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Quantum Series®
ENGINEERING

QUANTUM Series

Semester - 3 Civil Engineering

Fluid Mechanics

- Topic-wise coverage of entire syllabus in Question-Answer form.
- Short Questions (2 Marks)

Session 2019-20 Odd Semester

Includes solution of following AKTU Question Papers
2014-15 • 2015-16 • 2016-17 • 2017-18 • 2018-19

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1 UNIT Fluid and Continuum

Part-1 ..... **(1–2A to 1–15A)**

- *Fluid and Continuum*
- *Physical Properties of Fluids*
- *Rheology of Fluid*

A. Concept Outline : Part-1 ..... 1–2A
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Part-2 ..... **(1–15A to 1–26A)**

- *Pressure-Density-Height Relationship*
- *Manometers*
- *Pressure Transducers*

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Part-3 ..... **(1–27A to 1–47A)**

- *Pressure on Plane and Curved Surfaces*
- *Centre of Pressure*
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- *Fluid Masses Subjected to Linear Acceleration and Uniform Rotation About an Axis*

A. Concept Outline : Part-4 ..... 1–47A
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PART-1

Fluid and Continuum, Physical Properties of Fluid, Rheology of Fluid

CONCEPT OUTLINE : PART-1

Fluid : A fluid is a substance that deforms continuously under the application of shear (tangential) stress no matter how small the shear stress may be.

Continuum : A continuous and homogeneous medium is called continuum.

Newton's Law of Viscosity : This law states that the shear stress (τ) acting on a fluid element layer is directly proportional to the rate of shear strain.

Rheology : The study of non-Newtonian fluids is termed as rheology.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.1. **What is fluid ? Discuss continuum in brief.**

Answer

A. Fluid :

1. It is a substance that deforms continuously under the application of shear (tangential) stress no matter how small the shear stress may be.

B. Continuum :

1. A fluid is composed of molecules which may be widely spaced apart, especially in the gas phase. Yet it is convenient to disregard the atomic nature of the fluid and view it as a continuous, homogeneous matter with no holes, that is, a continuum.
2. The continuum idealization allows us to treat properties as point functions and to assume that the properties vary continually in space with no jump discontinuities.
3. This idealization is valid as long as the size of the system we deal with is large relative to the space between the molecules.

Que 1.2. **Discuss some physical properties of fluids in brief.**

AKTU 2014-15, Marks 05

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Answer

Some physical properties of fluids are as follows :

A. Density :

1. It may be defined as the mass per unit volume at a standard temperature and pressure. It is also known as mass density or specific mass.

$$\rho = \frac{m}{V}, \quad \text{where, } m = \text{mass, in kg}$$
$$V = \text{volume, in m}^3.$$

2. The unit of ρ is kg/m³.

B. Weight Density :

1. It can be defined as the weight per unit volume at the standard temperature and pressure. It is also known as specific weight.

$$w = \frac{\text{Weight}}{\text{Volume}} = \frac{m \times g}{V} = \rho g \qquad \left(\because \frac{m}{V} = \rho\right)$$

2. The unit of w is N/m³.

C. Specific Volume :

1. It is defined as the volume per unit mass of fluid.

Mathematically, $v = \frac{V}{m} = \frac{1}{\rho}$

D. Specific Gravity :

1. It is the ratio of the specific weight of the given fluid to the specific weight of a standard fluid.

$$S = \frac{\text{Specific weight of given fluid}}{\text{Specific weight of standard fluid}}$$

2. For liquids, standard fluid is pure water at 4 °C and air is standard fluid, for gases.

Que 1.3. **State Newton's law of viscosity and derive the same. What are its applications ?**

Answer

A. Newton's Law of Viscosity :

1. This law states that the shear stress (τ) acting on a fluid element layer is directly proportional to the rate of shear strain.

$$\tau = \mu \cdot \frac{du}{dy}$$

B. Derivation :

1. From Fig. 1.3.1, when two layers of fluid, at a distance dy apart, move one over the other at different velocities u and $u + du$.
2. The viscosity together with relative velocity causes shear stress acting between fluid layers.
3. This shear stress is proportional to the rate of change of velocity with respect to y. It is denoted by τ.

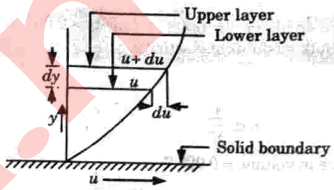

Fig. 1.3.1. Velocity variation near a solid boundary.

Mathematically, $\tau \propto \frac{du}{dy}$ or $\tau = \mu \cdot \frac{du}{dy}$

Where, μ = Constant of proportionality and is known as coefficient of dynamic viscosity or viscosity.

$\frac{du}{dy}$ = Rate of shear deformation or velocity gradient.

C. Applications :

1. Lubrication in bearings.
2. Relative movement between two plates.

Que 1.4. **What is the difference between dynamic viscosity and kinematic viscosity ? State their units of measurements. Determine the bulk modulus of elasticity of a fluid that has a density increase of 0.002 percent for a pressure increase of 45 kN/m².**

AKTU 2013-14, Marks 05

Answer

A. Difference between Dynamic Viscosity and Kinematic Viscosity :

S. No.	Dynamic Viscosity	Kinematic Viscosity
1.	It is defined as the property of a fluid which determines its resistance to shearing stresses.	It is defined as the ratio between the dynamic viscosity and density of fluid.
2.	It is denoted by μ.	It is denoted by ν.
3.	Mathematically, $\mu = \frac{\tau}{\left[\frac{du}{dy}\right]}$	Mathematically, $\nu = \frac{\mu}{\rho}$
4.	The unit of μ is N.s/m².	The unit of ν is m²/s.

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B. Numerical :

Given : $dp = 45$ kN/m², Increase in density = 0.002 %
To Find : Bulk modulus of elasticity.

1. We know,

$$\rho \propto \frac{1}{V}$$

So, decrease in volume = 0.002 %

2. Bulk modulus, $K = \dfrac{dp}{\dfrac{dV}{V}} = \dfrac{45}{\dfrac{0.002}{100}}$

$$= 22.5 \times 10^5 \text{ kN/m}^2$$

Que 1.5. **Through a very narrow gap of height 'h' a thin plate of large extent is pulled at velocity 'v'. On one side of the plate is oil of viscosity 'μ_1' and on the other side of the plate the oil of the viscosity 'μ_2' lie. Calculate the position of the plate so that**
a. The shear force on the two sides of the plate is equal.
b. The pull required to drag the plate is minimum.

AKTU 2011-12, Marks 05

Answer

Given : Height of gap = h, Pulled velocity of the plate = v,
Viscosity above thin plate = μ_1, Viscosity below thin plate = μ_2
To Find : Position of plate :
a. When the shear force on the two sides of the plate is equal.
b. Pull required to drag the plate is minimum.

1. Let A = Cross-section area of the plate, and
y = Distance of the thin plate from the lower surface of the gap.

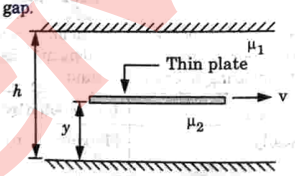

Fig. 1.5.1

2. Force on the upper side of the plate,

$$F_{upper} = \mu \frac{du}{dy} \cdot A = \mu_1 \times \frac{v}{(h-y)} \cdot A$$

3. Force on the lower side of the plate, $F_{lower} = \mu_2 \cdot \dfrac{du}{dy} \cdot A = \mu_2 \times \dfrac{v}{y} \cdot A$

a. When the Shear Force on the Two Sides of the Plate are Equal :

$$F_{upper} = F_{lower}$$

$$\mu_1 \cdot \frac{v}{(h-y)} A = \mu_2 \frac{v}{y} A$$

or

$$y \cdot \mu_1 = \mu_2 (h-y)$$

$$y = \frac{\mu_2 h}{(\mu_1 + \mu_2)}$$

b. When the Pull Required to Drag the plate is Minimum :

1. Total drag = Sum of the forces on the upper and lower surface of the plate

$$F = F_{upper} + F_{lower}$$

$$F = \mu_1 \cdot \frac{vA}{h-y} + \mu_2 \frac{vA}{y}$$

2. For the drag force to be minimum, $\dfrac{dF}{dy} = 0$

$$\frac{dF}{dy} = \frac{d}{dy}\left[\mu_1 \cdot \frac{v}{h-y} \cdot A + \mu_2 \cdot \frac{vA}{y}\right] = 0$$

$$\frac{\mu_1}{(h-y)^2} - \frac{\mu_2}{y^2} = 0$$

$$\frac{\mu_1}{(h-y)^2} = \frac{\mu_2}{y^2}$$

$$\left(\frac{y}{h-y}\right)^2 = \frac{\mu_2}{\mu_1}$$

$$\frac{y}{h-y} = \sqrt{\frac{\mu_2}{\mu_1}}$$

On solving we get,

$$y = \frac{h}{1 + \sqrt{\frac{\mu_2}{\mu_1}}}$$

Que 1.6. **A Newtonian fluid is filled in the clearance between a shaft and a concentric sleeve. The sleeve attains a speed of 50 cm/s, when a force of 40 N is applied to the sleeve parallel to the shaft. Determine the speed if a force of 200 N is applied.**

AKTU 2013-14, Marks 05

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Answer

Given : Speed of sleeve, $u_1 = 50$ cm/s, when force, $F_1 = 40$ N
To Find : Speed of sleeve at the force 200 N

1. Using relation $\tau = \mu \dfrac{du}{dy}$

$$\therefore \quad \frac{F}{A} = \mu \frac{(u-0)}{y} \qquad (\because \ F = \tau \times A)$$

$$F = \frac{A\mu u}{y}$$

$$\therefore \quad \frac{F_1}{u_1} = \frac{F_2}{u_2} \qquad (\because \ A, \mu \text{ and } y \text{ are constant})$$

2. Substituting the given values,

$$\frac{40}{50} = \frac{200}{u_2}$$

$$\therefore \quad u_2 = \frac{50 \times 200}{40} = 50 \times 5 = 250 \text{ cm/s.}$$

Que 1.7. **An oil of viscosity 5 poise is used for lubrication between a shaft and sleeve. The diameter of the shaft is 0.5 m and it rotates at 200 rpm. Calculate the power lost in oil for the sleeve length of 100 mm, the thickness of the oil film is 1 mm.**

AKTU 2014-15, Marks 05

Answer

Given : Viscosity, $\mu = 5$ poise $= \dfrac{5}{10} = 0.5$ N-s/m^2,
Diameter of shaft, $D = 0.5$ m, Speed of shaft, $N = 200$ rpm
Sleeve length, $L = 100$ mm $= 100 \times 10^{-3}$ m $= 0.1$ m,
Thickness of oil film, $t = 1.0$ mm $= 1 \times 10^{-3}$ m
To Find : Power lost in the oil.

1. Tangential velocity of shaft,

$$u = \frac{\pi DN}{60} = \frac{\pi \times 0.5 \times 200}{60} = 5.236 \text{ m/s}$$

2. Using the relation, $\tau = \mu \dfrac{du}{dy}$

where, du = Change of velocity $= u - 0 = u = 5.236$ m/s
dy = Change of distance $= t = 1 \times 10^{-3}$ m

$$\therefore \quad \tau = \frac{0.5 \times 5.236}{1 \times 10^{-3}} = 2618 \text{ N/m}^2$$

3. Shear force on the shaft,

$$F = \tau \times A = 2618 \times \pi D \times L \qquad (\because \ A = \pi D \times L)$$
$$= 2618 \times \pi \times 0.5 \times 0.1 = 411.23 \text{ N}$$

4. Torque on the shaft,

$$T = \text{Force} \times \frac{D}{2} = 411.23 \times \frac{0.5}{2} = 102.81 \text{ N-m}$$

5. Power lost $= T \times \omega = T \times \dfrac{2\pi N}{60}$

$$= 102.81 \times \frac{2\pi \times 200}{60} = 2153 \text{ W} = 2.15 \text{ kW}$$

Que 1.8. **A sphere of diameter 0.50 m and S.G. 7.0 when submerged completely in a liquid causes a tension of 3.465 kN in a string from which it is suspended. What is the specific gravity of the liquid ?**

AKTU 2012-13, Marks 05

Answer

Given : Diameter = 0.50 m, Specific gravity, $S = 7$, Tension, $T = 3.465$
To find : Specific gravity of the liquid.

1. We know,

$$S = \frac{\text{density of material}}{\text{density of water}} = \frac{\rho}{1000}$$

$$7 = \frac{\rho}{1000}$$
$$\rho = 7000 \text{ kg/m}^2$$

2. In equilibrium condition,

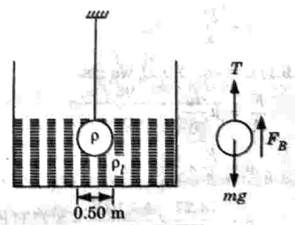

Fig. 1.8.1.

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$$T + F_B = mg$$

$$T + \rho_l \times g \times \frac{4}{3}\pi(0.25)^3 = \frac{4}{3}\pi(0.25)^3 \times 7000 \times 9.81$$

$$3465 + \frac{4}{3}\pi \times 0.25^3 \times 9.81 \times \rho_l = \frac{4}{3}\pi \times 0.25^3 \times 7000 \times 9.81$$

$$\rho_l = 1603.3 \text{ kg/m}^3$$

3. $$S_l = \frac{\text{Density of liquid}}{\text{Density of water}} = \frac{1603.3}{1000}$$

$$S_l = 1.6$$

Que 1.9. **A square plate 50 cm × 50 cm weighing 200 N slides down an inclined plane of slope 1 vertical : 2.5 horizontal with a uniform velocity of 0.40 m/s. If a thin layer of oil of thickness 0.5 cm fills the space between the plate and the inclined plane. Determine the coefficient of viscosity of oil.** **AKTU 2014-15, Marks 05**

Answer

Given : $A = 50 \times 50 = 2500 \text{ cm}^2 = 0.25 \text{ m}^2$, Velocity, $u = 0.40$ m/s
Layer thickness, $dy = 0.5 \text{ cm} = 5 \times 10^{-3}$ m,
To Find : Coefficient of viscosity of oil.

1. Component of weight along the plane = $W \sin\theta$

where, $$\sin\theta = \frac{BC}{AC} = \frac{1}{\sqrt{2.5^2 + 1^2}} = \frac{1}{2.693}$$

$$\therefore \quad F = W \sin\theta = 200 \times \frac{1}{2.693} = 74.27\text{N}$$

2. Now $$\tau = \mu \frac{du}{dy} \quad ...(1.9.1)$$

where $du = u - 0 = u = 0.4$ m/s and $dy = 5 \times 10^{-3}$ m

4. We also know, $$\tau = \frac{F}{A} \quad ...(1.9.2)$$

5. Equating eq. (1.9.1) and eq. (1.9.2), we get

$$\frac{F}{A} = \mu \frac{du}{dy}$$

$$\therefore \quad \mu = \frac{F}{A} \times \frac{dy}{du}$$

$$\mu = \frac{74.27}{0.25} \times \frac{5 \times 10^{-3}}{0.40} = 3.7135 \text{ Pa-s or } 37.135 \text{ Poise}$$

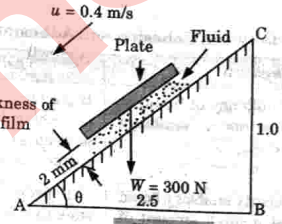

Fig. 1.9.1.

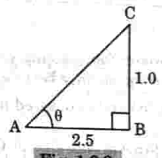

Fig. 1.9.2.

Que 1.10. **Explain the concept of fluid continuum. A rectangular plate of 0.50 m × 0.50 m weighing 500 N slides down an inclined plane making 30° angle with horizontal at a velocity of 1.75 m/s. If the 2 mm gap between the plate and the inclined surface is filled with lubricating oil, find its dynamic viscosity in poise.**

AKTU 2012-13, Marks 05

Answer

A. **Continuum :** Refer Q. 1.1, Page 1–2A, Unit-1.
B. **Numerical :** Same as Q. 1.9, Page 1–9A, Unit-1.
Ans. $\mu = 1.14 \text{ Ns/m}^2$ or 11.4 poise.

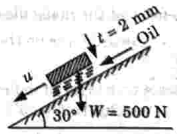

Fig. 1.10.1.

Que 1.11. **Differentiate between cohesion and adhesion properties and discuss about surface tension.**

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Answer

A. Difference between Cohesion and Adhesion :

S. No.	Cohesion	Adhesion
1.	It is a tendency of liquid to remain as one as semblage of particles.	It is a force of attraction between the molecules of a liquid and the molecules of a solid boundary surface in contact with the liquid.
2.	This property enables a liquid to resist a small amount of tensile stress.	This property makes a liquid to stick to another body.

B. Surface Tension :

1. It is defined as the tensile force acting on the surface of a liquid in contact with a gas or on the surface between two immiscible liquids.
2. It is denoted by sigma (σ) and expressed as kgf/m while in SI unit as N/m.
3. This occurs due to the force of cohesion at the free surface as shown in Fig. 1.11.1.

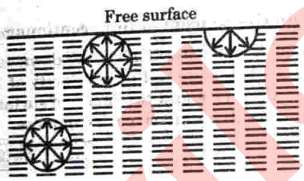

Fig. 1.11.1.

4. Consider a liquid molecule in the interior of liquid mass, surrounded by other molecules all around and is in equilibrium.
5. At the free surface of the liquid, there are no liquid molecules above the surface to balance the force of the molecules below it.
6. As a result, there is a net inward force on the molecule and this force is normal to the surface.
7. Thus at the free surface a thin layer of molecules is formed which acts as membrane because of which a thin small needle can float on the free surface.

Que 1.12. Give the expressions for pressure inside a water droplet, soap bubbles and liquid jet.

Answer

A. Water Droplet :

1. As shown in Fig. 1.12.1 consider a water droplet upon which pressure force and surface tension acts.

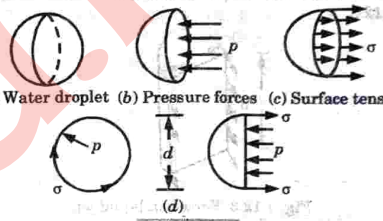

Fig. 1.12.1.

2. Let p = Pressure inside the droplet above outside pressure
   d = Diameter of the droplet, and
   σ = Surface tension of the liquid.
3. From the free body diagram [Fig. 1.12.1(*d*)] it is clear that two forces are acting on the droplet.
   a. Pressure force $= p \times \frac{\pi}{4} d^2$
   b. Surface tension force acting around the circumference $= \sigma \times \pi d$
4. Under equilibrium conditions these two forces will be equal and opposite *i.e.*,

$$p \times \frac{\pi}{4} d^2 = \sigma \pi d$$

$$\therefore \quad p = \frac{4\sigma}{d}$$

B. Hollow Soap Bubble :

1. Soap bubbles being hollow have two surfaces on which surface tension σ acts.

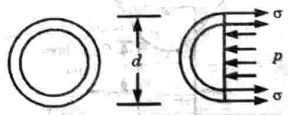

Fig. 1.12.2. Free body diagram.

2. From the free body diagram Fig. 1.12.2.

$$p \times \frac{\pi d^2}{4} = 2 \times (\sigma \times \pi d)$$

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$$\therefore \quad p = \frac{8\sigma}{d}$$

C. A Liquid Jet :

1. Let us consider cylindrical liquid jet of diameter d and length l, as shown in Fig. 1.12.3.

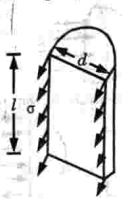

Fig. 1.12.3. Forces on liquid jet.

2. Pressure force = $p \times l \times d$
3. Surface tension force = $\sigma \times 2l$
4. On equating the two forces,

$$p \times l \times d = \sigma \times 2l$$

$$\therefore \quad p = \frac{2\sigma}{d}$$

Que 1.13. **What is capillarity ? What is its significance in fluid flow problems ?**

Answer

A. Capillarity :

1. Capillarity is a phenomenon by which a liquid rises into a thin glass tube above or below its general level.
2. This occurred due to the combined effect of cohesion and adhesion of liquid particles.
3. The Fig. 1.13.1 shows the phenomenon of rising water in the tube of smaller diameter.

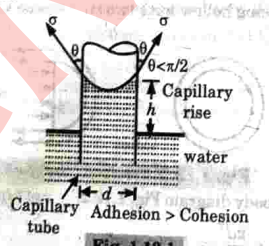

Fig. 1.13.1.

4. Let
   d = Diameter of capillary tube,
   θ = Angle of contact of the water surface,
   h = Height of capillary rise,
   σ = Surface tension force per unit length, and
   w = Weight density ($\rho.g$).
5. Upward surface tension force is,
   Lifting force = Weight of the water column in the tube (gravity force)

i.e., $$\pi d.\sigma \cos\theta = \frac{\pi}{4} \times d^2 \times h \times w$$

$$\therefore \quad h = \frac{4\sigma \cos\theta}{wd}$$

B. Significance :

1. Water rises in the tube of smaller diameters above or below its level.

Que 1.14. **What should be the diameter of a droplet of water, if the pressure inside is to be 0.0018 kg(f)/cm² greater than the outside ? Given the value of surface tension of water in contact with air at 20 °C as 0.0075 kg(f)/m.** **AKTU 2015-16, Marks 05**

Answer

Given :
Pressure inside the droplet above outside pressure, $p = 0.0018$ kg(f)/cm²,
Surface tension, $\sigma = 0.0075$ kg(f)/m $= 7.5 \times 10^{-5}$ kg(f)/cm
To Find : Diameter of a droplet of water.

1. For water droplet,

$$p = \frac{4\sigma}{d}$$

$$d = \frac{4\sigma}{p}$$

2. On putting the given values,

$$d = \frac{4 \times 7.5 \times 10^{-5} \text{ kg(f)/cm}}{0.0018 \text{ kg(f)cm}^2} = 0.1667 \text{ cm}$$

Que 1.15. **Draw and explain the Rheological diagram.** **AKTU 2014-15, Marks 05**

Answer

1. Rheology is the study of deformation of flowing fluids.

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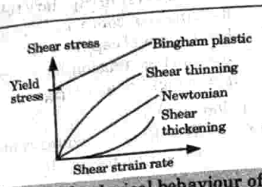

Fig. 1.15.1. Rheological behaviour of fluids.

Explanation of Rheological Diagram :

1. Fluids for which the shear stress is not linearly related to the shear strain rate are called non-Newtonian fluids.
2. Some non-Newtonian fluids exhibit a property *i.e.*, a fluid that returns (either fully or partially) to its original shape after the applied stress is released is known as viscoelastic. The shear stress for these types of fluid depends not only on the local strain rate, but also on its history.
3. Some non-Newtonian fluids are called shear thinning fluids or pseudo-plastic fluids, because the more the fluid is sheared, the less viscous it becomes.
4. Plastic fluids are those in which the shear thinning effect is extreme.
5. In some fluids a finite stress called the yield stress is required before the fluid begins to flow at all; such fluids are called Bingham plastic fluids. Example : Certain pastes like tooth paste.
6. The fluids if sheared more, the more viscous it becomes are known as shear thickening fluids or dilatant fluids. Example : a thick mixture of sand and water.

PART-2

Pressure-Density-Height Relationship, Manometers, Pressure Transducers.

CONCEPT OUTLINE : PART-2

Absolute Pressure : The actual pressure at a given position is called the absolute pressure.

Gauge Pressure : It is defined as the difference between absolute pressure and local atmospheric pressure.

Relationship between Absolute, Gauge and Vacuum Pressure :

$$P_{gauge} = P_{abs} - P_{atm}$$

$$P_{vac} = P_{atm} - P_{abs}$$

Manometers : These are the devices used for measuring pressure at a point in a fluid by balancing the column of fluid by the same or another column of the fluid.

Mechanical Gauges : These are defined as the devices used for measuring the pressure by balancing the fluid column by the spring or dead weight.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.16. **Explain atmospheric pressure, gauge pressure, vacuum pressure and absolute pressure with suitable sketch. Also, give the relationship between vacuum, gauge and absolute pressure.**

Answer

A. Absolute Pressure :

1. The actual pressure at a given position is called the absolute pressure, and it is measured relative to absolute vacuum (*i.e.*, absolute zero pressure).

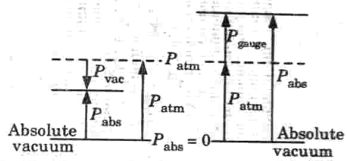

Fig. 1.16.1. Absolute, gauge and vacuum pressures.

B. Atmospheric Pressure :

1. The atmospheric air exerts a normal pressure upon all surfaces with which it is in contact, and it is known as atmospheric pressure.
2. The atmospheric pressure varies with the altitude and it can be measured by means of a barometer. As such it is also called the barometric pressure.

C. Gauge Pressure :

1. Gauge pressure is defined as the difference between absolute pressure and the local atmospheric pressure.
2. Gauge pressure can be positive or negative.

D. Vacuum Pressure :

1. Pressures below atmospheric pressure are called as vacuum pressures.

E. Relationship Between Absolute, Gauge and Vacuum Pressure :

1. $$P_{gauge} = P_{abs} - P_{atm}$$
2. $$P_{vac} = P_{atm} - P_{abs}$$

Que 1.17. **Derive an expression for pressure variation in a fluid at rest.**

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Answer

1. As shown in Fig. 1.17.1 consider a small fluid element.
2. Let
   ΔA = Cross sectional area of element,
   Δz = Height of fluid element,
   P = Pressure on face AB, and
   z = Distance of fluid element from free surface.
3. The various force acting on the fluid elements are :
   a. Pressure force on face $AB = p \times \Delta A$ which is acting perpendicular to face AB in the downward direction.
   b. Pressure force on face $CD = \left(P + \frac{\partial P}{\partial z}\Delta z\right) \times \Delta A$, it is acting perpendicular to face CD in vertically upward direction.
   c. Weight of fluid element = Weight density × Volume
   $= W \times (\Delta A \times \Delta z)$

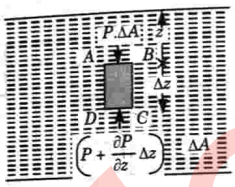

Fig. 1.17.1. Force acting on fluid element.

4. Pressure forces on surfaces BC and AD are equal and opposite.
5. For an equilibrium of fluid elements,

$$P\Delta A - \left(P + \frac{\partial P}{\partial z}\Delta z\right)\Delta A + W \times (\Delta A \times \Delta z) = 0$$

or $$P\Delta A - P\Delta A - \frac{\partial P}{\partial z}\Delta z\Delta A + W \times (\Delta A \times \Delta z) = 0$$

or $$-\frac{\partial P}{\partial z}\Delta z\Delta A + W\Delta A\,\Delta z = 0$$

or $$\left[\frac{\partial P}{\partial z} - W\right]\Delta z\,\Delta A = 0$$

$$\frac{\partial P}{\partial z} = W$$

$$\therefore \quad \frac{\partial P}{\partial z} = \rho g \qquad \text{...(1.17.1)}$$

6. Eq. (1.17.1) is the hydrostatic law and states that the rate of increase of pressure in a vertically downward direction is equal to weight density of the fluid at that point.

7. By integrating the eq. (1.17.1) for liquids,

$$\int dP = \int \rho g dz$$

$$P = \rho g z$$

where, P = Pressure above atmospheric pressure and
z = Distance of the point from free surface.

8. $z = \frac{P}{\rho g}$, here z is called pressure head.

Que 1.18. **Write the name of some pressure measurement devices.**

Answer

The pressure of a fluid is measured by the following devices :

A. Manometers :

1. Manometers are defined as the devices used for measuring the pressure at a point in a fluid by balancing the column of fluid by the same or another column of the fluid.
2. They are classified as :
   a. Simple manometers, and
   b. Differential manometers.

B. Mechanical Gauges :

1. Mechanical gauges are defined as the devices used for measuring the pressure by balancing the fluid column by the spring or dead weight.
2. The commonly used mechanical pressure gauges are :
   a. Diaphragm pressure gauge,
   b. Bourdon tube pressure gauge,
   c. Dead-weight pressure gauge, and
   d. Bellows pressure gauge.

Que 1.19. **Describe with sketches the simple manometers and its types.**

Answer

1. A simple manometer consists of a glass tube having one of its end connected to a point where pressure is to be measured and other end remains open to atmosphere.
2. Common types of simple manometers are :

A. Piezometer :

1. It is the simplest form of manometer used for measuring gauge pressure.
2. One end of this manometer is connected to the point where pressure is to be measured and other end is kept open to the atmosphere.

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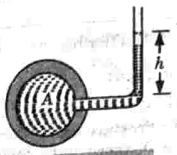

Fig. 1.19.1.

3. The rise of liquid level gives the pressure head at that point.
4. Let the height of liquid be h in piezometer tube then the pressure at point A is,

$$P_A = \rho \times g \times h \text{ N/m}^2$$

B. *U*-tube Manometer :

1. It is made up of glass tube bent in U-shape, one end of which is connected to a point at which pressure is to be measured and other end remains open to the atmosphere.
2. The tube generally contains mercury or any other liquid whose specific gravity is greater than the specific gravity of the liquid whose pressure is to be measured.

a. For Gauge Pressure :

1. Let B is the point at which pressure is to be measured, whose value is P.
2. The datum line is $A - A$.
3. Let h_1 = Height of light liquid above the datum line,
   h_2 = Height of heavy liquid above the datum line,
   S_1 = Specific gravity of light liquid,
   ρ_1 = Density of light liquid = $1000 \times S_1$,
   S_2 = Specific gravity of heavy liquid,
   ρ_2 = Density of heavy liquid = $1000 \times S_2$.

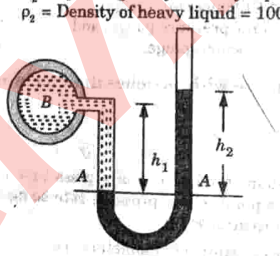

Fig. 1.19.2.

4. As the pressure is same for the horizontal surface, hence pressure above the horizontal datum line $A - A$ in the right column of U-tube manometer should be the same.

5. Pressure above $A - A$ in the left column $= P + \rho_1 \times g \times h_1$
6. Pressure above $A - A$ in right column $= \rho_2 \times g \times h_2$
7. Hence equating the two pressures,

$$P + \rho_1 g h_1 = \rho_2 g h_2$$

$$P = (\rho_2 g h_2 - \rho_1 g h_1) = (\rho_2 h_2 - \rho_1 h_1) g$$

b. For Vacuum Pressure :

1. For measuring vacuum pressure, the level of the heavy liquid in the manometer is shown in Fig. 1.19.3.
2. Pressure above $A - A$ in the left column

$$= \rho_2 g h_2 + \rho_1 g h_1 + P$$

3. Pressure head in the right column above $A - A = 0$

$$\rho_2 g h_2 + \rho_1 g h_1 + P = 0$$

$$P = -(\rho_2 g h_2 + \rho_1 g h_1)$$

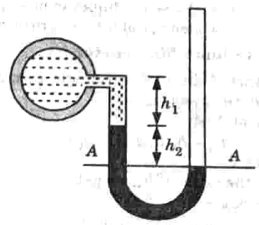

Fig. 1.19.3.

C. Single Column Manometer :

1. It is a modified form of a U-tube manometer in which a reservoir having a large cross sectional area (near about 100 times) as compared to the area of the tube is connected to one of the limbs of the manometer.
2. Because of large cross sectional area of the reservoir, for any variation in pressure, the change in the liquid level in the reservoir will be very small which may be neglected.
3. Hence, the pressure is given by the height of the liquid in the other limb. The other limb may be vertical or inclined.

a. Vertical Single Column Manometer :

1. Pressure at A can be given as,

$$P_A = h_2 \rho_2 g - h_1 \rho_1 g$$

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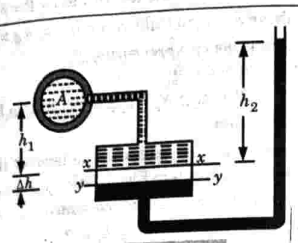

Fig. 1.19.4.

where, h_2 = Rise of heavy liquid in right limb,
h_1 = Height of centre of pipe above $x - x$,
P_A = Pressure at A, which is to be measured,
ρ_1 = Density of liquid in pipe, and
ρ_2 = Density of liquid in reservoir.

b. Inclined Single Column Manometer :

1. Fig. 1.19.5 shows the inclined single column manometer. This manometer is more sensitive.
2. The pressure at A is,
$$P_A = h_2 \rho_2 g - h_1 \rho_1 g$$
3. But, $h_2 = L \sin \theta$.
Substituting the value of h_2, we get
$P_A = L \sin \theta \times \rho_2 g - h_1 \rho_1 g$.

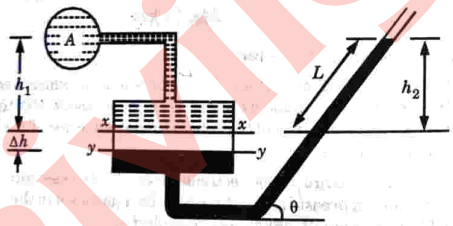

Fig. 1.19.5.

Que 1.20. What are the differential manometers ? Discuss them in brief.

Answer

1. These are the devices used for measurement of the difference of pressures between two points in a pipe or in two different pipes.

2. Thus, differential manometer consists of a U-tube, containing a heavy liquid, whose two ends are connected to the points, whose difference of pressure is to be measured.
3. Most common type of differential manometers are :

A. U-tube Differential Manometer :

1. From Fig. 1.20.1, let the two points A and B are at two different levels and also contain liquids of different specific gravity.
2. Let the pressure at A and B is P_A and P_B respectively.
3. Let h = Difference of mercury level in the U-tube,
y = Distance of centre B, from the mercury level in the right limb,
x = Distance of the centre A, from the mercury level in the right limb,

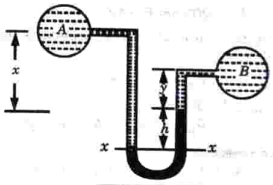

Fig. 1.20.1.

ρ_1 = Density of liquid at A
ρ_2 = Density of liquid at B, and
ρ_g = Density of heavy liquid or mercury.

4. Pressure above $x - x$ in the left limb $= \rho_1 g(h + x) + P_A$
5. Pressure above $x - x$ in the right limb
$$= \rho_g \times g \times h + \rho_2 \times g \times y + P_B$$
6. Equating the two pressures,
$$\rho_1 g (h + x) + P_A = \rho_g \times g \times h + \rho_2 gy + P_B$$
$$\therefore \quad P_A - P_B = \rho_g \times g \times h + \rho_2 gy - \rho_1 g (h + x)$$
7. Difference of pressure at A and B
$$= h \times g (\rho_g - \rho_1) + \rho_2 gy - \rho_1 gx$$

B. Inverted U-tube Differential Manometer :

1. It is used for measuring the difference of low pressures.
2. Fig. 1.20.2 shows an inverted U-tube differential manometer connected to the two points A and B.
3. Let the pressure at A be more than the pressure at B.
4. Let h_1 = Height of liquid in left limb
h_2 = Height of the liquid in right limb,

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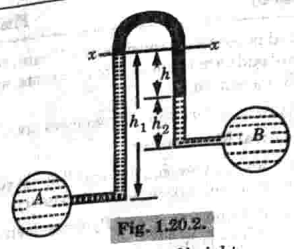

Fig. 1.20.2.

h = Difference of height,
ρ_1 = Density of liquid at A,
ρ_2 = Density of liquid at B,
ρ_s = Density of light liquid,
P_A = Pressure at A, and
P_B = Pressure at B.

5. Taking $x-x$ as datum line, then the pressure in the left limb below $x-x$

$$= P_A - \rho_1 \times g \times h_1$$

6. Pressure in the right limb below $x-x$

$$= P_B - \rho_2 \times g \times h_2 - \rho_s \times g \times h$$

7. Equating the two pressures,

$$P_A - \rho_1 \times g \times h_1 = P_B - \rho_2 \times g \times h_2 - \rho_s \times g \times h$$

or

$$P_A - P_B = \rho_1 \times g \times h_1 - \rho_2 \times g \times h_2 - \rho_s \times g \times h$$

Que 1.21. Determine the reading of the pressure gauge 'B' shown in the Fig. 1.21.1.

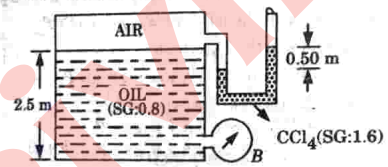

Fig. 1.21.1.

AKTU 2012-13, Marks 03

Answer

Given : SG of oil = 0.8, SG of CCl_4 = 1.6
Height of oil = 2.5 m, Height of CCl_4 = 0.50 m
To Find : Reading of the pressure gauge 'B'.

1. $$P_{air} = P_{atm} + \rho g h$$
$$= \left(\frac{760}{1000} \times 13.6 \times 9.81\right) + (1.6 \times 9.81 \times 0.5)$$
$$P_{air} = 109.24 \text{ kN/m}^2$$

2. $$P_B = P_{air} + P_{oil} = 109.24 + (0.8 \times 9.81 \times 2.5)$$
$$P_B = 128.86 \text{ kN/m}^2$$

3. $$P_B = P_{guage} + P_{atm}$$
$$P_{guage} = 128.86 - \left(\frac{760}{1000} \times 13.6 \times 9.81\right)$$
$$P_{guage} = 27.46 \text{ kN/m}^2$$

4. Hence, the pressure reading at B = 27.46 kN/m²

Que 1.22. Write short note on the following :

A. Pressure transducers,
B. Strain gauge pressure transducers, and
C. Piezoelectric transducers.

Answer

A. Pressure Transducers :

1. Modern pressure sensors, called pressure transducers, use various techniques to convert the pressure effect to an electrical effect such as a change in voltage, resistance, or capacitance.
2. Pressure transducers are smaller and faster, and they can be more sensitive, reliable, and precise than their mechanical counterparts.
3. They can measure pressures from less than a millionth of 1 atm to several thousands of atm.
4. Gauge pressure transducers use the atmospheric pressure as a reference by venting the back side of the pressure-sensing diaphragm to the atmosphere, and they give a zero signal output at atmospheric pressure regardless of altitude.
5. Absolute pressure transducers are calibrated to have a zero signal output at full vacuum.
6. Differential pressure transducers measure the pressure difference between two locations directly instead of using two pressure transducers and taking their difference.

B. Strain Gauge Pressure Transducers :

1. These pressure transducers works by having a diaphragm deflect between two chambers open to the pressure inputs.
2. As the diaphragm stretches in response to a change difference across it, the strain gauge stretches and a Wheatstone bridge circuit amplifies the output.

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3. A capacitance transducers work similarly, but capacitance change is measured instead of resistance change as the diaphragm stretches.

C. Piezoelectric Transducers :

1. These are also called solid-state pressure transducers, work on the principle that an electric potential is generated in a crystalline substance when it is subjected to mechanical pressure. This phenomenon is called the piezoelectric (or press-electric) effect.
2. These have a much faster frequency response compared to diaphragm units and are very suitable for high-pressure applications.

Que 1.23. **Fig. 1.23.1 shows a conical vessel having its outlet at *A* to which a *U*-tube manometer is connected. The reading of the manometer given in the Fig. 1.23.1 shows when the vessel is empty. Find the reading of the manometer when vessel is completely filled with water.**

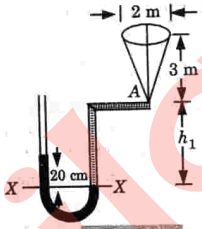

Fig. 1.23.1.

Answer

Given : Difference of mercury level, $h_2 = 20$ cm
To Find : Reading of the manometer when the vessel is completely filled with water.

Case (*i*) : When vessel is empty.

1. Let, h_1 = Height of water above *X-X*
   Specific gravity of mercury, $S_2 = 13.6$
   Specific gravity of water, $S_1 = 1.0$
   Density of mercury, $\rho_2 = 13.6 \times 1000$
   Density of water, $\rho_1 = 1000$
2. Equating the pressure above datum line *X-X*, we have

$$\rho_2 \times g \times h_2 = \rho_1 \times g \times h_1$$

or $$13.6 \times 1000 \times 9.81 \times 0.2 = 1000 \times 9.81 \times h_1$$

$$h_1 = 2.72 \text{ m of water.}$$

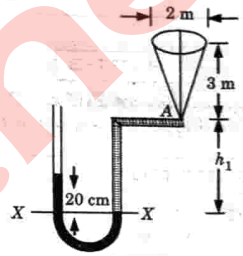

Fig. 1.23.2.

Case (*ii*) : Vessel is full of water.

1. Consider the vessel to be completely filled with water.
2. As a result of this let the mercury level go down by *y* cm in the right limb, and the mercury level go up by the same amount in the left limb.
3. Now the datum line is *Z-Z*.
4. Equating the pressure above the datum line *Z-Z*.
   Pressure in left limb = Pressure in right limb

$$13.6 \times 1000 \times 9.81 \times (0.2 + 2y/100) = 1000 \times 9.81 \times (3 + h_1 + y/100)$$

$$13.6 \times (0.2 + 2y/100) = (3 + 2.72 + y/100) \qquad (\because h_1 = 2.72 \text{ m})$$

$$(2.72 + 27.2y/100) = 3 + 2.72 + y/100$$

$$(27.2y - y)/100 = 3.0$$

$$26.2y = 3 \times 100 = 300$$

$$\therefore \quad y = \frac{300}{26.2} = 11.45 \text{ cm}$$

5. The difference of mercury level in two limbs
   $= (20 + 2y)$ cm of mercury
   $= 20 + 2 \times 11.45 = 20 + 22.90$
   $= 42.90$ cm of mercury
6. Reading of manometer = 42.90 cm

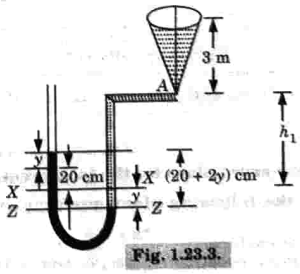

Fig. 1.23.3.

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PART-3

Pressure on Plane and Curved Surfaces, Centre of Pressure, Buoyancy and Stability of Immersed and Floating Bodies.

CONCEPT OUTLINE : PART-3

Total Pressure (F) :

1. Total pressure on a vertical plane surface submerged in liquid

$$F = \rho A g \bar{h}$$

2. Total pressure on a inclined plane surface submerged in liquid

$$F = \rho A g \bar{y} \sin\theta = \rho A g \bar{h} \qquad (\because \bar{h} = \bar{y} \sin\theta)$$

Centre of Pressure (h^*) :

1. For a vertical plane surface submerged in liquid

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

2. For an inclined plane surface submerged in liquid

$$h^* = \frac{I_G \sin^2\theta}{A\bar{h}} + \bar{h}$$

Where, $\bar{h}$ = Distance of C.G. from free surface, and

$\bar{y}$ = Distance of C.G. of the inclined plane surface from a plane which is perpendicular to the inclined plane.

Buoyancy : Whenever a body is immersed fully or partially in a fluid it is subjected to an upward force which tends to lift it up. This tendency for an immersed body to be lifted up in the fluid, due to an upward force opposite to action of gravity is known as buoyancy.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.24. **Derive expressions for the total pressure and centre of pressure when the following plane surfaces are submerged in liquids :**

A. Vertical plane surface,
B. Horizontal plane surface,
C. Inclined plane surface, and

D. Curved surface.

Answer

A. Vertical Plane Surface Submerged in Liquid :

1. As shown in Fig. 1.24.1, consider a plane vertical surface of an arbitrary shape immersed in a liquid.

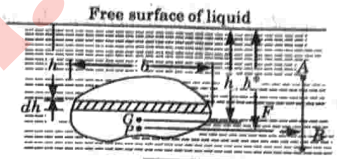

Fig. 1.24.1.

2. Let A = Total area of the surface,

$\bar{h}$ = Distance of C.G. of the area from free surface of liquid,
G = Centre of gravity of plane surface,
P = Centre of pressure, and
h^* = Distance of centre of pressure from free surface of liquid.

a. Total Pressure :

1. Consider a strip of thickness dh and width b at a depth of h from free surface of liquid.
2. Pressure intensity on the strip, $p = \rho g h$
3. Area of the strip, $dA = b \times dh$
4. Total pressure force on strip, $dF = p \times dA = \rho g h \times b \times dh$
5. Total pressure force on the whole surface

$$F = \int dF = \int \rho g h \times b \times dh = \rho g \int b \times h \times dh$$

6. But $\int b \times h \times dh = \int h \times dA$

= Moment of surface area about the free surface of liquid.
= Area of surface × distance of C.G. from free surface
$= A \times \bar{h}$

7. Total pressure, $F = \rho g A \bar{h}$ ...(1.24.1)

b. Centre of Pressure (h^*) :

1. Moment of the force F about free surface of the liquid

$= F \times h^*$ ...(1.24.2)

2. Moment of force dF, acting on a strip about free surface of liquid

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$= dF \times h$ $(\because dF = \rho gh \times b \times dh)$

$= \rho gh \times b \times dh \times h$

3. Sum of moments of all such forces about free surface of liquid

$$= \int \rho gh \times b \times dh \times h = \rho g \int b \times h \times h dh$$

$$= \rho g \int bh^2 dh = \rho g \int h^2 dA \qquad (\because bdh = dA)$$

4. But $\int h^2 dA$ = moment of inertia of the surface about free surface of liquid

$= I_o$

5. Sum of moments about free surface $= \rho g I_o$ ...(1.24.3)

6. Equating eq. (1.24.2) and eq. (1.24.3),

$$F \times h^* = \rho g I_o, \text{ but } F = \rho g A \bar{h}$$

$$\therefore \quad \rho g A \bar{h} . h^* = \rho g I_o$$

$$h^* = \frac{\rho g I_o}{\rho g A \bar{h}} = \frac{I_o}{A \bar{h}} \qquad ...(1.24.4)$$

7. By the theorem of parallel axis,

$$I_o = I_G + A \times \bar{h}^2$$

Where, I_G = Moment of Inertia of area about an axis passing through the C.G. of the area and parallel to the free surface of liquid.

8. Substituting I_0 in eq. (1.24.4), give

$$h^* = \frac{I_G + A\bar{h}^2}{A\bar{h}} = \frac{I_G}{A\bar{h}} + \bar{h} \qquad ...(1.24.5)$$

9. In eq. (1.24.5), $\bar{h}$ is the distance of C.G. of the area of the vertical surface from the free surface of the liquid.

