1. HAPPY BIRTHDAY

2. XLYAXLOLAXLYARZVU ;

3. a. E => Q T => J

04 => 16 19 => 79

Linear cipher : $C \equiv aP + b \pmod{26}$

$16 \equiv 4a + b \pmod{26}$

$9 \equiv 19a + b \pmod{26} \Rightarrow$

$b \equiv 9 - 19a \pmod{26}$

$16 \equiv 4a + (9 - 19a) \equiv -15a + 9 \pmod{26}$

$7 \equiv -15a \equiv -15(3) \pmod{26}$

$\therefore a \equiv 3 \pmod{26}$

$16 \equiv 4(3) + 6 \equiv 12 + 6 \pmod{26}$

$\therefore b \equiv 16 - 12 \equiv 4$

b. see attached javascript program.

4.

5. TH => +10 +IE => PP

19, 7 => 7, 14 7, 4 => 15, 15

$C_1 \equiv aP_1 + bP_2 \pmod{26}$

$C_2 \equiv cP_1 + dP_2 \pmod{26}$

a. solved via Theorem 4.9

$7 \equiv a \cdot 19 + b \cdot 7 \pmod{26}$

$14 \equiv c \cdot 19 + d \cdot 7 \pmod{26}$

$15 \equiv a \cdot 7 + b \cdot 4 \pmod{26}$

$15 \equiv c \cdot 7 + d \cdot 4 \pmod{26}$

$7 \equiv a \cdot 19 + b \cdot 7 \pmod{26}$ ×4×7

$-15 \equiv a \cdot 7 + b \cdot 4 \pmod{26}$ ×7×19

$-77 \equiv 25 \equiv 1 \equiv 27a \equiv a \pmod{26}$

$\therefore a \equiv 1 \pmod{26}$

$14 \equiv c \cdot 19 + d \cdot 7 \pmod{26}$ ×4×7

$-15 \equiv c \cdot 7 + d \cdot 4 \pmod{26}$ ×7×19

$(7 \cdot 7 - 15 \cdot 19) \equiv -236 \equiv -2 \equiv -27b \equiv -b \pmod{26}$

$\therefore b \equiv 2 \pmod{26}$

$-49 \equiv 3 \equiv 27c \equiv c \pmod{26}$

$\therefore c \equiv 3 \pmod{26}$

$(14 \cdot 7 - 15 \cdot 19) \equiv -187 \equiv -5 \equiv -27d \equiv -d \pmod{26}$

$\therefore d \equiv 5 \pmod{26}$

$a \equiv 1 \pmod{26}$

$b \equiv 2$

$c \equiv 3$

$d \equiv 5$

b. The expressions

$P_1 \equiv -5C_1 + 2C_2 \pmod{26}$

$P_2 \equiv 3C_1 - C_2 \pmod{26}$

were put into a javascript function "Hill()" see attached files

produces

"HE" "AR" "TH" "EB" "EL" "LS"

or Hear the bells

7. see 9.7.sage file