

10. The sum of integers from 1 to n that are relatively prime to n is congruent to 0 (mod n) $\forall n \geq 2$

$$\sum_{i=1}^n \begin{cases} i & \text{if } \gcd(n, i) = 1 \\ 0 & \text{else} \end{cases} \equiv 0 \pmod{n}$$

Theorem from class: The sum of integers relatively prime to n $\in \mathbb{Z}'_n$ is $\frac{1}{2} n \phi(n) \quad \forall n \geq 2$

via \uparrow

$$\frac{1}{2} n \phi(n) \equiv 0 \pmod{n}$$

$$\frac{1}{2} (0) \phi(n) \equiv 0 \pmod{n}$$

$$\text{The sum } \equiv 0 \pmod{n} \quad \#$$