B. Horizontal Plane Surface Submerged in Liquid :

1. As shown in Fig. 1.24.2 consider a plane horizontal surface immersed in a static liquid.
2. Every point of the surface is at the same depth from the free surface.

$p = \rho gh$, where h is depth of surface.

3. Then total force, F on the surface

$= p \times \text{Area} = \rho g \bar{h} \times A = \rho g A \bar{h}$

Where $\bar{h}$ = Depth of C.G. from free surface of liquid $= h$

also h^* = Depth of centre of pressure from free surface $= h$

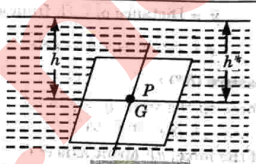

Fig. 1.24.2.

C. Inclined Plane Surface Submerged in Liquid :

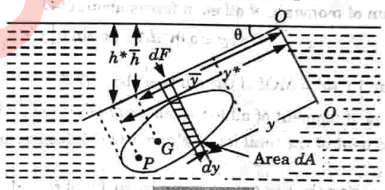

Fig. 1.24.3.

1. As shown in Fig. 1.24.3, consider a plane surface of an arbitrary shape immersed in a liquid in such a way that the plane of the surface makes an angle θ with the free surface of the liquid.

a. Total Pressure :

1. Let the plane of the surface, if produced, meet the free liquid surface at O.
2. Then O-O is the axis perpendicular to the plane of the surface.
3. Let $\bar{y}$ = Distance of the C.G. of the inclined surface from O-O, and

y^* = Distance of the centre of pressure from O-O.

4. Taking a small strip of area dA at a depth 'h' from free surface and at a distance y from the axis O-O.
5. Pressure intensity on the strip, $p = \rho gh$
6. Pressure force, dF on the strip, $dF = p \times$ area of strip

$= \rho gh \times dA$

7. Total pressure force on the whole area,

$$F = \int dF = \int \rho gh dA$$

8. As clear from the Fig. 1.24.3,

$$\frac{h}{y} = \frac{\bar{h}}{\bar{y}} = \frac{h^*}{y^*} = \sin\theta \qquad [\because h = y \sin\theta]$$

$$\therefore \quad F = \int \rho g \times y \times \sin\theta \times dA = \rho g \sin\theta \int y dA$$

9. But $\int y dA = A\bar{y}$

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Where $\bar{y}$ = Distance of C.G. from axis O-O

10. $F = \rho g \sin\theta \bar{y} \times A = \rho g A \bar{h}$ $(\because \bar{h} = \bar{y}\sin\theta)$

b. Centre of Pressure (h^*) :

1. Pressure force on the strip $dF = \rho g h\, dA$
$= \rho g y \sin\theta\, dA$ $(\because h = y\sin\theta)$
2. Moment of the force, dF about axis O-O
$= dF \times y = \rho g y \sin\theta\, dA \times y$
$= \rho g \sin\theta y^2\, dA$
3. Sum of moments of all such forces about O-O
$= \int \rho g \sin\theta y^2 dA = \rho g \sin\theta \int y^2 dA$
4. But $\int y^2 dA$ = MOI of the surface about O-$O = I_0$
5. Sum of moment of all forces about O-$O = \rho g \sin\theta\, I_0$ ...(1.24.6)
6. Moment of the total force, F about O-O is also given by
$= F \times y^*$ ...(1.24.7)
7. Equating the two values given by eq. (1.24.6) and eq. (1.24.7),
$F \times y^* = \rho g \sin\theta I_0$
or $y^* = \dfrac{\rho g \sin\theta I_0}{F}$ ...(1.24.8)
8. Since $y^* = \dfrac{h^*}{\sin\theta}$, $F = \rho g A \bar{h}$
and $I_0 = I_G + A\bar{y}^2$
9. Substituting these values in eq. (1.24.8),
$$\frac{h^*}{\sin\theta} = \frac{\rho g \sin\theta}{\rho g A \bar{h}}[I_G + A\bar{y}^2]$$
$$h^* = \frac{\sin^2\theta}{A\bar{h}}[I_G + A\bar{y}^2]$$
10. But $\dfrac{\bar{h}}{\bar{y}} = \sin\theta$ or $\bar{y} = \dfrac{\bar{h}}{\sin\theta}$
$$h^* = \frac{\sin^2\theta}{A\bar{h}}\left[I_G + A \times \left(\frac{\bar{h}}{\sin\theta}\right)^2\right]$$
$$h^* = \frac{I_G \sin^2\theta}{A\bar{h}} + \bar{h}$$

D. Curved Surface Submerged in Liquid :

1. As shown in Fig. 1.24.4(a) consider a curved surface AB, submerged in a static fluid.

2. Let dA be the area of a small strip at a depth of h from water surface.

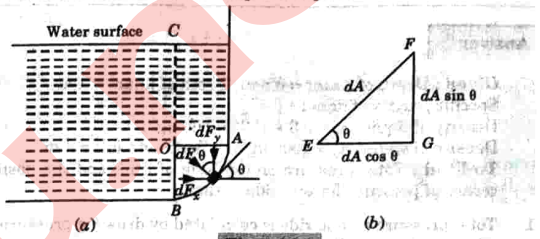

Fig. 1.24.4.

3. Then pressure intensity on the area $dA = \rho g h$
4. Pressure force, $dF = p \times \text{area} = \rho g h \times dA$
5. This force dF acts normal to the surface.
6. In this case, as the direction of the forces on the small areas vary from point to point.
7. On resolving the force dF into two components dF_x and dF_y in the x and y directions respectively.
8. Then, the total force on the curved surface is
$$F = \sqrt{F_x^2 + F_y^2}$$
9. Inclination of resultant with horizontal is
$$\tan\theta = \frac{F_y}{F_x}$$
10. After resolving the force dF,
$dF_x = dF \sin\theta = \rho g h\, dA \sin\theta$ $(\because dF = \rho g h dA)$
and $dF_y = dF \cos\theta = \rho g h\, dA \cos\theta$
11. Hence, total forces in the x and y direction :
$$F_x = \int dF_x = \int \rho g h dA \sin\theta = \rho g \int h dA \sin\theta$$
and $$F_y = \int dF_y = \int \rho g h dA \cos\theta = \rho g \int h dA \cos\theta$$

Que 1.25. **A tank contains water upto a height of 0.5 m above the base. An immiscible liquid of specific gravity 0.8 is filled on the top of water upto 1 m height. Calculate total pressure on one side of tank and the position of centre of pressure for one side of the tank which is 2 m wide.** **AKTU 2014-15, Marks 05**

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Answer

Given : Depth of water = 0.5 m, Depth of liquid = 1 m
Specific gravity of liquid = 0.8
Density of liquid, $\rho_1 = 0.8 \times 1000 = 800$ kg/m^3
Density of water, $\rho_2 = 1000$ kg/m^3, Width of tank = 2 m
To Find : Total pressure on one side of tank and the position of centre of pressure for one side of tank.

1. Total pressure on one side is calculated by drawing pressure diagram, which is shown in Fig. 1.25.1.
2. Intensity of pressure on top,
$$p_A = 0$$
3. Intensity of pressure on D (or DE),
$$p_D = \rho_1 g h_1$$

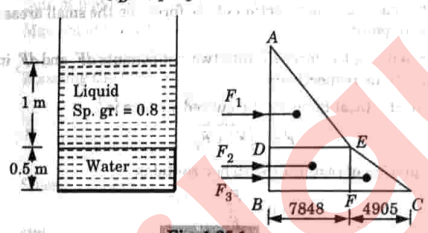

Fig. 1.25.1.

$$= 800 \times 9.81 \times 1.0 = 7848 \text{ N/m}^2$$

4. Intensity of pressure on base (or BC),
$$p_B = \rho_1 g h_1 + \rho_2 g \times 0.5$$
$$= 7848 + 1000 \times 9.81 \times 0.5 = 7848 + 4905$$
$$= 12753 \text{ N/m}^2$$
5. Now force, F_1 = Area of ΔADE × Width of tank
$$= \frac{1}{2} \times AD \times DE \times 2.0$$
$$= \frac{1}{2} \times 1 \times 7848 \times 2.0 = 7848 \text{ N}$$
F_2 = Area of rectangle $DBFE$ × Width of tank
$$= 0.5 \times 7848 \times 2 = 7848 \text{ N}$$
F_3 = Area of ΔEFC × Width of tank
$$= \frac{1}{2} \times EF \times FC \times 2.0$$
$$= \frac{1}{2} \times 0.5 \times 4905 \times 2.0 = 2452.5 \text{ N}$$

6. Total force,
$$F = F_1 + F_2 + F_3$$
$$= 7848 + 7848 + 2452.5 = 18148.5 \text{ N}$$
7. Taking the moments of all forces about A,
$$F \times h^* = F_1 \times \frac{2}{3} AD + F_2\left(AD + \frac{1}{2} BD\right) + F_3\left[AD + \frac{2}{3} BD\right]$$
$$18148.5 \times h^* = 7848 \times \frac{2}{3} \times 1 + 7848\left(1.0 + \frac{0.5}{2}\right) + 2452.5\left(1.0 + \frac{2}{3} \times 0.5\right)$$
$$= 5232 + 9810 + 3270 = 18312$$
Centre of pressure,
$$h^* = \frac{18312}{18148.5} = 1.009 \text{ m from top.}$$

Que 1.26. **An annular circular plate of 2 m external diameter and 1 m internal diameter is immersed vertically in water so that its lowest edge is 5 m below the free water surface. Determine the total force and position of centre of pressure.**

AKTU 2012-13, Marks 05

Answer

Given : $d_o = 2$ m, $d_i = 1$ m
To Find : Total force and position of centre of pressure.

1. $$\bar{h} = 5 - \frac{d_o}{2} = 5 - 1 = 4 \text{ m}$$
2. $$A = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(2^2 - 1^2) = 2.36 \text{ m}^2$$
3. Total force on the plate,
$$F = WA\bar{h} = 1000 \times 9.81 \times 2.36 \times 4 = 92.61 \text{ kN}$$
4. Centre of pressure, $h^* = \frac{I_G}{A\bar{h}} + \bar{h}$
$$I_G = \frac{\pi}{64}[d_o^4 - d_i^4] = \frac{\pi}{64}[2^4 - 1^4] = 0.7363 \text{ m}^4$$

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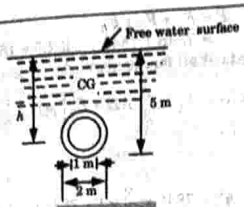

Fig. 1.26.1.

5. Therefore, $h^* = \frac{0.7363}{2.36 \times 4} + 4 = 4.08$ m

Que 1.27. **A trapezoidal plate having its parallel sides equal to '20' and '0' at a distance of 'h' apart; is immersed vertically in a liquid with '20' side uppermost and at a distance 'h' below the surface of the liquid. Find the thrust on the surface and depth of centre of pressure.**

AKTU 2011-12, Marks 05

Answer

Given : In ΔABC, $AB = 20$
To Find : Thrust on the surface and depth of centre of pressure.

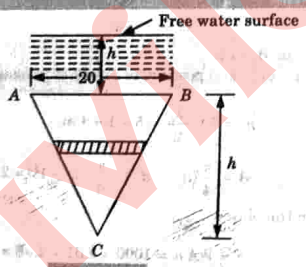

Fig. 1.27.1.

a. Thrust on the Surface :

1. We know,

$$F = \rho g A \bar{h} \quad \text{(vertical immersed body)}$$

$$F = \rho g \times \frac{1}{2} \times (h) \times 20 \times \left(\frac{h}{3} + h\right) \quad \left(\because \bar{h} = \frac{h}{3} + h = \frac{4h}{3}\right)$$

$$F = \rho g \times \frac{1}{2} \times h \times 20 \times \frac{4h}{3}$$

$$F = \frac{40}{3} \rho g h^2$$

b. Depth of Centre of Pressure :

1. We know

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

2. $$I_G = \frac{bh^3}{36} = \frac{20 \times h^3}{36}$$

$$I_G = \frac{5}{9} h^3$$

3. So, $$h^* = \frac{\frac{5}{9} h^3}{\frac{1}{2} \times 20 \times h \times \frac{4h}{3}} + \frac{4h}{3}$$

$$h^* = \frac{\frac{5}{9} h^3}{\frac{40}{3} h^2} + \frac{4h}{3}$$

$$h^* = \frac{h}{24} + \frac{4h}{3}$$

$$h^* = \frac{33h}{24} = \frac{11h}{8}$$

Que 1.28. **A piece of wood of specific gravity 0.6 and size 10 cm square in cross section and 2.5 m long floats in water. How much lead (of specific gravity 12) needs to be fastened at the lower end of the stick so that it flows up right with 0.5 m length out of water ?**

AKTU 2013-14, Marks 05

Answer

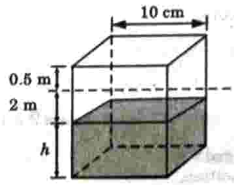

Fig. 1.28.1.

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Given : Area $=10^{-2}$ m², Length of block = 2.5 m
To Find : Amount of lead needed to be fastened at lower end.

1. Volume of block immersed in water $= 10^{-2} \times (2 + h)$
2. Weight of block $= 0.6 \times 1000 \times 9.81 \times 10^{-2} \times 2.5 = 147.15$ N
3. Weight of lead $= 12 \times 1000 \times 9.81 \times 10^{-2} \times h = 1177.2\,h$ N
4. Weight of block + weight of lead = weight of water displaced

$$147.15 + 1177.2\,h = 1000 \times 9.81 \times 10^{-2} \times (2 + h)$$
$$147.15 + 1177.2\,h = 196.2 + 98.1\,h$$
$$107.9\,h = 49.05$$
$$h = 0.045 \text{ m}$$

Que 1.29. **A 3.6 m by 1.5 m wide rectangular gate is vertical and is hinged at point 0.5 m below the centre of gravity of the gate. The total depth of water is 6 m. What horizontal force must be applied at the bottom of the gate to keep the gate closed ?**

AKTU 2015-16, Marks 05

Answer

Given : Height of gate = 3.6 m, Width of gate = 1.5 m
Depth of water = 6 m
To Find : Horizontal force.

1. The gate is hinged at point 0.5 m below the centre of gravity of the gate.
2. Centre of gravity from free liquid surface,

$$\bar{h} = 6 - \frac{3.6}{2}$$
$$\bar{h} = 4.2 \text{ m}$$

3. Now, the pressure force acting on the gate,

$$F = \rho g A\,\bar{h}$$
$$F = 1000 \times 9.81 \times 1.5 \times 3.6 \times 4.2 = 222.49 \text{ kN}$$

4. Now, centre of pressure,

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$
$$= \frac{\frac{1.5 \times 3.6^3}{12}}{1.5 \times 3.6 \times 4.2} + 4.2 = 4.457 \text{ m}$$

5. *Let, reaction at hinged = R*
6. *For equilibrium condition,*

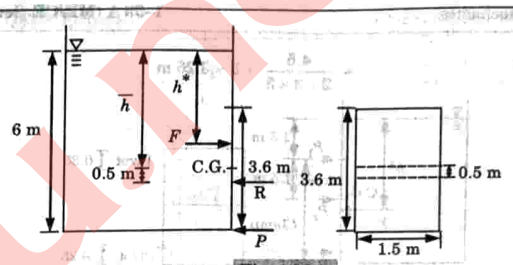

Fig. 1.29.1.

$$F = R + P$$
$$R + P = 222.49 \text{ kN} \quad \text{...(1.29.1)}$$

where, P = horizontal force acting on bottom of gate.

7. Taking moment about free surface,

$$F \times h^* = R \times (4.2 + 0.5) + P \times 6$$
$$222.49 \times 4.457 = 4.7\,R + 6\,P$$
$$4.7\,R + 6\,P = 991.64 \text{ kN} \quad \text{...(1.29.2)}$$

8. From eq. (1.29.1) and eq. (1.29.2),

$$R = 264.076 \text{ kN}$$
$$P = -41.5869 \text{ kN}$$

9. Hence, force required to keep the gate closed is 41.5869 kN in direction of total pressure force.

Que 1.30. **A rectangular door covering an opening 3 m wide and 2 m high in a vertical wall is hinged about its vertical edge by two pivots placed symmetrically 0.25 m from either end. The door is locked by a clamp placed at the centre of the vertical edge. Determine the reactions at the two hinges and the clamp, when the height of water is 1.5 m above the top edge of the opening.**

AKTU 2015-16, Marks 05

Answer

Given : Width = 3 m, Height = 2 m, Hinged from either end = 0.25 m
Height of water above the top edge = 1.5 m
To Find : Reactions at two hinges and the clamp.

1. $$F = \rho g A\,\bar{h} = 1000 \times 9.81 \times 2 \times 3 \times 3 = 176.58 \text{ kN}$$
2. $$I_G = \frac{bd^3}{12} = \frac{2 \times 3^3}{12} = 4.5 \text{ m}^4$$
3. $$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

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$$= \frac{4.5}{2 \times 3 \times 3} + 3 = 3.25 \text{ m}$$

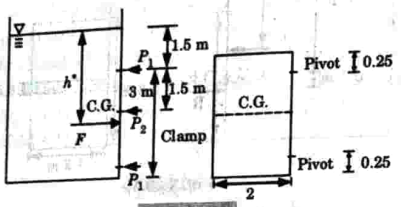

Fig. 1.30.1.

4. From equilibrium condition,

$$F = 2P_1 + P_2$$

$$P_2 + 2P_1 = 176.58 \text{ kN} \quad ...(1.30.1)$$

5. Taking moment about free surface,

$$F \times 3.25 - 3P_2 - 1.75P_1 - 4.25P_1 = 0$$

$$F \times 3.25 - 3P_2 - 6P_1 = 0$$

$$3P_2 + 6P_1 = 573.885 \text{ kN}$$

$$P_2 + 2P_1 = 191.29 \quad ...(1.30.2)$$

Since the left hand side of both the equation is same so it can not be solved.

Que 1.31. **A circular plate 3 meters in diameter is submerged in water in such a way that the greatest and least depths of the surface, (below water surface) are 2 m, and 1 m respectively, calculate :**

i. The total pressure on front face of the plate, and
ii. The position of centre of pressure.

AKTU 2016-17, Marks 10

Answer

Given : Diameter of circular plate, $d = 3$ m, $DC = 1$ m, $BE = 2$ m
To Find : i. The total pressure on front face of the plate, and
ii. The position of centre of pressure.

1. Area of circular plate, $A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(3)^2 = 7.068 \text{ m}^2$
2. Distance of C.G. from free surface,

$$\bar{h} = CD + GC \sin\theta$$

3. From Fig. 1.31.1

$$\sin\theta = \frac{AB}{BC} = \frac{BE - AE}{BC}$$

$$= \frac{2-1}{3} \quad \left\{\because \begin{matrix} AE = CD = 1 \text{ m} \\ BC = d = 3 \text{ m} \end{matrix}\right\}$$

$$\sin\theta = \frac{1}{3}$$

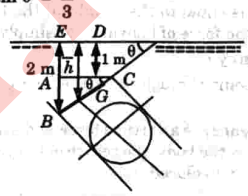

Fig. 1.31.1.

4. So, $\bar{h} = 2 + 2 \times \frac{1}{3} = 2.66$ m

5. Total pressure $(F) = \rho g A \bar{h} = 1000 \times 9.81 \times 7.068 \times 2.66$
$= 184852.655$ N

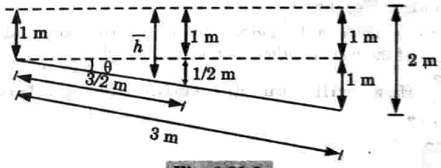

Fig. 1.31.2.

6. Moment of inertia of the Fig. 1.31.2 about horizontal axis through its centre of gravity

$$I_G = \frac{\pi d^4}{64} = \frac{\pi}{64}(3)^4$$

$$I_G = 3.976 \text{ m}^4$$

7. Centre of pressure $(h^*) = \frac{I_G \sin^2\theta}{A\bar{h}} + \bar{h}$

$$h^* = \frac{3.976 \times \left(\frac{1}{3}\right)^2}{7.069 \times 1.5} + 1.5 = 1.5416 \text{ m}$$

Que 1.32. **Define the following terms :**

a. Buoyancy
b. Centre of buoyancy
c. Metacentre
d. Metacentric height

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Answer

A. Buoyancy :

1. When a body is immersed in a fluid, an upward force is exerted by the fluid on the body.
2. This upward force is equal to the weight of the fluid displaced by the body and is called the force of buoyancy or simply buoyancy.

B. Centre of Buoyancy :

1. It is defined as the point, through which the force of buoyancy is supposed to act.
2. As the force of buoyancy is a vertical force and is equal to the weight of the fluid displaced by the body, the centre of buoyancy will be the centre of gravity of the fluid displaced.

C. Metacentre :

1. It is defined as the point about which a body starts oscillating when the body is tilted by a small angle.
2. The metacentre may also be defined as the point at which the line of action of the force of buoyancy will meet the normal axis of the body when the body is given a small angular displacement.

D. Metacentric Height :

1. The distance between the metacentre of a floating body and the centre of gravity of the body is called metacentric height.

Que 1.33. How will you determine metacentric height analytically ?

Answer

A. Determination of Metacentric Height Analytically :

1. Fig. 1.33.1(*a*) shows the position of a floating body in equilibrium.

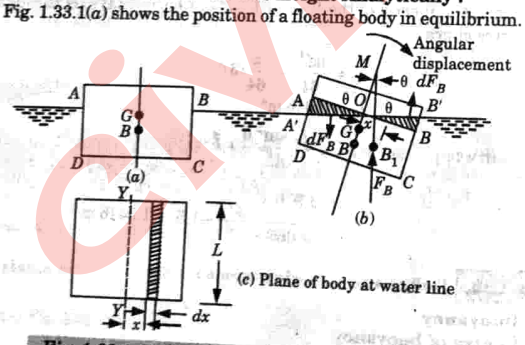

Fig. 1.33.1. Metacentre height of floating body.

2. The location of centre of gravity and centre of buoyancy in this position is at G and B.
3. Let the floating body be given a small angular displacement in the clockwise direction as shown in Fig. 1.33.1(*b*).
4. Due to angular displacement the centre of buoyancy is shifted and the new centre of buoyancy is at B_1.
5. The vertical line through B_1 cuts the normal axis at M. Hence M is the metacentre and GM is metacentric height.

a. Couple due to Wedges :

1. Considering a small strip of thickness dx at a distance x from O towards the right of the axis.
2. The height of strip = x, $\angle BOB' = x\,\theta$

   $\therefore \quad \angle BOB' = \angle AOA' = \angle BMB_1 = \theta$

   $\therefore$ Area of strip = Height × Thickness = $x.\theta.dx$
3. If L is the length of the floating body, then

   Volume of strip = Area × $L = x\theta Ldx$
4. Weight of the strip = ρg × volume = $\rho gx\theta Ldx$
5. Similarly, if a small strip of thickness dx at a distance x from O towards the left of the axis is considered, the weight of the strip will be $wx\theta Ldx$.
6. The two weights are acting in the opposite direction and hence constitute a couple.
7. Moment of this couple

   = Weight of each strip × Distance between these two weights

   $= \rho gx\theta Ldx\,[x + x] = \rho gx\theta Ldx.2x$

   $= 2\,\rho gx^2\,\theta\,L\,dx$
8. Moment of the couple for the whole wedge

   $$= \int 2\rho gx^2\theta Ldx \quad \text{...(1.33.1)}$$
9. Moment of couple due to shifting of centre of buoyancy from B to B_1

   $$= F_B \times BB_1 = F_B \times BM \times \theta$$

   $[\because BB_1 = BM \times \theta$ if θ is very small$]$

   $$= W \times BM \times \theta \qquad [\because F_B = W] \quad \text{...(1.33.2)}$$
10. But these two couples are the same. Hence equating eq. (1.33.1) and eq. (1.33.2),

    $$W \times BM \times \theta = \int 2\rho gx^2\theta\,L\,dx$$

    $$W \times BM = 2\rho g\int x^2\,L\,dx$$
11. Now $\quad Ldx = dA$, Elemental area on the water line

    $$W \times BM = 2\rho g\int x^2\,dA$$

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12. From the Fig. 1.33.1(*c*) it is clear that $2\int x^2\, dA$ is the second moment of area of the plane of the body at water surface about the axis *Y-Y*. Therefore,

$$W \times BM = \rho g I$$

$$BM = \frac{\rho g I}{W}$$

13. W = Weight of the body
= Weight of the fluid displaced by the body
= ρg × Volume of the fluid displaced by the body
= ρg × Volume of the body submerged in water
= $\rho g \times \forall_{sm}$

Therefore, $$BM = \frac{\rho g I}{\rho g \forall_{sm}} = \frac{I}{\forall_{sm}} \quad ...(1.33.3)$$

14. Metacentric height,

$$GM = BM - BG = \frac{I}{\forall_{sm}} - BG$$

Que 1.34. Explain the stability of floating and submerged bodies or write a short note on the stability of floating and submerged bodies.

Answer

A. Stability of Floating Body :

1. The position of metacentre (*M*) determines the stability of a floating body.
2. In case of a floating body, the weight of the body is equal to the weight of the fluid displaced.
3. Depending upon the relative position of *M* with respect to *G*, the body may be in stable equilibrium, unstable equilibrium or neutral equilibrium.

a. Stable Equilibrium :

1. If the point *M* lies above *G*, the floating body will be in stable equilibrium as shown in Fig. 1.34.1(*b*).

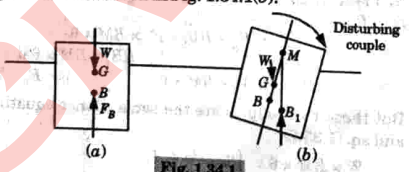

Fig. 1.34.1.

2. If a slight angular displacement is given to the floating body in the clockwise direction, the centre of buoyancy shifts from *B* to B_1 such that the vertical line through B_1 cuts at *M*.

3. Then the buoyant force F_B through B_1 and weight *W* through *G* constitute a couple acting in the anti-clockwise direction and thus bringing the floating body in the original position.

b. Unstable Equilibrium :

1. If the point *M* lies below *G* as shown in Fig. 1.34.2(*b*), then the floating body will be in an unstable equilibrium.
2. The disturbing couple is acting in the clockwise direction.
3. The couple due to the buoyant force F_B and *W* is also acting in the clockwise direction and thus overturning the floating body.

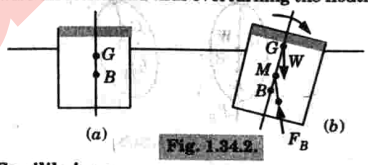

Fig. 1.34.2.

c. Neutral Equilibrium :

1. If the point *M* coincides with the centre of gravity *G*, of the body, the floating body will be in neutral equilibrium.

B. Stability of Submerged Body :

1. The positions of centre of gravity and centre of buoyancy in the case of a completely submerged body are fixed.
2. For example, consider a balloon, completely submerged in air and let the lower portion of the balloon contains heavier material, so that its centre of gravity be lower than its centre of buoyancy.
3. Let the weight of the balloon is *W* acting through *G*, vertically in the downward direction, while the buoyant force F_B is acting vertically up, through *B*.

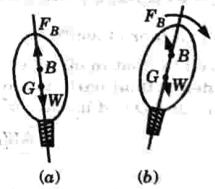

Fig. 1.34.3. Stability of submerged body.

a. Stable Equilibrium :

1. If the balloon is given an angular displacement in the clockwise direction as shown in Fig. 1.34.3(*b*), then *W* and F_B constitute a couple acting in the anticlockwise direction and brings the balloon in the original position. Hence the balloon is in stable equilibrium.

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2. When $W = F_B$ and point B is above G, the body is said to be in stable equilibrium.

b. Unstable Equilibrium :

1. If $W = F_B$, but the centre of buoyancy (B) is below centre of gravity (G), the body is in unstable equilibrium.

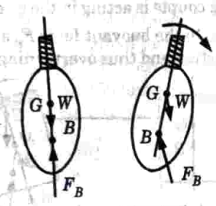

Fig. 1.34.4.

2. A slight displacement of the body, in the clockwise direction, gives the couple due to W and F_B also in the clockwise direction.
3. Thus the body does not restore back to its original position and hence the body is in unstable equilibrium.

c. Neutral Equilibrium :

1. If $F_B = W$ and B and G are at the same point, then the body is said to be in neutral equilibrium.

Fig. 1.34.5. Neutral equilibrium.

Que 1.35. **A wooden cylinder (specific gravity = 0.6) and circular cross-section is required to float in oil (specific gravity = 0.8). Find L/D ratio for the cylinder to float with its longitudinal axis vertical in oil. Where L is height and D is diameter of cylinder.**

AKTU 2011-12, Marks 10

Answer

Given : Specific gravity of wooden cylinder = 0.6
Specific gravity of oil = 0.8, Diameter of cylinder = D
Height of cylinder = L
To Find : L/D Ratio for the cylinder to float with longitudinal axis vertical in oil.

1. Let the depth of cylinder within oil = l

2. Principle of buoyancy,
Weight of cylinder = Weight of oil displaced

$$\left(\frac{\pi}{4}D^2L \times 0.6 \times 1000 \times 9.81\right) = \left(\frac{\pi}{4}D^2 \cdot l \times 0.8 \times 1000 \times 9.81\right)$$

$$0.6L = 0.8l$$

$$l = \frac{0.6}{0.8}L = \frac{3}{4}L$$

3. Now position of centre of gravity, $AG = \frac{L}{2}$

4. Position of centre of buoyancy, $AB = \frac{l}{2} = \frac{3}{8}L$

$$BG = AG - AB = \frac{L}{2} - \frac{3}{8}L = \frac{L}{8}$$

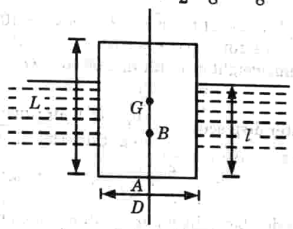

Fig. 1.35.1.

5. Therefore, metacentric height, $GM = \frac{I}{\forall} - BG$

$$\frac{I}{\forall} = \frac{\frac{\pi}{64}D^4}{\frac{\pi}{4}D^2 \cdot l} = \frac{D^2}{16l} = \frac{D^2}{16\left(\frac{3}{4}L\right)} = \frac{D^2}{12L}$$

$$GM = \frac{D^2}{12L} - \frac{L}{8}$$

6. For stable equilibrium, $GM > 0$

$$\frac{D^2}{12L} - \frac{L}{8} > 0$$

$$\frac{D^2}{L^2} > \frac{12}{8}$$

$$\frac{D}{L} > \sqrt{\frac{3}{2}}$$

$$\Rightarrow \quad \frac{L}{D} < 0.8164$$

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Que 1.36. **A wooden block of rectangular section 1.25 m wide, 2 m deep and 4 m long floats horizontally in sea water. If the specific weight of wood is 0.64 and water weighs 1.025 kg (f)/m³, find the volume of the water displaced and the position of the centre of buoyancy.** **AKTU 2011-12, Marks 10**

Answer

Given : Width of block = 1.25 m, Depth of block = 2 m
Length of block = 4 m, Specific weight of wood = 0.64 kg(f)/m³,
Specific water weight = 1.025 kg(f)/m²
To Find : Volume of water displaced and position of centre of buoyancy

1. Volume of block = $1.25 \times 2 \times 4 = 10\ m^3$
2. Weight of block = $0.64 \times 9.81 \times 10 = 62.72$ kN
3. For equilibrium, weight of water displaced = weight of block = 62.72 kN
4. Volume of water displaced = $\dfrac{\text{weight of water displaced}}{\text{weight density of water}}$

$$= \frac{62.72}{1.025 \times 9.81} = 6.2375\ m^3$$

5. Volume of wooden block in water = volume of water displaced
$1.25 \times h \times 4 = 6.2375$
$h = 1.25$ m
6. Hence, centre of buoyancy = $\dfrac{1.25}{2} = 0.625$ m

PART-4

Fluid Masses Subjected to Linear Acceleration and Uniform Rotation.

CONCEPT OUTLINE : PART-4

1. If a liquid container subjected to a constant horizontal acceleration a, the slope for the surface of constant pressure is given by

$$\frac{dz}{dx} = -\frac{a}{g}$$

2. If a liquid container subjected to a constant vertical acceleration a, the surface of constant pressure is horizontal.
3. If a cylindrical vessel containing liquid is rotated at a constant angular velocity ω with its axis vertical, then in this case the free surface of liquid developed is a paraboloid of revolution in shape with its section being a parabola represented by the following equation,

$$z = \frac{\omega^2 x^2}{2g}$$

Where, (x, z) = Coordinates of any point on the free surface of liquid with origin at the vortex of the parabola.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.37. **Give pressure variation in a fluid subjected to acceleration.**

Answer

A. Pressure Variation due to Constant Horizontal Acceleration :

1. The constant horizontal acceleration a_x causes the level of the liquid on the front side of the tank to fall and that on the back side to rise.
2. This makes the free liquid surface to slope upward in a direction opposite to a_x as shown in Fig. 1.37.1.

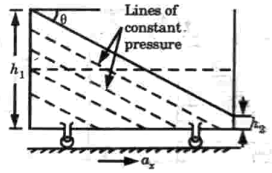

Fig. 1.37.1.

2. Considering the equilibrium of entire mass of liquid as shown in Fig. 1.37.2.

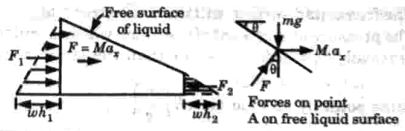

Fig. 1.37.2. Free body diagram of fluid mass.

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3. If F_1 and F_2 are respectively the hydrostatic pressure forces on the left and right hand end faces of the container, then on applying Newton's second law of motion,

$$\Sigma F_x = M.a_x$$
$$\text{or } F_1 - F_2 = M.a_x$$

B. Pressure Variation due to Uniform Vertical Acceleration :

1. Consider an open container filled with liquid which moves vertically upward with a uniform linear acceleration say a_z.
2. Consider a small column of liquid which is acted upon by pressure force F, W and acceleration force ma_z.

Where, F = Pressure force acting on the bottom of the liquid column = $p.dA$

W = Weight of liquid column = $w.\,h\,.\,dA$

m = Mass of liquid column = $\frac{w}{g}(h.dA)$

3. Applying Newton's second law of motion

$$\Sigma F_z = m.a_z,$$
$$F - W = ma_z$$

or $$p.dA - w\,(h.dA) = \frac{w}{g}(h.dA).a_z$$

or $$p = wh + wh.\frac{a_z}{g} \quad ...(1.37.1)$$

4. From this equation, it is found that
   a. The pressure varies linearly in a vertical direction.

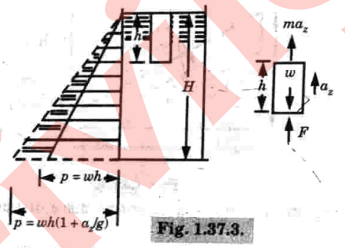

Fig. 1.37.3.

   b. The free liquid surface will remain horizontal.
   c. The pressure at any point of the liquid which is uniformly accelerated vertically upward, is greater than the static pressure wh at the same point by an amount $wh\left(\frac{a_z}{g}\right)$.
5. If the liquid mass is uniformly accelerated in a vertically downward direction, a_z shall be negative.

6. Therefore eq. (1.37.1) will change to

$$p = wh\left(1 - \frac{a_z}{g}\right) = wh - wh\left(\frac{a_z}{g}\right)$$

7. This equation shows that the pressure at a point is less than the static pressure wh by an amount $wh\frac{a_z}{g}$.

Que 1.38. Give an expression for pressure distribution in a liquid subjected to rotation.

Answer

1. As shown in Fig. 1.38.1, consider a fluid element $ABCD$ rotating at a uniform velocity in a horizontal plane about an axis perpendicular to the plane of paper and passing through O.

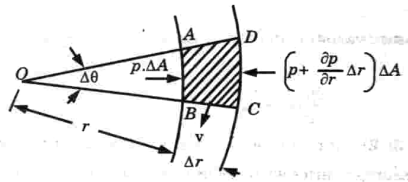

Fig. 1.38.1.

2. Let r = Radius of the element from O,
   $\Delta\theta$ = Angle subtended by the element at O,
   Δr = Radial thickness of the element, and
   ΔA = Area of cross-section of element.
3. The forces acting on the element are :
   a. Pressure force, $p\Delta A$, on the face AB.
   b. Pressure force, $\left(p + \frac{\partial p}{\partial r}\Delta r\right)\Delta A$ on the face CD.
   c. Centrifugal force, $\frac{m\,v^2}{r}$ acting in the direction away from the centre O.
4. Now, the mass of the element = Mass density × volume
   $= \rho \times \Delta A \times \Delta r$
5. Centrifugal force $= \rho\Delta A\Delta r\frac{v^2}{r}$
6. Equating the forces in the radial direction,

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$$\left(p+\frac{\partial p}{\partial r}\Delta r\right)\Delta A - p\Delta A = \rho\Delta A\,\Delta r\frac{v^2}{r}$$

or
$$\frac{\partial p}{\partial r}\Delta r\Delta A = \rho\Delta A\,\Delta r\frac{v^2}{r}$$

$$\frac{\partial p}{\partial r} = \rho\frac{v^2}{r} \quad ...(1.38.1)$$

7. Eq. (1.38.1) gives the pressure variation along the radial direction for a forced or free vortex flow in a horizontal plane.

8. The expression $\frac{\partial p}{\partial r}$ is called pressure gradient in the radial direction.

9. As $\frac{\partial p}{\partial r}$ is positive, hence pressure increases with the increase of radius 'r'.

10. The pressure variation in the vertical plane is given by the hydrostatic law, *i.e.*,

$$\frac{\partial p}{\partial z} = -\rho g \quad ...(1.38.2)$$

11. In eq. (1.38.2), z is measured vertically in the upward direction.

12. The pressure, p varies with respect to r and z or p is a function of r and z and hence total derivatives of p is

$$dp = \frac{\partial p}{\partial r}dr + \frac{\partial p}{\partial z}dz$$

13. Substituting the values of $\frac{\partial p}{\partial r}$ from eq. (1.38.1) and $\frac{\partial p}{\partial z}$ from eq. (1.38.2),

$$dp = \rho\frac{v^2}{r}dr - \rho g dz \quad ...(1.38.3)$$

Eq. (1.38.3) gives the variation of pressure of a rotating fluid in any plane.

Que 1.39. **A tank 3 m in length, 2 m wide and 2 m deep containing 1.5 m depth of water is given a constant horizontal acceleration of 3 m/s². Determine the water surface slope. Will the water spill out ?**

AKTU 2012-13, Marks 05

Answer

Given : Tank dimensions = 3 m (length) × 2 m (width) × 2 m (depth)
$a_x = 3$ m/s²,
To Find : i. Water surface slope.
ii. Will the water spill out ?

1. Water surface slope,

$$\tan\theta = \frac{a_x}{g} = \frac{3}{9.81} = 0.3058$$
$$\theta = 17°$$

2. Now, in ΔDOE,

$$\tan\theta = \frac{DE}{DO}$$
$$0.3058 = \frac{DE}{1.5}$$
$$DE = 1.5 \times 0.3058 = 0.4587$$
$$AE = AD + DE = 1.5 + 0.4587 = 1.9587 \text{ m}$$

3. Since for spilling out of water AE should be greater than the depth of tank. But this is not the case hence water will not spill out.

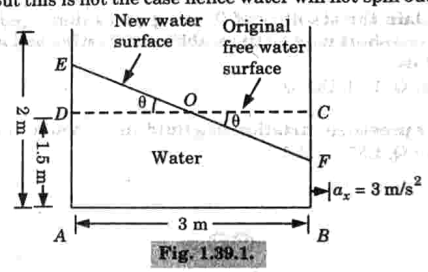

Fig. 1.39.1.

VERY IMPORTANT QUESTIONS

Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.

Q. 1. Discuss some physical properties of fluids in brief.

Ans. Refer Q. 1.2, Unit-1.

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Q. 2. State Newton's law of viscosity and derive the same. What are its applications ?

Ans. Refer Q. 1.3, Unit-1.

Q. 3. Derive an expression for pressure variation in a fluid at rest.

Ans. Refer Q. 1.17, Unit-1.

Q. 4. Derive expressions for the total pressure and centre of pressure when the following plane surfaces are submerged in liquids :

1. **Vertical plane surface,**
2. **Horizontal plane surface,**
3. **Inclined plane surface, and**
4. **Curved surface.**

Ans. Refer Q. 1.24, Unit-1.

Q. 5. How will you determine metacentric height analytically ?

Ans. Refer Q. 1.33, Unit-1.

Q. 6. Explain the stability of floating and submerged bodies or write a short note on the stability of floating and submerged bodies.

Ans. Refer Q. 1.34, Unit-1.

Q. 7. Give pressure variation in a fluid subjected to acceleration.

Ans. Refer Q. 1.37, Unit-1.

☺☺☺

2 UNIT — Types of Fluid and Continuity Equation

Part-1 ..... **(2–2A to 2–10A)**

- *Continuum and Free Molecular Flows*
- *Types of Fluid Flows*
- *Streamlines*
- *Continuity Equation for 3D and 1D Flows*

A. Concept Outline : Part-1 ..... 2–2A
B. Long and Medium Answer Type Questions 2–2A

Part-2 ..... **(2–10A to 2–29A)**

- *Circulation*
- *Stream Function*
- *Velocity Potential Function*

A. Concept Outline : Part-2 ..... 2–11A
B. Long and Medium Answer Type Questions 2–11A

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PART-1

Continuum and Free Molecular Flows, Types of Fluid Flows, Streamlines and Continuity Equation for 3D and 1D Flows.

CONCEPT OUTLINE : PART-1

Continuum : It is convenient to disregard the atomic nature of the fluid and view it as a continuous, homogeneous matter with no holes that is a continuum.

Types of Flows :

i. Continuum and free molecular flows.
ii. Steady and unsteady flows.
iii. Uniform and non-uniform flows.
iv. Laminar and turbulent flows.
v. Rotational and irrotational flows.
vi. Compressible and incompressible flows.
vii. Subsonic, sonic and supersonic flows.
viii. Sub-critical, critical and supercritical flows.
ix. One, two and three dimensional flows.

Continuity Theorem : It states that "Within a fixed region the rate of increase of fluid mass contained within the region must be equal to the difference between the rate at which the fluid mass enters the region and the rate at which the fluid mass leaves the region."

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.1. **Explain the following with example :**

a. **Steady and unsteady flows,**
b. **Laminar and turbulent flows,**
c. **Rotational and irrotational flows,**
d. **Compressible and incompressible flows, and**
e. **Uniform and non-uniform flows.**

Answer

a. **Steady and Unsteady Flows :**

1. Steady flow is defined as that type of flow in which the fluid characteristics like velocity, pressure, density, etc., at a point do not change with time.

Mathematically,

$$\left(\frac{\partial v}{\partial t}\right)_{\text{at fixed point}} = 0, \quad \left(\frac{\partial p}{\partial t}\right)_{\text{at fixed point}} = 0, \quad \left(\frac{\partial \rho}{\partial t}\right)_{\text{at fixed point}} = 0$$

Example : Flow of liquid through a long pipe of constant diameter at a constant rate.

2. Unsteady flow is that type of flow, in which the velocity, pressure density at a point changes with respect to time.

Mathematically,

$$\left(\frac{\partial v}{\partial t}\right)_{\text{at fixed point}} \neq 0, \quad \left(\frac{\partial \rho}{\partial t}\right)_{\text{at fixed point}} \neq 0$$

Example : Flow of liquid through a long pipe of constant diameter at either increasing or decreasing rate.

b. **Laminar and Turbulent Flows :**

1. Laminar flow is defined as those types of flow in which the fluid particles move along well-defined paths or stream line and all the stream-lines are straight and parallel.

Example : Flow through a capillary tube.

2. Turbulent flow is that type of flow in which the particles move in a zig-zag way.

Example : Flow in natural streams, artificial channels, sewers etc.

3. If the Reynold number is less than 2000, the flow is called laminar. If the Reynold number is more than 4000, it is called turbulent flow.

c. **Rotational and Irrotational Flows :**

1. Rotational flow is that type of flow in which the fluid particles while flowing along stream lines, also rotate about their own axis.

Example : Flow of liquid in the rotating tanks.

2. If the fluid particles while flowing along stream lines, do not rotate about their own axis that type of flow is called irrotational flow

Example : Flow over a drain hole of a stationary tank or a wash basin.

d. **Compressible and Incompressible Flows :**

1. Compressible flow is that type of flow in which the density of the fluid changes from point to point, or in other words the density (ρ) is not constant for the fluid.

Mathematically,

$$\rho \neq \text{constant}$$

Examples : Flow of gases through orifices nozzles, gas turbines etc.

2. Incompressible flow is that type of flow in which the density is constant for the fluid flow.

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Mathematically,

$$\rho = \text{constant}.$$

Examples : Subsonic aerodynamics.

e. Uniform and Non-uniform Flows :

1. Uniform flow is defined as that type of flow in which the velocity at any given time does not change with respect to space.

   Mathematically,

$$\left(\frac{\partial v}{\partial S}\right)_{t = \text{constant}} = 0$$

   Where, ∂v = Change of velocity, and
   ∂S = Length of flow in the direction S.

   Example : Flow through a straight pipe of constant diameter.

2. Non-uniform flow is that type of flow in which the velocity at any given time changes with respect to space.

   Mathematically,

$$\left(\frac{\partial v}{\partial S}\right)_{t = \text{constant}} \neq 0$$

   Example : Flow around a uniform diameter pipe-bend or a canal bend and flow through a non-prismatic pipe or channel.

Que 2.2. **Write short note on**

A. Subsonic, sonic and supersonic flows.
B. Subcritical, critical and supercritical flows.
C. One, two and three dimensional flows.

Answer

A. Subsonic, Sonic and Supersonic flows :

1. Subsonic flow, sonic flow and supersonic flows are defined on the basis of a dimensionless number called Mach number.
2. When Mach number is less than 1 ($M < 1$), flow is to be called subsonic flow.
3. When Mach number is equal to 1 ($M = 1$), flow is to be called sonic flow.
4. When Mach number is greater than 1 ($M > 1$), flow is to be called supersonic flow.

B. Subcritical, Critical and Supercritical flow :

1. When Froude number is less than one ($Fe < 1$), the flow is subcritical flow.
2. When Froude number is equal to one ($Fe = 1$), the flow is critical flow.
3. When Froude number is greater than one ($Fe > 1$), the flow is supercritical flow.

