap B')P(A\cap B') = 0.80\cdot 0.075 = 0.06$

$P(A'\cap B'\cap C) = P(C|A'\cap B')P(A'\cap B') = 0.90\cdot 0.56 = 0.504$

$P(B'\cap C) = P(A\cap B'\cap C) + P(A'\cap B'\cap C) = 0.06 + 0.504 = \boxed{0.564}$

c. $P(A|C\cap B') = \dfrac{P(A\cap C\cap B')}{P(C\cap B')} = \dfrac{0.06}{0.564} = \boxed{0.106}$

7. $P(L) = [0.4, 0.3, 0.2, 0.3]$

$P(S) = [0.2, 0.1, 0.5, 0.2]$

$P(C) = 0.4\cdot 0.2 + 0.3\cdot 0.1 + 0.2\cdot 0.5 + 0.3\cdot 0.2 - (0.4\cdot 0.3\cdot 0.2\cdot 0.7\cdot 0.2\cdot 0.1\cdot 0.5\cdot 0.2)$

$= 0.26999 \approx 0.27$

8. John: 20% 1/200 err

Tom: 60% 1/100 err

Jeff: 15% 1/90

Pat: 5% 1/200

$\dfrac{\frac{1}{200}\cdot 0.20}{\frac{1}{200}\cdot 0.20 + \frac{1}{100}\cdot 0.60 + \frac{1}{90}\cdot 0.15 + \frac{1}{200}\cdot 0.05} = \boxed{0.112}$

9. $P(D|H) = 0.9$ $P(D'|H') = 0.90$ $P(H) = 0.01$

$P(D) =$ Diagnosed

$P(H) =$ Hag

$P(D\cap H) = P(D|H)P(H) = 0.9\cdot 0.01 = 0.009$

$P(D\cap H') = P(D|H')P(H') = (1-P(D'|H'))(1-P(H)) = (1-0.90)(1-0.01) = 0.099$

$P(H|D) = \dfrac{P(H\cap D)}{P(D)}$

$P(D) = P(D\cap H) + P(D\cap H') = 0.108$

$P(H|D) = \dfrac{P(H\cap D)}{P(D)} = \dfrac{0.009}{0.108} = \underline{0.08\overline{3}}$

this result - only 8.3% chance of a true positive - did surprise me and indicates that this test will likely cause more harm than good unless disease is extremely dangerous and/or cases increased among those being tested.

10. $P(A) = 0.4$, $P(B) = 0.35$ $P(C) = 0.25$ $P(A_0) = 0.05$ $P(B_0) = 0.03$ $P(C_0) = 0.15$

a. $\dfrac{0.15\cdot 0.25}{0.4\cdot 0.05 + 0.35\cdot 0.03 + 0.25\cdot 0.15} = 0.551$

lowercase

$\dfrac{E(C)}{E(A)+E(B)+E(C)}$

b. $\dfrac{0.4\cdot 0.05}{E(A)+E(B)+E(C)} = 0.294$