1. $272x + 1479y = \gcd(272, 1479)$

Euclid's Algorithm for $\gcd(272, 1479)$

$272 = 0 \cdot 1479 + 272$

$1479 = 5 \cdot 272 + 119$

$272 = 2 \cdot 119 + 34$

$119 = 3 \cdot 34 + 17$

$34 = 2 \cdot 17 + 0$

$\hookrightarrow \gcd(272, 1479) = \underline{17}$

$$
\begin{aligned}
272x + 1479y = 17 &= 119 - 3 \cdot 34 \\
&= 119 - 3(272 - 2 \cdot 119) \\
&= (1479 - 5 \cdot 272) - 3 \cdot 272 + 6 \cdot 119 \\
&= 1479 - 8 \cdot 272 + 6 \cdot 119 \\
&= 1479 - 8 \cdot 272 + (1479 - 5 \cdot 272) \cdot 6 \\
&= \underline{7} \cdot 1479 - \underline{38} \cdot 272
\end{aligned}
$$

$x = -38$

$y = 7$

2. $\gcd(a, b) = 1 \Rightarrow \gcd(2a+b, a+2b) \in \{1, 3\}$

tough ~~~

3. $\text{lcm}(272, 1479) = \dfrac{272 \cdot 1479}{\gcd(272, 1479)} = 23664$

$\text{lcm}(94, 4747) = \dfrac{94 \cdot 4747}{\gcd(94, 4747)} = \dfrac{94 \cdot 4747}{47} = 9494$

$\gcd(94, 4747) = 4747(1) - 50(94) = 47$

$94 = 0 \cdot 4747 + 94$
$4747 = 50 \cdot 94 + 47$
$94 = 2 \cdot 47 + 0$

4. $\forall a, b \in \mathbb{Z}, k \in \mathbb{Z}^+.\ \text{lcm}(ka, kb) = k\,\text{lcm}(a, b)$

Theorem 2-8 states that lcm can be defined in terms of gcd as such:

$$\gcd(a,b)\,\text{lcm}(a,b) = ab$$
$$\text{lcm}(a,b) = \frac{ab}{\gcd(a,b)}$$

This however only applies for positive a + b. Because lcm must always be positive (definition 2-4), we can make this formula apply to all integers a, b taking absolute value

$$\text{lcm}(a,b) = \frac{|ab|}{|\gcd(a,b)|}$$

always > 0 (definition 2-2)

$$\text{lcm}(a,b) = \frac{|ab|}{\gcd(a,b)}$$

could also use $\text{lcm}(|a|,|b|) = |\text{lcm}(a,b)|$

We can then plug in ka & kb into this formula

$$\text{lcm}(ka, kb) = \frac{|ka\,kb|}{\gcd(ka, kb)}$$
$$= \frac{k^2|ab|}{\gcd(ka, kb)}$$

via theorem 2-7

$$= \frac{k^2|a b|}{k\gcd(a,b)}$$
$$= k\frac{|ab|}{\gcd(a,b)} = \underline{k\,\text{lcm}(a,b)}$$ which is what we wanted to proove.

5. $\gcd(198, 288, 512) = \gcd(198, \gcd(288, 512)) = \gcd(198, 32) = 2$

using the following javascript definition of gcd ↓

```
gcd = (a,b) => (r = a%b, r ? gcd(b,r) : b)
```

~~they hold~~ this approach works as the gcd is the maximum item in the set of overlapping factors and intersections of sets are commutative + associative

6. a. $54x - 21y = 906$

$$54 = 2 \cdot 21 + 12$$
$$21 = 12 + 9$$
$$12 = 9 + 3$$
$$9 = 3 \cdot 3 + 0$$
$$\gcd(54, 21) = 3$$

Because $3 \mid 906$ there exist solutions

$$3 = 12 - 9$$
$$3 = (54 - 2 \cdot 21) - (21 - 2 \cdot 12)$$
$$= 54 - 3 \cdot 21 + (54 - 2 \cdot 21)$$
$$3 = \underline{2} \cdot 54 - \underline{5} \cdot 21$$

$$302(3) = 906 = 302(2)54 - 302(5)21$$
$$906 = 604 \cdot 54 - 1510 \cdot 21$$

thus $x = \underline{604}$ and $y = \underline{1510}$ are one solution

$$x' = x_0 + \left(\frac{b}{d}\right)t = 604 + \frac{21}{3}t = \underline{604 + 7t}$$
$$y' = y_0 + \left(\frac{a}{d}\right)t = 1510 + \frac{54}{3}t = \underline{1510 + 18t}$$

> general solutions

b. $14x + 35y = 93$ $\quad \gcd(14, 35) = 7$

because $7 \nmid 93$ there are no solutions $\in \mathbb{Z}$

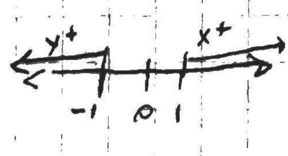
7. b. $-83x + 23y = 7$ $\quad$ 83 + 23 are relatively prime so solutions exist

$$\gcd(83, 23) = 1$$
$$83 = 3 \cdot 23 + 14$$
$$23 = 14 + 9$$
$$14 = 9 + 5$$
$$9 = 5 + 4$$
$$5 = 4 + 1$$
$$4 = 4 \cdot 1 + 0$$

$$1 = 5 - 4 = (14 - 9) - (9 - 5) = 14 - 2 \cdot 9 + (14 - 9) = 2 \cdot 14 - 3 \cdot 9$$
$$= 2(83 - 3 \cdot 23) - 3(23 - 14) = 2 \cdot 83 - 6 \cdot 23 - 3 \cdot 23 + 3 \cdot 83 - 9 \cdot 23$$
$$1 = 5 \cdot 83 - 18 \cdot 23 = (-5)(-83) + (-18)(23)$$
$$7 = -35(-83) + (-126)(23) \quad \therefore x = -35 \quad y = -126$$

$$x' = -35 + 23t \in \mathbb{Z}^+$$
$$y' = -126 - 83t \in \mathbb{Z}^+$$

$$t > \frac{35}{23}$$
$$t \geq 2$$



$$t < \frac{-126}{83}$$
$$<$$

There are no realistic solutions as both x and y would have to be positive

↙ and as you can see the ranging the

composite = !prime

8. $2, 7, 23, 47, 79, \ldots$ via brute force program

9. 1 because zero is not a prime

$p + (\geq 1)^2 > 1$

10. a. Because a prime number must be odd, k must be a multiple of 2, thus

$$3(2k) + 1 = 6k + 1$$

b. $\forall$ primes $p \geq 5 \Rightarrow p^2 + 2$ is composite

All integers can be represented as following via the division algorithm

$6k \quad 6k+1 \quad 6k+2 \quad 6k+3 \quad 6k+4 \quad 6k+5$

Of those only $6k+1$ and $6k+5$ can be prime

$(6k+1)^2 + 2$

$= 36k^2 + 12k + 1 + 2$

$= 3(12k^2 + 4k + 1)$

$\in 3K$

$(6k+5)^2 + 2$

$= 36k^2 + 60k + 25 + 2$

$= 3(12k^2 + 20k + 9)$

$\in 3k$

Because $p^2 + 2$ must be divisible by 3 it cannot be prime