C. One, Two and Three Dimensional flows :

1. One dimensional flow is that type of flow in which the flow parameter such as velocity is a function of time and one space co-ordinate only.

   Mathematically,

$$u = f(x),\ v = 0 \text{ and } w = 0.$$

   Where u, v and w are velocity components in x, y and z directions respectively.

2. Two-dimensional flow is that type of flow in which the velocity is a function of time and two rectangular space co-ordinates.

   Mathematically,

$$u = f_1(x, y),\ v = f_2(x, y) \text{ and } w = 0.$$

3. Three-dimensional flow is that type of flow in which the velocity is a function of time and three mutually perpendicular directions.

   Mathematically,

$$u = f_1(x, y, z),\ v = f_2(x, y, z) \text{ and } w = f_3(x, y, z).$$

Que 2.3. **Streamlines, path lines and streak lines are identical for steady flow. Explain.**

Answer

1. A path line is the path or trajectory traced out by a moving fluid particle.
2. This approach might be used to study, the trajectory of a contaminant leaving a smoke stack.
3. Sometimes, we might choose to focus our attention on a fixed location in space and identify, again by the use of dye or smoke, all fluid particles passing through this point.
4. After a short period of time we would have a number of identifiable fluid particles in the flow, all of which had, at some time, passed through one fixed location in the space.
5. The line joining these fluid particles is defined as a streak line.
6. Streamlines are the lines drawn in the flow field so that at a given instant they are tangent to the direction of flow at every point in the flow field.
7. In a steady flow, the velocity at each point in the flow field remains constant with time, and consequently, the streamline shape do not vary from one instant to the next. This implies that a particle located on a given streamline will always move along the same streamline.
8. Furthermore, consecutive particles passing through a fixed point in space will be on the same streamline and subsequently, will remain on this streamline. Thus in a steady flow, path lines, streak lines and stream lines are identical lines in the flow field.

Que 2.4. **Write down continuity theorem and derive the continuity equation for 1-D.**

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Answer

A. Continuity Theorem :

It states that "Within a fixed region the rate of increase of fluid mass contained within the region must be equal to the difference between the rate at which the fluid mass enters the region and the rate at which the fluid mass leaves the region."

B. Continuity Equation :

1. Consider two cross-section of a pipe as shown in Fig. 2.4.1.

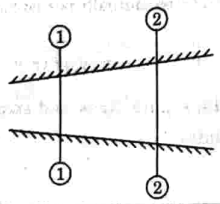

Fig. 2.4.1. Fluid flow through a pipe.

Let, A_1 = Area of the pipe at section 1–1,
V_1 = Velocity of the fluid at section 1–1,
ρ_1 = Density of the fluid at section 1–1,
and, A_2, V_2, ρ_2 are corresponding values at section 2–2.

2. The total quantity of fluid passing through section 1–1 = $\rho_1 A_1 V_1$
The total quantity of fluid passing through section 2–2 = $\rho_2 A_2 V_2$
3. From the law of conservation of matter (theorem of continuity), we have

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2 \quad ...(2.4.1)$$

4. Eq. (2.4.1) is applicable to the compressible as well as incompressible fluids and is called continuity equation. In case of incompressible fluids, $\rho_1 = \rho_2$ and the continuity eq. (2.4.1) reduces to

$$A_1 V_1 = A_2 V_2 \quad ...(2.4.2)$$

Que 2.5. **Derive the continuity equation for steady irrotational flows in cartesian coordinates for incompressible fluids.**

AKTU 2012-13, Marks 05

OR

Derive continuity equation for a 3-D steady or unsteady flow in a cartesian coordinate system.

AKTU 2015-16, Marks 05

Answer

1. Consider an elementary rectangular parallelopiped with sides of length δx, δy and δz as shown in Fig. 2.5.1.
2. Let the centre of the parallelopiped be at a point $P(x, y, z)$ where the velocity components in the x, y and z directions are u, v and w respectively and ρ be the mass density of the fluid.
3. The mass of fluid passing per unit time through the face of area $\delta y \delta z$ normal to the X-axis through point P, is

$$(\rho u \delta y \delta z)$$

4. Then the mass of fluid flowing per unit time into the parallelopiped through the face $ABCD$ is

$$(\rho u\, \delta y\, \delta z) + \frac{\partial}{\partial x}(\rho u\, \delta y\, \delta z)\left(-\frac{\delta x}{2}\right) \quad ...(2.5.1)$$

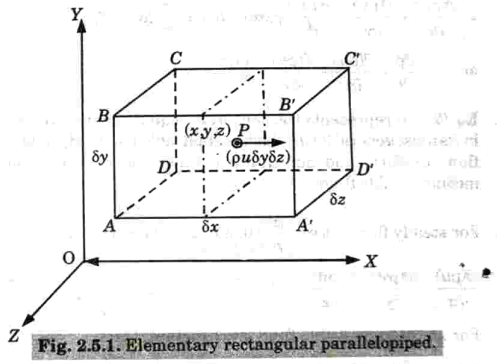

Fig. 2.5.1. Elementary rectangular parallelopiped.

5. Similarly the mass of fluid per unit time out of the parallelopiped through the face $A'B'C'D'$ is

$$(\rho u\, \delta y\, \delta z) + \frac{\partial}{\partial x}(\rho u\, \delta y\, \delta z)\left(\frac{\delta x}{2}\right) \quad ...(2.5.2)$$

6. Therefore, the net mass of fluid from eq. (2.5.1) and eq. (2.5.2)

$$\left[(\rho u\, \delta y\, \delta z) - \frac{\partial}{\partial x}(\rho u\, \delta y\, \delta z)\frac{\delta x}{2}\right] - \left[(\rho u\, \delta y\, \delta z) + \frac{\partial}{\partial x}(\rho u\, \delta y\, \delta z)\frac{\delta x}{2}\right]$$

$$= -\frac{\partial}{\partial x}(\rho u)\, \delta x\, \delta y\, \delta z$$

7. Similarly the net mass of fluid that remains in the parallelopiped per unit time,

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$= -\frac{\partial}{\partial y}(\rho v)\,\delta x\,\delta y\,\delta z$, through pair of faces $AA'D'D$ and $BB'C'C$

$= -\frac{\partial}{\partial z}(\rho w)\,\delta x\,\delta y\,\delta z$, through pair of faces $DD'C'C$ and $AA'B'B$

8. By adding all these expressions the net total mass of fluid that has remained in the parallelopiped per unit time is obtained as

$$-\left[\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z}\right]\delta x\,\delta y\,\delta z \quad ...(2.5.3)$$

9. The mass of the fluid in the paralleopiped is $(\rho\,\delta x\,\delta y\,\delta z)$ and its rate of increase with time is

$$\frac{\partial}{\partial t}(\rho\,\delta x\,\delta y\,\delta z) = \frac{\partial \rho}{\partial t}(\delta x\,\delta y\,\delta z) \quad ...(2.5.4)$$

10. According to law of conservation of mass, equating the eq. (2.5.3) and eq. (2.5.4),

$$-\left[\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z}\right](\delta x\,\delta y\,\delta z) = \frac{\partial \rho}{\partial t}(\delta x\,\delta y\,\delta z)$$

or
$$\frac{\partial \rho}{\partial t}+\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z} = 0 \quad ...(2.5.5)$$

11. Eq. (2.5.5) represents the continuity equation in cartesian coordinates in its most general form which is applicable for steady as well as unsteady flow, uniform and non-uniform flow, and compressible as well as incompressible fluids.

12. For steady flow since, $\frac{\partial \rho}{\partial t} = 0$, eq. (2.5.5) reduces to

$$\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z} = 0 \quad ...(2.5.6)$$

13. For an incompressible fluid, ρ = constant, then,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z} = 0$$

Que 2.6. **A jet of water from a 25 mm diameter nozzle is directed vertically upwards. Assuming that the jet remains circular and neglecting any loss of energy, what will be the diameter at a point 4.5 m above the nozzle, if the velocity with which the jet leaves the nozzle is 12 m/s ?**

AKTU 2014-15, Marks 05

Answer

Given : Diameter of nozzle, D_1 = 25 mm = 0.025 m
Velocity of jet at nozzle, v_1 = 12 m/s
Height of point A, h = 4.5 m
To Find : Diameter at a point 4.5 m above the nozzle.

1. Let the velocity of the jet at a height of 4.5 m = v_2
2. Consider the vertical motion of the jet from the outlet of the nozzle to the point A (neglecting any loss of energy)
3. Using $v^2 - u^2 = 2gh$

$$v_2^2 - 12^2 = 2 \times (-9.81) \times 4.5$$

$$v_2 = \sqrt{12^2 - 2 \times 9.81 \times 4.5} = \sqrt{144 - 88.29}$$
$$= 7.46 \text{ ms}$$

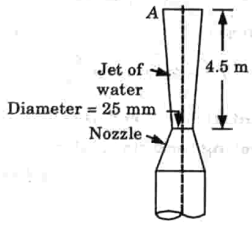

Fig. 2.6.1.

4. Now applying continuity equation to the outlet of nozzle and at point A,

$$A_1 v_1 = A_2 v_2$$

or
$$A_2 = \frac{A_1 v_1}{v_2} = \frac{\frac{\pi}{4} D_1^2 \times v_1}{v_2} = \frac{\pi \times (0.025)^2 \times 12}{4 \times 7.46}$$
$$= 0.0007896$$

5. Let D_2 = Diameter of jet at point A.

Then
$$A_2 = \frac{\pi}{2} D_2^2 \text{ or } 0.0007896 = \frac{\pi}{4} \times D_2^2$$

$$\therefore \quad D_2 = \sqrt{\frac{0.0007896 \times 4}{\pi}} = 0.0317 \text{ m} = 31.7 \text{ mm}$$

Que 2.7. **Two velocity components are given in the following equations, find the third component such that they satisfy the continuity equation :**

$$u = x^3 + y^2 + 2z^2,\ v = -x^2 y - yz - xy$$

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Answer

Given : $u = x^3 + y^2 + 2z^2$, $v = -x^2y - yz - xy$
To Find : Third component that satisfy the continuity equation.

1. From continuity equation,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$3x^2 + (-x^2 - z - x) + \frac{\partial w}{\partial z} = 0$$

$$2x^2 - z - x + \frac{\partial w}{\partial z} = 0$$

$$\frac{\partial w}{\partial z} = -2x^2 + z + x$$

2. On integrating, we get

$$w = -2x^2z + xz + \frac{z^2}{2}$$

Que 2.8. **Differentiate between the Eulerian and Lagrangian methods of representing fluid flow. Also derive the 3-D continuity equation.**

AKTU 2013-14, Marks 05

Answer

A. Difference between the Lagrangian and Eulerian Methods :

S. No.	Lagrangian Method	Eulerian Method
1.	Observer concentrates on the movement of a single particle.	Observer concentrates on a point in the fluid system.
2.	The path taken by the particle and the changes in its velocity and acceleration are studied.	Velocity, acceleration and other characteristics of the fluid at that particular point are studied.

B. 3-D Continuity Equation : Refer Q. 2.5, Page 2–6A, Unit-2.

PART-2

Circulation, Stream Function and Velocity Potential Function.

CONCEPT OUTLINE : PART-2

Circulation : The flow along a closed curve is called circulation (Γ).
Stream Function (ψ) : It is a scalar function of space and time such that its partial derivative with respect to any direction gives the velocity component at right angles to that direction.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.9. **Write a short note on circulation.**

Answer

1. Let us consider a closed curve in a two-dimensional flow field as shown in Fig. 2.9.1, the curve being cut by the streamlines.

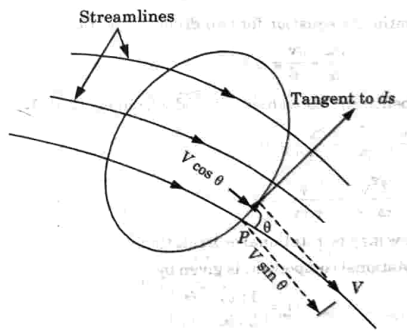

Fig. 2.9.1. Circulation in a two-deimensinal flow.

2. Let P be the point of intersection of the curve with one streamline, θ be the angle which the streamline makes with the curve.
3. The component of velocity along the closed curve at the point of intersection is equal $V \cos \theta$.
4. Circulation Γ is defined mathematically as the line integral of the tangential velocity about a closed path (contour).

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Thus, $\Gamma = \oint V \cos\theta . ds$

Where, V = Velocity in the flow field at the element ds, and

θ = Angle between V and tangent to the path (in the positive anticlockwise direction along the path) at the point.

Que 2.10. What is stream function ? Give its properties.

Answer

A. Stream Function :

1. Stream function is the scalar function of space and time such that its partial derivative with respect to any direction gives the velocity component at right angle to that direction.
2. It is denoted by ψ (psi) and defined only for two dimensional flow.

   Mathematically, for steady clockwise flow,

   $\psi = f(x, y)$ such that

   $$\frac{\partial \psi}{\partial x} = -v \text{ and } \frac{\partial \psi}{\partial y} = u \quad ...(2.10.1)$$
3. The continuity equation for two-dimensional flow is

   $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$
4. On substituting the values of u and v from eq. (2.10.1),

   $$\frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial y}\right) + \frac{\partial}{\partial y}\left(-\frac{\partial \psi}{\partial x}\right) = 0$$

   or $$\frac{\partial^2 \psi}{\partial x\, \partial y} - \frac{\partial^2 \psi}{\partial x\, \partial y} = 0$$

   The flow may be rotational or irrotational.
5. The rotational component, is given by

   $$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$
6. Substituting the values of u and v from eq. (2.10.1) in the above rotational component

   $$\omega_z = \frac{1}{2}\left[\frac{\partial}{\partial x}\left(-\frac{\partial \psi}{\partial x}\right) - \frac{\partial}{\partial y}\left(\frac{\partial \psi}{\partial y}\right)\right] = -\frac{1}{2}\left[\frac{\partial^2 \psi}{\partial x^2} + \frac{d^2 \psi}{\partial y^2}\right]$$
7. For irrotational flow $\omega_z = 0$. Hence the above equation becomes

   $$\frac{\partial^2 \psi}{\partial x^2} + \frac{d^2 \psi}{\partial y^2} = 0$$

   This is Laplace equation for Ψ.

B. Properties of Stream Function :

1. Existence of stream function (Ψ) represents a possible case of fluid flow which may be rotational or irrotational.
2. In case it satisfies the Laplace equation, it is a possible case of an irrotational flow.

Que 2.11. Derive the equation of a streamline for a 2-D flow. Prove that the discharge between two streamlines is the difference in their stream function values.

Answer

A. Equation of a Streamline for 2-D Flow :

1. For constant stream function,

   i.e., $$\frac{\partial \psi}{\partial x} dx + \frac{\partial \psi}{\partial y} dy = 0$$

   $$-v dx + u dy = 0 \qquad \left\{\because \frac{\partial \psi}{\partial x} = -v, \frac{\partial \psi}{\partial y} = u\right\}$$

   $$\Rightarrow \quad v dx = u dy$$

   or $$\frac{dx}{u} = \frac{dy}{v}$$
2. Above equation represent the equation of a streamline in x-y plane.

B. Discharge Between Two Streamlines :

1. Let $\psi(x, y)$ represent the streamline L.
2. The adjacent stream line M has stream function $\psi + d\psi$.
3. Let the velocity vector V perpendicular to the line AB has components u and v in the direction of X and Y axes respectively.
4. From continuity equation,

   Flow across AB = flow across AO + flow across OB

   $$V.ds = -v.dx + u.dy$$

   Negative sign shows that the v is acting in downward direction.
5. Putting $v = -\frac{\partial \psi}{\partial x}$, $u = \frac{\partial \psi}{\partial y}$ and $V.ds = dq$, we get

   $$V.ds = \frac{\partial \psi}{\partial x} \cdot dx + \frac{\partial \psi}{\partial y} \cdot dy$$

   $$dq = d\psi$$

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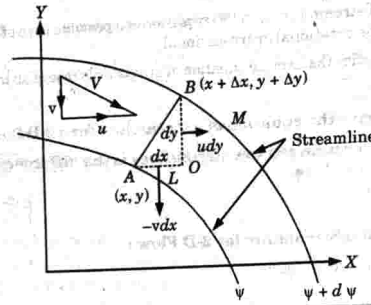

Fig. 2.11.1. Flow between two points and its relation to stream function.

6. Hence discharge between two streamlines is the difference in their stream function values.

Que 2.12. **What is the relationship between equipotential line and line of constant stream function at the point of intersection ?**

OR

Prove that stream function (ψ) and potential function (ϕ) are orthogonal to each other.

Answer

1. For equipotential line, $d\phi = 0$

$$\frac{\partial \phi}{\partial x}.dx + \frac{\partial \phi}{\partial y}.dy = 0$$

$$-u.dx + (-v).dy = 0 \qquad \left\{\because \frac{\partial \phi}{\partial x} = -u \text{ and } \frac{\partial \phi}{\partial y} = -v\right\}$$

$$\frac{dy}{dx} = -\frac{u}{v} = \text{Slope of equipotential line}$$

2. For constant stream function, $d\psi = 0$

$$\frac{\partial \Psi}{\partial x}.dx + \frac{\partial \Psi}{\partial y}.dy = 0$$

$$-v.dx + u.dy = 0 \qquad \left\{\because \frac{\partial \Psi}{\partial x} = -v, \ \frac{\partial \Psi}{\partial y} = u\right\}$$

$$\frac{dy}{dx} = \frac{v}{u} = \text{Slope of streamline}$$

3. Now, slope of streamline × slope of equipotential line

$$= \left(\frac{v}{u}\right) \times \left(-\frac{u}{v}\right) = -1$$

4. The product of the slope of the equipotential line and the slope of the stream of the point of intersection is equal to – 1.
5. Thus the equipotential lines are orthogonal to the streamlines at all points of intersection.

Que 2.13. **What is velocity potential ? Also derive the Laplace equation.**

Answer

A. Velocity Potential :

1. The velocity potential is defined as a scalar function of space and time such that its negative derivative with respect to any direction gives the fluid velocity in that direction. It is denoted by ϕ (phi).
2. Thus mathematically the velocity potential is defined as :

$$\phi = f(x, y, z, t) \quad \text{... for unsteady flow}$$

and

$$\phi = f(x, y, z) \quad \text{... for steady flow}$$

such that

$$\left.\begin{aligned} u &= -\frac{\partial \phi}{\partial x} \\ v &= -\frac{\partial \phi}{\partial y} \\ w &= -\frac{\partial \phi}{\partial z} \end{aligned}\right\} \quad \text{...(2.13.1)}$$

Where u, v, and w are the components of velocity in the x, y, and z directions respectively.

3. The negative sign signifies that ϕ decreases with an increase in the values of x, y and z. In other words it indicates that the flow is always in the direction of decreasing ϕ.

B. Laplace Equation :

1. For an incompressible steady flow the continuity equation is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

2. By substituting the values of u, v and w in eq. (2.13.1), we get

$$\frac{\partial}{\partial x}\left(-\frac{\partial \phi}{\partial x}\right) + \frac{\partial}{\partial y}\left(-\frac{\partial \phi}{\partial y}\right) + \frac{\partial}{\partial z}\left(-\frac{\partial \phi}{\partial z}\right) = 0$$

$$\frac{d^2\phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$$

This equation is known as Laplace equation.

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Que 2.14. **What is the relation between stream function and velocity potential function ?**

Answer

1. From velocity potential function,

$$u = -\frac{\partial \phi}{\partial x} \text{ and } v = -\frac{\partial \phi}{\partial y}$$

2. Stream function gives, $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$

Thus $$u = -\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y} \text{ and } v = -\frac{\partial \psi}{\partial x} = -\frac{\partial \phi}{\partial y}$$

3. Hence $$\frac{\partial \phi}{\partial x} = -\frac{\partial \psi}{\partial y} \text{ and } \frac{\partial \phi}{\partial y} = \frac{\partial \psi}{\partial x}$$

Que 2.15. **Explain the uniform flow with source and sink. Obtain the expressions for stream function and velocity potential functions.**

AKTU 2013-14, Marks 05

Answer

A. Uniform Flow with Source and Sink :

1. Fig. 2.15.1(*a*) shows a uniform flow of velocity U and Fig. 2.15.1(*b*) shows a source sink pair of equal strength.
2. When this uniform flow is flowing over the source sink pair, a resultant flow will be obtained as shown in Fig. 2.15.1(*c*). This resultant flow is also known as the flow past a Rankine oval body.
3. Let
   U = Velocity of uniform flow along x-axis,
   q = Strength of source,
   $(-q)$ = Strength of sink, and
   $2a$ = Distance between source and sink which is along x-axis.
4. The origin O of the x-y co-ordinates is mid-way between source and sink. Consider a point $P(x, y)$ lying in the resultant flow field.
5. The stream function (ψ) for the resultant flow field is obtained as,

ψ = Stream function due to uniform flow
+ Stream function due to source
+ Stream function due to sink

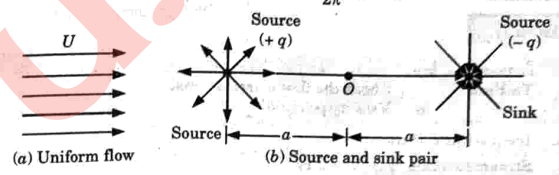
$$= \psi_{\text{uniform flow}} + \psi_{\text{source}} + \psi_{\text{sink}}$$

$$= U \times y + \frac{q}{2\pi}\theta_1 + \frac{(-q)}{2\pi} \times \theta_2$$

Where θ_1 is the angle made by P with source along X-axis and θ_2 with sink

$$= U \times y + \frac{q\theta_1}{2\pi} - \frac{q\theta_2}{2\pi} = U \times y + \frac{q}{2\pi}(\theta_1 - \theta_2)$$

$$= U \times r \sin\theta + \frac{q}{2\pi}(\theta_1 - \theta_2) \qquad (\because \ y = r \sin\theta)$$



(*a*) Uniform flow (*b*) Source and sink pair

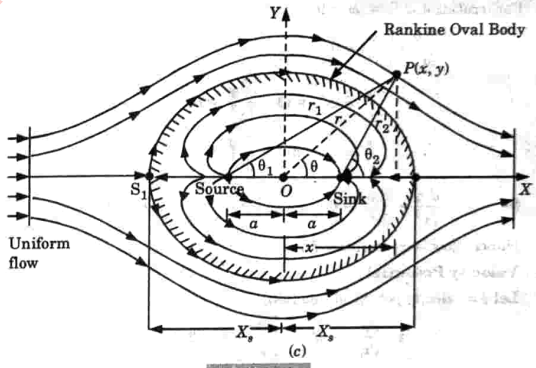


Fig. 2.15.1.

6. The velocity potential function (ϕ) for the resultant flow field is obtained as,

ϕ = Potential function due to uniform flow
+ Potential function due to source
+ Potential function due to sink

$$\phi = \phi_{\text{uniform flow}} + \phi_{\text{source}} + \phi_{\text{sink}}$$

$$= U \times x + \frac{q}{2\pi}\log_e r_1 + \frac{(-q)}{2\pi}\log_e r_2$$

$$= U \times r\cos\theta + \frac{q}{2\pi}[\log_e r_1 - \log_e r_2] \qquad (\because \ x = r\cos\theta)$$

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$$= U \times r \cos\theta + \frac{q}{2\pi}\left[\log_e \frac{r_1}{r_2}\right]$$

Que 2.16. **Check whether the flow defined by the stream function $\psi = 2xy$ is irrotational ? If so, determine the corresponding velocity potential.**

Answer

Given : $\psi = 2xy$
To Find : i. Check the flow is irrotational
ii. Velocity potential

A. Irrotational Flow :

1. Stream function, $\psi(x, y) = 2xy$
2. For irrotational flow, $\omega_z = 0$

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = 0$$

3. $$\frac{\partial\psi}{\partial x} = 2y \quad \text{and} \quad \frac{\partial^2\psi}{\partial x^2} = 0$$

$$\frac{\partial\psi}{\partial y} = 2x \quad \text{and} \quad \frac{\partial^2\psi}{\partial y^2} = 0$$

4. So $$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = 0$$
5. Hence, flow is irrotational flow.

B. Velocity Potential :

1. Let ϕ = velocity potential function

$$\frac{\partial\phi}{\partial x} = -u = -\left[\frac{\partial\psi}{\partial y}\right] = -(2x)$$

$$\frac{\partial\phi}{\partial x} = -2x \qquad \text{...(2.16.1)}$$

$$\frac{\partial\phi}{\partial y} = -v = -\left[-\frac{\partial\psi}{\partial x}\right]$$

$$\frac{\partial\phi}{\partial y} = 2y \qquad \text{...(2.16.2)}$$

2. On integrating the eq. (2.16.1),

$$\int d\phi = -\int 2x\, dx$$

$$\phi = -x^2 + C \qquad \text{...(2.16.3)}$$

Where, C is a constant which is a function of y (*i.e.*, $C = f(y)$).

3. On differentiating eq. (2.16.3) w.r.t. y,

$$\frac{\partial\phi}{\partial y} = \frac{\partial C}{\partial y}$$

$$\frac{\partial C}{\partial y} = 2y \qquad \text{(From eq. (2.16.2))}$$

On integration, $\int dC = \int 2y\, dy$

$$C = y^2$$

4. Substituting the value of C in eq. (2.16.3),

$$\phi = -x^2 + y^2$$

Que 2.17. **If the velocity field is given by $u = (16y - 8x)$, $v = (8y - 7x)$ find the circulation around the closed curve defined by $x = 2$, $y = 1$, $x = 4$, $y = 4$.** **AKTU 2012-13, Marks 05**

Answer

Given : Velocity field,
$u = 16y - 8x$
$v = 8y - 7x$
Closed curve defined by
$x = 2, \quad y = 1$
$x = 4, \quad y = 4$
To Find : Circulation around the closed curve.

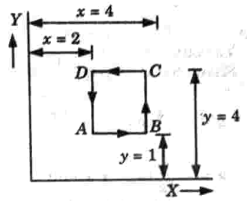

Fig. 2.17.1.

1. Circulation $\Gamma_{ABCD} = \int_{ABCD}(udx + vdy)$

$$= \int_{AB}(udx + vdy) + \int_{BC}(udx + vdy) + \int_{CD}(udx + vdy) + \int_{DA}(udx + vdy)$$

$$\Gamma_{ABCD} = \int_2^4 (16y - 8x)dx + \int_1^4 (8y - 7x)dy + \int_4^2 (16y - 8x)dx + \int_4^1 (8y - 7x)dy$$

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$$= \underset{(i)}{[16yx - 4x^2]_2^4} + \underset{(ii)}{[4y^2 - 7xy]_1^4} + \underset{(iii)}{[16yx - 4x^2]_4^2} + \underset{(iv)}{[4y^2 - 7xy]_4^1}$$

2. For (*i*) Integral : $y = 1$
   For (*ii*) Integral : $x = 4$
   For (*iii*) Integral : $y = 4$
   For (*iv*) Integral : $x = 2$
   Using the above values we have

$$\Gamma_{ABCD} = [(16 \times 1 \times 4) - (4 \times 4^2) - (16 \times 1 \times 2) + (4 \times 2^2)] + [4 \times 4^2 - (7 \times 4 \times 4) - (4 \times 1^2) + (7 \times 4 \times 1)] + [(16 \times 4 \times 2) - (4 \times 2^2) - (16 \times 4 \times 4) + (4 \times 4^2)] + [(4 \times 1^2) - (7 \times 2 \times 1) - (4 \times 4^2) + (7 \times 2 \times 4)]$$

$$= [64 - 64 - 32 + 16] + [64 - 112 - 4 + 28] + [128 - 16 - 256 + 64] + [4 - 14 - 64 + 56]$$

$$= -16 - 24 - 80 - 18 = -138$$

3. Area of the curve $ABCD = (4 - 2) \times (4 - 1) = 6$ square unit

$$\therefore \text{Circulation per unit area} = \frac{-138}{6} = -23 \text{ unit}$$

Que 2.18. **Calculate the velocity component '*u*' and 'v' for the following velocity potential function : $\phi = x^2 - y^2$. Also show that it satisfies continuity equation for two-dimensional flow.**

AKTU 2011-12, Marks 05

Answer

Given : Velocity potential function, $\phi = x^2 - y^2$
To Find : i. Velocity component
ii. Show that it satisfy continuity equation for 2-D

1. We know, $$u = -\frac{\partial \phi}{\partial x} \text{ and } v = -\frac{\partial \phi}{\partial y}$$

$$u = -\frac{\partial}{\partial x}(x^2 - y^2) \text{ and } v = -\frac{\partial}{\partial y}(x^2 - y^2)$$

$$u = -2x \text{ and } v = +2y$$

2. For a 2-D, steady and incompressible fluid flow, continuity equation is

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

$$\frac{\partial^2}{\partial x^2}(x^2 - y^2) + \frac{\partial^2}{\partial y^2}(x^2 - y^2) = 0$$

$$2 - 2 = 0$$

Hence it satisfies continuity equation for a 2-D flow.

Que 2.19. **If the stream function $\psi = x^3 - 3xy^2$, indicate the flow is irrotational flow. Determine ϕ.**

AKTU 2011-12, Marks 05

Answer

Given : $\psi = x^3 - 3xy^2$
To Find : Velocity potential function ϕ

1. A 2-D flow in *x*-*y* plane will be irrotational if the vorticity vector in the *z*-direction is zero.

i.e., $$\omega_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0 \quad \text{...(2.19.1)}$$

2. We know, $$u = \frac{\partial \psi}{\partial y} = \frac{\partial}{\partial y}(x^3 - 3xy^2) = -6xy$$

and $$v = -\frac{\partial \psi}{\partial x} = -\frac{\partial}{\partial x}(x^3 - 3xy^2)$$

$$= -(3x^2 - 3y^2) = -3(x^2 - y^2)$$

$$\frac{\partial u}{\partial y} = -6x \text{ and } \frac{\partial v}{\partial x} = -6x$$

3. From eq. (2.19.1),

$$-6x - (-6x) = 0$$

Hence the flow is irrotational.

Calculation of ϕ :

1. $$\psi = x^3 - 3xy^2$$

2. $$u = -\frac{\partial \phi}{\partial x} \text{ and } v = -\frac{\partial \phi}{\partial y}$$

3. From CR equation,

$$\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}$$

$$-\frac{\partial \phi}{\partial y} = \frac{\partial \psi}{\partial x}$$

4. So, $$\frac{\partial \phi}{\partial x} = \frac{\partial}{\partial y}(x^3 - 3xy^2) = -6xy \quad \text{...(2.19.2)}$$

$$-\frac{\partial \phi}{\partial y} = \frac{\partial}{\partial x}(x^3 - 3xy^2) = 3x^2 - 3y^2$$

$$\frac{\partial \phi}{\partial y} = -(3x^2 - 3y^2) \quad \text{...(2.19.3)}$$

5. Integrate eq. (2.19.2),

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$$\phi = -\frac{6x^2}{2}y + f(y) = -3x^2y + f(y) \quad ...(2.19.4)$$

6. Now differentiate eq. (2.19.4) with respect to y, we get

$$\frac{\partial \phi}{\partial y} = -3x^2 + f'(y) \quad ...(2.19.5)$$

7. Now equating the RHS part of eq. (2.19.3) and eq. (2.19.5),

$$-3x^2 + 3y^2 = -3x^2 + f'(y)$$
$$f'(y) = 3y^2$$

8. Now integrating both the sides,

$$f(y) = \frac{3y^3}{3} + C$$

Here C = integration constant

$$f(y) = y^3 + C$$

9. From eq. (2.19.4),

$$\phi = -3x^2y + y^3 + C$$

Que 2.20. **In a steady flow two points 'A' and 'B' are 0.5 m apart on a straight streamline. If the velocity of flow varies linearly between 'A' and 'B'; what is the acceleration at each point if the velocity at A is 2 m/s and at B is 6 m/s.**

AKTU 2011-12, Marks 05

Answer

Given : Distance between A and B = 0.5 m
velocity at A = 2 m/s
velocity at B = 6 m/s
To Find : Acceleration at A and B

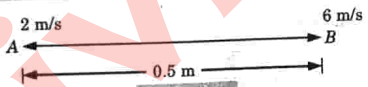

Fig. 2.20.1.

1. Velocity at distance x from point A

$$u = 2 + \frac{4}{0.5}x$$

2. We know

$$a_x = \frac{du}{dt} = u\frac{\partial u}{\partial x}$$

3. $$\frac{\partial u}{\partial x} = \frac{4}{0.5}$$

and $$u\frac{\partial u}{\partial x} = 8(2 + 8x)$$
$$= 16 + 64x$$

So, $$a_x = 16 + 64x$$

4. Acceleration at point A, put $x = 0$

$$a_A = 16 \text{ m/s}^2$$

5. Acceleration at point B, put $x = 0.5$

$$a_B = 16 + 64(0.5)$$
$$= 48 \text{ m/s}^2$$

Que 2.21. **Sketch the streamlines represented by $\Psi = x^2 + y^2$. Also find out the velocity and its direction at point (1, 2).**

AKTU 2014-15, Marks 10

Answer

Given : $\psi = x^2 + y^2$
To Find : i. Sketch of streamlines
ii. Velocity and its direction

i. **Sketch of Streamlines :**

$$\psi = x^2 + y^2$$

1. Let ψ = 1, 2, 3 and so on.
2. Then we have
$$1 = x^2 + y^2$$
$$2 = x^2 + y^2$$
$$3 = x^2 + y^2$$
and so on.
3. Each equation is an equation of a circle.
4. Thus we shall get concentric circles of different diameters shown in Fig. 2.21.1.

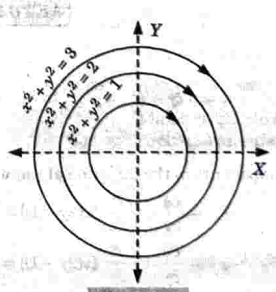

Fig. 2.21.1.

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ii. Velocity and Direction :

1. The velocity components u and v are

$$u = \frac{\partial \psi}{\partial y} = \frac{\partial}{\partial y}(x^2 + y^2) = 2y$$

$$v = -\frac{\partial \psi}{\partial x} = -\frac{\partial}{\partial x}(x^2 + y^2) = -2x$$

2. At the point (1, 2), the velocity components are

$$u = 2 \times 2 = 4 \text{ units/s}$$
$$v = -2 \times 1 = -2 \text{ units/s}$$

3. Resultant velocity $= \sqrt{u^2 + v^2} = \sqrt{4^2 + (-2)^2}$

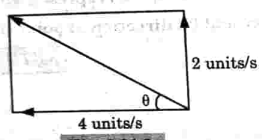

Fig. 2.21.2.

$$= \sqrt{20} = 4.47 \text{ units/s}$$

5. $$\tan\theta = \frac{v}{u} = \frac{2}{4} = \frac{1}{2}$$

$$\therefore \quad \theta = \tan^{-1} 0.5 = 26° \, 34'$$

Thus resultant velocity makes an angle of 26° 34′ with x-axis in clockwise direction.

Que 2.22. **If for a 2-D potential flow, the velocity potential is given by $\phi = x(2y - 1)$. Determine the velocity at the point $P(4, 5)$. Determine also the value of stream function at the point P.**

AKTU 2014-15, Marks 10

Answer

Given : $\phi = x(2y - 1)$
To Find : i. Velocity at point P
ii. Stream function at point P

1. The velocity components in the direction of x and y are

$$u = -\frac{\partial \phi}{\partial x} = -\frac{\partial}{\partial x}[x(2y - 1)] = -[2y - 1] = 1 - 2y$$

$$v = -\frac{\partial \phi}{\partial y} = -\frac{\partial}{\partial y}[x(2y - 1)] = -[2x] = -2x$$

2. At the point $P(4, 5)$, *i.e.*, at $x = 4$, $y = 5$

$$u = 1 - 2 \times 5 = -9 \text{ units/s}$$
$$v = -2 \times 4 = -8 \text{ units/s}$$

3. Resultant velocity at $P = \sqrt{9^2 + 8^2} = \sqrt{81 + 64}$
$= 12.04$ units/s

ii. Stream Function at P :

1. We know that $$\frac{\partial \psi}{\partial y} = u = 1 - 2y \quad \text{...(2.22.1)}$$

and $$\frac{\partial \psi}{\partial x} = -v = 2x \quad \text{...(2.22.2)}$$

2. Integrating eq. (2.22.1) w.r.t. 'y', we get

$$\int d\psi = -\int (2y - 1)\, dy$$

or $$\psi = -\frac{2y^2}{2} + y + K$$

$$\psi = -y^2 + y + K \quad \text{...(2.22.3)}$$

The constant of integration K is not a function of y but it can be a function of x.

3. Differentiating the eq. (2.22.3), w.r.t. 'x',

$$\frac{\partial \psi}{\partial x} = \frac{\partial K}{\partial x}$$

But from eq. (2.22.2)

$$\frac{\partial \psi}{\partial x} = 2x$$

4. Equating the value of $\frac{\partial \psi}{\partial x}$,

$$\frac{\partial K}{\partial x} = 2x$$

Integrating this equation,

$$K = \int 2x\,dx = \frac{2x^2}{2} = x^2$$

5. Substituting this value of K in eq. (2.22.3), we get

$$\psi = -y^2 + y + x^2.$$

6. Stream function ψ at $P(4, 5) = -5^2 + 5 + 4^2 = -25 + 5 + 16 = -4$ units.

Que 2.23. **Explain whether the velocity components given by $u = 2x^3$ and $v = -6x^2y$ represent a possible steady two dimensional incompressible flow. State whether the flow is irrotational and if so, work out the velocity potential function. If rotational, determine the vorticity, shear strain and dilatancy.**

AKTU 2013-14, Marks 05

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Answer

Given : $u = 2x^3$, $v = -6x^2y$

To Find :

i. Check whether flow is irrotational and if so work out velocity potential function.

ii. If rotational, determine the vorticity, shear strain and dilatancy.

1. We know,
$$u = \frac{\partial \psi}{\partial y} = 2x^3 \quad ...(2.23.1)$$
$$v = -\frac{\partial \psi}{\partial x} = -6x^2y \quad ...(2.23.2)$$

2. Integrating eq. (2.23.2),
$$\psi = 2x^3y + f(y) \quad ...(2.23.3)$$

3. Now differentiating eq. (2.23.3) w.r.t y,
$$\frac{\partial \psi}{\partial y} = 2x^3 + f'(y) \quad ...(2.23.4)$$

4. Equating eq. (2.23.1) and eq. (2.23.4),
$$2x^3 = 2x^3 + f'(y)$$
$$f'(y) = 0$$

5. On integrating, $f(y) = c$ ...(2.23.5)

6. Now putting value of eq. (2.23.5) in eq. (2.23.3)
$$\psi = 2x^3y + c$$

7. For irrotational flow, $\omega_z = 0$
$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0 \quad ...(2.23.6)$$
$$\frac{\partial \psi}{\partial x} = 6x^2y \text{ and } \frac{\partial^2 \psi}{\partial x^2} = 12xy$$
$$\frac{\partial \psi}{\partial y} = 2x^3 \text{ and } \frac{\partial^2 \psi}{\partial y^2} = 0$$

On putting these values in eq. (2.23.6), we get
$$\frac{\partial^2 \psi}{\partial^2 x} + \frac{\partial^2 \psi}{\partial^2 y} \neq 0$$

Hence, flow is rotational

A. Vorticity :
$$\omega = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]$$
$$\omega = \frac{1}{2}[-12xy - 0]$$
$$\omega = -6xy$$

B. Shear Strain :
$$\gamma_{xy} = \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) = \frac{1}{2}[-12xy + 0]$$
$$= -6xy$$

C. Dilatancy :
$$e_x = \frac{\partial u}{\partial x} = 6x^2$$
$$e_y = \frac{\partial v}{\partial y} = -6x^2$$

Que 2.24. **A flow is described by the stream function $\psi = 3\sqrt{2}\,xy$. Locate the point at which the velocity vector has a magnitude of 6 units and makes an angle of 145° with the x-axis.**

AKTU 2013-14, Marks 05

Answer

Given : $\psi = 3\sqrt{2}\,xy$, Magnitude of velocity vector, $R = 6$, $\theta = 145°$

To Find : Locate the point for velocity vector.

1. The velocity components are given as,
$$u = \frac{\partial \psi}{\partial y} = \frac{\partial}{\partial y} 3\sqrt{2}xy = 3\sqrt{2}x$$
$$v = -\frac{\partial \psi}{\partial x} = -\frac{\partial}{\partial x} 3\sqrt{2}xy = -3\sqrt{2}y$$

2. We know,
$$R = \sqrt{u^2 + v^2}$$
$$6 = \sqrt{(3\sqrt{2}x)^2 + (-3\sqrt{2}y)^2}$$
$$36 = 18x^2 + 18y^2$$
$$x^2 + y^2 = 2 \quad ...(2.24.1)$$

3.
$$\tan\theta = \frac{v}{u}$$
$$\tan 145° = \frac{-3\sqrt{2}y}{3\sqrt{2}x}$$
$$\frac{y}{x} = 0.7$$
$$y = 0.7x \quad ...(2.24.2)$$

4. On solving eq. (2.24.1) and eq. (2.24.2)
$$x = 1.158$$
$$y = 0.810$$

Que 2.25. **What is flownet ? Describe anyone method of drawing flownet. List the assumptions in the flownet study.**

Answer

A. Flownet :

1. Flownet is a grid obtained by drawing a series of stream lines and equipotential lines.
2. Flownet is used to study the two dimensional irrotational flows for which the mathematical relations of stream function and velocity function are difficult to find or solve.

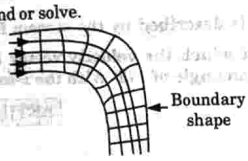

Fig. 2.25.1. Flownet.

B. Methods of Drawing Flownets :

1. There are four methods to draw a flownet and one method is given below :

a. Analytical Method :

1. First find the equations of velocity function (ϕ) and stream function (ψ).
2. Plot these curves on a graph to give the flownet pattern for the flow of fluid between the given boundary shape.
3. This method is used for the problems with simple and ideal boundary conditions.

C. Assumptions in the Flownet Study :

1. The flow should be steady.
2. The flow should be irrotational.
3. The flow is not governed by the gravity force.

Que 2.26. **If $\phi = -\frac{A}{2\pi}\ln r$; where A is a positive constant, determine ψ and plot typical equipotential lines and stream lines. Identify the flow pattern.**

AKTU 2012-13, Marks 06

Answer

Given : $\phi = -\frac{A}{2\pi}\ln r$

To Find :
i. Determine ψ
ii. Plot typical equipotential lines and stream lines.
iii. Identify the flow pattern.

1. $$\frac{\partial\phi}{\partial r} = -\frac{A}{2\pi r} = \frac{\partial\psi}{r\partial\theta}$$

2. On integration, $$\psi = -\int\frac{A}{2\pi}d\theta + f(r) = -\frac{A\theta}{2\pi} + f(r)$$

3. $$-\frac{\partial\psi}{\partial r} = f'(r) \text{ and } -\frac{\partial\psi}{\partial r} = \frac{1}{r}\frac{\partial\phi}{\partial\theta} \text{ and } \frac{\partial\phi}{\partial\theta} = 0$$

4. $$-\frac{\partial\psi}{\partial r} = 0 = f'(r), f(r) = C,$$
Where C is a numerical constant

5. $$\psi = -\frac{A\theta}{2\pi} + C \text{ or } -\frac{A\theta}{2\pi} \text{ if } C \text{ is included in } \psi.$$
$$\psi = \text{constant} \Rightarrow \frac{A\theta}{2\pi} = \text{constant } i.e., \theta = \text{constant}$$
This will give radial lines as streamlines.

6. Since, $v_r = -\frac{\partial\psi}{r\partial\theta} = \frac{A}{2\pi r}$, is positive, flow is outwards. It is known as the source, ϕ = constant means r = constant.
7. Hence equipotential lines will be circles.

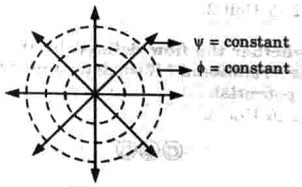

Fig. 2.26.1.

8. $$Q = v_r \times 2\pi r \times 1 = \frac{A}{2\pi r} \times 2\pi r = A$$
It is also known as the source strength.
9. Further since $r = 0$, v_r is infinite, the origin is a singular point.

VERY IMPORTANT QUESTIONS

Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.

Q. 1. Explain with example compressible, incompressible flows and uniform, non-uniform flows.

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Ans. Refer Q. 2.1, Unit-2.

Q. 2. Write short note on
A. Subsonic flow, sonic flow and supersonic flow.
B. Subcritical, Critical and Super Critical flows.

Ans. Refer Q. 2.2, Unit-2.

Q. 3. Streamlines, path lines and streak lines are identical for steady flow. Explain.

Ans. Refer Q. 2.3, Unit-2.

Q. 4. Derive continuity equation for a 3-D steady or unsteady flow in a cartesian coordinate system.

Ans. Refer Q. 2.5, Unit-2.

Q. 5. What is stream function ? Give its properties.

Ans. Refer Q. 2.10, Unit-2.

Q. 6. Explain the uniform flow with source and sink. Obtain the expressions for stream function and velocity potential functions.

Ans. Refer Q. 2.15, Unit-2.

Q. 7. Check whether the flow defined by the stream function $\psi = 2xy$ is irrotational ? If so, determine the corresponding velocity potential.

Ans. Refer Q. 2.16, Unit-2.

☺☺☺

3 UNIT — Potential Flow, Bernoulli's Equation and Its Applications

Part-1 ..... **(3–2A to 3–5A)**

- *Potential Flow*
- *Source and Sink*
- *Doublet*
- *Half Body*

A. Concept Outline : Part-1 ..... 3–2A
B. Long and Medium Answer Type Questions 3–2A

Part-2 ..... **(3–5A to 3–24A)**

- *Equation of Motion along a Streamline and its Integration*
- *Bernoulli's Equation and its Applications*

A. Concept Outline : Part-2 ..... 3–5A
B. Long and Medium Answer Type Questions 3–6A

Part-3 ..... **(3–24A to 3–44A)**

- *Momentum Equation and its Application*
- *Minor Energy Losses in Pipes in Series and Parallel*
- *Power Transmission Through a Pipe*
- *Three Reservoir Problems and Pipe Networks*

A. Concept Outline : Part-3 ..... 3–25A
B. Long and Medium Answer Type Questions 3–25A

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PART-1

Potential Flow, Source, Sink, Doublet and Half Body.

