Input: Select positive constants: $p, \beta, b_1, b_2, \sigma \in (0,1)$. Select initial values ρ_0, $\kappa \in (0,1]$ and $(x_0, y_0, s_0, z_0) > 0$ such that $(x_0, y_0, s_0, z_0) \in \mathcal{N}_\beta(\rho_0)$

Step 1

If $F_{\rho_k}(x_k, y_k, s_k, z_k) = 0$

set $(x_{k+1}, y_{k+1}, s_{k+1}, z_{k+1}) = (x_k, y_k, s_k, z_k)$ and go to Step 3

Otherwise solve the linear system

$$\begin{pmatrix} \operatorname{diag}(s_k) + \kappa\rho_k^p \operatorname{diag}(x_k) B^T B & -\operatorname{diag}(x_k) A^T \\ \operatorname{diag}(y_k) A & \operatorname{diag}(z_k) + \kappa\rho_k^p \operatorname{diag}(y_k) \end{pmatrix} \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix}$$
$$= \begin{pmatrix} \rho_k e - \operatorname{diag}(x_k)\left(\kappa\rho_k^p \operatorname{diag}(x_k) B^T B (x_k - \hat{x}) - A^T y_k + c\right) \\ \rho_k e - \operatorname{diag}(y_k)(A x_k + \kappa\rho_k^p y_k - b) \end{pmatrix}$$

and set

$$\begin{pmatrix} \Delta s \\ \Delta z \end{pmatrix} = \begin{pmatrix} \kappa\rho_k^p B^T B & -A^T \\ A & \kappa\rho_k^p \mathbf{1} \end{pmatrix} \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} + \begin{pmatrix} \kappa\rho_k^p B^T B (x_k - \hat{x}) - A^T y_k - s_k + c \\ A x_k + \kappa\rho_k^p y_k - z_k - b \end{pmatrix}.$$

Step 2

Find the step size λ_k such that

$(x_{k+1}, y_{k+1}, s_{k+1}, z_{k+1}) = (x_k + \lambda_k \Delta x, y_k + \lambda_k \Delta y, s_k + \lambda_k \Delta s, z_k + \lambda_k \Delta z) \in \mathcal{N}_\beta(\rho_k)$

This is achieved in 2 substeps

substep 1

Find α such that $\forall \lambda \in (0, \alpha)$ $(x_k + \lambda\Delta x, y_k + \lambda\Delta y, s_k + \lambda\Delta s, z_k + \lambda\Delta z) > 0$

substep 2

Let $\lambda_k = \alpha b_1^j$, where j is the smallest integer such that

$$\begin{aligned} &\left\| F_{\rho_k}(x_k + \lambda_k \Delta x, y_k + \lambda_k \Delta y, s_k + \lambda_k \Delta s, z_k + \lambda_k \Delta z) \right\|_\infty \\ &\quad \le (1 - \sigma\lambda_k) \left\| F_{\rho_k}(x_k, y_k, s_k, z_k) \right\|_\infty . \end{aligned}$$

Step 3

Find the smallest $\rho_{k+1} < \rho_k$ such that $(x_{k+1}, y_{k+1}, s_{k+1}, z_{k+1}) \in \mathcal{N}_\beta(\rho_{k+1})$.

Let $\rho_{k+1} = (1 - b_2^j)\rho_k$ where j is the smallest integer for which we have

$$\left\| F_{\rho_{k+1}}(x_{k+1}, y_{k+1}, s_{k+1}, z_{k+1}) \right\|_\infty \le \beta\rho_{k+1}.$$

Stopping criterion: $\rho_{k+1} \le tol$ or $\|F_{\rho_{k+1}}(x_{k+1}, y_{k+1}, s_{k+1}, z_{k+1})\|_\infty \le tol$.