CONCEPT OUTLINE : PART-1

Potential Flow : The inviscid, incompressible, irrotational flow is often called as potential flow.
Source : It is a point from which flow is moving radially in all directions of a plane at uniform rate.
Sink : It is a point in which fluid moves radially inwards and it disappears at a constant rate.
Half Body : It is formed due to a particular combination of a uniform flow and a plane source. The body is known as half body because it has only the leading point, it trails to infinity at the downstream end.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.1. **What do you understand by the term ideal fluid flow or potential flow ? Discuss its importance.**

Answer

A. Potential Flow or Ideal Fluid Flow :

1. It refers to an incompressible and inviscid flow, *i.e.*,

$$\rho = \text{constant and } \mu = 0$$

2. As viscosity is zero, *i.e.*, there cannot be any shear stresses in a potential flow. Hence,
   a. The only stress at a point in an ideal fluid flow must be the pressure.
   b. The only stress the fluid may act on the solid boundary must be normal to it at that point.
3. An ideal fluid flow may be steady or unsteady and uniform or non-uniform.

B. Importance of Ideal Fluid Flow or Potential Flow :

1. An engineer deals with flows of various types in different situations *e.g.*, flow through confined passages, fluid machines, control systems, etc.
2. In reality, the flow may be complex due to such phenomenon as boundary layers, separation, recirculation etc., and an exact analysis of flow may be difficult. This is the reason that the simple technique of ideal fluid flow analysis assumes considerable importance in engineering.

Que 3.2. **Write short note on source, sink and doublet.**

Answer

A. Source :

1. It is a point from which flow is moving radially in all directions of a plane at uniform rate.

B. Sink :

1. It is a point in which fluid moves radially inwards and it disappears at a constant rate.

C. Doublet :

1. It is a special case of a source and sink pair (both of them are of equal strength, q) when the two approach each other in such a way that the distance $2a$ between them approaches zero and the product $2a.q$ remains constant.
2. This product $2a.q$ is known as doublet strength and is denoted by μ.
   $\therefore$ Doublet strength, $\mu = 2a.q$

Que 3.3. **What is a half body ? Why is it called a half body ?**

Answer

A. Half Body :

1. It is a particular combination of a uniform flow and a plane source.

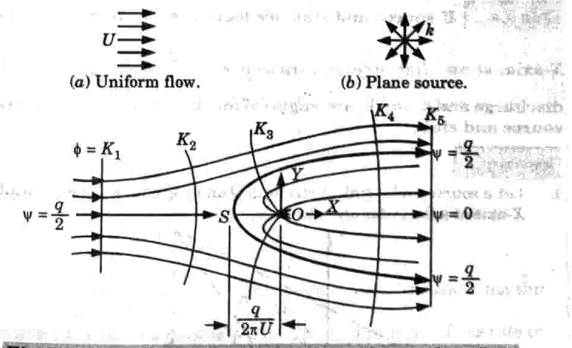

Fig. 3.3.1. Flow pattern due to plane source in a uniform flow.

2. The flow pattern resulting from the combination of a uniform flow and a plane source would be such that the stream function is given by

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$$\psi = Uy + k\theta = U r \sin\theta + k\theta$$

Where $k = \frac{q}{2\pi}$

And the origin is taken at the source.

3. A plot of (ψ = constant) lines reveals that the composite flow resembles

   a. Flow over a plane half-body, outside $\psi = \frac{q}{2}$

   b. Source flow within a plane half-body as shown in Fig. 3.3.1.

4. The plane half-body is described by the dividing streamline, $\psi = \frac{q}{2}$ which is

$$Uy + \frac{q\theta}{2\pi} = \frac{q}{2}$$

or
$$y = \frac{q\left(1 - \frac{\theta}{\pi}\right)}{2U} \quad ...(3.3.1)$$

5. At $\theta = 0, y_{max} = \frac{q}{2U}$, the maximum ordinate, ...(3 3.2)

   and at $\theta = \frac{\pi}{2}, y = \frac{q}{4U}$, the ordinate above the origin. ...(3.3.3)

6. At $\theta = \pi, y = 0$, the leading point of the half body.

   and at $\theta = \frac{3\pi}{2}, y = \frac{q}{4U}$, the ordinate below the origin.

7. The reason why it is called a half-body is that it has only the leading point; it trails to infinity at the downstream end.

Que 3.4. **If source and sink are located at finite distance along X-axis, show that stream function $\psi = \frac{Q}{2\pi}(\theta_1 - \theta_2)$. Where Q is discharge and θ_1 and θ_2 are angles of any point $P(x, y)$ from X-axis at source and sink.**

Answer

1. Let a source and a sink of strength Q and $-Q$ placed symmetrically on X-axis at a finite distance $2a$.

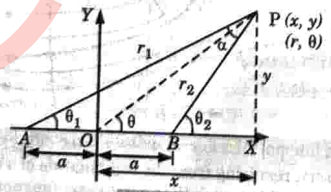

Fig. 3.4.1.

Where, Q = Discharge (or strength) of source, and

$-Q$ = Discharge (or strength) of sink.

2. Let P be the point in the resultant flownet of source and sink.

3. Let, r, θ = Cylindrical coordinates of point P with respect to origin O,

   x, y = Corresponding coordinates of point P,

   r_1, θ_1 = Position of P w.r.t. source at A,

   r_2, θ_2 = Position of P w.r.t. sink at B, and

   α = Angle subtended at P by the joining of source and sink *i.e.*, $\angle APB$.

4. The equation for stream function due to source is given by

$$\psi_1 = \frac{Q}{2\pi} \cdot \theta_1$$

5. The equation for stream function due to sink is given by

$$\psi_2 = \frac{-Q}{2\pi} \cdot \theta_2$$

5. The equation for resultant stream function (ψ) will be the sum of ψ_1 and ψ_2.

So,
$$\psi = \psi_1 + \psi_2 = \frac{Q}{2\pi} \cdot \theta_1 - \frac{Q}{2\pi} \cdot \theta_2$$

$$\psi = \frac{Q}{2\pi}(\theta_1 - \theta_2)$$

PART-2

Equation of Motion along Streamline and its Integrations, Bernoulli's Equation and its Applications-Pitot Tube, Orifice Meter and Bend Meter, Notches and Weir.

CONCEPT OUTLINE : PART-2

Bernoulli's Equation :
Pressure energy + Kinetic energy + Potential energy = Constant

$$\frac{p}{\rho g} + \frac{v^2}{2g} + z = \text{constant}$$

Venturimeter : A venturimeter is a device used for measuring the rate of flow of a fluid flowing through a pipe.

Notch : A notch is a device used for the measurement of the rate of flow of a liquid through a small channel or tank.

Weir : A weir is any regular obstruction in an open stream over which the flow takes place.

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Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.5. Derive Euler's equation of motion.

Answer

1. As shown in Fig. 3.5.1, consider a stream-line in which flow is taking place in S-direction.

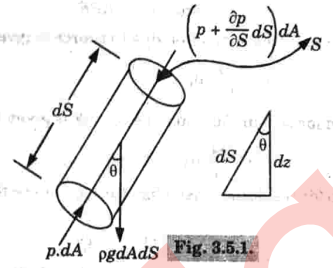

Fig. 3.5.1.

2. Consider a cylindrical element of cross section dA, length dS and Let θ is the angle between the direction of flow and the line of action of the weight of element.
3. The forces acting on the cylindrical elements are :
   a. Pressure force pdA in the direction of flow.
   b. Pressure force $\left(p+\frac{\partial p}{\partial S}dS\right)dA$ opposite to the direction of flow.
   c. Weight of element $\rho g\, dA\, dS$.
4. For equilibrium,

$$p\,dA - \left(p+\frac{\partial p}{\partial S}dS\right)dA - \rho g\, dA\, dS\cos\theta = \rho\, dA.dS \times a_s \quad ...(3.5.1)$$

Where a_s is the acceleration in the direction of S.

5. As we know, $a_s = \frac{dv}{dt}$, where v is a function of S and t

$$a_s = \frac{\partial v}{\partial S}\frac{dS}{dt} + \frac{\partial v}{\partial t} = v\frac{\partial v}{\partial S} + \frac{\partial v}{\partial t} \quad \left(\because \frac{dS}{dt} = v\right)$$

If the flow is steady, then the term $\frac{\partial v}{\partial t}$ will be zero.

$$\therefore \quad a_s = v\frac{\partial v}{\partial S}$$

6. Substituting the value of a_s in eq. (3.5.1).

$$-\frac{\partial p}{\partial S}dS\,dA - \rho g\, dA\, dS\cos\theta = \rho\, dA\, dS \times v\frac{\partial v}{\partial S} \quad ...(3.5.2)$$

7. Dividing by $\rho dS\, dA$ in eq. (3.5.2),

$$-\frac{\partial p}{\rho\partial S} - g\cos\theta = v\frac{\partial v}{\partial S}$$

$$\text{or} \quad \frac{\partial p}{\rho\partial S} + g\cos\theta + v\frac{\partial v}{\partial S} = 0 \quad ...(3.5.3)$$

8. From geometric relation between dS and dz,

$$\cos\theta = \frac{dz}{dS}$$

9. Eq. (3.5.3) becomes

$$\frac{1}{\rho}\frac{\partial p}{\partial S} + g\frac{dz}{dS} + v\frac{\partial v}{\partial S} = 0$$

$$\text{or} \quad \frac{\partial p}{\rho} + g\,dz + v\,\partial v = 0 \quad ...(3.5.4)$$

The eq. (3.5.4) is known as Euler's equation of motion.

Que 3.6. State Bernoulli's theorem for steady flow of an incompressible fluid.

Answer

1. Bernoulli's theorem states that, "In a steady, ideal flow of an incompressible fluid, the total energy at any point of the fluid is constant".
2. Pressure energy + Kinetic energy + Potential energy = Constant

$$\frac{p}{\rho g} + \frac{v^2}{2g} + z = \text{constant}$$

3. Bernoulli's equation for real fluids

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_L$$

Where, h_L = Loss of energy between points 1 and 2.

Que 3.7. How will you obtain Bernoulli's equation from Euler's equation of motion along a streamline ? Write assumptions of Bernoulli's equation.

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Answer

A. Assumptions :

1. The fluid is ideal, *i.e.*, viscosity is zero.
2. The flow is steady.
3. The flow is incompressible.
4. The flow is irrotational.

B. Bernoulli's Equation from Euler's Equation :

1. Euler's equation of motion is

$$\frac{\partial p}{\rho} + g.dz + v.dv = 0$$

2. On integrating,

$$\int \frac{dp}{\rho} + \int g.dz + \int v.dv = \text{constant}$$

$$\frac{p}{\rho} + g.z + \frac{v^2}{2} = \text{constant}$$

$$\frac{p}{\rho g} + z + \frac{v^2}{2g} = \text{constant} \quad ...(3.7.1)$$

3. The eq. (3.7.1) is known as Bernoulli's equation in which

$\frac{p}{\rho g}$ = Pressure head, $\frac{v^2}{2g}$ = Kinetic head and z = Potential head.

Que 3.8. **What is Euler's equation of motion and how will you obtain the Bernoulli's equation from it ? The rate of water flow through a vertical conical draft tube of a Kaplan turbine is 17.5 m³/s. The diameter of the draft tube on the side connected to the outlet of the turbine runner is 2.5 m and the average velocity at exit is 1.5 m/s. If the pressure at the inlet of the tube is not to be less than – 0.7 bar ; how far the tube should extend above the tail race ? Neglect the frictional effect and presume that exit of the draft tube lies 1.2 m below the water level.** **AKTU 2013-14, Marks 05**

Answer

A. Euler's Equation : Refer Q. 3.5, Page 3–6A, Unit-3.

B. Bernoulli's Equation from Euler's Equation : Refer Q. 3.7, Page 3–7A, Unit-3.

C. Numerical :

Given : Discharge at inlet = 17.5 m³/sec, Diameter at inlet = 2.5 m
Velocity at exit = 1.5 m sec
Pressure at inlet = – 0.7 bar = – 0.7 × 10⁵ N/m²
Pressure at outlet = 1.2 m below water level.
Say height of tube above the tail race is h.
To Find : Height of tube above the tail race.

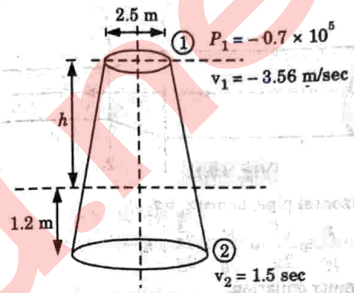

Fig. 3.8.1.

1. Velocity at inlet $= \frac{\text{Discharge at inlet}}{\text{Area at inlet}} = \frac{17.5}{\frac{\pi}{4}(2.5)^2}$

$= 3.56$ m/sec

2. Applying Bernoulli's equation between sections (1) and (2),

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

$$\frac{-0.7 \times 10^5}{\rho g} + \frac{(3.56)^2}{2g} + (h + 1.2) = 1.2 + \frac{(1.5)^2}{2g} + 0$$

$$h = 6.60 \text{ m}$$

Que 3.9. **Water flows through a 0.9 m diameter pipe at the end of which there is a reducer connecting to a 0.6 m diameter pipe. If the gauge pressure at the entrance to the reducer is 412.02 kN/m² and the velocity is 2 m/s, determine the resultant thrust on the reducer, assuming that the frictional loss of head in the reducer is 1.5 m.** **AKTU 2015-16, Marks 05**

Answer

Given : $d_1 = 0.9$ m, $d_2 = 0.6$ m, $P_1 = 412.02$ kN/m²
$v_1 = 2$ m/sec, $h_f = 1.5$ m
To Find : Resultant thrust.

1. Applying Bernoulli's theorem at section (1) and (2), we get

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_f$$

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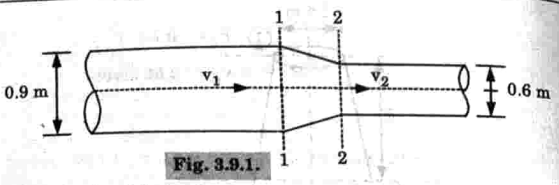

Fig. 3.9.1.

2. As it is horizontal pipe, hence $z_1 = z_2$

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + h_f \quad ...(3.9.1)$$

3. From continuity equation,

$$v_1 A_1 = v_2 A_2$$

$$v_2 = \frac{v_1 A_1}{A_2} = \left(\frac{d_1}{d_2}\right)^2 \times v_1 = \left(\frac{0.9}{0.6}\right)^2 \times 2 = \frac{9}{2} \text{ m/sec.}$$

4. Putting the values of P_1, v_1, P_2, v_2 and h_f in eq (3.9.1),

$$\frac{412.02 \times 10^3}{1000 \times 9.81} + \frac{2^2}{2 \times 9.81} = \frac{P_2}{1000 \times 9.18} + \frac{(9/2)^2}{2 \times 9.18} + 1.5$$

$$\frac{412.02}{9.18} + \frac{2}{9.81} = \frac{P_2}{1000 \times 9.18} + \frac{81}{8 \times 9.18} + 1.5$$

$$412.02 + 2 = \frac{P_2}{1000} + \frac{81}{8} + 1.5 \times 9.81$$

$$414.02 = \frac{P_2}{1000} + 10.125 + 14.715$$

$$414.02 = \frac{P_2}{1000} + 24.84$$

$$\frac{P_2}{1000} = 389.18 \text{ N/m}^2$$

$$P_2 = 389.18 \text{ kN/m}^2$$

Hence, resultant thrust on the reducer = 389.18 kN/m².

Que 3.10. **Suggest the device used for the measurement of fluid flow through ducts or pipes. Explain them.**

OR

What are the various applications of Bernoulli's equation ? Explain them.

Answer

Some of the simple applications of Bernoulli's equation are discussed in following section given below :

A. Venturimeter :

1. A venturimeter is a device used for measuring the rate of flow of a fluid flowing through a pipe.
2. It consists of three parts, as follows :
   a. A short converging part,
   b. Throat, and
   c. Diverging part.
3. It works on the principle of Bernoulli's theorem.

a. Mathematical Expression for Rate of Flow Through Venturimeter :

1. As shown in Fig. 3.10.1, a venturimeter is fitted in a horizontal pipe through which a fluid is flowing.

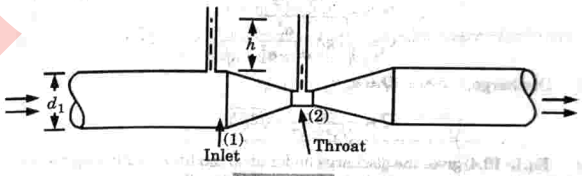

Fig. 3.10.1.

2. Let, d_1 = Diameter of pipe at section (1),
   p_1 = Pressure at section (1),
   v_1 = Velocity of fluid at section (1), and
   a_1 = Area at section (1) = $\frac{\pi}{4} d_1^2$
   and d_2, p_2, v_2, a_2 are corresponding values at section (2).
3. By applying Bernoulli's theorem at section (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 \quad ...(3.10.1)$$

4. As it is horizontal pipe, hence $z_1 = z_2$

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + \frac{v_2^2}{2g}$$

or

$$\frac{p_1 - p_2}{\rho g} = \frac{v_2^2}{2g} - \frac{v_1^2}{2g} \quad ...(3.10.2)$$

5. But $\frac{p_1 - p_2}{\rho g}$ is the difference of pressure heads at sections (1) and (2) and it is equal to h,

$$\frac{p_1 - p_2}{\rho g} = h$$

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$$h = \frac{v_2^2}{2g} - \frac{v_1^2}{2g} \quad ...(3.10.3)$$

6. Applying continuity equation at sections (1) and (2),

$$a_1 v_1 = a_2 v_2 \text{ or } v_1 = \frac{a_2 v_2}{a_1}$$

7. Substituting this value of v_1 in eq. (3.10.3), gives

$$h = \frac{v_2^2}{2g} - \frac{\left(\frac{a_2 v_2}{a_1}\right)^2}{2g} = \frac{v_2^2}{2g}\left[1 - \frac{a_2^2}{a_1^2}\right]$$

or $$v_2^2 = 2gh\frac{a_1^2}{a_1^2 - a_2^2}$$

$$\therefore \quad v_2 = \sqrt{2gh\frac{a_1^2}{a_1^2 - a_2^2}} = \frac{a_1}{\sqrt{a_1^2 - a_2^2}}\sqrt{2gh}$$

8. Discharge, $Q = a_2 v_2$

$$Q = \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh} \quad ...(3.10.4)$$

9. Eq. (3.10.4) gives the discharge under ideal conditions called as theoretical discharge whereas actual discharge will be less than theoretical discharge.

$$\therefore \quad Q_{act} = C_d \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

Where C_d is the coefficient of discharge for venturimeter and its value is less than unity.

Case I :

1. Pipe is horizontal and the differential manometer contains liquid heavier than the liquid flowing through the pipe.
2. Let, S_h = Specific gravity of the heavier liquid,
   S_o = Specific gravity of the liquid flowing through pipe, and
   x = Difference of the heavier liquid column in U-tube.

$$h = x\left[\frac{S_h}{S_o} - 1\right]$$

Case II :

1. Pipe is horizontal and the differential manometer contains liquid lighter than the liquid flowing through the pipe.
2. $$h = x\left[1 - \frac{S_l}{S_o}\right]$$

Where, S_l = Specific gravity of lighter liquid in U-tube,
S_o = Specific gravity of fluid flowing through pipe, and
x = Difference of the lighter liquid columns in U-tube.

Case III :

1. Pipe is inclined and the differential manometer contains liquid heavier than the liquid flowing through the pipe.
2. $$h = \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = x\left[\frac{S_h}{S_o} - 1\right]$$

Case IV :

1. Pipe is inclined and the differential manometer contains liquid lighter than the liquid flowing through the pipe.
2. $$h = \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = x\left[1 - \frac{S_l}{S_o}\right]$$

B. Orifice Meter :

1. It works on the Bernoulli's principle and is a device used for measuring the rate of flow of a fluid flowing through a pipe.
2. It consists of a flat circular plate which has a circular sharp edged hole called orifice, which is concentric with the pipe.

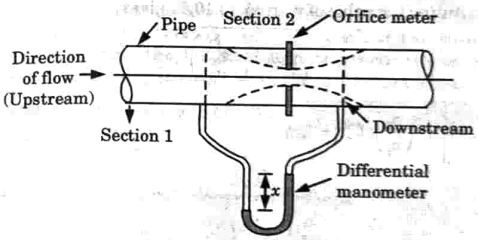

Fig. 3.10.2.

3. As shown in Fig. 3.10.2, let,
   p_1 = Pressure at section (1),
   v_1 = Velocity of flow at section (1),
   a_1 = Area of pipe at section (1), and
   p_2, v_2, a_2 are corresponding values at section (2).
4. Applying Bernoulli's equation at section (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

or $$\left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

Where, $\left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = h$ = Differential head.

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$$h = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

$$v_2 = \sqrt{2gh + v_1^2} \quad ...(3.10.5)$$

5. Now section (2) is at the vena-contracta and a_2 represents the area at the vena-contracta.

6. If a_0 is the area of orifice then,

$$C_C = \frac{a_2}{a_0}$$

Where, C_C = Coefficient of contraction

$$a_2 = a_0 \times C_C \quad ...(3.10.6)$$

7. By continuity equation,

$$a_1 v_1 = a_2 v_2$$

$$v_1 = \frac{a_0 C_C}{a_1} v_2 \quad ...(3.10.7)$$

8. Substituting the value of v_1 in eq. (3.10.5), gives

$$v_2 = \sqrt{2gh + \left(\frac{a_0 C_C}{a_1}\right)^2 . v_2^2}$$

or $$v_2^2\left[1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2\right] = 2gh$$

$$\therefore \quad v_2 = \frac{\sqrt{2gh}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2}}$$

9. The discharge, $Q = v_2 \times a_2 = v_2 \times a_0 C_C \quad (\because a_2 = a_0 C_C)$

i.e., $$Q = \frac{a_0 C_c \sqrt{2gh}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2}} \quad ...(3.10.8)$$

10. The above expression can be simplified by using,

$$C_d = C_C \frac{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_C^2}} \Rightarrow C_C = C_d \frac{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_C^2}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2}}$$

11. Substituting the value of C_C in eq. (3.10.8), gives

$$Q = a_0 \times C_d \frac{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_C^2}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2}} \times \frac{\sqrt{2gh}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_C^2}}$$

$$= \frac{C_d a_0 \sqrt{2gh}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2}} = \frac{C_d a_0 a_1 \sqrt{2gh}}{\sqrt{a_1^2 - a_0^2}}$$

Where, C_d = Coefficient of discharge for orifice meter.

12. The coefficient of discharge for orifice meter is much smaller than that for a venturimeter.

Que 3.11. **A horizontal venturimeter with a discharge coefficient of 0.98 is being used to measure the low rate of a liquid of density 1030 kg/m³. The pipe diameter at entry to the venturi is 75 mm and the venturi throat has an area of 1000 mm². If the flow rate is 0.011 m³/s. Determine the height difference recorded on a *U*-tube manometer connecting the throat to the upstream pipe. Take the relative density of mercury to be 13.6.** **AKTU 2016-17, Marks 05**

Answer

Given : $C_d = 0.98$, Liquid density = 1030 kg/m³

Relative density of liquid = $\frac{1030}{1000}$ = 1.03 kg/m³

Relative density of mercury = 13.6

Pipe diameter at inlet d_1 = 75 mm = 0.075 m, Flow rate Q = 0.011 m³/s

$a_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4} \times (.075)^2 = 4.4178 \times 10^{-3}$ m²

Throat area a_2 = 1000 mm² = 1×10^{-3} m²

To find : Height difference

1. As we know flow rate $Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$

$$0.011 = 0.98 \times \frac{4.4178 \times 10^{-3} \times 1 \times 10^{-3}}{\sqrt{(4.4178 \times 10^{-3})^2 - (1 \times 10^{-3})^2}} \times \sqrt{2 \times 9.81 \times h}$$

$$h = 329019.86 \times (19.517 \times 10^{-6} - 1 \times 10^{-6})$$

$$h = 329019.86 \times 18.517 \times 10^{-6}$$

$$h = 6.092 \text{ m}$$

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2. $$h = y\left[\frac{S_{Hg}}{S_{fluid}} - 1\right]$$

$$6.092 = y\left[\frac{13.6}{1.03} - 1\right]$$

$$6.092 = y \times \frac{12.57}{1.03}$$

$$y = 0.4992 \text{ m}$$

Que 3.12. **A venturimetre having a diameter of 75 mm at the throat and 150 mm diameter at the enlarged end is installed in a horizontal pipeline 150 mm in diameter carrying oil of SG 0.9. The difference of pressure in between enlarged end and the throat is 175 mm mercury as recorded by a *U*-tube differential manometer. Determine the discharge through the pipe. Assume the coefficient of discharge of the meter is 0.97.** **AKTU 2011-12, Marks 05**

Answer

Given : Dia. at inlet, $d_1 = 150$ mm $= 0.15$ m
Dia. at throat, $d_2 = 75$ mm $= 0.075$ m, $C_d = 0.97$
Specific gravity of oil, $S_0 = 0.9$
Specific gravity of mercury, $S_1 = 13.6$
Reading of differential manometer, $x = 175$ mm $= 0.175$m
To Find : Discharge through the pipe

1. Area at inlet, $$a_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4} \times (0.15)^2 = 0.01767 \text{ m}^2$$

2. Area at throat, $$a_2 = \frac{\pi}{4} d_2^2 = \frac{\pi}{4} \times (0.075)^2 = 4.4178 \times 10^{-3} \text{ m}^2$$

3. Difference of pressure head, $$h = x\left(\frac{S_h}{S_0} - 1\right) = 0.175\left[\frac{13.6}{0.9} - 1\right] = 2.4694$$

4. Discharge through venturimeter, $$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

$$= 0.97 \times \frac{0.01767 \times 4.4178 \times 10^{-3}}{\sqrt{(0.01767)^2 - (4.4178 \times 10^{-3})^2}} \times \sqrt{2 \times 9.81 \times 2.4694}$$

$$= 0.030806 \text{ m}^3/\text{s} = 30.806 \text{ litres/s.}$$

Que 3.13. **In an experiment of flow through orifices the following data were recorded :**
Diameter of sharp orifice = 100 mm
Diameter of jet at vena-contracta = 78.42 mm
Height of water tank maintained = constant = 3.60 m
Discharge measured (Tank-method) = 0.0385 m³/s
Determine C_c, C_v and C_d. **AKTU 2012-13, Marks 10**

Answer

Given : Diameter of orifice, $d = 100$ mm $= 0.1$ m
Jet diameter at vena-contracta $= 78.42$ mm $= 0.07842$ m
Height of water tank, $H = 3.60$ m, Discharge, $Q_{act} = 0.0385$ m³/s
To Find : C_c, C_v and C_d

1. $$Q_{th} = v_{th} \times \text{Area of orifice}$$

$$v_{th} = \sqrt{2gH} = \sqrt{2 \times 9.81 \times 3.60} = 8.40 \text{ m/s}$$

$$\therefore \quad Q_{th} = 8.40 \times \frac{\pi}{4} d^2 = 8.40 \times \frac{\pi}{4} \times (0.1)^2 = 0.066 \text{ m}^3/\text{s}$$

2. Coefficient of discharge, $$C_d = \frac{\text{Actual discharge } (Q_{act})}{\text{Theoretical discharge } (Q_{th})}$$

$$C_d = \frac{0.0385}{0.066} = 0.58$$

3. Coefficient of contraction, $$C_c = \frac{a_c}{a}$$

$$a_c = \frac{\pi}{4} (\text{jet diameter})^2 = \frac{\pi}{4} \times (0.07842)^2 = 0.0048$$

$$a = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times (0.1)^2 = 0.0079$$

$$C_c = \frac{0.0048}{0.0079} = 0.61$$

4. We know that, $$C_d = C_v \times C_c$$

$$\therefore \quad C_v = \frac{C_d}{C_c} = \frac{0.58}{0.61}$$

$$C_v = 0.95$$

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Que 3.14. **In a vertical conveying oil of specific gravity 0.8, two pressure gauges have been installed at *A* and *B* where the diameters are 16 cm and 8 cm respectively. *A* is 2 m above *B*. The pressure gauge readings have shown that the pressure at *B* is greater than at *A* by 0.981 N/cm². Neglecting all losses, calculate the flow rate. If the gauges at *A* and *B* are replaced by tubes filled with the same liquid and connected to a *U*-tube containing mercury. Calculate the difference of level of mercury in the two limbs of *U*-tube.**

AKTU 2014-15, Marks 05

Answer

Given : Specific gravity of oil, $S_o = 0.8$

$\therefore$ Density, $\rho = 0.8 \times 1000 = 800$ kg/m³

Diameter at *A*, $D_A = 16$ cm $= 0.16$ m

Diameter at *B*, $D_B = 8$ cm $= 0.08$ m

To Find : Flow rate and difference in the level of mercury

1. Area at *A*, $A_1 = \frac{\pi}{4}(0.16)^2 = 0.0201$ m² and

   Area at *B*, $A_2 = \frac{\pi}{4}(0.08)^2 = 0.005026$ m²

2. Difference of pressures,

$$P_B - P_A = 0.981 \text{ N/cm}^2 = 0.981 \times 10^4 \text{ N/m}^2 = 9810 \text{ N/m}^2$$

Difference of pressure head

$$\therefore \quad \frac{P_B - P_A}{\rho g} = \frac{9810}{800 \times 9.81} = 1.25 \text{ m} \qquad (\because \rho = 800 \text{ kg/m}^3)$$

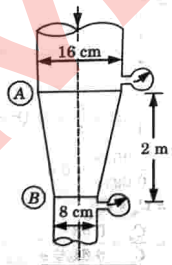

Fig. 3.14.1.

3. Applying Bernoulli's theorem at *A* and *B* taking the reference line passing through section *B*,

$$\frac{P_A}{\rho g} + \frac{v_A^2}{2g} + z_A = \frac{P_B}{\rho g} + \frac{v_B^2}{2g} + z_B$$

$$\text{or} \quad \frac{P_A}{\rho g} - \frac{P_B}{\rho g} + z_A - z_B = \frac{v_B^2}{2g} - \frac{v_A^2}{2g}$$

$$\left(\frac{P_A - P_B}{\rho g}\right) + 2.0 - 0.0 = \frac{v_B^2}{2g} - \frac{v_A^2}{2g}$$

$$0.75 = \frac{v_B^2}{2g} - \frac{v_A^2}{2g} \qquad \left(\because \frac{P_B - P_A}{\rho g} = 1.25\right) \quad \text{... (3.14.1)}$$

4. Now applying continuity equation at *A* and *B*, we get

$$v_A \times A_1 = v_B \times A_2$$

$$v_B = \frac{v_A \times A_1}{A_2} = \frac{v_A \times \frac{\pi}{4}(0.16)^2}{\frac{\pi}{4}(0.08)^2} = 4v_A$$

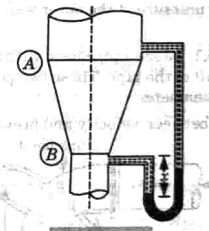

Fig. 3.14.2.

5. Substituting the value of v_B in eq. (3.14.1), we get

$$0.75 = \frac{16v_A^2}{2g} - \frac{v_A^2}{2g} = \frac{15v_A^2}{2g}$$

$$\therefore \quad v_A = \sqrt{\frac{0.75 \times 2 \times 9.81}{15}} = = 0.99 \text{ m/s}$$

6. Rate of flow, $Q = v_A \times A_1$

$$= 0.99 \times 0.0201 = 0.01989 \text{ m}^3/\text{s}$$

7. Let, x = Difference of mercury level.

Then $$h = x\left(\frac{S_g}{S_o} - 1\right)$$

Where $$h = \left(\frac{P_A}{\rho g} + z_A\right) - \left(\frac{P_B}{\rho g} + z_B\right) = \frac{P_A - P_B}{\rho g} + z_A - z_B$$

$$= -1.25 + 2.0 - 0$$

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$$= 0.75 \qquad \left(\because \frac{P_B - P_A}{\rho g} = 1.25\right)$$

$$\therefore \quad 0.75 = x\left[\frac{13.6}{0.8} - 1\right] = x \times 16$$

Difference of level of mercury in the U-tube,

$$x = 0.04687 \text{ m} = 4.687 \text{ cm}$$

Que 3.15. Explain elbow meter with neat sketch and give its application.

Answer

A. Elbow Meter :

1. When a liquid flows in a pipe bend, there exists a difference of pressure between the outside and inside of the bend, this difference of pressure is used to measure the discharge in pipeline.
2. In a pipe bend, the pressure at the outer wall of bend is more than that at the inner wall.
3. Now from Fig. 3.15.1, we see a pipe bend with two pressure p_o at outside wall p_i at inside wall of the pipe. These two points are connected to the limbs of U-tube manometer.
4. From the relation between velocity and pressure difference,

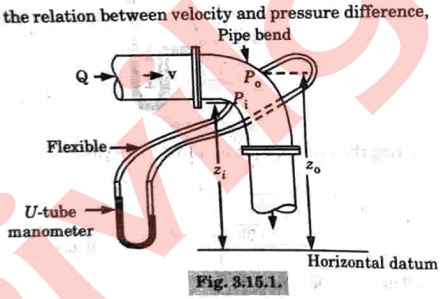

Fig. 3.15.1.

$$K \cdot \frac{v^2}{2g} = \left(\frac{p_o}{w} + z_o\right) - \left(\frac{p_i}{w} + z_i\right)$$

$$v = \frac{\sqrt{2g}}{\sqrt{K}}\sqrt{\left(\frac{p_o}{w} + z_o\right) - \left(\frac{p_i}{w} + z_i\right)}$$

Where, K = Constant (1.3 to 3.2 depends upon size and shape of the bend).
v = Velocity of flow.

5. Discharge, $Q = A.v = C_d \cdot A\sqrt{2g\left[\left(\frac{p_o}{w} + z_o\right) - \left(\frac{p_i}{w} + z_i\right)\right]}$

Where, C_d = Co-efficient of discharge.

$$= \frac{1}{\sqrt{K}} \quad (0.56 \le C_d \le 0.88)$$

and $C_d = \sqrt{\frac{R_b}{D}}$

Where, R_b = Radius of pipe bend, and
D = Diameter of pipe.

B. Applications :

1. An elbow meter can be used for the measurement of discharge in pipes which are fitted with bends or elbow.

Que 3.16. What is notch ? What are the different types of notches ?

Answer

1. A notch is a device used for the measurement of the rate of flow of a liquid through a small channel or a tank.
2. It may be defined as an opening in the side of a tank or a small channel in such a way that the liquid surface in the tank or channel is below the top edge of the opening.
3. The different types of notches are :
   (i) Rectangular notch (ii) Triangular notch (iii) Trapezoidal notch.

Que 3.17. Derive the expression for discharge over the following notches : A. Rectangular, and B. Triangular.

Answer

A. Rectangular Notch :

1. As shown in Fig. 3.17.1 consider a rectangular notch provided in a channel carrying water.
2. Let, H = Head of water over the crest, and
   L = Length of notch.

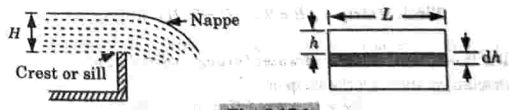

Fig. 3.17.1.

3. In order to find the discharge of water flowing over the notch, consider an elementary horizontal strip of water of thickness dh and length L at a depth h from the free surface of water.

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4. The area of strip = $L \times dh$ and theoretical velocity of water flowing through strip = $\sqrt{2gh}$.
5. The discharge dQ, through strip is

$$dQ = C_d \times \text{area of strip} \times \text{theoretical velocity}$$

$$= C_d \times L \times dh \times \sqrt{2gh} \quad \text{...(3.17.1)}$$

Where, C_d = Coefficient of discharge.

6. The total discharge, Q for the whole notch is determined by integrating eq. (3.17.1) between the limits 0 and H.

$$Q = \int_0^H C_d \times L\sqrt{2gh}\, dh = C_d \times L \times \sqrt{2g}\int_0^H h^{1/2} dh$$

$$= C_d \times L \times \sqrt{2g}\left[\frac{h^{3/2}}{3/2}\right]_0^H$$

$$Q = \frac{2}{3} \times C_d \times \sqrt{2g}\, H^{3/2}$$

B. Triangular Notch :

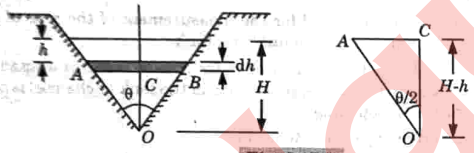

Fig. 3.17.2.

1. Let, H = Head of water above the notch, and
   θ = Angle of notch.
2. Consider a horizontal strip of water having thickness 'dh' at a depth of h from the free surface of water.
3. From the geometry of notch,

$$\tan \theta/2 = \frac{AC}{OC} = \frac{AC}{H-h}$$

$$\therefore \quad AC = (H-h)\tan\left(\frac{\theta}{2}\right)$$

$$\text{Width of strip} = AB = 2 \times AC = 2(H-h)\tan\left(\frac{\theta}{2}\right)$$

4. The theoretical velocity of water through strip = $\sqrt{2gh}$
5. Discharge, through the strip is,

$$dQ = C_d \times \text{area of strip} \times \text{velocity}$$

$$= C_d \times 2(H-h)\tan\left(\frac{\theta}{2}\right) \times dh \times \sqrt{2gh}$$

$$= 2\, C_d (H-h)\tan\left(\frac{\theta}{2}\right) \times \sqrt{2gh} \times dh$$

6. Total discharge is

$$Q = \int_0^H 2C_d (H-h)\tan\left(\frac{\theta}{2}\right) \times \sqrt{2gh}\, dh$$

$$= 2C_d \times \tan\left(\frac{\theta}{2}\right) \times \sqrt{2g}\int_0^H (H-h)\, h^{1/2} dh$$

$$= 2 \times C_d \times \tan\left(\frac{\theta}{2}\right) \times \sqrt{2g}\left[\frac{Hh^{3/2}}{3/2} - \frac{h^{5/2}}{5/2}\right]_0^H$$

$$Q = \frac{8}{15} \times C_d \times \tan\left(\frac{\theta}{2}\right) \times \sqrt{2g}\, H^{5/2}$$

Que 3.18. **Define weir and give its classification. Differentiate between notch and weir.**

Answer

A. Weir :

1. It is any regular obstruction in an open stream over which the flow takes place.

B. Classification of Weirs :

a. On the Basis of Shape

1. Rectangular weir, and
2. Cipoletti weir.

b. On the Basis of Nature of Discharge :

1. Ordinary weir, and
2. Submerged weir.

c. On the Basis of the Width of Crest :

1. Narrow crested, and
2. Broad crested.

d. According to the Nature of Crest :

1. Sharp crested weir, and
2. Ogee weir.

e. On the Basis of the Effect of Sides on the Emerging Nappe :

1. Weir with end contraction, and
2. Weir without end contraction.

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C. Difference Between Notch and Weir :

S. No.	Notch	Weir
1.	The size of notch is very small.	The size of weir is large.
2.	It is used to determine the flow through small tanks or pipes.	It is used to measure the flow of rivers.

Que 3.19. **Find out the depth and top width of a *U*-notch discharging 0.7 m³/s. The head over the notch is 10 cm when the discharge is 0.009 m³/s. Take $C_d = 0.6$.**

Answer

Given : Head over the notch, $H_1 = 10$ cm $= 0.1$ m, $Q = 0.009$ m³/s
To Find : Depth and top width.

1. Discharge over the *U*-notch,

$$Q = \frac{2}{3} C_d \times L\sqrt{2g} \cdot H_1^{3/2}$$

$$0.009 = \frac{2}{3} \times 0.6 \times L\sqrt{2 \times 9.81}(0.1)^{3/2}$$

$$L = \frac{0.009 \times 3}{2 \times 0.6 \times \sqrt{2 \times 9.81} \times (0.1)^{3/2}}$$

Length of the notch, $L = 0.16$ m

Length of the notch, L = Top width of notch = 0.16 m

2. Now,

$$Q = \frac{2}{3} C_d \times L\sqrt{2g}.\, H^{3/2}$$

$$0.7 = \frac{2}{3} \times 0.6 \times 0.16\sqrt{2 \times 9.81}\,.\, H^{3/2}$$

$$H^{3/2} = \frac{0.7 \times 3}{2 \times 0.6 \times 0.16\sqrt{2 \times 9.81}}$$

$$H = 1.826 \text{ m}$$

3. Depth of *U*-notch $= H - H_1 = 1.826 - 0.1 = 1.726$ m

PART-3

Momentum Equation and its Applications to Pipe Bends, Resistance to Flow and Minor Energy Losses in Pipe in Series and Parallel, Power Transmission Through a Pipe, Siphon, Water Hammer, Three Reservoir Problems and Pipe Networks.

CONCEPT OUTLINE : PART-3

Momentum Equation : According to law of conservation of momentum, the net force acting on a fluid mass is equal to the change in momentum of flow per unit time in the direction of force.

$$F = \frac{d(m.v)}{dt}$$

This is known as the momentum principle or momentum equation.

Minor Energy Losses : Minor energy losses are due to :

i. Sudden enlargement of pipe.
ii. Sudden contraction of pipe.
iii. Bend in pipe.
iv. An obstruction in pipe.
v. Pipe fittings etc.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.20. **Describe momentum equation. Where this equation is used ?**

Answer

A. Momentum Equation :

1. This equation is based on the law of conservation of momentum.
2. According to law of conservation of momentum, the net force acting on a fluid mass is equal to the change in momentum of flow per unit time in the direction of force.
3. According to Newton's second law of motion,

$$F = m \times a$$

Where, m = Mass of fluid,
a = Acceleration in direction of force, and
F = Force acting on fluid.

$$F = m.\frac{dv}{dt} \qquad \left\{\because a = \frac{dv}{dt}\right\}$$

$$F = \frac{d(m.v)}{dt} \qquad (\because m \text{ is constant})$$

This is known as the momentum principle or momentum equation.

B. Uses :

1. This equation is used to determine the force or impulse acting at the bend in the bend pipes, reducers, moving vanes and jet propulsion etc.

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2. This equation is used to determine the characteristics of flow in sudden enlargement in a pipe.

Que 3.21. **Derive an expression for the force exerted by a flowing fluid on a pipe bend.**

Answer

A. Force Exerted by a Flowing Fluid on a Pipe Bend :

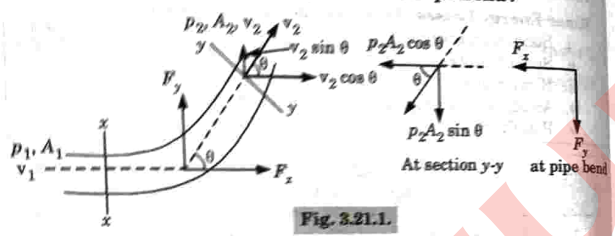

Fig. 3.21.1.

1. Consider section x-x and y-y in a bend pipe having pressure, cross section area and velocity as p_1, A_1, v_1 at x-x section and p_2, A_2, v_2 at y-y section.
2. Forces F_x and F_y are acting on the pipe bend due to fluid flow but force exerted by the pipe bend F_x and F_y are acting in opposite direction.
3. Using impulse momentum equation in X-direction,

$$p_1A_1 - p_2A_2 \cos\theta - F_x = \frac{d}{dt}(m.v)$$

$$p_1A_1 - p_2A_2 \cos\theta - F_x = \rho Q (v_2 \cos\theta - v_1)$$

Where, ρQ = Mass of fluid flowing per second, and

$v_2 \cos\theta - v_1$ = Change in velocity in X-direction.

$$F_x = \rho Q (v_1 - v_2 \cos\theta) + p_1A_1 - p_2A_2 \cos\theta$$

4. Now, using impulse momentum equation in Y-direction.

$$0 - p_2A_2 \sin\theta - F_y = \rho Q (v_2 \sin\theta - 0)$$

$$F_y = -\rho Q v_2 \sin\theta - p_2A_2 \sin\theta$$

5. Now the resultant force F_R acting on the bend.

$$F_R = \sqrt{F_x^2 + F_y^2}$$

6. Direction of resultant force,

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$$

Que 3.22. **Water is flowing in a 300 mm pipeline fitted with a 45° bend in the vertical plane. The diameter at the outlet of the bend is 150 mm. The pipe axis at the inlet is horizontal and the outlet is 1.5 m above the inlet. If the flow through the bend is 0.4 m³/s and a head-loss of 0.5 m occurs in the bend, calculate the magnitude and direction of the resultant force the bend support must withstand. The volume of the bend is 0.075 m³ and the pressure at the inlet is 300 kN/m².** **AKTU 2016-17, Marks 05**

Answer

Given : $d_1 = 300$ mm, $d_2 = 150$ mm, $\theta_1 = 0$, $\theta_2 = 45°$, $Q = 0.4$ m³/s, $h_f = 0.5$ m, $V = 0.075$ m³, $P_1 = 300$ kN/m², $z_1 = 0$, $z_2 = 1.5$ m

To Find : Magnitude and direction of resultant force.

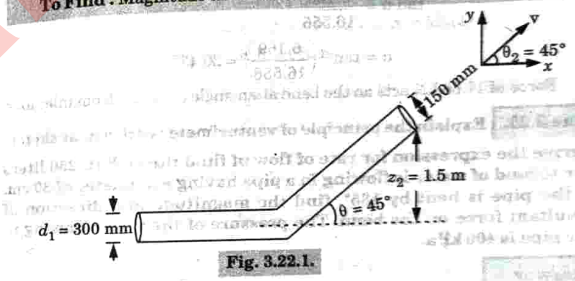

Fig. 3.22.1.

1. For continuity of flow :

$$Q = A_1v_1 = A_2v_2$$

$$0.4 = \frac{\pi}{4}(0.3)^2 v_1 = \frac{\pi}{4}(0.15)^2 v_2$$

$$v_1 = 5.66 \text{ m/s}$$

$$v_2 = 22.64 \text{ m/s}$$

2. By applying Bernoulli's equation,

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_f$$

$$\frac{300 \times 10^3}{1000 \times 9.81} + \frac{(5.66)^2}{2 \times 9.81} + 0 = \frac{P_2}{1000 \times 9.81} + \frac{(22.64)^2}{2 \times 9.81} + 1.5 + 0.5$$

$$P_2 = 40113.09 \text{ N/m}^2 = 40.113 \text{ kN/m}^2$$

3. By applying the impulse momentum equation in both X and Y direction.

i. For X-direction :

$$P_1A_1 \cos\theta_1 - P_2A_2 \cos\theta_2 - F_x = \rho Q (v_2 \cos\theta_2 - v_1 \cos\theta_1)$$

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$$300 \times 10^3 \times \frac{\pi}{4}(0.3)^2 \times \cos 0° - 40.113 \times 10^3 \times \frac{\pi}{4} \times (0.15)^2 \times \cos 45° - F_x$$
$$= 1000 \times 0.4\,(22.64 \cos 45° - 5.64 \cos 0°)$$
$$F_x = 16.556 \text{ kN}$$

ii. For Y-direction :

$$P_1A_1 \sin\theta_1 - P_2A_2 \sin\theta_2 + F_y + w = \rho Q\,(v_2 \sin\theta_2 + v_1 \sin\theta_1)$$
$$F_y - 40.113 \times 10^3 \times \frac{\pi}{4}(0.15)^2 \sin 45° + (0.075 \times 9810)$$
$$= 1000 \times 0.4\,(22.64 \sin 45°)$$
$$F_y = 6.169 \text{ kN}$$

3. Resultant Force :

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(16.556)^2 + (6.169)^2}$$
$$F = 17.66 \text{ kN}$$

4. $$\tan\alpha = \frac{6.169}{16.556}$$
$$\alpha = \tan^{-1}\left(\frac{6.169}{16.556}\right) = 20.43°$$

Force of 17.66 kN acts on the bend at an angle of 20.43° from inlet axis.

Que 3.23. Explain the principle of venturimeter with a neat sketch. Derive the expression for rate of flow of fluid through it. 250 liters per second of water is flowing in a pipe having a diameter of 30 cm. If the pipe is bent by 135°, find the magnitude and direction of resultant force on the bend. The pressure of the water flowing in the pipe is 400 kPa.

Answer

A. Venturimeter : Refer Q. 3.10, Page 3–10A, Unit-3.

B. Numerical :

Given : Diameter of the bend at inlet, $D_1 = 300$ mm $= 0.3$ m
Diameter of the bend at outlet, $D_2 = 300$ mm $= 0.3$ m
Discharge, $Q = 250$ litres/sec $= 0.25$ m³/s
Pressure, $p_1 = p_2 = 400$ kN/m²
To Find : Magnitude and direction of resultant force on the bend.

1. Area $A_1 = A_2 = \pi/4 \times 0.3^2 = 0.07068$ m²
2. Velocity at section 1-1, $v_1 = \frac{Q}{A_1} = \frac{0.25}{0.07068} = 3.54$ m/s
3. Velocity at section 2-2, $v_2 = v_1 = 3.54$ m/s $(\because A_1 = A_2)$
4. Force along X-axis;

$$F_x = \frac{wQ}{g}[v_1 - (-v_2 \cos 45°)] + p_1A_1 + p_2A_2 \cos 45°$$
$$= \frac{9.81 \times 0.25}{9.81}[3.54 - (-3.54 \times 0.707)]$$
$$+ (400 \times 0.07068) + (400 \times 0.07068 \times 0.707)$$
$$= 0.25 \times (3.54 + 3.54 \times 0.707) + 28.27 + 19.98$$
$$= 49.76 \text{ kN } (\rightarrow)$$

5. Force along Y-axis :

$$F_y = \frac{wQ}{g}[0 - v_2 \sin 45°] - p_2A_2 \sin 45°$$
$$= \frac{9.81 \times 0.25}{9.81}(0 - 3.54 \times 0.707)$$
$$- 400 \times 0.07068 \times 0.707$$
$$= -0.625 - 19.98 = -20.6 \text{ kN}(\downarrow)$$

6. The magnitude of the resultant force,

$$F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{49.76^2 + 20.6^2} = 53.85 \text{ kN}$$

7. The direction of F_R with X-axis is given as

$$\tan\theta = \frac{F_y}{F_x} = \frac{20.6}{49.76} = 0.414$$
$$\theta = \tan^{-1} 0.414 = 22.5°$$

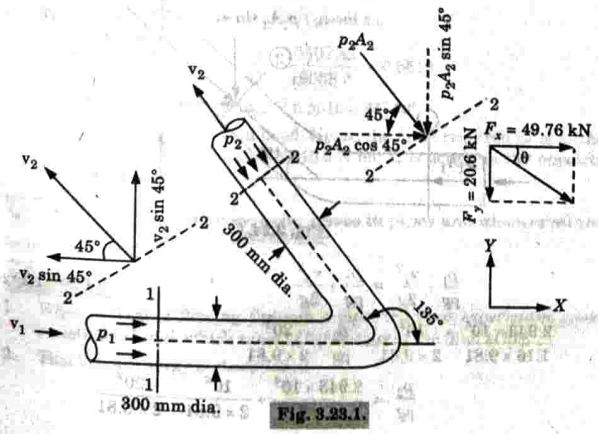

Fig. 3.23.1.

Que 3.24. **In a 45° bend a rectangular air duct of 1 m² cross-sectional area is gradually reduces to 0.5 m² area. Find the magnitude and direction of the force required to hold the duct in position if the velocity of flow at the 1 m² section is 10 m/s and pressure is 2.943 N/cm². Take density of air as 1.16 kg/m³.**

UPTU 2014-15, Marks 05

Answer

Given : Area at section (1), $A_1 = 1$ m, Area at section (2), $A_2 = 0.5$ m²
Velocity at section (1), $v_1 = 10$ m/s
Pressure at section (1), $p_1 = 2.943$ N/cm² $= 2.943 \times 10^4$ N/m²
$= 29430$ N/m², Density of air, $\rho = 1.16$ kg/m³
To Find : Magnitude and direction of the force.

1. Applying continuity equation at sections (1) and (2)

$$A_1 v_1 = A_2 v_2$$

$$\therefore \quad v_2 = \frac{A_1 v_1}{A_2} = \frac{1}{0.5} \times 10 = 20 \text{ m/s}$$

2. Discharge, $Q = A_1 v_1 = 1 \times 10 = 10$ m³/s
3. Applying Bernoulli's equation at (1) and (2)

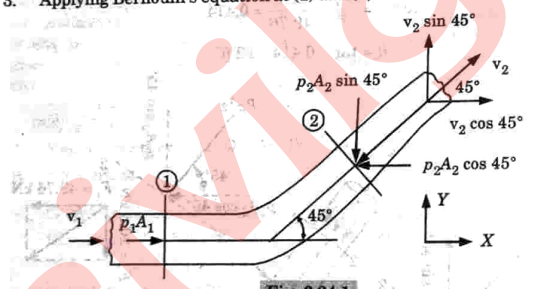

Fig. 3.24.1.

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} \qquad [\because z_1 = z_2]$$

$$\frac{2.943 \times 10^4}{1.16 \times 9.81} + \frac{10^2}{2 \times 9.81} = \frac{p_2}{\rho g} + \frac{20^2}{2 \times 9.81}$$

$$\therefore \quad \frac{p_2}{\rho g} = \frac{2.943 \times 10^4}{1.16 \times 9.81} + \frac{10^2}{2 \times 9.81} - \frac{20^2}{2 \times 9.81}$$

$$= 2586.2 + 5.0968 - 20.387 = 2570.90 \text{ m}$$

$$\therefore \quad p_2 = 2570.90 \times 1.16 \times 9.81 = 29255.8 \text{ N}$$

5. Force along X-axis, $F_x = \rho Q\,[v_{1x} - v_{2x}] + (p_1A_1)_x + (p_2A_2)_x$

where, $A_{1x} = 1$ m², $v_{2x} = v_2 \cos 45° = 20 \times 0.7071$,

$(p_1A_1)_x = p_1A_1 = 29430 \times 1 = 29430$ N

and $(p_2A_2)_x = -p_2A_2 \cos 45° = -p_2A_2 \cos 45°$

$= -29255.8 \times 0.5 \times 0.7071$

$$\therefore \quad F_x = 1.16 \times 10 \times [10 - 20 \times 0.7071] + 29430 \times 1 - 29255.8 \times 0.5 \times 0.7071$$

$$= -48.04 + 29430 - 10343.37$$

$$= 19038.59 \text{ N}$$

6. Similarly force along Y-axis,

$$F_y = \rho Q[v_{1y} - v_{2y}] + (p_1A_1)_y + (p_2A_2)_y$$

where $v_{1y} = 0$, $v_{2y} = v_2 \sin 45° = 20 \times 0.7071 = 14.142$

$(p_1A_1)_y = 0$, $(p_2A_2)_y = -p_2A_2 \sin 45° = -29255.8 \times 0.5 \times 0.7071$

$= -10343.37$

$$F_y = 1.16 \times 10[0 - 14.142] + 0 - 10343.37$$

$$= -164.05 - 10343.37 = -10507.42 \text{ N}$$

7. Resultant force, $F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{(19038.6)^2 + (10507.42)^2}$

$= 21746.6$ N

8. The direction of F_R with X-axis is given as,

$$\tan\theta = \frac{F_y}{F_x} = \frac{10507.42}{19038.6} = 0.5519$$

$$\therefore \quad \theta = \tan^{-1} 0.5519 = 28° 53'.$$

9. F_R is the force exerted on bend. Hence the force required to hold the duct in position is equal to 21746.6 N but it is acting in the opposite direction of F_R.

Que 3.25. **Write a short note on losses in pipes and discuss minor losses in detail.**

Answer

1. When a fluid is flowing through a pipe, the fluid experiences some resistance due to which some of the energy of the fluid is lost.
2. This loss of energy is classified as :

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Energy Losses

A. Major Energy Losses	**B. Minor Energy Losses**
This is due to friction and it is calculated by the following formulae	a. Sudden expansion of pipe b. Sudden contraction of pipe c. Bend in pipe d. Pipe fittings e. An obstruction in pipe
a. Darcy - Weisbach formula b. Chezy's formula	

A. Major Losses or Loss of Energy or Head due to Friction :

a. Darcy - Weisbach Formula for Head Loss due to Friction :

1. The equation is, $h_f = \dfrac{4fLv^2}{2g \times d}$

Where, h_f = Loss of head due to friction,

f = Coefficient of friction and it is a function of Reynold's number

$= \dfrac{16}{Re}$ for $Re < 2000$ (laminar flow)

$= \dfrac{0.079}{Re^{1/4}}$ for Re varying from 4000 to 10^8

L = Length of pipe,

v = Mean velocity of flow, and

d = Diameter of pipe.

b. Chezy's Formula for Loss of Head due to Friction in Pipes :

1. The equation is, $h_f = \dfrac{f'}{\rho g} \times \dfrac{P}{A} \times L \times v^2$

Where, h_f = Loss of head due to friction,

P = Wetted perimeter of pipe,

A = Area of cross section of pipe,

L = Length of pipe, and

v = Mean velocity of flow.

2. The ratio of $\dfrac{A}{P} = \dfrac{\text{Area of flow}}{\text{Perimeter (wetted)}}$ is called hydraulic mean depth or hydraulic radius and is denoted by m.

B. Minor Energy or Head Losses :

1. The loss of energy due to change of velocity of the flowing fluid in magnitude or direction is called minor loss of energy.
2. The minor loss of energy includes the following :

a. Loss of Head due to Sudden Enlargement :

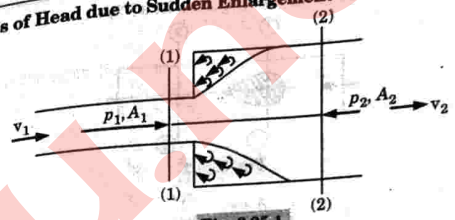

Fig. 3.25.1.

$$\therefore \quad h_e = \frac{(v_1 - v_2)^2}{2g}$$

b. Loss of Head due to Sudden Contraction :

$$h_c = \frac{v_2^2}{2g}\left[\frac{1}{C_c} - 1\right]^2 = K.\frac{v_2^2}{2g}$$

Where $K = \left[\dfrac{1}{C_c} - 1\right]^2$

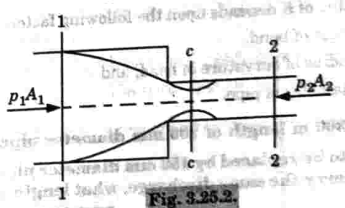

Fig. 3.25.2.

c. Loss of Head at the Entrance of a Pipe :

1. This type of loss occurs when a liquid enters a pipe which is connected to a large tank or reservoir.
2. Loss of head at the entrance (or inlet) of a pipe with sharp cornered entrance is taken as $0.5\dfrac{v^2}{2g}$.

Where, v = Velocity of liquid in pipe.

d. Loss of Head at the Exit of Pipe :

1. This loss is denoted by h_o.

$$h_o = \frac{v^2}{2g}, \text{ Where v = Velocity at outlet of pipe.}$$

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e. Loss of Head due to an Obstruction in a Pipe :

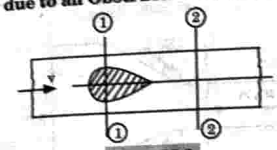

Fig. 3.25.3.

$$h_e = \frac{\left(\frac{A \times v}{C_c(A-a)} - v\right)^2}{2g} = \frac{v^2}{2g}\left[\frac{A}{C_c(A-a)} - 1\right]^2$$

f. Loss of head in pipe due to bend :

1. $$h_b = \frac{Kv^2}{2g}$$

Where, h_b = Loss of head due to bend,
v = Velocity of flow, and
K = Coefficient of bend.

2. The value of K depends upon the following factors :
   a. Angle of bend,
   b. Radius of curvature of bend, and
   c. Diameter of pipe.

Que 3.26. If 300 m length of 200 mm diameter pipe with friction factor 0.018 is to be replaced by 150 mm diameter pipe with friction factor 0.02 to carry the same discharge, what length will have to be provided ? **AKTU 2012-13, Marks 10**

Answer

Given : $L_1 = 0.3$ m, $d_1 = 0.2$ m, $f_1 = 0.018$, $d_2 = 0.15$ m, $f_2 = 0.02$
To Find : Length of the pipe

1. Here, $h_{f1} = h_{f2}$

$$\frac{4f_1L_1v_1^2}{2gd_1} = \frac{4f_2L_2v_2^2}{2gd_2}$$

$$\frac{0.018 \times 0.3 \times v_1^2}{0.2} = \frac{0.02 \times L_2 \times v_2^2}{0.15}$$

$$\frac{v_1}{v_2} = \sqrt{\frac{0.02 \times L_2 \times 0.2}{0.018 \times 0.3}}$$

$$\frac{v_1}{v_2} = \sqrt{0.74 \times L_2}$$

2. Now for same discharge

$$Q_1 = Q_2$$
$$A_1v_1 = A_2v_2$$
$$\frac{v_1}{v_2} = \frac{A_2}{A_1}$$
$$\frac{v_1}{v_2} = \left(\frac{d_2}{d_1}\right)^2$$
$$\sqrt{0.74 \times L_2} = \left(\frac{0.15}{0.20}\right)^2 = 0.5625$$
$$0.74 \times L_2 = 0.3164$$
$$L_2 = 0.428 \text{ m}$$

So, the length of the pipe be provided = 0.428 m

Que 3.27. Derive the head loss expressions for the pipes in series and parallel.

Answer

A. Pipe in Series :

1. Let, L_1, L_2, L_3 = Length of pipes 1, 2, and 3 respectively,
d_1, d_2, d_3 = Diameter of pipes 1, 2, and 3 respectively,

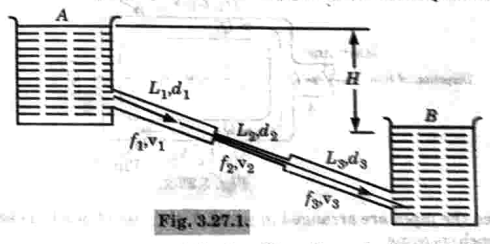

Fig. 3.27.1.

v_1, v_2, v_3 = Velocity of flow through pipes 1, 2, 3, and
f_1, f_2, f_3 = Coefficient of friction for pipes 1, 2, and 3.
H = Difference of water level in two tanks.

2. Pipes are in series, as shown in Fig. 3.27.1, hence the discharge passing through each pipe is same.

$$\therefore \quad Q = A_1v_1 = A_2v_2 = A_3v_3$$

3. The difference in liquid surface levels is equal to the sum of the total head loss in the pipes.

$$\therefore \quad H = \frac{0.5v_1^2}{2g} + \frac{4f_1L_1v_1^2}{d_1 \times 2g} + \frac{0.5v_2^2}{2g} + \frac{4f_2L_2v_2^2}{d_2 \times 2g} + \frac{(v_2 - v_3)^2}{2g} + \frac{4f_3L_3v_3^2}{d_3 \times 2g} + \frac{v_2^2}{2g} \quad ...(3.27.1)$$

4. If minor losses are neglected, then the eq. (3.27.1) becomes

$$H = \frac{4f_1L_1v_1^2}{d_1 \times 2g} + \frac{4f_2L_2v_2^2}{d_2 \times 2g} + \frac{4f_3L_3v_3^2}{d_3 \times 2g} \quad ...(3.27.2)$$

5. If the coefficient of friction is same for all pipes, *i.e.*, $f_1 = f_2 = f_3 = f$ then eq. (3.27.2) becomes as

$$H = \frac{4f_1L_1v_1^2}{d_1 \times 2g} + \frac{4f_2L_2v_2^2}{d_2 \times 2g} + \frac{4f_3L_3v_3^2}{d_3 \times 2g}$$

$$H = \frac{4f}{2g}\left[\frac{L_1v_1^2}{d_1} + \frac{L_2v_2^2}{d_2} + \frac{L_3v_3^2}{d_3}\right] \quad ...(3.27.3)$$

B. Pipe in Parallel :

1. The pipes are said to be in parallel (Fig. 3.27.2) when a main line divides into two or more parallel pipes which again join together downstream and continues as a main line.
2. It may be seen from Fig. 3.27.2, the rate of discharge in the main line is equal to the pipes.

Thus, $Q = Q_1 + Q_2$ ...(3.27.4)

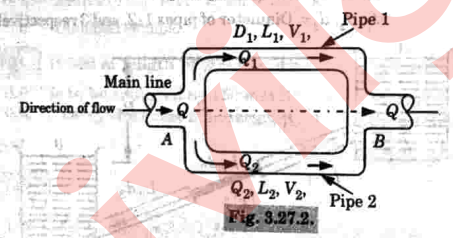

Fig. 3.27.2.

3. When the pipes are arranged in parallel, the loss of head in each pipe (branch) is same.

∴ Loss of head in pipe 1 = Loss of head in pipe 2.

or $h_f = \frac{4f_1L_1V_1^2}{D_1 \times 2g} = \frac{4f_2L_2V_2^2}{D_2 \times 2g}$

When, $f_1 = f_2$, then ...(3.27.5)

$$\frac{L_1V_1^2}{D_1} = \frac{L_2V_2^2}{D_2} \quad ...(3.27.6)$$

Que 3.28. **A main pipe divides into two parallel pipes which again forms one pipe. The length and diameter of the first parallel pipe are 2000 m and 1.0 m respectively, while the length and diameter of second parallel pipe are 2000 m and 0.8 m. Find the rate of flow in each parallel pipe, if total flow in the main is 3.5 m³/s. The coefficient of friction for each parallel pipe is same and equal to 0.005.**

Answer

Given :

Discharge through main pipe, $Q = 3.5$ m³/s

Coefficient of friction, $f = 0.005$

Length of first pipe, $L_1 = 2000$ m and diameter $d_1 = 1.0$ m

Length of second pipe, $L_2 = 2000$ m and diameter $d_2 = 0.8$ m

To Find : Rate of Flow in each Parallel Pipe.

1. For parallel pipes, $Q = Q_1 + Q_2 = A_1v_1 + A_2v_2$ ....(3.28.1)
2. Loss of head for branch pipe (1) = Loss of head for branch pipe (2)

$$\frac{4f_1L_1v_1^2}{d_1 \times 2g} = \frac{4f_2L_2v_2^2}{d_2 \times 2g}$$

$$\frac{L_1v_1^2}{d_1} = \frac{L_2v_2^2}{d_2} \quad (\text{As } f_1 = f_2 = f)$$

$$\frac{2000.v_1^2}{1} = \frac{2000.v_2^2}{0.8}$$

$$v_1^2 = \frac{v_2^2}{0.8} \Rightarrow \left(\frac{v_1}{v_2}\right)^2 = \frac{1}{0.8}$$

3. From eq. (3.28.1)

$$3.5 = \frac{\pi}{4}d_1^2 \cdot v_1 + \frac{\pi}{4}d_2^2 \cdot v_2$$

$$3.5 = \frac{\pi}{4}(1.0)^2 \cdot v_1 + \frac{\pi}{4}(0.8)^2 \cdot v_2$$

$$4.456 = v_1 + 0.64\,v_2$$

$$v_1 = 4.456 - 0.64v_2$$

$$\frac{v_1}{v_2} = \frac{4.456}{v_2} - 0.64$$

$$\sqrt{\frac{1}{0.8}} = \frac{4.456}{v_2} - 0.64$$

$$v_2 = \frac{4.456}{\left(0.64 + \sqrt{\frac{1}{0.8}}\right)} = 2.5346 \text{ m/s}$$

$$v_1 = 4.456 - 0.64 \times 2.5346$$
$$v_1 = 2.8338 \text{ m/s}$$

4. Flow rate for pipe (1),

$$Q_1 = A_1 v_1 = \frac{\pi}{4} d_1^2 \ (v_1) = \frac{\pi}{4} (1.0)^2 (2.8338) = 2.23 \text{ m}^3/\text{s}$$

5. Flow rate for pipe (2),

$$Q_2 = A_2 v_2 = \frac{\pi}{4} d_2^2 v_2 = \frac{\pi}{4} (0.8)^2 (2.5346) = 1.274 \text{ m}^3/\text{s}$$

Que 3.29. **A compound piping system consists of 1800 m of 0.50 m, 1200 m of 0.40 m and 600 m of 0.30 m new cast iron pipes connected in series. Convert the system to (a) an equivalent length of 0.40 m pipe, and (b) equivalent size pipe of 3600 m long.**

AKTU 2015-16, Marks 05

Answer

Given : $L_1 = 1800$ m, $D_1 = 0.50$ m, $L_2 = 1200$ m, $D_2 = 0.40$ m, $L_3 = 600$ m, $D_3 = 0.30$ m

To Find : a. Equivalent length of 0.40 m pipe, and
b. Equivalent size pipe of 3600 m long

1. From equivalent pipe size equation,

$$\frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5} = \frac{L}{D^5}$$

$$\frac{L}{D^5} = \frac{1800}{0.50^5} + \frac{1200}{0.40^5} + \frac{600}{0.30^5}$$

$$\frac{L}{D^5} = 421701.08 \qquad \text{...(3.29.1)}$$

2. Equivalent length of 0.40 m pipe,
Putting $D = 0.40$ m in eq. (3.29.1), we get
$L = 4318.21$ m

3. Equivalent size of 3600 m long pipe,
Putting $L = 3600$ m in eq. (3.29.1), we get
$D = 0.3857$ m

Que 3.30. **Derive an expression for power transmission through pipes.**

Answer

1. Power transmitted by a pipe depends upon
   a. The discharge passing through pipe.
   b. The total head of water.

2. Consider a tank and pipe connected system as shown in Fig. 3.30.1.

Fig. 3.30.1.

3. Let,
H = Head of water at inlet of pipe,
L = Length of pipe,
D = Diameter of pipe,
v = Velocity of water in pipe,
f = Coefficient of friction, and
h_f = Head loss in pipe due to friction.

4. The head available at outlet of pipe
= total head at inlet – head loss due to friction in pipe

$$= H - h_f$$

$$= H - \frac{4fLv^2}{d \times 2g} \qquad \left(\because h_f = \frac{4fLv^2}{2dg}\right)$$

5. Weight of water flowing through pipe per second
$W = \rho g \times$ volume of water/s
$= \rho g \times$ area $\times$ velocity

$$= \rho g \frac{\pi}{4} d^2 \times v$$

6. Power transmitted at outlet of pipe
= Weight of water/s × head at outlet

$$= \rho g \frac{\pi}{4} d^2 v \left(H - \frac{4fLv^2}{d \times 2g}\right) \text{ W}$$

Que 3.31. **What is meant by water hammer ? Give expression for the rise of pressure due to water hammer.**

Answer

A. Water Hammer :

1. In a long pipe, when flowing water is suddenly brought to rest by closing the value or by any similar cause, there will be a sudden rise in pressure due to the momentum of water being destroyed. This phenomenon of sudden rise in pressure is known as water hammer or hammer blow.
2. A sudden rise in pressure has the effect of hammering action on the walls of the pipe.

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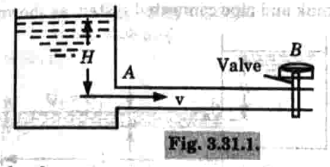

Fig. 3.31.1.

3. The magnitude of pressure rise as a result of water hammer depends upon the following factors :
   a. The velocity of flow of water in pipe.
   b. The length of pipe.
   c. Time taken to close the valve.
   d. Elastic properties of the material of the pipe.
4. The following cases of water hammer in pipes will be considered :

a. Gradual Closure of Valve :

1. Let, A = area of cross section of the pipe AB,
   L = length of pipe,
   v = velocity of flow of water through pipe,
   t = time (in second) required to close the valve, and
   p = intensity of pressure wave produced.
2. Mass of water in pipe
   $$AB = \rho \times \text{volume of water} = \rho \times A \times L$$
3. The valve is gradually closed in time 't' seconds and hence the water is brought from initial velocity v to zero velocity in time 't' seconds.
4. Retardation of water
   $$= \frac{\text{change of velocity}}{\text{time}} = \frac{v - 0}{t} = \frac{v}{t}$$
5. Retarding force = Mass × retardation = $\rho AL \times \frac{v}{t}$ ...(3.31.1)
6. If p is the intensity of pressure wave produced due to closure of the valve, the force due to pressure wave
   $$= p \times \text{area of pipe} = p \times A \quad ...(3.31.2)$$
7. Equating the two forces, given by eq. (3.31.1) and eq. (3.31.2),
   $$\rho AL \times \frac{v}{t} = p \times A$$
   $$p = \frac{\rho L v}{t} \quad ...(3.31.3)$$
8. Head of pressure,
   $$H = \frac{p}{\rho g} = \frac{\rho L v}{\rho g \times t}$$
   or
   $$H = \frac{Lv}{gt} \quad ...(3.31.4)$$

i. The valve closure is said to gradual if $t > \frac{2L}{C}$
   Where t = time in second, C = velocity of pressure wave.

ii. The valve closure is said to be sudden if $t < \frac{2L}{C}$

b. Sudden Closure of Valve and Pipe is Rigid :

1. Let the pipe is rigid and valve fitted at the end B is closed suddenly.
2. Let, K = bulk modulus of water.
3. When the valve is closed suddenly, the kinetic energy of the flowing water is converted into strain energy of water if the effect of friction is neglected and pipe wall is assumed perfectly rigid.
4. Loss of kinetic energy = $\frac{1}{2}$ × mass of water in pipe × v^2
   $$= \frac{1}{2} \times \rho AL \times v^2$$
5. Gain of strain energy
   $$= \frac{1}{2}\left(\frac{p^2}{K}\right) \times \text{volume} = \frac{1}{2}\frac{p^2}{K} \times AL$$
6. On equating loss of kinetic energy to gain of strain energy,
   $$\frac{1}{2}\rho AL \times v^2 = \frac{1}{2}\frac{p^2}{K} \times AL$$
   $$p^2 = \frac{1}{2}\rho AL \times v^2 \times \frac{2K}{AL} = \rho K v^2$$
   $$p = \sqrt{\rho K v^2} = v\sqrt{K\rho}$$
   $$= v\sqrt{\frac{K\rho^2}{\rho}} = \rho v \times C \quad (\because \sqrt{K/\rho} = C)$$

   Where, C = velocity of pressure wave.

Que 3.32. What is pipe network ? Give the necessary conditions of pipe network.

Answer

1. A pipe network is a system in which many pipes are interconnected and they form several loops or circuits of pipes.
2. Example :
   a. Water supply system in a city is the very commonly used pipe network system.
   b. Supply of steam from boiler to other machineries is done by pipe network system, etc.

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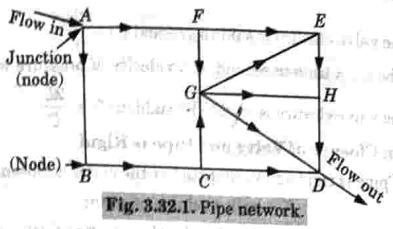

Fig. 3.32.1. Pipe network.

3. In such system, it is required to determine the distribution of flow through the various pipes of the network.

A. Necessary Conditions for any Pipe Network :

1. The system should follow the continuity equation *i.e.*,
   Flow into the junction = Flow out of the junction
2. For a loop of pipe circuit,
   $$\Sigma h_f = 0$$
3. The head loss in each pipe is expressed as,
   $$h_f = r.\,Q^n$$
   Where, r = constant and n = 2 for turbulent flow.

Que 3.33. **Three reservoirs, *A*, *B* and *C* are connected by a pipe system having lengths 700 m, 1200 m and 500 m and diameters 400 mm, 300 mm and 200 mm respectively. The water levels in reservoir *A* and *B* from a datum line are 50 m and 45 m respectively. The level of water in reservoir *C* is below the level of water in reservoir *B*. Find the discharge into or from the reservoirs *B* and *C*. If the rate of flow from reservoir *A* is 150, find the height of water level in the reservoir *C*. (Take f = 0.005, for all pipes)**

Answer

Given : L_1 = 700 m, d_1 = 400 mm, Q_1 = 150 litre/s
L_2 = 1200 mm, d_2 = 300 mm, L_3 = 500 mm, d_3 = 200 mm, z_A = 50 m
z_B = 45 m
To Find : i. Discharge into or from the reservoirs *B* and *C*.
ii. Height of water level in the reservoir *C*.

1. Applying Bernoulli's equation to point *E* and *D*,
   $$z_A = z_D + \frac{p_D}{\rho g} + h_{f1}$$
   Where, $$h_{f1} = \frac{4fL_1v_1^2}{2d_1g}$$

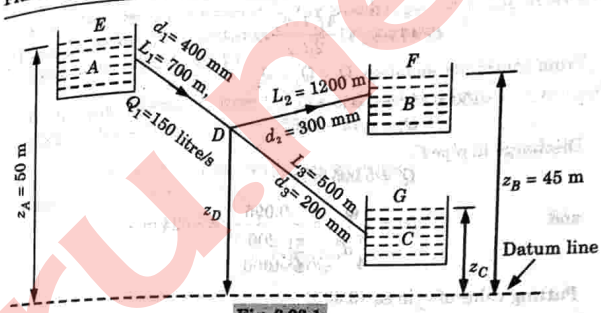

Fig. 3.33.1.

$$v_1 = \frac{Q_1}{\text{Area}} = \frac{(150\times10^{-3})}{\frac{\pi}{4}(0.4)^2} = 1.19 \text{ m/s}$$

$$h_{f1} = \frac{4\times0.005\times700(1.19)^2}{2\times\left(\frac{400}{1000}\right)\times9.81} = 2.53 \text{ m}$$

So, $$z_D + \frac{p_D}{\rho g} = z_A - h_{f1} = 50 - 2.53 = 47.47 \text{ m}$$

Here, piezometric head at D = 47.47. But z_B = 45 m, hence water flows from D to B.

2. Applying Bernoulli's equation to point *B* and *D*,
   $$z_D + \frac{p_D}{\rho g} = z_B + h_{f2}$$
   $$47.47 = 45 + h_{f2}$$
   $$h_{f2} = 2.47 \text{ m}$$
3. $$h_{f2} = \frac{4f\,L_2v_2^2}{d_2\times2g} \Rightarrow v_2 = \sqrt{\frac{2gh_{f2}\cdot d_2}{4f\,L_2}}$$
   $$v_2 = \sqrt{\frac{2\times9.81\times2.47}{4\times0.005\times1200}\left(\frac{300}{1000}\right)} = 0.778 \text{ m/s}$$
4. Discharge in pipe (2), $Q_2 = A_2 \cdot v_2$
   $$= \frac{\pi}{4}d_2^2\cdot v_2$$
   $$= \frac{\pi}{4}\left(\frac{300}{1000}\right)^2(0.778)$$
   $$Q_2 = 0.055 \text{ m}^3/\text{s} = 55 \text{ litres/s}$$
5. Apply Bernoulli's equation to *D* and *C*,
   $$z_D + \frac{p_D}{\rho g} = z_C + h_{f3}$$

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$$47.47 = z_C + \frac{4f\,L_3 v_3^2}{2d_3 g} \quad ...(3.33.1)$$

6. From continuity equation, $Q_1 = Q_2 + Q_3$

$$150 \times 10^{-3} = 0.055 + Q_3$$

$$Q_3 = 0.15 - 0.055$$

7. Discharge in pipe C,

$$Q_3 = 0.095 \text{ m}^3/\text{s}$$

and

$$v_3 = \frac{Q}{\frac{\pi}{4} d_3^2} = \frac{0.095}{\frac{\pi}{4}\left(\frac{200}{1000}\right)^2} = 3.024 \text{ m/s}$$

8. Putting value of v_3 in eq. (3.33.1)

$$47.47 = z_C + \frac{4 \times 0.005 \times 500 \times (3.024)^2}{2 \times \left(\frac{200}{1000}\right) \times 9.81}$$

$$z_C = 24.165 \text{ m}$$

VERY IMPORTANT QUESTIONS

Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.

Q. 1. What do you understand by the term ideal fluid flow or potential flow ? Discuss its importance.

Ans. Refer Q. 3.1, Unit-3.

Q. 2. Derive Euler's equation of motion.

Ans. Refer Q. 3.5, Unit-3.

Q. 3. How will you obtain Bernoulli's equation from Euler's equation of motion along a streamline ? Write assumptions of Bernoulli's equation.

Ans. Refer Q. 3.7, Unit-3.

Q. 4. Suggest the device used for the measurement of fluid flow through ducts or pipes. Explain them.

Ans. Refer Q. 3.10, Unit-3.

Q. 5. Derive the expression for discharge over the following notches : A. Rectangular, B. Triangular.

Ans. Refer Q. 3.17, Unit-3.

Q. 6. Describe momentum equation. Where this equation is used ?

Ans. Refer Q. 3.20, Unit-3.

Q. 7. Write a short note on losses in pipes.

Ans. Refer Q. 3.25, Unit-3.

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4 UNIT

Flow Through Pipes

Part-1 ..... **(4–2A to 4–17A)**

- *Equation of Motion for Laminar Flow through Pipes*
- *Stoke's Law*
- *Transition from Laminar to Turbulent Flow*

A. Concept Outline : Part-1 ..... 4–2A
B. Long and Medium Answer Type Questions 4–3A

Part-2 ..... **(4–18A to 4–29A)**

- *Turbulent flow • Types of Turbulent Flow • Isotropic, Homogeneous Turbulence • Scale and Intensity of Turbulence*
- *Measurement of Turbulence • Eddy Viscosity • Mixing Length Concept and Velocity Distribution in Turbulent Flow Over Smooth and Rough Surfaces*

A. Concept Outline : Part-2 ..... 4–18A
B. Long and Medium Answer Type Questions 4–18A

Part-3 ..... **(4–29A to 4–48A)**

- *Boundary Layer Thickness • Boundary Layer Over a Flat Plate*
- *Laminar Boundary Layer • Application of Momentum Equation*
- *Turbulent Boundary Layer • Laminar Sub-layer*
- *Separation and its Control*

A. Concept Outline : Part-3 ..... 4–29A
B. Long and Medium Answer Type Questions 4–30A

PART-1

Equation of Motion for Laminar Flow Through Pipes, Stoke's Law and Transition from Laminar to Turbulent Flow.

CONCEPT OUTLINE : PART-1

Relation Between Pressure Gradient and Shear Stress :

$$\frac{\partial \tau}{\partial y} = \frac{\partial p}{\partial x}$$

Reynold's number, $Re = \frac{\rho v D}{\mu} = \frac{\rho v L}{\mu}$

For a circular pipe, $Re < 2000 \Rightarrow$ Laminar flow.
$Re > 4000 \Rightarrow$ Turbulent flow.

For Laminar or Viscous Flow Through Circular Pipes :

a. Shear stress, $\tau = -\frac{\partial p}{\partial x}\frac{r}{2}$

b. Velocity, $u = -\frac{1}{4\mu}\cdot\frac{\partial p}{\partial x}(R^2 - r^2)$

Maximum velocity, $u_{max} = -\frac{1}{4\mu}\cdot\frac{\partial p}{\partial x}R^2$

Average velocity, $\bar{u} = \frac{u_{max}}{2}$

c. Loss of pressure head,

$$h_f = \frac{32\,\mu\bar{u}L}{\rho g.D^2}$$

Viscous or Laminar Flow Between Two Parallel Plates :

I. Couette Flow : It is a kind of flow between two parallel plates when one plate is moving and other at rest.

Velocity distribution, $u = \frac{U}{b}y - \frac{1}{2\mu}\frac{\partial p}{\partial x}(by - y^2)$

Discharge per unit width, $q = \frac{U.b}{2} - \frac{b^3}{12\mu}\cdot\frac{\partial p}{\partial x}$

Shear stress distribution, $\tau = \mu\frac{U}{b} - \frac{1}{2}\frac{\partial p}{\partial x}(b - 2y)$

II. When Both Plates are at Rest :

$$u = -\frac{1}{2\mu}\frac{\partial p}{\partial x}(by - y^2)$$

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$$q = -\frac{b^3}{2\mu}\cdot\frac{\partial p}{\partial x}$$

$$\tau = -\frac{1}{2}\cdot\frac{\partial p}{\partial x}(b-2y)$$

The Coefficient of Viscosity by Falling Sphere Method :

$$\mu = \frac{gd^2}{18v}(\rho_s - \rho_f)$$

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.1. **What are the characteristics of laminar flow ? Give some examples of laminar (or viscous) flow.**

Answer

A. Characteristics :

1. There exists a shear stress in laminar flow, which is given by Newton's law of viscosity,

$$\tau = \mu\cdot\frac{du}{dy}$$

2. The laminar flow is rotational.
3. There is a continuous dissipation of energy and for maintaining the flow we must supply the energy externally.
4. Loss of energy (due to shear) $\propto$ v.μ

   Where, v = velocity of flowing fluid, and

   μ = viscosity of flowing fluid.
5. No slip will occur at the boundary.
6. There will be no mixing of layers occur.
7. For laminar flow, Reynold's number < 2000.

B. Examples of Laminar Flow :

1. Movement of blood in veins and arteries of body.
2. Rise of water in plants through their roots.
3. Flow past tiny bodies.

Que 4.2. **Sketch the shear stress and velocity profiles across a section of a circular pipe, for the viscous flow. Derive the expression governing the shear stress and velocity profiles.**

OR

Derive the equation of motion for laminar flow through pipes. Also derive the expression for velocity and shear stress distribution across the pipe with neat sketches.

Answer

1. Let us consider a horizontal pipe having diameter '*d*' and radius '*R*'.
2. Direction of fluid is shown in Fig. 4.2.1.
3. Take a fluid element in between the radius *r* and *r* + *dr* and length of the fluid element be Δ*x*.
4. If *p* is the pressure on the face *AB*, then pressure on face *CD* will be

$$p + \frac{\partial p}{\partial x}.\Delta x.$$

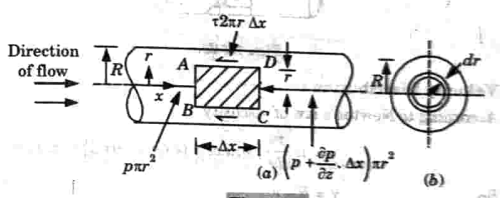

Fig. 4.2.1.

5. Total pressure force = pressure force at face *AB* – pressure force at face *CD*

$$= p.\pi r^2 - \left(p + \frac{\partial p}{\partial x}.\Delta x\right)\pi r^2$$

$$= -\frac{\partial p}{\partial x}.\Delta x.\pi r^2$$

6. The shear force acting on the surface *AD* and *BC*

$$= -\tau.2\pi r.\Delta x \text{ (opposite to the direction of flow)}$$

A. Shear Stress Distribution :

1. Now, $\Sigma F = 0$

$$-\frac{\partial p}{\partial x}.\Delta x.\pi r^2 - \tau.2\pi r.\Delta x = 0$$

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$$\frac{\partial p}{\partial x} \cdot r = -2.\tau$$

$$\tau = -\frac{r}{2} \cdot \frac{\partial p}{\partial x}, \text{ shear stress distribution}$$

2. At $r = R$,

Wall shear stress, $\tau_w = \frac{-R}{2} \frac{\partial p}{\partial x}$

3. As $\frac{\partial p}{\partial x} = \text{constant, so } \tau \propto r$

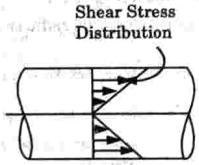


Fig. 4.2.2.

B. Velocity Distribution :

1. According to Newton's law of viscosity

$$\tau = \mu . \frac{du}{dy}, \text{ where } y \text{ is measured from pipe wall.}$$

2. So, $y = R - r$

Differentiating both the sides,

$$dy = -dr \qquad (dR = 0, \text{ as } R \text{ is constant})$$

3. Therefore, $\tau = -\mu \frac{du}{dr}$

4. Since,

$$\tau = -\mu \frac{du}{dr} = -\frac{r}{2} \frac{\partial p}{\partial x}$$

$$\frac{du}{dr} = \frac{1}{2\mu} \frac{\partial p}{\partial x} r$$

$$du = \frac{1}{2\mu} \cdot \frac{\partial p}{\partial x} \cdot r \cdot dr$$

$\left(\text{As we know that } \frac{\partial p}{\partial x} \text{ and } \frac{1}{2\mu} \text{ are constants}\right)$

6. On integrating both the sides,

$$u = \frac{1}{4\mu} \frac{\partial p}{\partial x} r^2$$

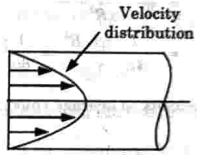
7. For $r = R, u = 0$

and $r = r, u = u$

$$[u]_u^0 = \frac{1}{4\mu} \frac{\partial p}{\partial x} [r^2]_r^R$$

$$-u = \frac{1}{4\mu} \frac{\partial p}{\partial x} [R^2 - r^2]$$

$$u = -\frac{1}{4\mu} \frac{\partial p}{\partial x} [R^2 - r^2] \qquad \text{(velocity distribution)}$$



(b)

Fig. 4.2.3.

8. Hence velocity distribution is parabolic in nature.

9. When $r = 0$,

$$u_{max.} = -\frac{1}{4\mu} \frac{\partial p}{\partial x} \cdot R^2$$

Que 4.3. **Prove that the maximum velocity in a circular pipe for viscous flow is equal to two times the average velocity of flow.**

Answer

1. Average velocity, $\bar{u} = \frac{\text{discharge } (Q)}{\text{area } (\pi R^2)}$

2. Discharge through an elemental ring of radius r,

$$dQ = \text{Velocity at radius } r\ (u) \times \text{Area of ring element}$$

$$= -\frac{1}{4\mu} \frac{\partial p}{\partial x} [R^2 - r^2] \times (2\pi r\, dr)$$

3. Total discharge, $Q = \int dQ = \int_0^R -\frac{1}{4\mu} \frac{\partial p}{\partial x} [R^2 - r^2] \times (2\pi r\, dr)$

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$$= -\frac{2\pi}{4\mu}\frac{\partial p}{\partial x}\int_0^R r(R^2 - r^2)dr$$

$$Q = -\frac{2\pi}{4\mu}\frac{\partial p}{\partial x}\int_0^R (R^2 \cdot r - r^3)dr$$

$$= -\frac{2\pi}{4\mu}\frac{\partial p}{\partial x}\left[R^2 \cdot \frac{r^2}{2} - \frac{r^4}{4}\right]_0^R$$

$$= -\frac{2\pi}{4\mu}\frac{\partial p}{\partial x}\left[\frac{R^4}{2} - \frac{R^4}{4}\right] = -\frac{2\pi}{4\mu}\cdot\frac{\partial p}{\partial x}\frac{R^4}{4}$$

4. Now, $\bar{u} = \dfrac{Q}{\pi R^2}$

$$= \frac{\left(-\frac{2\pi}{4\mu}\cdot\frac{\partial p}{\partial x}\frac{R^4}{4}\right)}{\pi R^2}$$

$$= -\frac{2}{4\mu}\cdot\frac{\partial p}{\partial x}\cdot\frac{R^2}{4} = \frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right)\cdot R^2$$

5. Ratio of maximum velocity to average velocity $= \dfrac{u_{max.}}{\bar{u}}$

$$= \frac{\frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right)\cdot R^2}{\frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right)\cdot R^2}$$

$$\Rightarrow \quad \frac{u_{max.}}{\bar{u}} = 2$$

6. So, ratio of maximum velocity to average velocity will be equal to 2.

Que 4.4. **For laminar flow between two parallel fixed plates, derive expressions for velocity and shear stress distributions.**

Answer

A. Velocity Distribution Between Fixed Parallel Plates :

1. Let us consider, two parallel plates are fixed having perpendicular distance between them 't'.
2. A viscous fluid is flowing in the direction as shown in Fig. 4.4.1.
3. For analysis consider a small fluid element of length Δx, thickness Δy at a distance y from the lower fixed plate and take width of element as unity.
4. Let the shear stress at lower face is τ and at the upper face be $\tau + \frac{\partial \tau}{\partial y}\cdot\Delta y$.

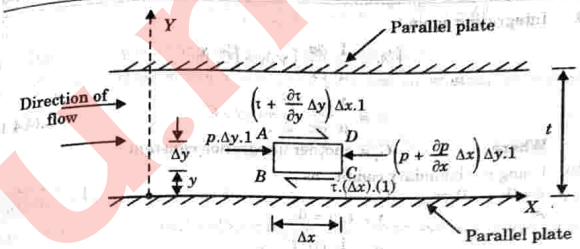

Fig. 4.4.1.

5. Forces acting on the fluid element,

a. Pressure force $= p\cdot(\Delta y\cdot 1) - \left(p + \frac{\partial p}{\partial x}\cdot\Delta x\right)\Delta y\cdot 1$

$$= -\frac{\partial p}{\partial x}\cdot\Delta x\cdot\Delta y$$

b. Total shear force acting on the fluid element

$$= \left(\tau + \frac{\partial \tau}{\partial y}\cdot\Delta y\right)\Delta x\cdot 1 - \tau\cdot\Delta x\cdot 1$$

$$= \frac{\partial \tau}{\partial y}\cdot\Delta y\cdot\Delta x$$

6. Now, $\Sigma F_x = 0$

$$-\frac{\partial p}{\partial x}\cdot\Delta x\cdot\Delta y + \frac{\partial \tau}{\partial y}\cdot\Delta y\cdot\Delta x = 0$$

$$\frac{\partial p}{\partial x} = \frac{\partial \tau}{\partial y}$$

7. From Newton's law of viscosity, $\tau = \mu\dfrac{du}{dy}$

So, $$\frac{\partial p}{\partial x} = \frac{\partial}{\partial y}\left(\mu\cdot\frac{\partial u}{\partial y}\right)$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu}\cdot\frac{\partial p}{\partial x}$$

8. On integration,

$$\frac{\partial u}{\partial y} = \frac{1}{\mu}\cdot\frac{\partial p}{\partial x}\cdot y + C_1 \quad \{\text{where } C_1 = \text{integration constant}\}$$

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9. Integrating again,

$$\int \partial u = \frac{1}{\mu}\cdot\frac{\partial p}{\partial x}\cdot\int y\,\partial y + \int C_1\,\partial y$$

$$u = \frac{1}{\mu}\cdot\frac{\partial p}{\partial x}\cdot\frac{y^2}{2} + C_1 y + C_2 \qquad ...(4.4.1)$$

Where C_2 = another integration constant.

10. Using the boundary conditions as

$y = 0, u = 0 \Rightarrow \quad C_2 = 0$

At $\quad y = t, u = 0$

$$0 = \frac{1}{\mu}\cdot\frac{\partial p}{\partial x}\cdot\frac{t^2}{2} + C_1 t$$

$$C_1 = -\frac{t}{2\mu}\frac{\partial p}{\partial x}$$

11. After putting the values of C_1 and C_2 in eq. (4.4.1),

$$u = \frac{1}{\mu}\cdot\frac{\partial p}{\partial x}\cdot\frac{y^2}{2} + \left(-\frac{t}{2\mu}\cdot\frac{\partial p}{\partial x}\right)\cdot y + 0$$

$$u = \frac{1}{2\mu}\cdot\frac{\partial p}{\partial x}\cdot y(y-t)$$

$$u = -\frac{1}{2\mu}\cdot\frac{\partial p}{\partial x}(ty - y^2)$$

Hence there is a parabolic distribution of velocity.

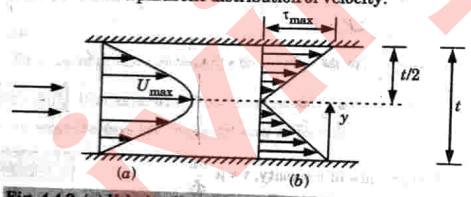

Fig. 4.4.2. (*a*) Velocity distribution, and (*b*) Shear stress distribution.

B. Shear Stress Distribution :

1. From Newton's law of viscosity,

$$\tau = \mu\cdot\frac{\partial u}{\partial y} \qquad ...(4.4.2)$$

2. On putting $u = -\frac{1}{2\mu}\frac{\partial p}{\partial x}(ty - y^2)$ in eq. (4.4.2), we get

$$\tau = \mu\frac{\partial}{\partial y}\left[-\frac{1}{2\mu}\frac{\partial p}{\partial x}(ty - y^2)\right]$$

$$\tau = -\frac{\mu}{2\mu}\left(\frac{\partial p}{\partial x}\right)(t - 2y) = -\frac{1}{2}\left(\frac{\partial p}{\partial x}\right)(t - 2y)$$

3. Hence, there is a linear distribution of shear stress.
4. Maximum shear stress at $y = 0$ and $y = t$ and shear stress is zero at $y = t/2$.

$$\tau_{max} = -\frac{1}{2}\left(\frac{\partial p}{\partial x}\right)\cdot t$$

Que 4.5. **Show that for the laminar flow of fluid in between two parallel plates, the ratio of maximum velocity to average velocity is equal to $\frac{3}{2}$.**

Answer

1. Velocity distribution,

$$u = -\frac{1}{2\mu}\frac{\partial p}{\partial x}[ty - y^2]$$

2. At $\quad y = \frac{t}{2}$

$u = U_{max}$

3. Then,

$$U_{max} = -\frac{1}{2\mu}\frac{\partial p}{\partial x}\left[t\left(\frac{t}{2}\right) - \left(\frac{t}{2}\right)^2\right]$$

$$= -\frac{1}{2\mu}\frac{\partial p}{\partial x}\left[\frac{t^2}{2} - \frac{t^2}{4}\right]$$

$$U_{max} = -\frac{1}{8\mu}\frac{\partial p}{\partial x}\cdot t^2$$

4. Let us consider that area of cross section = $t \times 1$
5. The rate of flow through small elemental strip dQ,

$dQ = u \times$ area of strip

$$= -\frac{1}{2\mu}\frac{\partial p}{\partial x}(ty - y^2)\times(dy.1) \qquad (\because \text{ Area of strip} = dy \times 1)$$

6. Total discharge, $\quad Q = \int dQ$

$$= \int_0^t -\frac{1}{2\mu}\frac{\partial p}{\partial x}(ty - y^2)dy$$

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$$= -\frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)\int_0^t (ty - y^2)dy$$

$$= -\frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)\left[\frac{ty^2}{2} - \frac{y^3}{3}\right]_0^t$$

$$= -\frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)\left[\frac{t^3}{2} - \frac{t^3}{3}\right] = -\frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)\frac{t^3}{6}$$

$$Q = -\frac{1}{12\mu}\left(\frac{\partial p}{\partial x}\right)t^3$$

6. Average velocity, $\bar{u} = \dfrac{Q}{\text{Area of cross section}}$ $\quad [\because \; Q = A \times \bar{u}]$

$$= \frac{-\frac{1}{12\mu}\left(\frac{\partial p}{\partial x}\right)t^3}{t \times 1}$$

$$\bar{u} = -\frac{1}{12\mu}\left(\frac{\partial p}{\partial x}\right)t^2$$

9. Now, ratio of maximum velocity to average velocity is

$$\frac{\text{Maximum velocity}}{\text{Average velocity}} = \frac{U_{max.}}{\bar{u}}$$

$$= \frac{-\frac{1}{8\mu}\left(\frac{\partial p}{\partial x}\right)\cdot t^2}{-\frac{1}{12\mu}\left(\frac{\partial p}{\partial x}\right)\cdot t^2} = \frac{12}{8} = \frac{3}{2}$$

Que 4.6. **Prove that for laminar flow through a circular pipe, energy correction factor (α) = 2.**

Answer

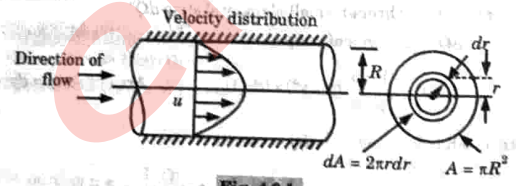

Fig. 4.6.1.

1. Kinetic energy per second of the fluid flowing through an elementary ring of radius 'r' and of width 'dr',

$$\text{K.E.} = \frac{1}{2} \times \text{mass per second} \times u^2$$

$$= \frac{1}{2}\rho dQ.u^2 \qquad [\text{mass per second} = \rho.dQ]$$

$$= \frac{1}{2}\rho(u \times 2\pi r\, dr)\cdot u^2$$

$$= \frac{1}{2}\rho \cdot 2\pi r\, u^3\, dr = \pi\rho \cdot r u^3\, dr$$

2. Total K.E. per second $= \int_0^R \pi\rho \cdot r\, u^3\, dr$

3. On putting

$$u = \frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right)(R^2 - r^2)$$

Total K.E. per second,

$$= \int_0^R \pi \cdot \rho \cdot \left[\frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right)(R^2 - r^2)\right]^3 \cdot r\, dr$$

$$= \pi\rho\left[\frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right)\right]^3 \int_0^R [R^2 - r^2]^3 \cdot r\, dr$$

$$= \pi\rho\left[\frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right)\right]^3 \int_0^R (R^6 \cdot r - r^7 - 3R^4 \cdot r^3 + 3R^2 \cdot r^5)\, dr$$

$$= \pi\rho\left[\frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right)\right]^3 \left[R^6 \cdot \frac{r^2}{2} - \frac{r^8}{8} - 3R^4 \cdot \frac{r^4}{4} + 3R^2 \cdot \frac{r^6}{6}\right]_0^R$$

$$= -\frac{\pi\rho}{64\mu^3}\left(\frac{\partial p}{\partial x}\right)^3 \frac{R^8}{8}$$

4. Kinetic energy of the flow for average velocity per second

$$= \frac{1}{2} \times \left(\frac{\text{mass}}{\text{sec}}\right) \times \bar{u}^2 \quad \frac{1}{2} \times (\rho A\bar{u}) \times \bar{u}^2$$

{mass per second = area × density × avg. velocity}

$$= \frac{1}{2}\rho A\bar{u}^3$$

5. On putting, $A = \pi R^2$, and $\bar{u} = \dfrac{1}{8\mu}\left(-\dfrac{\partial p}{\partial x}\right)\cdot R^2$

K.E. of the flow per second for average velocity

$$= \frac{1}{2}\rho\pi R^2\left[\frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right)R^2\right]^3$$

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$$= -\frac{1}{2}\left(\frac{\pi\rho}{64\mu^3}\left(\frac{\partial p}{\partial x}\right)^3 \cdot \frac{R^8}{8}\right)$$

6. Energy correction factor,

$$\alpha = \frac{\text{K.E. of flow / s for actual velocity}}{\text{K.E. of flow / s for average velocity}}$$

$$\alpha = \frac{-\frac{\pi\rho}{64\mu^3}\left(\frac{\partial p}{\partial x}\right)^3 \frac{R^8}{8}}{-\frac{1}{2}\left(\frac{\pi\rho}{64\mu^3}\left(\frac{\partial p}{\partial x}\right)^3 \cdot \frac{R^8}{8}\right)}$$

$$\Rightarrow \quad \alpha = 2$$

Que 4.7. State the Stoke's law for finding drag force of a sphere moving in an infinite medium. How this law can be applied to determine the viscosity of fluid ?

Answer

1. When a small and heavy sphere is to be dropped in a viscous fluid after a certain depth, the velocity of sphere becomes constant or sphere moves with a uniform velocity.
2. According to Stoke's law a drag force, F acting on the sphere moving with a constant velocity in a viscous fluid of viscosity μ is given by

$$F = 3\pi\mu U d$$

Where, F = Drag force,
U = Velocity of sphere (limiting speed of sphere), and
d = Diameter of sphere.

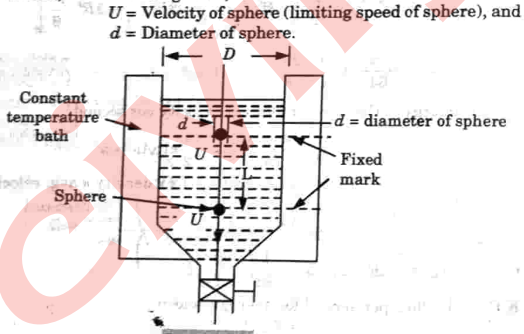

Fig. 4.7.1.

3. When a sphere is moving in a viscous fluid two forces act on the sphere :
   a. Weight of sphere, and
   b. Buoyant force.
4. In limiting case or attaining constant velocity (terminal velocity),
   Drag force = Weight of sphere – Buoyant force
5. Let us consider sphere of density ρ_s and weight w will travel a distance L in time t in the fluid of density ρ_f.
6. Weight of sphere, W = (volume × density of sphere) × g

$$= \left(\frac{4}{3}\pi r^3 \times \rho_s\right) g$$

$$= \frac{\pi}{6} d^3 \cdot \rho_s g \qquad \left\{\because r = \frac{d}{2}\right\}$$

7. Buoyant force, F_B = Weight of liquid displaced
   = Volume of displaced liquid × density of fluid × g

$$F_B = \frac{\pi}{6} d^3 \times \rho_f \times g$$

[∵ Volume of sphere = Volume of liquid displaced]

8. For limiting condition,
   Drag force = weight of sphere – buoyant force

$$F = W - F_B$$

$$3\pi\mu U d = \frac{\pi}{6} d^3 \rho_s \cdot g - \frac{\pi}{6} d^3 \rho_f \cdot g$$

$$\mu = \frac{\pi}{6} d^3 \frac{(\rho_s - \rho_f)}{3\pi U d} \cdot g$$

$$\mu = \frac{g d^2}{18U}(\rho_s - \rho_f)$$

9. Hence, coefficient of viscosity for fluid can be founded by knowing the density of fluid and sphere, diameter and limiting speed of sphere.

Que 4.8. Discuss transition from laminar to turbulent flow.

Answer

1. For turbulent flow over a flat plate, the boundary layer starts out as laminar flow at the leading edge and subsequently, the flow turns into transition flow and very shortly thereafter turns into turbulent flow.
2. The turbulent boundary layer continues to grow in thickness, with a small region below it called a viscous sub-layer.
3. In this sub-layer, the flow is well behaved, just as the laminar boundary layer (Fig. 4.8.1).

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4. A careful observation further suggests that at a certain axial location, the laminar boundary layer tends to become unstable. Physically this means that the disturbances in the flow grow in amplitude at this location.

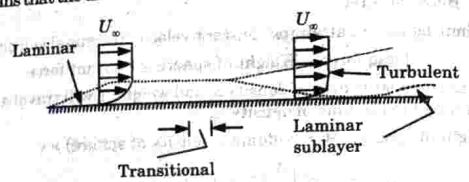

Fig. 4.8.1. Laminar-turbulent transition.

5. Free stream turbulence, wall roughness and acoustic signals may be among the sources of such disturbances. Transition to turbulent flow is thus initiated with the instability in laminar flow. The Reynold's number was defined as

$$Re = U_\infty \delta^* / v$$

where v is the free stream velocity and δ^* is the displacement thickness.

6. Taylor developed an alternate theory, which assumed that the transition is caused by a momentary separation at the boundary layer associated with the free stream turbulence.
7. In a pipe flow the initiation of turbulence is usually observed at Reynold's numbers ($U_\infty D/V$) in the range of 2000 to 2700.
8. The development starts with a laminar profile, undergoes a transition, changes over to turbulent profile and then stays turbulent thereafter (Fig. 4.8.1).
9. The length of development is of the order of 25 to 40 diameters of the pipe.

Que 4.9. **For laminar flow of an oil having dynamic viscosity μ = 1.766 Pa.s in a 0.3 m diameter pipe, the velocity distribution is parabolic with a maximum point velocity of 3 m/s at the centre of the pipe. Calculate the shear stresses at the pipe wall and within the fluid 50 mm from the pipe wall.**

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Answer

Given : μ = 1.766 Pa.s, D = 0.3 m, μ_{max} = 3 m/s
To Find : Shear stresses at the pipe wall and within the fluid 50 mm from the pipe wall.

1. Since, $$U = \frac{1}{2} u_{max} = \frac{3}{2} = 1.5 \text{ m/s}$$

2. $$\left(-\frac{\partial p}{\partial x}\right) = \frac{p_1 - p_2}{L} = \frac{32\mu U}{D^2}$$

Thus, $$\left(-\frac{\partial p}{\partial x}\right) = \frac{32 \times 1.766 \times 1.5}{(0.3)^2} = 941.87 \text{ Pa/m}$$

3. The shear stress at the pipe wall,

$$\tau_0 = \left(-\frac{\partial p}{\partial x}\right)\frac{R}{2} = \frac{(941.87 \times 0.3)}{2 \times 2} = 70.64 \text{ Pa}$$

4. The shear stress at 50 mm from the pipe wall is

$$\tau_0 = \left(-\frac{\partial p}{\partial x}\right)\frac{r}{2}$$

$$= 941.87 \times \frac{(0.15 - 0.05)}{2} = 47.09 \text{ Pa}$$

Que 4.10. **Find the loss of head due to friction and power required to pump an oil of specific gravity 0.85 and absolute viscosity 1.5 poise through a 25 cm diameter and 10 km long pipe laid at a slope of 1 in 200. The rate of flow of oil is 0.022 m³/s.**

Answer

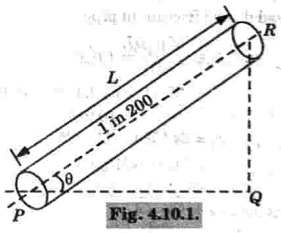

Fig. 4.10.1.

Given : Specific gravity of oil = 0.85
Density of oil, ρ_o = 0.85 × 1000 = 850 kg/m³
Absolute viscosity, μ_o = 1.5 poise = $\frac{1.5}{10}$ = 0.15 N-s/m²
Diameter of pipe, D = 25 cm = 0.25 m
Length of pipe, L = 10 km = 10000 m
Slope of pipe, $\tan\theta = \frac{1}{200}$, Rate of flow of oil, Q = 0.022 m³/s
To Find : i. Loss of head due to friction, and
ii. Power required to pump an oil.

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1. Check whether the flow is laminar or not.

We know, $v = \frac{Q}{A} = \frac{0.022}{\frac{\pi}{4}D^2} = \frac{0.022 \times 4}{\pi (0.25)^2}$

Reynold's number, $Re = \frac{\rho_o v D}{\mu}$

$v = 0.448$ m/s

$$Re = 850 \times 0.448 \times \frac{0.25}{0.15} = 634.67 < 2000$$

Hence, flow is laminar.

2. From ΔPQR, $\tan\theta = \frac{1}{200} \Rightarrow \sin\theta = \frac{1}{200}$

{as θ is very small so $\sin\theta = \tan\theta$}

$$\sin\theta = \frac{RQ}{PR}$$

$$RQ = PR.\sin\theta = 10000 \cdot \frac{1}{200}$$

$$= 50 \text{ m}$$

3. For loss of head due to friction in pipe,

$$p_1 - p_2 = \frac{32\mu_o \bar{u} L}{D^2} + \rho_o g h \qquad \{\because h = RQ \text{ and } \bar{u} = v\}$$

$$= \frac{32 \times 0.15 \times 0.448 \times 10000}{(0.25)^2} + (850 \times 9.81 \times 50)$$

$$p_1 - p_2 = 344064 + 416925$$

$$= 760989 \text{ N/m}^2$$

4. Head loss $= \frac{p_1 - p_2}{\rho_o g}$

$$h_f = \frac{760989}{850 \times 9.81} = 91.26 \text{ m}$$

5. Weight of oil flowing per second,

$$w = \rho_o g \times Q$$

$$= 850 \times 9.81 \times 0.022 = 183.447 \text{ N/s}$$

6. Power required to pump an oil $= w.h_f$

$$= 91.26 \times 183.447$$

$$= 16741.37 \text{ W}$$

PART-2

Turbulent Flow Types of Turbulent flow, Isotropic, Homogeneous Turbulence, Scale and Intensity of Turbulence, Measurement of Turbulence, Eddy Viscosity, Mixing Length Concept and Velocity Distribution in Turbulent Flow Over Smooth and Rough Surfaces.

CONCEPT OUTLINE : PART-2

Turbulent Flow :

1. For Reynold's number more than 4000, flow is known as turbulent flow.
2. Shear in turbulent flow is mainly due to momentum transfer.
3. Loss of head due to friction in pipe flow is given by

$$h_f = \frac{4fLv^2}{2Dg},$$

where f = friction coefficient.

Eddy Viscosity : The viscosity which accounts for momentum transport by turbulent eddies is known as eddy viscosity.

Velocity Distribution for Turbulent Flow :

For smooth pipes, $\frac{u}{u_f} = 5.75 \log_{10}\left(\frac{u_f . y}{\nu}\right) + 5.5$

For rough pipes, $\frac{u}{u_f} = 5.75 \log_{10}\left(\frac{y}{k}\right) + 8.5$

Where, u_f = shear friction velocity $= \sqrt{\frac{\tau_0}{\rho}}$,

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.11. **What is turbulent flow ? Give its classification.**

Answer

1. In a pipe, turbulent flow occurs when $Re > 4000$.
2. In a turbulent flow, the fluid motion is irregular and chaotic and there is complete mixing of fluid due to collision of fluid masses with one another.
3. As the fluid masses in adjacent layers have different velocities, interchange of fluid masses between the adjacent layers is accompanied

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by a transfer of momentum which causes additional shear stresses of high magnitude between adjacent layers.

4. The contribution of fluid viscosity to total shear is small and is usually neglected.

5. The turbulent motion can be classified as follows :

   i. **Wall Turbulence :** It occurs in immediate vicinity of solid surfaces and in the boundary layer flows where the fluid has a negligible mean acceleration.

   ii. **Free Turbulence :** It occurs in jets, wakes, mixing layers etc.

   iii. **Convective Turbulence :** It takes place where there is conversion of P.E into K.E. by the process of mixing (*e.g.*, the turbulent flow in the annular space between the concentric rotating cylinder, conventional flow between parallel horizontal plates etc.).

Que 4.12. Write short note on isotropic and homogeneous turbulence.

Answer

A. Homogeneous Turbulence :

1. If the turbulence has the same structure quantitatively in all parts of the flow field, the turbulence is said to be homogeneous.

2. The term homogeneous turbulence implies that the velocity fluctuations in the system are random. The average turbulent characteristics are independent of the position in the fluid, *i.e.*, invariant to axis translation.

3. Consider the root mean square velocity fluctuations

$$u' = \sqrt{\overline{u^2}},\ v' = \sqrt{\overline{v^2}},\ w' = \sqrt{\overline{w^2}}$$

In homogeneous turbulence, the rms values of u', v' and w' can all be different, but each value must be constant over the entire turbulent field.

B. Isotropic Turbulence :

1. Turbulence is called isotropic if its statistical features have no directional preference and perfect disorder persists. Its velocity fluctuations are independent of the axis of reference, *i.e.* invariant to axis rotation and reflection.

2. In isotropic turbulence fluctuations are independent of the direction of reference and

$$\sqrt{\overline{u^2}} = \sqrt{\overline{v^2}} = \sqrt{\overline{w^2}}$$

or

$$u' = v' = w'$$

Que 4.13. Describe turbulence length scale and turbulence intensity.

Answer

A. Turbulence Length Scale :

1. The turbulence length scale, l, is a physical quantity describing the size of the large energy-containing eddies in a turbulent flow.

2. The turbulent length scale is often used to estimate the turbulent properties on the inlets of a CFD simulation.

3. The turbulent length scale should normally not be larger than the dimension of the problem, since that would mean that the turbulent eddies are larger than the problem size.

B. Turbulence Intensity :

1. Turbulence intensity is a scale characterizing turbulence expressed as a percent.

2. An idealized flow of air with absolutely no fluctuations in air speed or direction would have a turbulence intensity value of 0 %. This idealized case never occurs on earth.

3. However, due to how turbulence intensity is calculated, values greater than 100 % are possible. This can happen, for example, when the average air speed is small and there are large fluctuations present.

Que 4.14. Describe the measurement of turbulence with the help of hot-wire anemometer.

Answer

1. A hot-wire anemometer is instrument which is commonly used for measuring the velocity of flow of a compressible fluid such as gas.

2. The anemometer consists of a platinum, nickel or tungsten wire of about 5×10^{-3} to 8×10^{-3} mm diameter and 16 mm length.

3. The wire is mounted on the ends of two pointed prongs.

4. In the arrangement shown in Fig. 4.14.1(*a*), constant current is passed through wire by keeping the voltage across the bridge.

5. As the air or gas flows the hot-wire cools, its resistance changes and the galvanometer deflects.

6. The galvanometer deflection is correlated with the velocity of flow of air or gas by calibration. It is then termed as constant current hot-wire anemometer.

7. Fig. 4.14.1(*b*) illustrates another arrangement for hot-wire anemometer which is termed as constant temperature (or constant-resistance) hot-wire anemometer.

8. Initially when there is no flow and the hot-wire is in contact with air or gas at rest, a small current is passed through the hot-wire.

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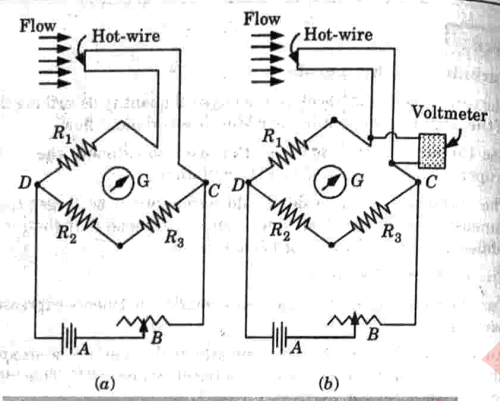

Fig. 4.14.1. (*a*) Constant-current hot-wire anemometer, (*b*) Constant-temperature hot-wire anemometer.

9. As the air or gas flows past the hot-wire, its temperature and hence its resistance will vary, which will cause the galvanometer needle to deflect from zero reading.
10. Now by adjusting the variable resistance *B* the current passing through the hot-wire is suitably adjusted so that its temperature and hence the resistance is maintained constant and the galvanometer reading is brought back to zero.
11. The reading of the voltmeter connected across the hot-wire will change which may be noted.

Que 4.15. **What is eddy viscosity ?**

Answer

1. The viscosity which accounts for momentum transport by turbulence eddies is known as eddy viscosity.
2. Similar to the expression for viscous shear, turbulent shear in mathematical form is expressed as

$$\tau_t = \eta \frac{d\bar{u}}{dy}$$

where, τ_t = Shear stress due to turbulence,

η = Eddy viscosity, and

$\bar{u}$ = Average velocity at a distance *y* from boundary.

3. The ratio of η (eddy viscosity) and ρ (mass density) is known as kinematic eddy viscosity and is denoted by ε (epsilon). Mathematically it is written as

$$\varepsilon = \frac{\eta}{\rho}$$

Que 4.16. **Discuss the significance of Reynold's stress and mixing length in the turbulent flow.**

Answer

A. Significance of Reynold's Stress :

1. When a fluid is in turbulent flow, the shear stress (turbulent),

$$\tau = \rho\, u'.v'$$

Where, u' = Fluctuating component of velocity in *X*- direction due to turbulence.

v' = Fluctuating component of velocity in *Y*-direction due to turbulence.

2. u' and v' are varying hence τ will also vary with time, therefore, taking the time average on both sides.

$$\bar{\tau} = \overline{\rho u' v'} \Rightarrow \text{Reynold's stress.}$$

3. Reynold's stress is used to determine the random orientation of fluid particles in turbulent flow.

B. Prandtl's Mixing Length Theory for Turbulent Shear Stress :

1. According to Prandtl, the mixing length *l*, is that distance between two layers in the transverse direction such that the lumps of fluid particles from one layer could reach the other layer and the particles are mixed in the other layer in such a way that the momentum of the particles in the *X*-direction remains same.
2. Assumptions in Prandtl's theory

$$u' = l \cdot \frac{du}{dy}$$

$$v' = l \cdot \frac{du}{dy}$$

3. Now, $$\overline{u' \times v'} = \left(l \frac{du}{dy}\right)\left(l \frac{du}{dy}\right)$$

$$= l^2 \left(\frac{du}{dy}\right)^2$$

4. Reynold's stress, $$\bar{\tau} = \overline{\rho u' v'}$$

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$$\bar{\tau} = \rho \cdot l^2 \left(\frac{du}{dy}\right)^2$$

5. Hence, the shear stress at any point in turbulent flow is

$$\bar{\tau} = \mu \cdot \frac{du}{dy} + \rho \cdot l^2 \left(\frac{du}{dy}\right)^2$$

Que 4.17. Obtain an expression for velocity distribution for turbulent flow in pipes.

Answer

1. In case of turbulent flow the total shear stress at any point is given by

$$\bar{\tau} = \mu \frac{du}{dy} + \rho\, l^2 \left(\frac{du}{dy}\right)^2$$

2. Since, viscous shear stress $\mu\left(\frac{du}{dy}\right)$ is negligible near the boundary.

Then, $$\bar{\tau} = \rho l^2 \left(\frac{du}{dy}\right)^2$$

where, l = Mixing length, and
y = Distance from the pipe wall.

3. But according to Prandtl's hypothesis,

$$l = ky$$

Where, k = Karman constant.

Then, $$\bar{\tau} = \rho (ky)^2 \left(\frac{du}{dy}\right)^2$$

$$\tau = \bar{\tau} = \rho\, k^2 y^2 \left(\frac{du}{dy}\right)^2$$

or, $$\frac{du}{dy} = \frac{1}{ky}\sqrt{\frac{\tau}{\rho}}$$

4. According to Prandtl, shear stress equals to turbulent shear stress at the pipe boundary, for small value of y.

$$\tau = \tau_o = \text{Shear stress at pipe boundary}$$

So, $$\frac{du}{dy} = \frac{1}{ky}\sqrt{\frac{\tau_o}{\rho}}$$

Here, $$\sqrt{\frac{\tau_o}{\rho}} = \text{shear velocity} = u_f$$

Then, $$du = u_f \cdot \frac{1}{ky} dy$$

5. On integrating the both sides,

$$u = \frac{u_f}{k} \log_e y + C \quad \text{...(4.17.1)}$$

(where C = integration constant)

8. Using boundary conditions, at $y = R$, $u = u_{max}$

$$u_{max} = \frac{u_f}{k} \log_e R + C$$

$$C = u_{max} - \frac{u_f}{k} \log_e R$$

9. Put the value of C in eq. (4.17.1),

$$u = \frac{u_f}{k} \log_e y + u_{max} - \frac{u_f}{k} \log_e R$$

$$u = u_{max} + \frac{u_f}{k} \log_e (y/R)$$

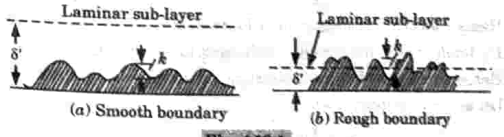
10. As k = Karman constant = 0.4

So, $$u = u_{max} + 2.5\, u_f \log_e (y/R) \quad \text{...(4.17.2)}$$

11. The eq. (4.17.2) is called Prandtl's universal distribution equation for turbulent flow in pipes.

Que 4.18. Write down the analysis of turbulent flow along the boundary and how would you distinguish between hydrodynamically smooth and rough boundaries ?

Answer



(a) Smooth boundary (b) Rough boundary

Fig. 4.18.1.

1. For analysis of turbulent flow along boundary, the flow is divided into two portions :

a. **Laminar Sub-layer :**

1. It is the thin layer of fluid very close to the boundary.
2. In this portion viscous shear stress predominates while the shear stress due to turbulence is negligible.

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3. The height up to which the effort of viscosity predominates in this zone is denoted by δ'.

b. Turbulent Zone :

1. In turbulent zone, shear stress due to turbulence is large as compared to viscous stress.
2. Let k = average height of irregularities.
3. When k is much less than δ', the boundary is called smooth boundary or hydrodynamically smooth boundary.
4. Due to large thickness of laminar sublayer ($\delta' >> k$) the eddies of turbulent flow are unable to reach the surface irregularities.
5. If k is greater than δ', the eddies of turbulent flow will come in contact with the irregularities of the surface and lot of energy will be lost. Such a boundary is called hydrodynamically rough boundary.
6. From Nikuradse's experiment :

   i. If $\frac{k}{\delta'} < 0.25$, the boundary is smooth boundary.

   ii. If $0.25 < \frac{k}{\delta'} < 6.0$, the boundary is in transition state.

   iii. If $\frac{k}{\delta'} > 6.0$, the boundary is rough boundary.

Que 4.19. Obtain an expression for the velocity distribution for turbulent flow in smooth pipe.

Answer

1. The velocity distribution for turbulent flow in a pipe is given by

$$u = \frac{1}{k} u_f \cdot \log_e y + C \qquad ...(4.19.1)$$

Hence, velocity u is a logarithmic function of y.

2. If y tends to zero the velocity u will tends to $-\infty$ (minus infinity). Hence, there must be a certain distance at which velocity becomes zero.
3. Let at $y = y'$ velocity $u = 0$

$$0 = \frac{1}{k} u_f \cdot \log_e y' + C$$

$$\Rightarrow \quad C = -\frac{1}{k} u_f \log_e y'$$

4. Substitute the value of 'C' in eq. (4.19.1),

$$u = \frac{1}{k} u_f \log_e y + \left(-\frac{1}{k} u_f \log_e y'\right)$$

$$u = \frac{1}{k} \cdot u_f \log_e (y / y')$$

$$u = \frac{1}{0.4} u_f \log_e (y / y')$$

$$(\because \quad k = 0.4 = \text{Karman constant})$$

$$u = 2.5 u_f \log_e (y / y')$$

5. On changing the base of log function from 'e' to 10,

$$\log_e (y/y') = 2.3 \log_{10} (y/y')$$

$$u = 2.5 \times 2.3\, u_f \log_{10} (y/y')$$

$$\frac{u}{u_f} = 5.75 \log_{10} (y/y') \qquad ...(4.19.2)$$

6. Since, for the smooth boundary there exists a laminar sublayer by Nikuradse's experimental observations.
7. For turbulent flow in smooth pipes (any size of pipe) :

Parameter, $u_f \cdot \frac{y}{\nu}$ = constant, for $y = \delta'$ and $y = y'$

For $y = \delta'$, $u_f \cdot \frac{\delta'}{\nu} = 11.6$, for $y = y'$, $u_f \cdot \frac{y'}{\nu} = 0.108$

On using these parameters

$$\delta' = 11.6 \left(\frac{\nu}{u_f}\right)$$

So,

$$\delta' = \frac{11.6}{0.108} y' \qquad \left\{\frac{\nu}{u_f} = \frac{y'}{0.108}\right\}$$

$$y' = \frac{\delta'}{107} = 0.108 \frac{\nu}{u_f}$$

8. Put the value of $y' = 0.108 \cdot \frac{\nu}{u_f}$ in eq. (4.19.2),

$$\frac{u}{u_f} = 5.75 \log_{10} \left(\frac{y}{\frac{0.108}{u_f} \cdot \nu}\right)$$

$$\frac{u}{u_f} = 5.75 \log_{10} \left(9.259\, u_f \frac{y}{\nu}\right)$$

$$= 5.75 \log_{10} 9.259 + 5.75 \log_{10} \left(u_f \frac{y}{\nu}\right)$$

$$\frac{u}{u_f} = 5.55 + 5.75 \log_{10} \left(\frac{u_f \cdot y}{\nu}\right) \qquad ...(4.19.3)$$

(ν = Kinematic viscosity)

9. The eq. (4.19.3) is known as Karman-Prandtl equation for the velocity distribution near hydrodynamically smooth boundaries.

Que 4.20. Derive Karman-Prandtl equation for velocity distribution near hydrodynamically rough boundaries.

OR

Derive an expression for turbulent flow in rough pipes.

Answer

1. For hydrodynamically rough pipes the value of k (average height of the irregularities projecting from the surface of a boundary) is greater than the value of δ' (thickness of laminar sub-layer). Hence laminar-sub-layer is completely destroyed.
2. According to the equation of velocity distribution for hydrodynamically smooth pipe,

$$\frac{u}{u_f} = 5.75 \log_{10} (y/y')$$

3. Since we know that from Nikuradse's experiment

$$y' = \frac{k}{30}$$

So,

$$\frac{u}{u_f} = 5.75 \log_{10}\left(\frac{30y}{k}\right)$$

$$= 5.75 \log_{10} 30 + 5.75 \log_{10} (y/k)$$

$$\frac{u}{u_f} = 8.5 + 5.75 \log_{10} (y/k)$$

4. The above expression is known as Karman-Prandtl equation for velocity distribution near hydrodynamically rough boundaries.

Que 4.21. Derive the following relationship for rough pipe, for the turbulent flow :

$$\frac{u-\overline{U}}{u_f} = 5.75 \log_{10}\left(\frac{y}{R}\right) + 3.75$$

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Answer

1. In case of rough pipes, the velocity at any point in the turbulent flow is given by,

$$\frac{u}{u_f} = 5.75 \log_{10} (y/k) + 8.5 \quad \text{... (4.21.1)}$$

$$= 5.75 \log_{10}\left(\frac{R-r}{k}\right) + 8.5 \quad (\because y = R - r)$$

or,

$$u = u_f\left[5.75 \log_{10}\left(\frac{R-r}{k}\right) + 8.5\right]$$

2. As we know, flow rate,

$$Q = \int_0^R u \times 2\pi\, r dr$$

3. Substituting the value of u,

$$Q = \int_0^R u_f\left[5.75 \log_{10}\left(\frac{R-r}{k}\right) + 8.5\right] 2\pi r.dr$$

4. Average velocity,

$$\overline{U} = \frac{Q}{\pi R^2}$$

$$= \frac{1}{\pi R^2}\int_0^R u_f\left[5.75 \log_{10}\left(\frac{R-r}{k}\right) + 8.5\right] 2\pi r.dr$$

5. On integration and simplification,

$$\frac{\overline{U}}{u_f} = 5.75 \log_{10}\left(\frac{R}{k}\right) + 4.75 \quad \text{... (4.21.2)}$$

6. On subtracting eq. (4.21.2) from eq. (4.21.1),

$$\frac{u}{u_f} - \frac{\overline{U}}{u_f} = \left[5.75 \log_{10}\left(\frac{y}{k}\right) + 8.5\right] - \left[5.75 \log_{10}\left(\frac{R}{k}\right) + 4.75\right]$$

$$\frac{u-\overline{U}}{u_f} = 3.75 + 5.75 \log_{10}\left(\frac{y}{k} \times \frac{k}{R}\right)$$

$$\frac{u-\overline{U}}{u_f} = 5.75 \log_{10}\left(\frac{y}{R}\right) + 3.75$$

Que 4.22. A pipe of 10 cm diameter carries water. The velocities at the pipe centre and 3 cm from the pipe centre are 2 m/s and 1.5 m/s respectively. If the flow in the pipe is turbulent, determine the shear stress at the pipe wall and at 3 cm from the pipe wall.

Answer

Given : Diameter of pipe, $d = 10$ cm $= 0.1$ m

Velocity at centre, $U_{max.} = 2$ m/s at $y = R = \frac{d}{2} = \frac{0.1}{2} = 0.05$ m

Velocity at 3 cm from pipe centre, $u = 1.5$ m/s at $r = 3$ cm $= 0.03$ m

To Find : i. Shear stress at the pipe wall

ii. Shear stress at 3 cm from pipe wall.

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1. Velocity distribution equation,

$$\frac{U_{max.} - u}{u_f} = 5.75 \log_{10}\left(\frac{R}{y}\right)$$

where $y = R - r = 0.05 - 0.03 = 0.02$ m

$$\frac{2 - 1.5}{u_f} = 5.75 \log_{10}\left(\frac{0.05}{0.02}\right)$$

$$u_f = 0.2185 \text{ m/s}$$

2. Shear stress at pipe wall (τ_o),

$$u_f = \sqrt{\frac{\tau_o}{\rho}}$$

$$\Rightarrow \quad \tau_o = u_f^2 \cdot \rho = (0.2185)^2(1000) = 47.74 \text{ N/m}^2$$

3. Shear stress τ, at a distance r from centre of pipe is given by

$$\tau = \tau_0 \frac{r}{R}$$

At a point 3 cm from the pipe wall,

$$r = R - 3 \text{ cm} = 5 - 3$$
$$r = 2 \text{ cm} = 0.02 \text{ m}$$

Shear stress, $\tau = 47.74\left(\frac{0.02}{0.05}\right) = 19.096 \text{ N/m}^2$

PART-3

Boundary Layer Thickness, Boundary Layer Over a Flat Plate, Laminar Boundary Layer, Application of Momentum Equation, Turbulent Boundary Layer, Laminar Sub-layer and Separation and its Control.

CONCEPT OUTLINE : PART-3

Boundary Layer Thickness : It is defined as the distance from the boundary in which the velocity reaches 99 percent of the free stream velocity. It is denoted by δ.

Displacement Thickness : $\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy$

Momentum Thickness : $\theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$

Energy Thickness : $\delta^{**} = \int_0^\delta \left(\frac{u}{U}\right)\left[1 - \left(\frac{u}{U}\right)^2\right] dy$

Von-Karman Momentum Integral Equation :

$$\frac{\tau_o}{\rho U^2} = \frac{d\theta}{dx}$$

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.23. **Explain boundary layer concept and wall shear.**

OR

What is a boundary layer ? Explain with a sketch development of boundary layer over a smooth plate.

OR

Define boundary layer and explain the fundamental causes of boundary layer development with suitable sketches.

Answer

1. When a real fluid flow over a solid wall, the fluid particles closed to the boundary get adhered to the boundary and as a result of this condition no slip occurs.
2. In other words the velocity of fluid close to the boundary will be the same as that of the boundary.
3. As we move farther away from the boundary, the velocity will be higher and as a result of this variation of velocity, the velocity gradient $\frac{du}{dy}$ will exist.
4. Thus the velocity of fluid increases from zero velocity on the stationary boundary to free-stream velocity (U) of the fluid in the direction normal to the boundary.
5. The variation of velocity from zero to free stream velocity in the direction normal to the boundary takes place in a narrow region in the vicinity of solid boundary.
6. This narrow region of the fluid is called boundary layer.

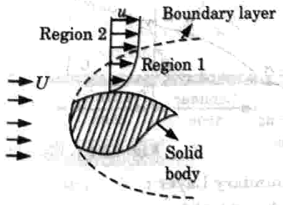

Fig. 4.23.1.

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7. Hence the flow of fluid in the neighbourhood of the solid boundary may be divided into two regions.

a. **Region 1 :**

1. A very thin layer of the fluid called the boundary layer, in the immediate neighbourhood of the solid boundary, where the variation of velocity from zero at the solid boundary to the free stream velocity in the direction normal to the boundary takes place.
2. In this region, the velocity gradient $\frac{du}{dy}$ exists and hence the fluid exerts a shear stress on the wall (wall shear) in the direction of motion.
3. The value of shear stress is given by

$$\tau = \mu . \frac{du}{dy}$$

b. **Region 2 :**

1. The remaining fluid, which is outside the boundary layer. The velocity outside the boundary layer is constant and equal to free stream velocity.
2. As there is no variation of velocity in this region the velocity gradient $\frac{du}{dy}$ becomes zero.
3. As a result of this the shear stress is zero.

Que 4.24. Explain laminar boundary layer, turbulent boundary layer and laminar sub-layer with sketch in case of flow over a plate.

Answer

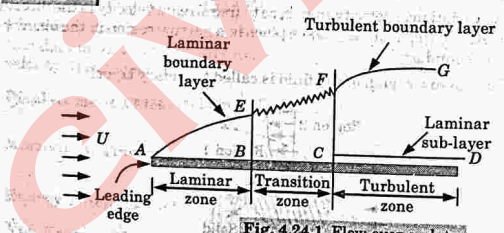

Fig. 4.24.1. Flow over a plate.

A. **Laminar Boundary Layer :**

1. Consider the flow of fluid, having free-stream velocity (U), over a smooth thin plate which is flat and placed parallel to the direction of free stream of fluid.

2. Considering the flow with zero pressure gradient on one side of the plate, which is stationary.
3. The velocity of fluid on the surface of the plate should be equal to the velocity of the plate.
4. As the plate is stationary and hence velocity of fluid on the surface of the plate is zero, but at a distance away from the plate, the fluid is having certain velocity.
5. Thus a velocity gradient sets up in the fluid near the surface of the plate. This velocity gradient develops shear resistance which retards the fluid.
6. The fluid with a uniform free stream velocity (U) is retarded in the vicinity of the solid surface of the plate and the boundary layer region begins at the sharp leading edge.
7. At subsequent points downstream the leading edge, the boundary layer region increases because the retarded fluid is further retarded. This is also referred as the growth of boundary layer.
8. Near the leading edge of the surface of the plate, where the thickness is small, the flow in the boundary layer is laminar though the main flow is turbulent.
9. This layer of the fluid is said to be laminar boundary layer shown by AE in Fig. 4.24.1. The length of the plate from the leading edge, upto which laminar boundary layer exists is called laminar zone shown by distance AB.
10. The distance of B from leading edge is obtained from Reynold number equal to 5×10^5 for a plate.

B. **Turbulent Boundary Layer :**

1. If the length of the plate is further increased, the thickness of boundary layer will go on increasing in the downstream direction.
2. Then the laminar boundary layer becomes unstable and motion of fluid within it, is disturbed and irregular which leads to a transition from laminar to turbulent boundary layer.
3. This short length over which the boundary layer flow changes from laminar to turbulent is called transition zone which is shown by distance BC.
4. Further downstream the transition zone, the boundary layer is turbulent and continues to grow in thickness.
5. This layer of boundary is called turbulent boundary layer, which is shown by the portion FG.

C. **Laminar Sub-Layer :**

1. This is the region in the turbulent boundary layer zone, adjacent to the solid surface of the plate.
2. In this zone, the velocity variation is influenced only by viscous effects.

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3. Though the velocity distribution would be a parabolic curve in the laminar sub-layer zone, but in view of the very small thickness one can reasonably assume that the velocity variation is linear and thus the velocity gradient can also be taken as constant.

4. Hence, the shear stress in the laminar sub-layer would be constant and equal to the boundary shear stress τ_o.

So, shear stress in the sub-layer is $\tau_o = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0} = \mu.\frac{U}{y}$ (As linear variation occurs).

Que 4.25. **What do you understand by momentum thickness and displacement thickness ?**

OR

Define displacement thickness. Derive the expression for momentum thickness for boundary layer flow.

Answer

A. Displacement Thickness :

1. It is defined as the distance perpendicular to the boundary, by which the free stream is displaced due to the formation of boundary layer.
2. It is denoted by δ^*.

Expression for δ^* :

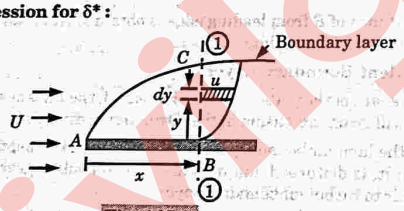

Fig. 4.25.1.

1. As shown in Fig. 4.25.1, consider the flow of a fluid having free-stream velocity equal to U over a thin smooth plate.
2. At a distance x from the leading edge consider a section 1-1.
3. The velocity of fluid at B is zero and at C, which lies on the boundary layer is U.
4. Thus velocity varies from zero at B to U at C, where BC is equal to the thickness of boundary layer *i.e.*,

   Distance $BC = \delta$
5. At the section 1-1, consider an elemental strip.
6. Let, y = Distance of elemental strip from the plate,

dy = Thickness of the elemental strip,
u = Velocity of fluid at the elemental strip, and
b = Width of plate.

Then area of elemental strip, $dA = b \times dy$

7. Mass of fluid per second flowing through elemental strip

$$= \rho \times \text{velocity} \times \text{area of elemental strip}$$
$$= \rho \times u \times dA = \rho u \times b \times dy \quad ...(4.25.1)$$

8. In the absence of the plate, the fluid would have been flowing with a constant velocity equal to free stream velocity (U) at section 1-1.
9. Then mass of fluid per second flowing through elemental strip would have been

$$\rho \times \text{velocity} \times \text{area} = \rho \times U \times b \times dy \quad ...(4.25.2)$$

10. As U is more than u, there will be a reduction in mass flowing per second through elemental strip.

This reduction in mass/s flowing through elemental strip

$$= \text{Mass/s given by eq. (4.25.2)} - \text{Mass/s given by eq. (4.25.1)}$$
$$= \rho \times U\, b\, dy - \rho\, u\, b\, dy = \rho \times b\,(U - u)\, dy$$

11. Hence, total reduction in mass of flowing fluid through BC due to presence of plate

$$= \int_0^\delta \rho b\,(U-u)\,dy = \rho b \int_0^\delta (U-u)\,dy \quad ...(4.25.3)$$

(if the fluid is incompressible, ρ is constant.)

12. Let the plate be displaced by a distance δ^* and velocity of the flow for the distance δ^* is equal to the free stream velocity.
13. Loss of the mass of the fluid/s flowing through the distance δ^*.

$$= \rho \times \text{velocity} \times \text{area} \quad (\because \text{ area} = \delta^* \times b)$$
$$= \rho \times U \times \delta^* \times b \quad ...(4.25.4)$$

14. Equating eq. (4.25.3) and eq. (4.25.4), we get

$$\rho b \int_0^\delta (U-u)\,dy = \rho \times U \times \delta^* \times b$$

$$\therefore \quad \int_0^\delta (U-u)\,dy = U \times \delta^*$$

or

$$\delta^* = \frac{1}{U}\int_0^\delta (U-u)\,dy = \int_0^\delta \left(1 - \frac{u}{U}\right) dy \quad ...(4.25.5)$$

B. Momentum Thickness :

1. It is defined as the distance measured perpendicular to the boundary of the solid body, by which the boundary should be displaced to compensate for the reduction in momentum of the flowing fluid on account of boundary layer formation.
2. It is denoted by θ.

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Expression for θ :

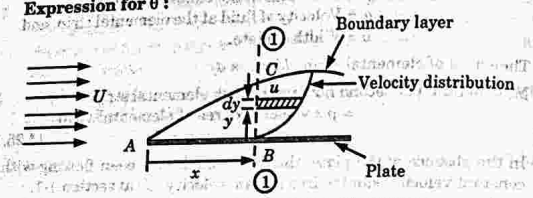

Fig. 4.25.2. Displacement thickness.

1. Considering the flow over a plate and let the section 1-1 is at a distance x from leading edge.
2. Consider an elemental strip at a distance y from the plate having thickness (dy).
3. The mass of fluid flowing per second through this elemental strip is given as,
$$= \rho\, u\, b\, dy.$$
4. Momentum of this fluid = Mass × velocity
$$= (\rho\, u\, b\, dy)\, u$$
5. Momentum of this fluid in the absence of boundary layer
$$= (\rho\, u\, b\, dy)\, U$$
6. Loss of momentum through elemental strip
$$= (\rho u b dy)\, U - (\rho u b dy) \times u = \rho b u\, (U - u)\, dy$$
7. Therefore, total loss of momentum/s through
$$BC = \int_0^{\delta} \rho b u (U - u) dy \quad ...(4.25.6)$$
8. Let θ = Distance by which the plate is displaced when the fluid is flowing with a constant velocity U.
9. Loss of momentum per sec of fluid flowing through distance θ with a velocity U
$$= \text{Mass of fluid through } \theta \times \text{velocity}$$
$$= (\rho \times \text{area} \times \text{velocity}) \times \text{velocity} \quad (\because \text{Area} = \theta \times b)$$
$$= (\rho \times \theta \times b \times U) \times U$$
$$= \rho\, b U^2 \theta \quad ...(4.25.7)$$
10. Equating eq. (4.24.6) and eq. (4.25.7), we get
$$\rho\, \theta\, b\, U^2 = \int_0^{\delta} \rho\, b\, u\, (U - u)\, dy$$
11. Considering incompressible fluid flow,
$$\theta U^2 = \int_0^{\delta} u\, (U - u)\, dy$$

$$\theta = \frac{1}{U^2} \int_0^{\delta} u\, (U - u)\, dy = \int_0^{\delta} \frac{u(U - u)}{U^2} dy$$
$$\theta = \int_0^{\delta} \frac{u}{U} \left[1 - \frac{u}{U}\right] dy$$

Que 4.26. Define the energy thickness and deduce its expression.

Answer

1. Due to formation of boundary layer, there is a reduction in the kinetic energy of fluid.
2. To compensate this loss of kinetic energy, boundary should be displaced by a certain distance.
3. This certain distance, when measured in perpendicular direction of boundary of solid body is called energy thickness. It is represented by δ^{**}.
4. Let the mass of fluid flowing per second through the strip of thickness dy
$$= \text{Density} \times \frac{\text{volume}}{\text{sec}} = \rho.ub.dy$$
Where, ρ = Density of fluid,
u = Velocity of fluid through strip,
b = Width of fluid, and
dy = Thickness of strip.
5. Kinetic energy of the fluid $= \frac{1}{2} mv^2 = \frac{1}{2} (\rho u b dy)\, u^2$
6. If the boundary layer is not present then kinetic energy of the fluid
$$= \frac{1}{2} (\rho\, u b dy)\, U^2$$
7. Loss of kinetic energy through strip
$$= \frac{1}{2} (\rho u b\, dy)\, U^2 - \frac{1}{2} (\rho u b\, dy) u^2$$
$$= \frac{1}{2} \rho u b [U^2 - u^2] dy$$
8. Now integrate it from 0 to δ to get total loss of kinetic energy of fluid,
$$= \int_0^{\delta} \frac{1}{2} \rho u b [U^2 - u^2] dy = \frac{1}{2} \rho b \int_0^{\delta} u [U^2 - u^2] dy \quad ...(4.26.1)$$
9. Let the plate is displaced by a distance δ^{**} to compensate this loss of kinetic energy.

10. Loss of kinetic energy through δ^{**}

$$= \frac{1}{2}(\text{density} \times \text{area} \times \text{velocity}) \times (\text{velocity})^2$$

$$= \frac{1}{2}(\rho b\,\delta^{**}U)U^2 = \frac{1}{2}\rho b\,\delta^{**}U^3 \quad \text{.....(4.26.2)}$$

11. On equating eq. (4.26.2) and eq. (4.26.1) we get

$$\frac{1}{2}\rho b\,\delta^{**}U^3 = \frac{1}{2}\rho b\int_0^\delta u[U^2 - u^2]dy$$

$$\delta^{**} = \int_0^\delta \frac{u}{U^3}[U^2 - u^2]dy = \int_0^\delta \left(\frac{u}{U}\right)\left[1 - \left(\frac{u}{U}\right)^2\right]dy$$

Where, U = Free stream velocity over a smooth thin plate.

Que 4.27. **The velocity distribution in the boundary layer is given as**

$$\frac{u}{v} = \frac{3}{2}\eta - \frac{1}{2}\eta^2$$

in which $\eta = y/\delta$. Compute δ^*/δ and θ/δ.

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Answer

Given : $\frac{u}{v} = \frac{3}{2}\eta - \frac{1}{2}\eta^2, \eta = \frac{y}{\delta}$

To Find : i. $\frac{\delta^*}{\delta}$, ii. $\frac{\theta}{\delta}$

1. Velocity profile, $\frac{u}{v} = \frac{3}{2}\eta - \frac{1}{2}\eta^2$
2. Displacement thickness

$$\delta^* = \int_0^\infty \left(1 - \frac{u}{v}\right)dy$$

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{v}\right)dy + \int_\delta^\infty \left(1 - \frac{u}{v}\right)dy$$

3. But outside the boundary layer $\frac{u}{v} = 1$ and hence

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{v}\right)dy$$

4. Now, $\frac{u}{v} = \frac{3}{2}\eta - \frac{1}{2}\eta^2$ and $\eta = \frac{y}{\delta}$

Then,

$$\delta^* = \int_0^\delta \left[1 - \frac{3}{\delta}\left(\frac{y}{\delta}\right) + \frac{1}{2}\left(\frac{y}{\delta}\right)^2\right]dy$$

$$= \left[y - \frac{3}{4\delta}y^2 + \frac{1}{6\delta}y^3\right]_0^\delta$$

$$\delta^* = \frac{5}{12}\delta$$

Hence, $\frac{\delta^*}{\delta} = \frac{5}{12}$

5. Similarly, momentum thickness,

$$\theta = \int_0^\delta \frac{u}{v}\left(1 - \frac{u}{v}\right)dy$$

$$\theta = \int_0^\delta \left[\frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^2\right]\left[1 - \frac{3}{2}\frac{y}{\delta} + \frac{1}{2}\left(\frac{y}{\delta}\right)^2\right]dy$$

$$\theta = \frac{19}{120}\delta$$

Hence, $\frac{\theta}{\delta} = \frac{19}{120}$

Que 4.28. **If velocity distribution in laminar boundary layer is given by :**

$\frac{u}{v} = \eta$ **where** $\eta = \frac{y}{\delta}$**; obtain values for** $\frac{\theta}{\delta}, \frac{\delta}{x}$ **and** $\frac{\delta^*}{\theta}$ **where the symbols appear for their usual meanings.**

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Answer

Given : $\frac{u}{v} = \eta$, where $\eta = \frac{y}{\delta}$; $\frac{u}{v} = \frac{y}{\delta}$

To Find : i. $\frac{\theta}{\delta}$, ii. $\frac{\delta^*}{\theta}$, iii. $\frac{\delta}{x}$

1. Momentum thickness,

$$\theta = \int_0^\delta \frac{u}{v}\left(1 - \frac{u}{v}\right)dy = \int_0^\delta \frac{y}{\delta}\left(1 - \frac{y}{\delta}\right)dy$$

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$$\theta = \int_0^\delta \left(\frac{y}{\delta} - \frac{y^2}{\delta^2}\right) dy = \left[\frac{y^2}{2\delta} - \frac{y^3}{3\delta^2}\right]_0^\delta$$

$$= \frac{\delta}{2} - \frac{\delta}{3}$$

$$\theta = \frac{\delta}{6}$$

Hence, $\frac{\theta}{\delta} = \frac{1}{6}$

2. Displacement thickness δ^*

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{v}\right) dy$$

$$\delta^* = \int_0^\delta \left(1 - \frac{y}{\delta}\right) dy$$

$$= \left[y - \frac{y^2}{2\delta}\right]_0^\delta = \delta - \frac{\delta}{2} = \frac{\delta}{2}$$

3. Therefore, $\frac{\delta^*}{\theta} = \frac{\delta/2}{\delta/6} = 3$

4. We know that,

$$\frac{\tau_o}{\rho v^2} = \frac{d}{dx}\left[\int_0^\delta \frac{u}{v}\left(1 - \frac{u}{v}\right) dy\right]$$

Substituting the value of $\frac{u}{v}$, we get

$$\frac{\tau_o}{\rho v^2} = \frac{d}{dx}\left[\int_0^\delta \frac{y}{\delta}\left(1 - \frac{y}{\delta}\right) dy\right] = \frac{d}{dx}\left[\int_0^\delta \left(\frac{y}{\delta} - \frac{y^2}{\delta^2}\right) dy\right]$$

$$= \frac{d}{dx}\left[\frac{y^2}{2\delta} - \frac{y^3}{3\delta^2}\right]_0^\delta = \frac{d}{dx}\left[\frac{\delta}{2} - \frac{\delta}{3}\right]$$

$$\tau_o = \frac{1}{6}\frac{d\delta}{dx}\rho v^2 \quad \text{...(4.28.1)}$$

5. According to Newton's law of viscosity,

$$\tau_o = \mu\left(\frac{du}{dy}\right)_{y=0} \quad \text{...(4.28.2)}$$

Since, $\left(\frac{du}{dy}\right)_{y=0} = \frac{v}{\delta}$

So, $\tau_o = \frac{\mu v}{\delta}$ ...(4.28.3)

6. From eq. (4.28.1) and eq. (4.28.3), we get

$$\frac{1}{6}\rho v^2 \frac{d\delta}{dx} = \frac{\mu v}{\delta}$$

$$\delta d\delta = \frac{6\mu}{\rho v} dx$$

7. Integrating both sides, we get

$$\frac{\delta^2}{2} = \frac{6\mu}{\rho v} x + C \quad \text{...(4.28.4)}$$

8. At $x = 0, \delta = 0, \therefore C = 0$

Therefore, eq. (4.28.4) becomes,

$$\frac{\delta^2}{2} = \frac{6\mu x}{\rho v}$$

$$\delta = \sqrt{\frac{12\mu x}{\rho v}} = 3.464\sqrt{\frac{\mu x \times x}{\rho v x}}$$

$$\delta = 3.464x\sqrt{\frac{1}{Re_x}} \quad \left(\because Re_x = \frac{\rho v x}{\mu}\right)$$

$$\frac{\delta}{x} = \frac{3.464}{\sqrt{Re_x}}$$

9. For laminar boundary layer, $Re_x = 5 \times 10^5$

$$\therefore \frac{\delta}{x} = \frac{3.464}{\sqrt{5 \times 10^5}} = 0.0049$$

Que 4.29. **Find the energy thickness for velocity distribution on the boundary layer is given by**

$$\frac{u}{v} = \frac{3}{2}\eta - \eta^2$$

where, $\eta = y/\delta$

AKTU 2013-14, Marks 05

Answer

Given : $\frac{u}{v} = \frac{3}{2}\eta - \eta^2, \eta = \frac{y}{\delta}$

To Find : Energy thickness

1. Energy thickness is given as,

$$\delta^{**} = \int_0^\delta \left(\frac{u}{v}\right)\left[1 - \left(\frac{u}{v}\right)^2\right] dy$$

2. Putting the value of $\frac{u}{v}$,

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$$\delta^{**} = \int_0^\delta \left[\frac{3}{2}\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2\right]\left[1 - \left\{\frac{3}{2}\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2\right\}^2\right] dy$$

$$= \int_0^\delta \left[\frac{3}{2}\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2\right]\left[1 - \frac{9}{4}\frac{y^2}{\delta^4} - \frac{y^4}{\delta^4} + \frac{3y^3}{3\delta^3}\right] dy$$

$$= \int_0^\delta \left[\frac{3}{2}\frac{y}{\delta} - \frac{27}{8}\frac{y^3}{\delta^3} - \frac{3}{2}\frac{y^5}{\delta^5} + \frac{9}{2}\frac{y^4}{\delta^4} - \frac{y^2}{\delta^2} + \frac{9}{4}\frac{y^4}{\delta^4} + \frac{y^6}{\delta^6} - \frac{3}{\delta^5}\frac{y^5}{\delta^5}\right] dy$$

$$= \int_0^\delta \left[\frac{3}{2}\frac{y}{\delta} - \frac{27}{8}\frac{y^3}{\delta^3} - \frac{9}{2}\frac{y^5}{\delta^5} + \frac{27}{4}\frac{y^4}{\delta^4} - \frac{y^2}{\delta^2} + \frac{y^6}{\delta^6}\right] dy$$

$$= \left[\frac{3}{4}\frac{y^2}{\delta} - \frac{y^3}{3\delta^2} - \frac{27}{32}\frac{y^4}{\delta^3} + \frac{27}{20}\frac{y^5}{\delta^4} - \frac{9}{12}\frac{y^6}{\delta^5} + \frac{y^7}{7\delta^6}\right]_0^\delta$$

$$= \frac{3}{4}\frac{\delta^2}{\delta} - \frac{\delta^3}{3\delta^2} - \frac{27}{32}\frac{y^4}{\delta^3} + \frac{27}{20}\frac{\delta^5}{\delta^4} - \frac{9}{12}\frac{\delta^6}{\delta^5} + \frac{\delta^7}{7\delta^6}$$

$$= \left[\frac{3}{4} - \frac{1}{3} - \frac{27}{32} + \frac{27}{20} - \frac{9}{12} + \frac{1}{7}\right]\delta$$

$$= \left[\frac{2520 - 1120 - 2835 + 4536 - 2520 + 480}{3360}\right]\delta$$

Hence, $\delta^{**} = \dfrac{1061\,\delta}{3360}$

Que 4.30. **Derive momentum integral equation for the boundary layer (Von-Karman).**

Answer

1. Let us consider a thin smooth flat plate with boundary layer as shown in Fig. 4.30.1.
2. Let the free stream velocity of flow be U over the plate.
3. Consider a small strip of dx at a distance x from one end of plate.
4. Now take the element $ABCD$, τ_o is the wall shear stress acting on the plate and velocity variation in strip $ABCD$ is also shown in the Fig. 4.30.1.

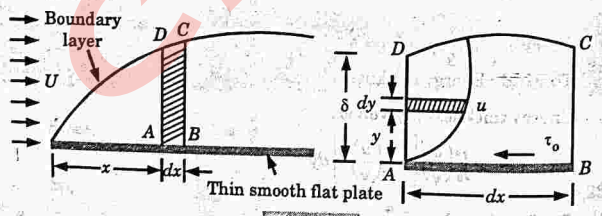

Fig. 4.30.1

5. In strip $ABCD$, CD is the outer edge of the boundary layer.
6. Consider unit width of plate perpendicular to the direction of flow.
7. Mass rate of fluid entering through $AD = \int_0^\delta \rho u dy$
8. Mass rate of fluid leaving through $BC = \int_0^\delta \rho u dy + \frac{d}{dx}\left[\int_0^\delta \rho u dy\right] dx$
9. Mass rate of fluid entering the control volume $ABCD$, through the surface CD = Mass rate of fluid through BC – Mass rate of fluid through AD

$$= \int_0^\delta \rho u dy + \frac{d}{dx}\left[\int_0^\delta \rho u dy\right] dx - \int_0^\delta \rho u dy = \frac{d}{dx}\left[\int_0^\delta \rho u dy\right] dx$$

10. The entering fluid through DC has uniform velocity U.
11. Momentum rate of fluid entering in $ABCD$ through AD (in x-direction)

$$P_{AD} = \int_0^\delta \rho u^2 dy$$

12. Momentum rate of fluid entering in $ABCD$ through BC (in x-direction)

$$P_{BC} = \int_0^\delta \rho \cdot u^2 dy + \frac{d}{dx}\left[\int_0^\delta \rho u^2 dy\right] dx$$

13. Momentum rate of fluid entering the $ABCD$ through CD (in x-direction)

$$P_{CD} = \text{mass} \times \text{velocity}$$

$$= \frac{d}{dx}\left[\int_0^\delta \rho u dy\right] dx \cdot U$$

$$P_{CD} = \frac{d}{dx}\left[\int_0^\delta \rho u U dy\right] dx$$

14. Rate of change of momentum = (momentum rate of fluid through BC) – (momentum rate of fluid through AD) – (momentum rate of fluid through DC)

$$= P_{BC} - P_{AD} - P_{DC}$$

$$= \int_0^\delta \rho \cdot u^2 dy + \frac{d}{dx}\left[\int_0^\delta \rho \cdot u^2 dy\right] dx - \int_0^\delta \rho \cdot u^2 dy - \frac{d}{dx}\left[\int_0^\delta \rho \cdot u \cdot U dy\right] dx$$

$$= \frac{d}{dx}\left[\int_0^\delta \rho \cdot u^2 dy - \int_0^\delta \rho \cdot u \cdot U dy\right] dx$$

15. Rate of change of momentum

$$= \rho \frac{d}{dx}\left[\int_0^\delta \left(u^2 - u \cdot U\right) dy\right] dx \quad \text{...(4.30.1)}$$

(ρ is constant, if fluid is incompressible.)

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16. According to momentum principle,

Force = rate of change of momentum

17. In control volume $ABCD$ only a shear force is acting on the side AB in the direction B to A.

18. So, drag force, $\Delta F_D = \tau_o \cdot dx$

Which is opposite to the direction of motion of fluid.

19. Thus, the total external force in the direction of rate of change of momentum $= -\tau_o \times dx$ (Negative sign indicates opposite direction) ...(4.30.2)

20. Now equating the eq. (4.30.1) and eq. (4.30.2) as

Rate of change of momentum = Force

$$\rho \frac{d}{dx}\left[\int_0^\delta (u^2 - u \cdot U)\, dy\right] dx = -\tau_o \times dx$$

$$\tau_o = -\rho \frac{d}{dx}\left[\int_0^\delta (u^2 - u \cdot U)\, dy\right]$$

$$\tau_o = -\rho \frac{d}{dx}\left[\int_0^\delta U^2 \left\{\left(\frac{u}{U}\right)^2 - \left(\frac{u}{U}\right)\right\} dy\right]$$

Hence, $$\tau_o = \rho \cdot U^2 \frac{d}{dx}\left[\int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy\right]$$

Here, $\int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$ = momentum thickness (θ).

So, $$\tau_o = \rho \cdot U^2 \frac{d\theta}{dx}$$

$$\frac{\tau_o}{\rho U^2} = \frac{d\theta}{dx}$$

21. The above equation is known as Von-Karman momentum equation for boundary layer flow.

Que 4.31. **An oil with density 900 kg/m³ and kinematic viscosity 10^{-5} m²/s is flowing over a plate of 3 m long and 2 m wide with a velocity of 3 m/s parallel to 3 m side. Find the boundary layer thickness at the point of transition and at the end of the plate.**

Answer

Given : Density of oil, $\rho = 900$ kg/m³, Free flow velocity, $U = 3$ m/s.
Kinematic viscosity of oil, $\nu = 10^{-5}$ m²/s,
Length of plate, $L = 3$ m, Width of plate, $b = 2$ m,
To Find : Boundary layer thickness at the point of transition and at the end of the plate.

1. Reynold's Number, $Re = \frac{U.L}{\nu} = \frac{3 \times 3}{10^{-5}} = 9 \times 10^5 > 5 \times 10^5$

Hence, upto a certain distance flow will be laminar then changes to turbulent flow.

2. Let, x be the distance upto which flow is laminar. Hence,

$$Re = \frac{U.x}{\nu}$$

$$5 \times 10^5 = \frac{3x}{10^{-5}}$$

$$x = \frac{5}{3} \text{m} = 1.67 \text{ m}$$

3. Using Blasius solution,

Boundary layer thickness at $x = 1.67$ or at transition equals to,

$$\delta = \frac{4.91x}{\sqrt{Re_x}} = \frac{4.91 \times 1.67}{\sqrt{5 \times 10^5}}$$

$$\delta = 0.01159 \text{ m}$$

4. Now, boundary layer thickness at the end of plate (*i.e.*, at $x = 3$ m)

$$\delta = \frac{4.91 \times 3}{\sqrt{9 \times 10^5}}$$

$$\delta = 0.0155 \text{ m}$$

Que 4.32. **A flat plate of 2 m width and 4 m length is kept parallel to air flow at 5.0 m/s velocity at 15 °C. Determine the length of plate over which the boundary layer is laminar, shear at the location where boundary layer ceases to be laminar, and total force on both sides on that portion of plate where the boundary layer is laminar.**

AKTU 2012-13, Marks 05

Answer

Given : Length of plate, $L = 4$ m, Width of plate, $b = 2$ m
Velocity of plate, $U = 5$ m/s
To Find : i. Length of plate over which the boundary layer is laminar.
ii. Shear at the location where boundary layer ceases to be laminar.
iii. Total force on both sides on that portion of plate where the boundary layer is laminar.

1. Reynold's number for laminar boundary layer

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$$Re_x = 5 \times 10^5$$

2. Let kinematic viscosity of air,

$$\nu = 0.15 \text{ stokes} = 0.15 \times 10^{-4} \text{ m}^2/\text{s}$$

3. Density of air, $\rho = 1.24 \text{ kg/m}^3$

4. Dynamic viscosity of air, $\mu = \nu \times \rho = 0.15 \times 10^{-4} \times 1.24$
$= 0.186 \times 10^{-4} \text{ kg/m-s}$

5. From equation, $Re_x = \dfrac{\rho U x}{\mu}$

$$5 \times 10^5 = \frac{1.24 \times 5 \times x}{0.186 \times 10^{-4}}$$

$$x = 1.5 \text{ m}$$

6. Shear at the location where boundary layer ceases to laminar,

$$\tau_x = 0.327 \frac{\mu U}{x} \sqrt{Re_x}$$

$$= \frac{0.327 \times 0.186 \times 10^{-4} \times 5}{1.5} \sqrt{5 \times 10^5}$$

$$= 0.0143 \text{ N/m}^2$$

7. Drag force on one side of the plate is given by,

$$F_D = 0.655 \times \mu U \times b \times \sqrt{\frac{\rho U x}{\mu}}$$

$$= 0.655 \times 0.186 \times 10^{-4} \times 5 \times 2 \sqrt{\frac{1.24 \times 5 \times 1.5}{0.186 \times 10^{-4}}}$$

$$= 0.0861 \text{ N}$$

Drag force on both sides = 2 × 0.0861 = 0.1722 N

Que 4.33. Explain the phenomenon of separation of boundary layer.

Answer

1. When a solid body is kept or immersed in a flowing fluid, boundary layer is formed adjacent to the solid body.
2. Within this thin layer of fluid, the velocity varies from zero to free stream velocity in the direction normal to the solid body.
3. Along the length of the solid body, the thickness of the boundary layer increases.
4. The fluid layer adjacent to the solid surface has to do work against surface friction at the expense of its kinetic energy.
5. This loss of the kinetic energy is recovered from the immediate fluid layer in contact with the layer adjacent to solid surface through momentum exchange process. Thus the velocity of layer goes on decreasing.

6. Along the length of the solid body, at a certain point a stage may come when the boundary layer may not be able to keep sticking to the solid body if it can not provide kinetic energy to overcome the resistance offered by the solid body. Thus, in other words, the boundary layer will get separated from the surface. This phenomenon is called the boundary layer separation.
7. The point on the body at which the boundary layer is on the verge of separation from the surface is called point of separation.

Que 4.34. Discuss the effect of pressure gradient on boundary layer separation. **AKTU 2014-15, Marks 05**

Answer

1. The effect of pressure gradient $\left(\dfrac{\partial p}{\partial x}\right)$ on boundary layer separation can be explained by considering the flow over a curved surface *ABCDE* as shown in Fig. 4.34.1.

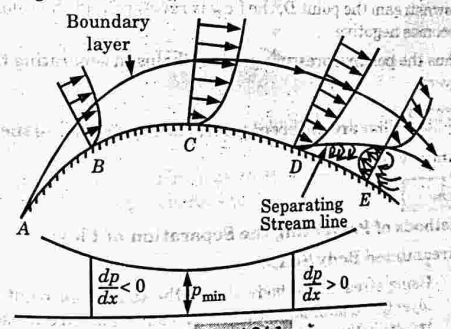

Fig. 4.34.1.

A. Region *ABC* of the Curved Surface :

1. In this region, the area of flow decreases and hence velocity increases. This means that flow gets accelerated in this region.
2. Due to increase of the velocity the pressure decreases in the direction of the flow and hence pressure gradient $\dfrac{dp}{dx}$ is negative in this region.
3. As long as $\dfrac{dp}{dx} < 0$, the entire boundary layer moves in forward direction.

B. Region *CDE* of the Curved Surface :

1. The pressure is minimum at point *C*.

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2. Along this region, the area of flow increases and hence velocity of flow along the direction of fluid decreases.
3. Due to decrease of velocity, the pressure increases in the direction of flow and hence pressure gradient $\frac{dp}{dx}$ is positive $\left(\frac{dp}{dx} > 0\right)$.
4. Thus in the region *CDE*, the pressure gradient is positive and velocity of fluid layers along the direction of flow decreases.
5. As explained in the Fig. 4.34.1 the velocity of the layer adjacent to the solid surface along the length of the solid surface goes on decreasing as the kinetic energy of the layer is used to overcome the frictional resistance of the surface.
6. Thus the combined effect of positive pressure gradient and surface resistance reduce the momentum of the fluid which is unable to overcome the surface resistance.
7. A stage comes, when the momentum of the fluid is unable to overcome the surface resistance and the boundary layer starts separating from the surface at the point *D*.
8. Downstream the point *D*, the flow is reversed and the velocity gradient becomes negative.
9. Thus the positive pressure gradient helps in separating the boundary layers.

Que 4.35. What are different methods of preventing the separation of boundary layers ?

Answer

A. Methods of Preventing the Separation of Flow :

i. Streamlined Body Shape :

1. Using streamlined body shape, the transition point of boundary layer (from laminar to turbulent) can be moved downstream which results in the reduction of the skin friction drag. Hence, separation of layers may be eliminated.

ii. Acceleration of Fluid in the Boundary Layer :

1. In this method, we supply additional energy to the particles of fluid which are being retarded in the boundary layer.
2. Energy can be transferred by
   a. Injecting the fluid into the region of boundary layer with the help of some device.
   b. Diverting a portion of fluid from high pressure region to the retarded region of boundary layer through a slot provided in the body.

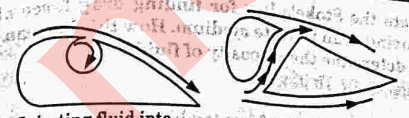

Fig. 4.35.1.

iii. Motion of Solid Boundary :

1. If we rotate a circular cylinder in a stream of fluid, such that the direction of motion of the upper side of cylinder is same in the direction of fluid.
2. But separation would occur on the lower side of cylinder as the direction of fluid motion is opposite to the motion of cylinder.

iv. By sucking the retarded flow.

v. By providing slots near the leading edge.

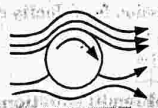

Fig. 4.35.1.

vi. Energize the flow by introducing optimum amount of swirl in the incoming flow.

vii. Remove the retarded or slow moving fluid particles in the boundary layer by suction through a porous surface.

VERY IMPORTANT QUESTIONS

Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.

Q. 1. Sketch the shear stress and velocity profiles across a section of a circular pipe, for the viscous flow. Derive the expression governing the shear stress and velocity profiles.

Ans. Refer Q. 4.2, Unit-4.

Q. 2. For laminar flow between two parallel fixed plates, derive expressions for velocity and shear stress distributions.

Ans. Refer Q. 4.4, Unit-4.

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5 UNIT

Drag and Lift, Similarity Laws and Dimensional Analysis

Part-1 ..... **(5–2A to 5–9A)**

- *Drag and Lift*
- *Drag on a Sphere*
- *Drag on a Two Dimensional Cylinder*
- *An Aerofoil*
- *Magnus Effect*

A. Concept Outline : Part-1 ..... 5–2A
B. Long and Medium Answer Type Questions 5–2A

Part-2 ..... **(5–9A to 5–15A)**

- *Similarity Laws : Geometric, Kinematics and Dynamic Similarity*
- *Undistorted and Distorted Model Studies*

A. Concept Outline : Part-2 ..... 5–9A
B. Long and Medium Answer Type Questions 5–10A

Part-3 ..... **(5–16A to 5–29A)**

- *Dimensional Analysis*
- *Buckingham's Pi Theorem*
- *Important Dimensionless Numbers and their Significance*

A. Concept Outline : Part-3 ..... 5–16A
B. Long and Medium Answer Type Questions 5–16A

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PART-1

Drag and Lift, Drag on a Sphere, Drag on a Two Dimensional Cylinder, an Aerofoil and Magnus Effect.

CONCEPT OUTLINE : PART-1

Drag and Lift : The force exerted by fluid in the direction of flow (free stream) on the body wholly immersed in a real fluid, is called drag force.

The force exerted by the fluid at the right angles to the direction of flow on the body wholly immersed in a real fluid, is called lift force.

Drag force, $F_D = C_D \times \frac{\rho U^2}{2} \times A$

Lift force, $F_L = C_L \times \frac{\rho U^2}{2} \times A$

Resultant force, $F = \sqrt{F_D^2 + F_L^2}$

For a Sphere : According to Stokes for $Re < 0.2$

Skin friction drag on sphere $= \frac{2}{3} \times$ Total drag on the sphere

Pressure drag on sphere $= \frac{1}{3} \times$ Total drag on the sphere

Total drag on sphere $= 3\pi\mu DU$

Magnus Effect : The generation of lift by spinning cylinder in a fluid stream is called Magnus effect.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.1. **Explain the force exerted by a flowing fluid on a stationary body.**

Answer

1. As shown in Fig. 5.1.1 consider a body held stationary in a real fluid, which is flowing at a uniform velocity U.
2. The fluid exerts a force on the stationary body.

3. The total force (F_R) exerted by the fluid on the body is perpendicular to the surface of the body. Thus the total force is inclined to the direction of motion.

Fig. 5.1.1.

A. Drag :

1. The component of the total force (F_R) in the direction of flow is called drag.
2. This component of force is denoted by F_D.

B. Lift :

1. The component of the total force (F_R) in the direction perpendicular to the direction of flow is known as lift.
2. This is denoted by F_L.
3. Lift force occurs only when the axis of the body is inclined to the direction of fluid flow.

Que 5.2. **Explain the term total drag on a body.**

Answer

1. Total drag acting on a body is the sum of pressure drag (form drag) and friction drag (shear drag or skin drag).
2. Friction drag acting on a body $= \int \tau_o \sin\theta \, dA$
3. Pressure drag acting on a body $= \int p \cos\theta \, dA$
4. Total drag acting on a body, $F_D = \int \tau_o \sin\theta \, dA + \int p \cos\theta \, dA$
5. The contribution of the pressure drag and friction drag to the total drag depends on the :
   a. Position of the body immersed in the fluid,
   b. Shape of the immersed body, and
   c. Fluid characteristics.

Que 5.3. **Explain drag on a sphere.**

Answer

A. Drag on a Sphere :

1. Consider a flow of real fluid passing with velocity U over a sphere having diameter D, density of fluid is ρ and viscosity is μ.

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2. Stokes developed a mathematical equation to determine the total drag acting on a sphere which is immersed in a fluid having Reynold's number less than 0.2.
3. According to stokes for Reynold's number less than 0.2, all the inertia forces acting on the fluid are assumed to be negligible; only viscous forces are to be considered.
4. According to Stokes,

   Total drag acting on sphere is, $F_D = 3\pi\mu DU$
5. In this total drag, two-third portion is contributed by skin-friction and the rest portion is contributed by pressure drag.
6. Hence, pressure drag, $F_{DP} = \frac{1}{3}F_D = \pi\mu DU$

   and skin friction drag, $F_{DF} = \frac{2}{3}F_D = 2\pi\mu DU$

Que 5.4. **Discuss about the drag on a cylinder and coefficient of drag for cylinder.**

Answer

1. Let us consider a cylinder having diameter, D and length, L is placed in a real fluid having kinematic viscosity ν and free flow velocity U.
2. If the Reynold's number for the flow is less than 0.2, the inertia forces are negligible as compared to viscous force.
3. If Reynold's number increases, inertia force also increases and flow pattern becomes unsymmetrical with respect to the axis perpendicular to flow direction.
4. From experiment following observations are to be made
   a. If $Re < 1$,

      Drag force $F_D \propto$ Velocity (U)

      And $C_D \propto \frac{1}{Re}$
   b. If $1 < Re < 2000$

      C_D will decreases and at $Re = 2000$, $C_D \approx 0.95$
   c. If $2000 < Re < 3 \times 10^4$

      C_D will start to increase up to a maximum value of 1.2 at $Re = 3 \times 10^4$
   d. If $3 \times 10^4 < Re < 3 \times 10^5$

      C_D again decreases and $C_D \approx 0.3$ at $Re = 3 \times 10^5$
   e. If $Re > 3 \times 10^5$

      C_D again increases and attains a maximum value of 0.7.

Que 5.5. **Write a short note on aerofoil / airfoil.**

Answer

1. An aerofoil or airfoil is a streamlined body which may be either symmetrical or unsymmetrical.
2. Following are the necessary and important definitions related to aerofoil :

A. Chord Line :

1. It is the line joining the leading and trailing edges of the aerofoil.
2. The length of the line is known as chord of aerofoil.

B. Profile Centreline :

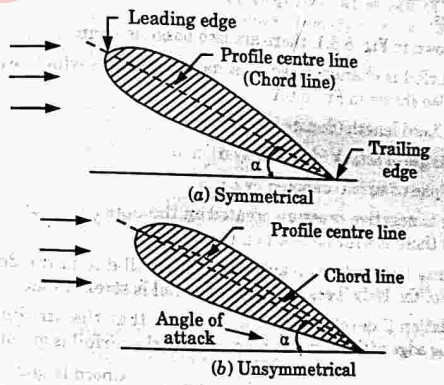

Fig. 5.5.1. Airfoil.

1. It is the line joining the midpoints of the profile.

C. Angle of Attack :

1. The angle between the chord line and direction of the fluid stream is known as angle of attack.

D. Camber :

1. It is the curvature of an airfoil.

E. Stall :

1. This is the condition when angle of attack (α) greater than angle of attack at maximum lift.
2. At stall the air separates from the airfoil or wing and eddies are formed as a consequence of which there is a considerable increase in the drag coefficient.

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F. Aspect Ratio (A.R.) :

1. The ratio of span of the wing to its mean chord is called the aspect ratio of a wing.

$$A.R. = \frac{l}{c}$$

Where, l = Span of the wing, and
c = Mean chord.

Que 5.6. Discuss the development of lift on an airfoil.

Answer

1. Airfoils are streamline bodies, it may or may not be symmetrical in shapes.
2. As shown in Fig. 5.6.1, there are two possible shapes of the airfoils.
3. The airfoil is characterized by some parameters which are as follows and also shown in Fig. 5.6.1.
   a. Chord length (denoted by C),
   b. Angle of attack (denoted by α), and
   c. Span of airfoil (denoted by L).
4. There is negative pressure created on the upper part of airfoil due to which there is a lift force act on the airfoil.
5. The drag force acting on airfoil is very small due to the design of the shape of the body (because shape of airfoil is streamlined).
6. Circulation Γ developed on the airfoil so that the streamline at the trailing edge of the airfoil is tangential to the airfoil is given as

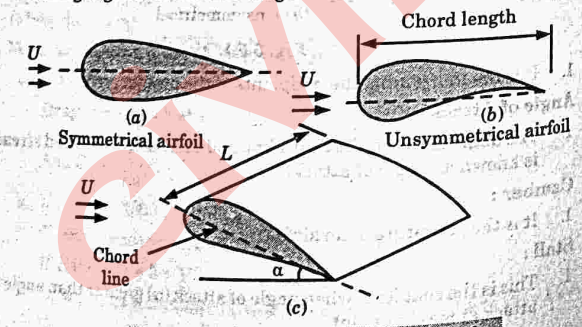

Fig. 5.6.1. Shapes of airfoils.

$$\Gamma = \pi CU \sin \alpha$$

7. Lift force acting on airfoil, $F_L = \rho UL\Gamma$

$$= \rho UL\,(\pi\, CU \sin \alpha)$$

$$= \pi \rho\, CU^2 L \sin \alpha \quad ...(5.6.1)$$

8. Lift force acting on airfoil in terms of coefficient of lift is given by

$$F_L = \frac{1}{2} C_L \cdot \rho A U^2 \quad ...(5.6.2)$$

9. On equating the eq. (5.6.1) and eq. (5.6.2) we get coefficient of lift, C_L as

$$C_L = 2\pi \sin\alpha \quad ...(5.6.3)$$

10. The eq. (5.6.3) shows that coefficient of lift depends upon the angle of attack.

Que 5.7. A flat plate of 400 m long, 120 m wide is covered through sea water. Determine the power required at speed of 3.0 m/s. If $\upsilon = 1.35 \times 10^{-6}$ m²/s and $\rho = 1020$ kg/m³. Justify the use of turbulent boundary layer formula for this case.

Answer

Given : Length of plate, $L = 400$ m, Width of plate, $b = 120$ m
Density of liquid, $\rho = 1020$ kg/m³,
Kinematic viscosity, $\nu = 1.35 \times 10^{-6}$ m²/s
Water is flowing with, $U = 3$ m/s
To Find : Power required at speed of 3.0 m/s

1. Reynold's number, $Re = \frac{UL}{\nu} = \frac{3 \times 400}{1.35 \times 10^{-6}} = 888.88 \times 10^6 > 5 \times 10^5$
2. To find the distance up to which the flow is laminar we have to calculate x.
   Where, x = Length of laminar boundary layer.
3. $$5 \times 10^5 = \frac{Ux}{\nu}$$
   $$5 \times 10^5 = \frac{3x}{1.35 \times 10^{-6}}$$
   $$x = \frac{5 \times 10^5 \times 1.35 \times 10^{-6}}{3} = 0.225 \text{ m}$$
4. Drag force for the portion of plate over which the flow is laminar boundary flow.
   Using Blasius solution
   $$C_D = \frac{0.455}{(\log_{10} R_{C_L})^{2.58}} = \frac{0.455}{(\log_{10} 8.88 \times 10^8)^{2.58}}$$
   $$= 1.6 \times 10^{-3}$$
5. Drag force, $$F_D = \frac{1}{2} \rho A U^2 \cdot C_D$$
   $$= \frac{1}{2} \times 1020 \times (400 \times 120) \times 3^2 \times 1.6 \times 10^{-3}$$

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$= 352512$ N

6. We know, $P = F_D \times U = 352512 \times 3 = 1057536$ W
$= 1057.536$ kW

Drag force on the both sides of plate $= 2F = 2 \times 352512 = 705024$ N

Que 5.8. **A kite 0.8 m × 0.8 m and weighing 4 N is maintained in air at an angle of 10° to the horizontal. The string attached to the kite makes an angle 45° to the horizontal and at this position the values of coefficient of drag and lift are 0.6 and 0.8 respectively. Find the speed of the wind, tension in the string. Take density of air = 1.25 kg/m³.**

Answer

Given : Projected Area, $A = 0.8 \times 0.8 = 0.64$ m²
Weight of kite, $W = 4$ N
Angle made by kite with horizontal, $\theta_1 = 10°$
Angle made by string with horizontal, $\theta_2 = 45°$
Coefficient of drag, $C_D = 0.6$, Coefficient of lift, $C_L = 0.8$
Density of air, $\rho_{air} = 1.25$ kg/m³

To Find : i. Speed of wind,
ii. Tension in the string

1. For equilibrium of kite

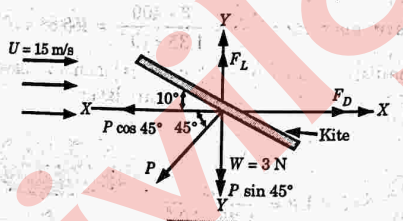

Fig. 5.8.1.

$$F_L = P \sin 45° + W$$

2. $$C_L \frac{1}{2}\rho A U^2 = \frac{P}{\sqrt{2}} + 4$$

$$\frac{1}{2} \times 1.25 \times 0.64 \times 0.8\, U^2 = 0.707\, P + 4$$

3. $$0.32\, U^2 = 0.707\, P + 4 \quad ...(5.8.1)$$

$$F_D = P \cos 45°$$

$$C_D\left(\frac{1}{2}\rho A U^2\right) = \frac{P}{\sqrt{2}}$$

$$U^2 = \frac{2P}{\sqrt{2}(0.6 \times 1.25 \times 0.64)} = 2.95P \quad ...(5.8.2)$$

4. From eq. (5.8.1) and eq. (5.8.2), we get
$0.32\,(2.95\,P) = 0.707\,P + 4$

5. Tension in the string,
$P = 16.877$ N

6. Speed of wind,
$U^2 = 2.95\,P = 49.787$
$U = 7.056$ m/s

Que 5.9. **Write down short note on Magnus effect.**

Answer

1. The generation of lift by spinning cylinder in a fluid stream is called Magnus effect.
2. Examples :
   a. This effect has been successfully employed in the propulsion of ships.
   b. The Magnus effect may also be used with advantage in the games like table tennis, golf, cricket etc.

PART-2

Similarity Laws : Geometric, Kinematics and Dynamic Similarity, Undistorted and Distorted Model Studies.

CONCEPT OUTLINE : PART-2

Similarity or Model Laws : The laws on which the models are designed for dynamic similarity are called model laws or similarity laws.

The model laws are as follows :
a. Reynold's model law, b. Froude model law,
c. Euler model law, d. Weber model law, and
e. Mach model law.

Classification of Hydraulic Models : Following are the two types of hydraulic models :
a. Undistorted models, and b. Distorted models.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

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Que 5.10. **Explain model (or similarity) laws, discuss in brief.**

Answer

1. The laws on which the models are designed for dynamic similarity are called model laws.
2. Some model laws are as follows :

A. Reynold's Model Law :

1. According to Reynold's model law,

$$(Re)_m = (Re)_p$$

$$\frac{\rho_m \cdot v_m \cdot L_m}{\mu_m} = \frac{\rho_p \cdot v_p \cdot L_p}{\mu_p}$$

Where, ρ = Density of fluid,
v = Velocity of fluid,
L = Length of model or prototype, and
μ = Viscosity of fluid.

'm' and 'p' are the subscripts for model and prototype respectively.

2. $$\frac{\rho_m}{\rho_p} \cdot \frac{v_m}{v_p} \cdot \frac{L_m}{L_p} \cdot \frac{\mu_p}{\mu_m} = 1$$

or $$\frac{\rho_p}{\rho_m} \cdot \frac{v_p}{v_m} \cdot \frac{L_p}{L_m} \cdot \frac{1}{(\mu_p / \mu_m)} = 1$$

$$\frac{\rho_r \cdot v_r \cdot L_r}{\mu_r} = 1$$

Where, ρ_r = Density scale ratio
v_r = Velocity scale ratio
L_r = Linear scale ratio, and
μ_r = Viscosity scale ratio,

3. Some scale ratios for time, acceleration, force and discharge for Reynold's model law are as follows :

t_r = Time scale ratio = $\frac{L_r}{v_r}$

a_r = Acceleration scale ratio = $\frac{v_r}{t_r}$

F_r = Force scale ratio
$= m_r \cdot a_r = \rho_r \cdot A_r \cdot v_r \cdot a_r$
$= \rho_r \cdot L_r^2 \cdot v_r \cdot a_r$ [$\because$ A_r = Area ratio = L_r^2]

Q_r = Discharge scale ratio
$= \rho_r \cdot A_r \cdot v_r = \rho_r \cdot L_r^2 \cdot v_r$

4. Models based on Reynold's number :
   a. Pipe flow model.
   b. Resistance experienced by sub-marines, airplanes, fully immersed bodies.

B. Froude Model Law :

1. According to Froude model law,

$$(F_e)_m = (F_e)_p$$

$$\frac{v_m}{\sqrt{g_m L_m}} = \frac{v_p}{\sqrt{g_p L_p}}$$

or $$\frac{v_m}{\sqrt{L_m}} = \frac{v_p}{\sqrt{L_p}}.$$

$$\frac{v_p}{v_m} \cdot \frac{1}{\sqrt{\frac{L_p}{L_m}}} = 1$$

$$\frac{v_r}{\sqrt{L_r}} = 1 \qquad \left\{\text{since } \frac{v_p}{v_m} = v_r \text{ and } \frac{L_p}{L_m} = L_r\right\}$$

2. Scale ratio for time,

$$T_r = \frac{T_p}{T_m} = \left(\frac{L_p}{v_p}\right)\left(\frac{v_m}{L_m}\right) = \left(\frac{L_p}{L_m}\right)\left(\frac{1}{v_p / v_m}\right)$$

$$\frac{T_p}{T_m} = \frac{L_r}{\sqrt{L_r}} = \sqrt{L_r} \qquad \left\{\because \frac{v_p}{v_m} = v_r = \sqrt{L_r}\right\} \qquad \text{...(5.10.1)}$$

3. Scale ratio for acceleration,

$$a_r = \frac{a_p}{a_m} = \left(\frac{v}{T}\right)_p \cdot \left(\frac{T}{v}\right)_m = \left(\frac{v_p}{v_m}\right) \cdot \left(\frac{1}{T_p / T_m}\right)$$

$$a_r = \frac{\sqrt{L_r}}{\sqrt{L_r}} = 1$$

4. Scale ratio for discharge,

$$Q = A \cdot v = L^2 \cdot \frac{L}{T} = \frac{L^3}{T}$$

$$Q_r = \frac{Q_p}{Q_m} = \frac{L_p^3}{L_m^3} \cdot \frac{T_m}{T_p}$$

$$= L_r^3 \times \frac{1}{(T_p / T_m)} = \frac{L_r^3}{\sqrt{L_r}} \qquad \text{[From eq. (5.10.1)]}$$

$$Q_r = L_r^{5/2}$$

5. Scale ratio for force, $F_r = L_r^3$
6. Models for Froude model law :
   a. Free surface flow (weirs, channels etc.) models, and
   b. Flow of jet from an orifice or nozzle.

C. Euler Model Law :

1. According to Euler model law,

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$$(E_u)_m = (E_u)_p$$

$$\frac{v_m}{\sqrt{p_m / \rho_m}} = \frac{v_p}{\sqrt{p_p / \rho_p}}$$

For same fluid $\rho_m = \rho_p = \rho$

So $$\frac{v_m}{\sqrt{p_m}} = \frac{v_p}{\sqrt{p_p}}$$

2. Application :
   a. Enclosed fluid system where the turbulence is fully developed.
   b. Fluid system where the phenomenon of cavitation occurs.

D. Weber Model Law :

1. According to this law,

$$(W_e)_m = (W_e)_p$$

$$\frac{v_m}{\sqrt{\sigma_m / (\rho_m \cdot L_m)}} = \frac{v_p}{\sqrt{\sigma_p / (\rho_p \cdot L_p)}}$$

2. Application :
   a. Capillary waves in channels.
   b. Capillary rise in narrow passages.
   c. Capillary movement of water in soil.
   d. Flow over weirs for very low heads.

E. Mach Model Law :

1. According to this law,

$$(M)_m = (M)_p$$

$$\frac{v_m}{\sqrt{K_m / \rho_m}} = \frac{v_p}{\sqrt{K_p / \rho_p}}$$

2. Application :
   a. Aerodynamic testing.
   b. Hydraulic model testing especially water hammer problems.
   c. Under water testing for torpedoes.

Que 5.11. **Describe in detail about geometric, kinematics and dynamic similarity.**

Answer

A. Geometric Similarity :

1. The geometric similarity is said to exist between the model and the prototype if the ratio of all corresponding linear dimension in the model and prototype are equal.
2. Let
   L_m = Length of model, b_m, = Breadth of model,
   D_m = Diameter of model, A_m = Area of model.
   $\forall_m$ = Volume of model,

and $L_p, b_p, D_p, A_p, \forall_p$ = Corresponding values of the prototype.

3. For geometric similarity between model and prototype, we must have the relation,

$$\frac{L_p}{L_m} = \frac{b_p}{b_m} = \frac{D_p}{D_m} = L_r$$

Where L_r is called the scale ratio.

4. For area's ratio and volume's ratio the relation should be as given below :

$$\frac{A_p}{A_m} = \frac{L_p \times b_p}{L_m \times b_m} = L_r \times L_r = L_r^2$$

and $$\frac{\forall_p}{\forall_m} = \left(\frac{L_p}{L_m}\right)^3 = \left(\frac{b_p}{b_m}\right)^3 = \left(\frac{D_p}{D_m}\right)^3 = L_r^3$$

B. Kinematic Similarity :

1. Kinematic similarity means the similarity of motion between model and prototype. Thus kinematic similarity is said to exist between the model and the prototype if the ratios of the velocity and acceleration at the corresponding points in the model and at the corresponding points in the prototype are the same.
2. Let
   V_{p1} = Velocity of fluid at point 1 in prototype,
   V_{p2} = Velocity of fluid at point 2 in prototype,
   a_{p1} = Acceleration of fluid at 1 in prototype, and
   a_{p2} = Acceleration of fluid at 2 in prototype
   $V_{m1}, V_{m2}, a_{m1}, a_{m2}$ = Corresponding values at the corresponding points of fluid velocity and acceleration in the model.
3. For kinematic similarity, we must have

$$\frac{V_{p_1}}{V_{m_1}} = \frac{V_{p_2}}{V_{m_2}} = V_r$$

Where V_r is the velocity ratio.

4. For acceleration, we must have $\frac{a_{p_1}}{a_{m_1}} = \frac{a_{p_2}}{a_{m_2}} = a_r$

Where a_r is the acceleration ratio.

C. Dynamic Similarity :

1. Dynamic similarity means the similarity of forces between the model and prototype. Thus dynamic similarity is said to exist between the model and the prototype if the ratios of the corresponding forces acting at the corresponding points are equal.
2. Let
   $(F_i)_p$ = Inertia force at a point in prototype,
   $(F_v)_p$ = Viscous force at the point in prototype,

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$(F_g)_p$ = Gravity force at the point in prototype, and

$(F_i)_m, (F_v)_m, (F_g)_m$ = Corresponding values of forces at the corresponding point in model.

3. For dynamic similarity, we have

$$\frac{(F_i)_p}{(F_i)_m} = \frac{(F_v)_p}{(F_v)_m} = \frac{(F_g)_p}{(F_g)_m} = F_r$$

Where F_r is the force ratio.

Que 5.12. Explain the terms : distorted models and undistorted models. What are the uses of distorted models ?

Answer

A. Distorted Models :

1. A model, which is not geometrically similar to its prototype is called distorted model.
2. For this model different scale ratios are adopted for the linear dimensions.
3. Use of distorted models :
   a. For maintaining accuracy in vertical dimensions.
   b. For maintaining turbulent flow.
   c. For reducing the cost of model.
   d. For obtaining suitable roughness condition.

B. Undistorted Models :

1. A model which is geometrically similar to its prototype is known as undistorted model.
2. In other words if the scale ratio for linear dimensions of model and its prototype is same, model is called undistorted model.
3. In such models the design and construction of the model and the interpretation of the model results are simpler.

Que 5.13. What are the advantages and disadvantages of distorted models ?

Answer

A. Advantages :

1. Due to increased scale ratio accurate measurements are possible.
2. Surface tension can be reduced.
3. Size of model can be reduced sufficiently which simplifies its operation and lowers the crests considerably.
4. Reynolds number of flow in a model can be increased to get better results.

B. Disadvantages :

1. The pressure and velocity distribution are not truly reproduced.

2. The model wave may or may not be same as in case of prototype.
3. Slopes of river bends, earth cuts and dikes cannot be truly reproduced.
4. To interpolate or extrapolate the results obtained from distorted models is difficult.

Que 5.14. A model boat $\frac{1}{50}$ of its prototype experienced 0.2 N of resistance when simulating a speed of 5 m/s of prototype. Find the corresponding resistance of the prototype considering resistance at free surface only. Water is used for model as well as prototype also.

Answer

Given : Linear scale ratio, $L_r = \frac{L_p}{L_m} = 50$

Resistance experienced by model, $(R_f)_m = 0.2$ N

Speed of prototype, $v_p = 5$ m/s

To Find : Resistance experienced by prototype.

1. Since, resistance, $R \propto v^2 . A$
2. Frictional drag on prototype,

$$(R_f)_p = f_p \times A_p \times v_p^2$$

3. Froude model clarify,

$$\frac{v_m}{\sqrt{L_m . g}} = \frac{v_p}{\sqrt{L_p . g}}$$

$$\frac{v_m}{\sqrt{L_m}} = \frac{v_p}{\sqrt{L_p}}$$

$$\frac{v_p}{v_m} = \sqrt{\frac{L_p}{L_m}} = \sqrt{50}$$

4. $$\frac{A_p}{A_m} = L^2_r = 50 \times 50$$

$$A_p = 2500\, A_m$$

5. $$\frac{(R_f)_p}{(R_f)_m} = \left(\frac{A_p}{A_m}\right) \cdot \left(\frac{v_p}{v_m}\right)^2$$

$$(R_f)_p = 2500(\sqrt{50})^2 \times (R_f)_m$$

$$(R_f)_p = 2500(\sqrt{50})^2 \times 0.2$$

$$(R_f)_p = 25000 \text{ N.}$$

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PART-3

Dimensional Analysis, Buckingham's Pi-Theorem, Important Dimensionless Numbers and their Significance.

CONCEPT OUTLINE : PART-3

Dimensional Analysis : It is a mathematical technique which makes use of the study of dimensions for solving several engineering problems.

π-terms : The dimensionless terms used in Buckingham's Pi-theorem are called π-terms.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.15. **What do you mean by dimensional analysis ? What are its uses ? What are advantages and limitations of dimensional analysis ?**

Answer

A. Dimensional Analysis :

1. It is a mathematical test or technique which makes use of the study of dimensions for solving several engineering problems.
2. It is based on the principle of dimensional homogeneity.

B. Uses of Dimensional Analysis :

1. The dimensional homogeneity of any equation can be tested by dimensional analysis.
2. A formula for flow phenomenon can be derived by using dimensional analysis.
3. To obtain equation to be expressed in terms of non-dimensional parameters in order to show the relative significance of each parameter.
4. To plan model tests to obtain experimental results which can be presented in a systematic manner.

C. Advantages of Dimensional Analysis :

1. It gives the functional relationship between the variables in dimensionless terms.
2. It reduces the number of variables involved in a physical phenomenon.

3. By using dimensional analysis design curves can be developed from experimental data or direct solution of the problem.
4. By the proper selection of variables the dimensionless parameters can be used to make certain logical deductions about the problem.

D. Limitations :

1. There are no certain or perfect rules for choosing the variables.
2. Dimensional analysis only indicates that there is some relationship between parameters but the value of coefficients and constants are not to be found.

Que 5.16. **Give Buckingham-Pi theorem and explain Buckingham's-Pi method.**

Answer

A. Buckingham-Pi Theorem :

1. The theorem states that if there are n variables (independent and dependent variables) in a physical phenomenon and if these variables contain m fundamental dimensions (M, L, T), then the variables are arranged into $(n - m)$ dimensionless terms.

B. Buckingham's-Pi method :

1. If there are n variables (both independent and dependent) in a physical phenomenon and the variables contain 'm' fundamental dimensions (M, L, T), then the variables are arranged into $(n - m)$ dimensionless terms. Each term is called a π-term.
2. Let $V_d, V_1, V_2, V_3 .... V_n$ are the variables involved in a physical problem.
3. Let V_d be the dependent variable and $V_1, V_2 ... V_n$ are the independent variables on which V_d depends.

   Then V_d is a function of $V_1, V_2 ... V_n$ and mathematically it is expressed as

   $$V_d = f(V_1, V_2 ..... V_n) \quad ...(5.16.1)$$
4. Eq. (5.16.1) can also be written as

   $$f_1 (V_d, V_1, V_2 ... V_n) = 0 \quad ...(5.16.2)$$

   Eq. (5.16.2) is a dimensionally homogeneous equation and it contains n variables.
5. If there are m fundamental dimensions (*i.e.*, M, L, T), then according to Buckingham's π-theorem eq. (5.16.2) can be written in terms of dimensionless groups or π-terms in which the number of π-terms is equal to $(n - m)$. Hence eq. (5.16.2) becomes

   $$f(\pi_1 \pi_2 ... \pi_{(n-m)}) = 0 \quad ...(5.16.3)$$

   Each of π-term is dimensionless and is independent of the systems.
6. Each π-term contains $m + 1$, number of variables, where m is the num of fundamental dimensions and is also called as repeating variable.

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7. Let in the above case V_1, V_2 and V_3 be the repeating variables if fundamental dimension is m $(M\,L\,T) = 3$. Then each π-term is written as

$$\left.\begin{aligned} \pi_1 &= V_1^{a_1}\, V_2^{b_1}\, V_3^{c_1}\, V_4 \\ \pi_2 &= V_1^{a_2}\, V_2^{b_2}\, V_3^{c_2}\, V_4 \\ \pi_{n-m} &= V_1^{a_{n-m}}\, V_2^{b_{n-m}}\, V_3^{c_{n-m}}\, V_4 \end{aligned}\right\} \quad ...(5.16.4)$$

8. Each equation is solved by the principle of dimensionless homogeneity and values of a_1, b_1, c_1 etc., are obtained.
9. These values are substituted in the eq. (5.16.4) and values of π_1, $\pi_2 ... \pi_{n-m}$ are obtained.
10. These values of π's are substituted in eq. (5.16.3) and the final equation for the phenomenon is obtained by expressing any one of the π-terms as a function of others as

$$\pi_1 = \phi[\pi_2, \pi_3 ... \pi_{n-m}]$$

or

$$\pi_2 = \phi[\pi_1, \pi_3 ... \pi_{n-m}]$$

Que 5.17. **Using Buckingham's π theorem, show that the discharge, Q consumed by an oil ring is given by**

$$Q = (Nd^3)\, f\,[\mu / (\rho N d^2), \sigma / (\rho N^2 d^3), \omega / (\rho N^2 d)]$$

Where, d is internal diameter of ring, N is rotational speed, ρ is density, μ is viscosity, σ is surface tension and ω is the specific weight of oil. **AKTU 2014-15, Marks 10**

Answer

Given : Discharge is a function of $d, N, \rho, \mu, \sigma, \omega$
$Q = f(d, N, \rho, \mu, \sigma, \omega)$ or $f_1(Q, d, N, \rho, \mu, \sigma, \omega) = 0$
To Find : $Q = (Nd^3)\, f\,[\mu / (\rho N d^2), \sigma / (\rho N^2 d^3), \omega / (\rho N^2 d)]$

1. Total number of variables, $n = 7$
2. Dimensions of each variable are

$$Q = L^3T^{-1},\ d = L,\ N = T^{-1},$$
$$\rho = ML^{-3},\ \mu = ML^{-1}T^{-1},\ \sigma = MT^{-2}$$

and

$$\omega = ML^{-2}T^{-2}$$

Total number of fundamental dimensions, $m = 3$

3. Total number of π-terms $= n - m = 7 - 3 = 4$
4. Now discharge function becomes as $f_1(\pi_1, \pi_2, \pi_3, \pi_4) = 0$ ...(5.17.1)
5. Choosing d, N, ρ as repeating variables, the π-terms are

$$\pi_1 = d^{a_1} \cdot N^{b_1} \cdot \rho^{c_1} \cdot Q$$
$$\pi_2 = d^{a_2} \cdot N^{b_2} \cdot \rho^{c_2} \cdot \mu$$
$$\pi_3 = d^{a_3} \cdot N^{b_3} \cdot \rho^{c_3} \cdot \sigma$$
$$\pi_4 = d^{a_4} \cdot N^{b_4} \cdot \rho^{c_4} \cdot \omega$$

6. **First π-term :** $\pi_1 = d^{a_1} \cdot N^{b_1} \cdot \rho^{c_1} \cdot Q$
Substituting dimensions on both sides,

$$M^0L^0T^0 = L^{a_1} \cdot (T^{-1})^{b_1} \cdot (ML^{-3})^{c_1} \cdot L^3T^{-1}$$

Equating the powers of M, L, T on both sides,

Power of M, $0 = c_1$, $\therefore$ $c_1 = 0$

Power of L, $0 = a_1 - 3c_1 + 3$, $\therefore$ $a_1 = 3c_1 - 3 = 0 - 3 = -3$

Power of T, $0 = -b_1 - 1$, $\therefore$ $b_1 = -1$

Substituting a_1, b_1, c_1 in π_1, $\pi_1 = d^{-3} \cdot N^{-1} \cdot \rho^0 \cdot Q = \dfrac{Q}{d^3N}$

7. **Second π-term :** $\pi_2 = d^{a_2} \cdot N^{b_2} \cdot \rho^{c_2} \cdot \mu$
Substituting the dimensions on both sides,

$$M^0L^0T^0 = L^{a_2} \cdot (T^{-1})^{b_2} \cdot (ML^{-3})^{c_2} \cdot ML^{-1}T^{-1}$$

Equating the powers of M, L, T on both sides,

Power of M, $0 = c_2 + 1$, $\therefore$ $c_2 = -1$

Power of L, $0 = a_2 - 3c_2 - 1$, $\therefore$ $a_2 = -2$

$\therefore$ $a_2 = 3c_2 + 1 = -3 + 1 = -2$

Power of T, $0 = -b_2 - 1$, $\therefore$ $b_2 = -1$

Substituting the values of a_2, b_2, c_2 in π_2,

$$\pi_2 = d^{-2} \cdot N^{-1} \cdot \rho^{-1} \cdot \mu = \frac{\mu}{d^2N\rho} \text{ or } \frac{\mu}{\rho N d^2}$$

8. **Third π-term :** $\pi_3 = d^{a_3} \cdot N^{b_3} \cdot \rho^{c_3} \cdot \sigma$
Substituting the dimensions on both sides,

$$M^0L^0T^0 = L^{a_3} \cdot (T^{-1})^{b_3} \cdot (ML^{-3})^{c_3} \cdot MT^{-2}$$

Equating the powers of M, L, T on both sides,

Power of M, $0 = c_3 + 1$, $\therefore$ $c_3 = -1$

Power of L, $0 = a_3 - 3c_3$, $\therefore$ $a_3 = 3c_3 = -3$

Power of T, $0 = -b_3 - 2$, $\therefore$ $b_3 = -2$

Substituting the values of a_3, b_3, c_3 in π_3,

$$\pi_3 = d^{-3} \cdot N^{-2} \cdot \rho^{-1} \cdot \sigma = \frac{\sigma}{d^3N^2\rho}$$

9. **Fourth π-term :** $\pi_4 = d^{a_4} \cdot N^{b_4} \cdot \rho^{c_4} \cdot \omega$
Substituting dimensions on both sides,

$$M^0L^0T^0 = L^{a_4} \cdot (T^{-1})^{b_4} \cdot (ML^{-3})^{c_4} \cdot ML^{-2}T^{-2}$$

Equating the powers of M, L, T on both sides,

Power of M, $0 = c_4 + 1$, $\therefore$ $c_4 = -1$

Power of L, $0 = a_4 - 3c_4 - 2$, $\therefore$ $a_4 = 3c_4 + 2 = -3 + 2 = -1$

Power of T, $0 = -b_4 - 2$, $\therefore$ $b_4 = -2$

Substituting the values of a_4, b_4, c_4 in π_4,

$$\pi_4 = d^{-1} \cdot N^{-2} \cdot \rho^{-1} \cdot \omega = \frac{\omega}{dN^2\rho}$$

10. Now substituting the values of $\pi_1, \pi_2, \pi_3, \pi_4$ in eq. (5.17.1),

$$f\left(\frac{Q}{d^3N}, \frac{\mu}{\rho N d^2}, \frac{\sigma}{d^3N^2\rho}, \frac{\omega}{dN^2\rho}\right) = 0$$

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or $$\frac{Q}{d^3N} = f_1\left[\frac{\mu}{\rho N d^2}, \frac{\sigma}{d^3N^2\rho}, \frac{\omega}{dN^2\rho}\right]$$

$$Q = d^3N\phi\left[\frac{\mu}{\rho N d^2}, \frac{\sigma}{d^3N^2\rho}, \frac{\omega}{dN^2\rho}\right]$$

Que 5.18. **Find the form of equation for discharge Q through a sharp edged triangular notch; assuming Q depends upon the central angle α of the notch, head H, gravitational acceleration 'g' and on the mass density ρ, viscosity μ and surface tension σ of the fluid.**

AKTU 2011-12, Marks 05

Answer

Given : Discharge Q is a function of $H, g, \alpha, \rho, \mu, \sigma$

$$f_1 = (Q, H, g, \alpha, \rho, \mu, \sigma)$$

To Find : Discharge equation.

1. Total number of variables $n = 7$.
2. Number of fundamental dimensions $m = 3$.
3. Number of π-terms $= 7 - 3 = 4$.
4. Now, discharge function becomes as $f_1(\pi_1, \pi_2, \pi_3, \pi_4) = 0$ ...(5.18.1)
5. Choosing g, H, ρ as the repeating variables.
6. We have four π-terms as

$$\pi_1 = H^{a1}g^{b1}\rho^{c1}Q$$
$$\pi_2 = H^{a2}g^{b2}\rho^{c2}\alpha$$
$$\pi_3 = H^{a3}g^{b3}\rho^{c3}\mu$$
$$\pi_4 = H^{a4}g^{b4}\rho^{c4}\sigma$$

7. **π_1-term :**
Substituting the dimensions on both sides,

$$M^0L^0T^0 = [L^{a1}]\,[LT^{-2}]^{b1}\,[ML^{-3}]^{c1}\,[L^3T^{-1}]$$
$$M^0L^0T^0 = M^{c1}L^{a1+b1-3c1+3}\,T^{-2b1-1}$$

On comparing the powers on both sides

$$\Rightarrow \quad c_1 = 0$$
$$-2b_1 - 1 = 0$$
$$b_1 = -\frac{1}{2}$$
$$a_1 + b_1 - 3c_1 + 3 = 0$$
$$a_1 - \frac{1}{2} - 3 \times 0 + 3 = 0$$
$$a_1 = \frac{1}{2} - 3$$

$$a_1 = -\frac{5}{2}$$

On substituting the values of a_1, b_1 and c_1

$$\pi_1 = H^{-\frac{5}{2}}\,g^{-1/2}\rho^0 Q$$
$$\pi_1 = \frac{Q}{H^{5/2}g^{1/2}}$$

8. **π_2-term :**
Substituting the dimensions on both sides,

$$M^0L^0T^0 = [L^{a2}]\,[LT^{-2}]^{b2}\,[ML^{-3}]^{c2}\,[M^0L^0T^0]$$
$$M^0L^0T^0 = M^{c2}L^{a2+b2-3c2}\,T^{-2b2}$$

On comparing the powers on both sides

$$\Rightarrow \quad c_2 = 0$$
$$-2b_2 = 0$$
$$\Rightarrow \quad b_2 = 0$$
$$a_2 + b_2 - 3c_2 = 0$$
$$\Rightarrow \quad a_2 = 0$$

On substituting the values of a_2, b_2 and c_2

$$\pi_2 = \alpha$$

9. **π_3-term :**
Substituting the dimensions on both sides,

$$M^0L^0T^0 = [L^{a3}]\,[LT^{-2}]^{b3}\,[ML^{-3}]^{c3}\,[ML^{-1}T^{-1}]$$
$$M^0L^0T^0 = M^{c3+1}L^{a3+b3-3c3-1}\,T^{-2b3-1}$$

On comparing the powers on the both sides

$$c_3 + 1 = 0$$
$$\Rightarrow \quad c_3 = -1$$
$$-2b_3 - 1 = 0$$
$$\Rightarrow \quad b_3 = -\frac{1}{2}$$
$$a_3 + b_3 - 3c_3 - 1 = 0$$
$$a_3 - \frac{1}{2} + 3 - 1 = 0$$
$$\Rightarrow \quad a_3 = -\frac{3}{2}$$

On substituting the values of a_3, b_3 and c_3

$$\pi_3 = H^{-3/2}g^{-1/2}\rho^{-1}\mu$$
$$\pi_3 = \frac{\mu}{H\rho\sqrt{gH}}$$

10. **π_4-term :**
Substituting the dimensions on both sides,

$$M^0L^0T^0 = L^{a4}\,[LT^{-2}]^{b4}\,[ML^{-3}]^{c4}\,[MT^{-2}]$$
$$M^0L^0T^0 = M^{c4+1}L^{a4+b4-3c4}\,T^{-2b4-2}$$

On comparing the powers on both sides

$$\Rightarrow \quad c_4 = -1$$

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$$-2b_4 - 2 = 0$$
$$b_4 = -1$$
$$\Rightarrow \quad a_4 + b_4 - 3c_4 = 0$$
$$a_4 - 1 + 3 = 0$$
$$a_4 = -2$$

On substituting the values of a_4, b_4 and c_4

$$\pi_4 = H^{-2} g^{-1} \rho^{-1} \sigma$$

$$\pi_4 = \frac{\sigma}{H^2 g \rho}$$

11. Substituting the values of π_1, π_2, π_3 and π_4 in eq. (5.18.1)

$$f_1\left(\frac{Q}{H^{5/2} g^{1/2}}, \alpha, \frac{\mu}{H\rho\sqrt{gH}}, \frac{\sigma}{H^2 g\rho}\right) = 0$$

Que 5.19. **The discharge over a spillway provided on the dam depends upon v (velocity of flow), L (depth of throat) H (water head on spillway) and g (acceleration due to gravity).**

Show that it is given by $\frac{Q}{vL^2} = f\left(\frac{\sqrt{gL}}{v}, \frac{H}{L}\right)$.

Answer

Given : $Q = f(v, L, H, g)$, or $f_1(Q, v, L, H, g) = 0$

To Find : $\frac{Q}{vL^2} = f\left(\frac{\sqrt{gL}}{v}, \frac{H}{L}\right)$

1. Total number of variables, $n = 5$
2. Dimensions of each variable are,
   $Q\ [L^3T^{-1}]$, $v\ [LT^{-1}]$, $L\ [L]$, $H\ [L]$, $g\ [LT^{-2}]$
3. The number of fundamental dimensions, $m = 2$
4. Number of π-terms $= n - m$
   $= 5 - 2 = 3$
   So, $f_1(\pi_1, \pi_2, \pi_3) = 0$ ...(5.19.1)
5. Each π-term contains $m + 1 = 2 + 1 = 3$ variables having two $(m = 2)$ repeating variables.
6. Take L and v as repeating variables and we get
   $$\pi_1 = L^{a1} . v^{b1} . Q$$
   $$\pi_2 = L^{a2} . v^{b2} . H$$
   $$\pi_3 = L^{a3} . v^{b3} . g$$
7. π_1-term :
   $[L^0T^0] = [L]^{a1}\ [LT^{-1}]^{b1} [L^3T^{-1}]$
   Comparing the powers of L and T on both sides, we have
   $$a_1 + b_1 + 3 = 0$$

and $\quad -b_1 - 1 = 0$

We get, $\quad a_1 = -2, \quad b_1 = -1$

$$\pi_1 = L^{-2} v^{-1} Q = \frac{Q}{vL^2}$$

8. π_2-term :
   $$[L^0T^0] = [L]^{a2}\ [LT^{-1}]^{b2} [L]$$
   Comparing the powers of L and T on both sides we have
   $$a_2 + b_2 + 1 = 0$$
   and $\quad -b_2 = 0$
   We get, $\quad a_2 = -1, \quad b_2 = 0$
   $$\pi_2 = L^{-1} v^0 H = \frac{H}{L}$$
9. π_3-term :
   $$[L^0T^0] = [L]^{a3}\ [LT^{-1}]^{b3} [LT^{-2}]$$
   Comparing the powers of L and T on both sides,
   $$a_3 + b_3 + 1 = 0$$
   and $\quad -b_3 - 2 = 0$
   We get, $\quad a_3 = 1, \quad b_3 = -2$
   $$\therefore \quad \pi_3 = L\, v^{-2} g = \frac{gL}{v^2} = \frac{\sqrt{gL}}{v}$$
13. On substituting the value of π_1, π_2, π_3 in eq. (5.19.1), we get
   $$f_1\left(\frac{Q}{vL^2}, \frac{H}{L}, \frac{\sqrt{gL}}{v}\right) = 0$$
14. Hence, $\quad \frac{Q}{vL^2} = f\left(\frac{\sqrt{gL}}{v}, \frac{H}{L}\right)$

Que 5.20. **Assuming the drag force, F, exerted on a body is a function of the following :**
Fluid density ρ, Fluid viscosity μ, Diameter d, Velocity u
Show the drag force can be expressed as,

$$F = d^2 u^2 \rho \phi\ (Re)$$

Where ϕ is some unknown function and Re is Reynold number.

AKTU 2016-17, Marks 10

Answer

Given : Drag force function, $F = f(d, u, \rho, \mu)$ or $F = C(d^a, u^b, \rho^c, \mu^d)$
where C is non-dimensional constant.

To Find : $F = d^2 u^2 \rho\phi(Re)$

1. Using M-L-T system the corresponding equation for dimensions is :

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$$MLT^{-2} = [CL^a \cdot (LT^{-1})^b \cdot (ML^{-3})^c \cdot (ML^{-1}T^{-1})^d]$$

2. For dimensional homogeneity the exponents of each dimension on both sides of the equation must be identical. Thus

For $M: 1 = c + d$ ...(5.20.1)

For $L: 1 = a + b - 3c - d$ ...(5.20.2)

For $T: -2 = -b - d$ ...(5.20.3)

3. There are four unknowns (a, b, c, d) but equations are three in number. Therefore, it is not possible to find the values of a, b, c and d. However, three of them can be expressed in terms of fourth variable which is most important.

4. There the role of viscosity is vital one and hence a, b, c are expressed in terms of d (*i.e.*, power to viscosity)

$\therefore$ $c = 1 - d$ ... from eq. (5.20.1)

$b = 2 - d$ ... from eq. (5.20.3)

Putting these values in eq. (5.20.2), we get

$$a = 1 - b + 3c + d = 1 - 2 + d + 3(1 - d) + d$$
$$= 1 - 2 + d + 3 - 3d + d = 2 - d$$

5. Substituting these values of exponents in Drag force function, we get

$$F = C[d^{2-d} \cdot u^{2-d} \cdot \rho^{1-d} \cdot \mu^d] = C\left[\rho\, d^2 u^2 \left(\frac{\mu}{\rho u d}\right)^d\right]$$

$$= \rho d^2 u^2 \phi\left(\frac{\mu}{\rho u d}\right)$$

Hence, $F = \rho d^2 u^2 \phi\,(Re)$

Que 5.21. **Use Buckingham's π-theorem to show that boundary shear stress τ_0, on turbulent flow through rough pipe is given by:**

$$\tau_0 = \rho . v^2 . \varphi\left(\frac{\rho v D}{\mu}, \frac{K}{D}\right).$$

AKTU 2013-14, Marks 10

Answer

Given : $\tau = f(\rho, v, D, K, \mu)$ or $f_1(\tau_o, \rho, v, D, K, \mu) = 0$

To Find : $\tau_o = \rho . v^2 \phi\left(\frac{\rho v D}{\mu}, \frac{K}{D}\right)$.

1. Total number of variables, $n = 6$
2. Total number of fundamental dimensions $= m = 3$
3. So total number of dimensionless functions $= n - m$
$= 6 - 3 = 3$
4. Every π-functions have $m + 1 (= 3 + 1 = 4)$ variables.
5. Choosing ρ, v, D as repeating variables we have

$$\pi_1 = \rho^{a_1} \cdot v^{b_1} D^{c_1} \cdot \tau_o$$
$$\pi_2 = \rho^{a_2} \cdot v^{b_2} D^{c_2} \cdot K$$
$$\pi_3 = \rho^{a_3} \cdot v^{b_3} D^{c_3} \cdot \mu$$

6. So $f_1(\pi_1, \pi_2, \pi_3) = 0$ ...(5.21.1)

7. Dimensions of all variables are as follows

$\rho = [ML^{-3}]$, $v = [LT^{-1}]$, $D = [L]$

$\mu = [ML^{-1}T^{-1}]$, $K = [L]$

$\tau_o = [ML^{-1}T^{-2}]$

8. **π_1-term :**

$$\pi_1 = \rho^{a_1} \cdot v^{b_1} D^{c_1} \cdot \tau_o$$

Putting the dimensions of the quantities in above π_1-term,

$$[M^0L^0T^0] = [M^{a_1} \cdot L^{-3a_1}][L^{b_1} T^{-b_1}][L^{c_1}][ML^{-1} T^{-2}]$$
$$[M^0L^0T^0] = [M^{a_1+1} \cdot L^{-3a_1+b_1+c_1-1} T^{-b_1-2}]$$

Comparing the powers of M, L and T on both sides,

Power of M, $a_1 + 1 = 0 \Rightarrow a_1 = -1$

Power of L, $-3a_1 + b_1 + c_1 - 1 = 0$

Power of T, $-b_1 - 2 = 0 \Rightarrow b_1 = -2$

$-3(-1) - 2 + c_1 - 1 = 0$

$c_1 = 0$

So $\pi_1 = \rho^{-1} \cdot v^{-2} D^0 \cdot \tau_o$

$$\pi_1 = \frac{\tau_o}{\rho v^2}$$

9. **π_2-term :**

$$\pi_2 = \rho^{a_2} \cdot v^{b_2} D^{c_2} \cdot K$$
$$[M^0L^0T^0] = [M^{a_2} \cdot L^{-3a_2}][L^{b_2} \cdot T^{-b_2}][L^{c_2}][L]$$
$$[M^0L^0T^0] = [M^{a_2} \cdot L^{-3a_2+b_2+c_2+1} T^{-b_2}]$$

Comparing the powers of M, L and T on both sides

Power of M, $a_2 = 0$

Power of T, $-b_2 = 0 \Rightarrow b_2 = 0$

Power of L, $-3a_2 + b_2 + c_2 + 1 = 0$

$c_2 = -1$

So, $\pi_2 = D^{-1} . K$

$$\pi_2 = \frac{K}{D}$$

10. **π_3-term :**

$$\pi_3 = \rho^{a_3} \cdot v^{b_3} \cdot D^{c_3} \cdot \mu$$
$$[M^0L^0T^0] = [M^{a_3} \cdot L^{-3a_3}][L^{b_3} \cdot T^{-b_3}][L^{c_3}][ML^{-1} T^{-1}]$$
$$[M^0L^0T^0] = [M^{a_3+1} \cdot L^{-3a_3+b_3+c_3-1} T^{-b_3-1}]$$

Comparing the powers of M, L and T on both sides,

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Power of M, $a_3 + 1 = 0 \Rightarrow a_3 = -1$

Power of T, $-b_3 - 1 = 0 \Rightarrow b_3 = -1$

Power of L, $-3a_3 + b_3 + c_3 - 1 = 0 \Rightarrow -3(-1) + (-1) + c_3 - 1 = 0$

$$c_3 = -1$$

So $$\pi_3 = \rho^{-1} \cdot v^{-1} \cdot D^{-1} \cdot \mu$$

$$\pi_3 = \frac{\mu}{\rho v D} \text{ or } \pi_3 = \frac{\rho v D}{\mu}$$

So π_3 is a function of $\frac{\rho v D}{\mu}$ or $\frac{\mu}{\rho v D}$.

11. Now put the value of π_1, π_2 and π_3 in eq. (5.21.1).

$$f_1\left(\frac{\tau_o}{\rho v^2}, \frac{K}{D}, \frac{\mu}{\rho v D}\right) = 0$$

$$\frac{\tau_o}{\rho v^2} = \phi\left(\frac{\mu}{\rho v D}, \frac{K}{D}\right)$$

$$\tau_o = \rho \cdot v^2 \phi\left(\frac{\mu}{\rho v D}, \frac{K}{D}\right)$$

Que 5.22. **Define the term dimensionless numbers and discuss some important dimensionless numbers and their significance.**

Answer

A. Dimensionless Numbers :

1. Dimensionless numbers are ratio of inertia force and a force, which may be a viscous force, gravity force, pressure force, surface tension force or elastic force.
2. Some important dimensionless numbers are :

a. Reynold's Number :

1. It is ratio of the inertia force to the viscous force.

Reynold's Number, $$(Re) = \frac{\text{Inertia force}}{\text{Viscous force}}$$

2. Inertia force = Mass × Acceleration

$$= \text{Density} \times \text{Volume} \times \frac{\text{Velocity}}{\text{Time}}$$

$= \rho . A.v.v$ {Volume per time = Area × Velocity = $A.v$}

$$= \rho . Av^2$$

Viscous force (F_v) = Shear stress × Area = $\tau \times A$

$$= \mu \cdot \frac{du}{dy} \times A = \frac{\mu \cdot v}{L} A$$

3. So, $$\text{Reynold's Number} = \frac{\text{Inertia force}}{\text{Viscous force}}$$

$$= \frac{\rho \cdot Av^2}{\left(\frac{\mu \cdot v \cdot A}{L}\right)}$$

$$Re = \frac{\rho v L}{\mu} \text{ or } \frac{\rho v d}{\mu} \quad \text{(for pipe flow)}$$

Significance :

1. Reynold's number is used to determine whether the flow is laminar or turbulent.
2. Reynold's number signifies the relative predominance of the inertia to the viscous forces occurring in the flow systems.

Applications :

1. Motion of submarine completely under water.
2. Incompressible flow through pipes of smaller size.
3. Flow through low speed turbo machines.

B. Froude's Number :

1. It is the square root of the ratio of inertia force to the gravity force of a flowing fluid.
2. Froude's Number, $Fe = \sqrt{\frac{F_i}{F_g}}$
3. $$F_i = \rho . Av^2$$

and $$F_g = mg = \rho . A . Lg$$

So, $$Fe = \sqrt{\frac{\rho A v^2}{\rho A L g}}$$

$$Fe = \frac{v}{\sqrt{Lg}}$$

Significance :

1. It signifies the dynamic similarity of the flow situation where gravitational force (F_g) is most significant.
2. Froude number differentiates the super critical, subcritical and critical flow.

Applications : Froude number is being used in

1. Flow over notches and weir.
2. Flow over the spillway of a dam.
3. Flow through open channels.
4. Motion of ship in rough and turbulent sea.

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C. Euler's Number (Eu) :

1. It is the square root of the ratio of the inertia force to the pressure force of a flowing fluid.

Mathematically,

$$Eu = \sqrt{\frac{F_i}{F_p}}$$

2. Pressure force (F_p) = pressure × area
$= p \times A$

and, Inertia force $(F_i) = \rho A v^2$

3. $$Eu = \sqrt{\frac{\rho A v^2}{pA}} = \frac{v}{\sqrt{p/\rho}}$$

Significance :

1. It signifies those flow problems or situations in which pressure gradient exists.

Applications :

1. Discharge through orifice and mouth piece.
2. Pressure rise due to sudden closure of valves.
3. Flow through pipes.
4. Water hammer created in penstocks.

D. Weber's Number (We) :

1. It is the square root of the ratio of inertia force of a flowing fluid to the surface tension force of a flowing fluid.

$$\text{Weber's Number } (We) = \sqrt{\frac{F_i}{F_s}}$$

2. Inertia force $(F_i) = \rho A v^2$
and, surface tension force (F_s) = surface tension × length $= \sigma \times L$

So $$We = \sqrt{\frac{\rho A v^2}{\sigma \times L}} = \frac{v}{\sqrt{\sigma / (\rho L)}} \quad \{\because A = L^2\}$$

Significance :

1. It signifies those flow problems in which surface tension force is dominant.

Applications : It is applicable in following situations :

1. Capillary movement.
2. Flow of blood in veins and arteries.
3. Liquid atomization.

E. Mach Number (M) :

1. It is defined as the square root of the ratio of inertia force to the elastic force.

Mathematically, $$M = \sqrt{\frac{F_i}{F_e}}$$

2. Inertia force $(F_i) = \rho A v^2$
and, Elastic force $(F_e) = K.A = K.L^2$ {Since Area $= L^2$}

3. Therefore, $$M = \sqrt{\frac{\rho A v^2}{K \cdot L^2}} = \frac{v}{\sqrt{K/\rho}}$$

$$M = \frac{v}{C}$$

$$\sqrt{\frac{K}{\rho}} = C \text{ (Velocity of sound in the fluid)}$$

Significance :

1. Mach number is used to differentiate the flow as subsonic flow, sonic flow and supersonic flow.

Applications :

1. High velocity flow in pipes.
2. Motion of missiles or high speed projectiles.

VERY IMPORTANT QUESTIONS

Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.

Q. 1. Explain the force exerted by a flowing fluid on a stationary body.

Ans. Refer Q. 5.1, Unit-5.

Q. 2. Discuss the development of lift on an airfoil.

Ans. Refer Q. 5.6, Unit-5.

Q. 3. Explain model (or similarity) laws, discuss in brief.

Ans. Refer Q. 5.10, Unit-5.

Q. 4. Explain the terms : distorted models and undistorted models. What are the uses of distorted models ?

Ans. Refer Q. 5.12, Unit-5.

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Q. 5. What do you mean by dimensional analysis ? What are its uses ? What are advantages and limitations of dimensional analysis ?

Ans. Refer Q. 5.15, Unit-5.

Q. 6. Explain Buckingham's-Pi method.

Ans. Refer Q. 5.16, Unit-5.

Q. 7. Define the term dimensionless numbers and discuss some important dimensionless numbers and their significance.

Ans. Refer Q. 5.22, Unit-5.

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1 UNIT Fluid and Continuum (2 Marks Questions)

Memory Based Questions

1.1. Define the term 'fluid'.

Ans. A fluid is a substance which deforms continuously when subjected to external shearing force.

1.2. What is an ideal fluid ?

Ans. An ideal fluid is one which has no viscosity, no surface tension and no compressibility.

1.3. Define real fluids.

Ans. A real practical fluid is one which has viscosity, surface tension and compressibility in addition to the density. The real fluids are actually available in nature.

1.4. What is continuum ?

Ans. A continuous and homogeneous medium is called continuum. From its view point, the overall properties and behaviour of fluids can be studied without regard for its atomic and molecular structure.

1.5. Define control volume. **AKTU 2016-17, Marks 02**

Ans. For applying basic principles of fluid flow usually control volume approach is adopted, in which a definite volume with fixed boundary shape is chosen in space along the fluid flow passage. This definite volume is called the control volume.

1.6. Enumerate some important properties of liquid.

Ans. Some important properties of liquid are :

1. Density,
2. Specific gravity,
3. Viscosity,
4. Cohesion,
5. Adhesion, and
6. Surface tension.

1.7. Define viscosity.

Ans. Viscosity may be defined as the property of a fluid which determines its resistance to shearing stresses. It is a measure of the internal fluid friction which causes resistance to flow.

1.8. What is kinematic viscosity ?

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Ans. Kinematic viscosity is defined as the ratio of the dynamic viscosity to the density of fluid. It is denoted by ν.

$$\nu = \frac{\mu}{\rho} \text{ m}^2\text{/sec or cm}^2\text{/sec}$$

1.9. State the Newton's law of viscosity.

Ans. The law states that the shear stress on a fluid element layer is directly proportional to the rate of shear strain. The constant of proportionality is called the coefficient of viscosity.
Mathematically,

$$\tau = \mu \cdot \frac{du}{dy}$$

1.10. What are Newtonian fluids ?

Ans. Newtonian fluids are those fluids which obey the Newton's law of viscosity. For such fluids, viscosity does not change with rate of deformation.

1.11. Define non-Newtonian fluids.

Ans. Fluids which do not follow the Newton's law of viscosity are known as non-Newtonian fluids.

1.12. Define surface tension. **AKTU 2016-17, Marks 02**

Ans. Surface tension is defined as the tensile force acting on the surface of a liquid in contact with a gas or on the surface between two immiscible liquids such that the contact surface behaves like a membrane under tension.

1.13. What is capillarity ?

Ans. Capillarity is defined as the phenomenon of rise or fall of a liquid surface in a small tube relative to the adjacent general level of liquid when the tube is held vertically in the liquid. This phenomenon is due to the combined effect of cohesion and adhesion of liquid particles.

1.14. Explain the term 'compressibility'.

Ans. The property by virtue of which fluids undergo a change in volume under the action of external pressure is known as compressibility.

1.15. State the hydrostatic law.

Ans. The law states that the rate of increase of pressure in a vertically downward direction is equal to weight density of the fluid. Thus,

$$\frac{dp}{dh} = \rho g$$

where, h = Distance of fluid element from free surface in vertical downward direction
ρ = Density of fluid, and
g = Acceleration due to gravity.

1.16. State the Pascal's law.

Ans. Pascal's law states that the intensity of pressure at any point in a liquid at rest is equal in all directions.

1.17. Define the term 'atmospheric pressure'.

Ans. The atmospheric air exerts a normal pressure upon all surfaces with which it is in contact, and it is known as atmospheric pressure. It is also called as barometric pressure.

1.18. Write down the definitions of gauge pressure and absolute pressure. **AKTU 2015-16, Marks 02**

Ans. **Gauge Pressure :** Gauge pressure is the pressure, measured with the help of pressure measuring instrument, in which the atmospheric pressure is taken as datum.
Absolute Pressure : Any pressure measured above the absolute zero of pressure is termed as an absolute pressure.

1.19. What is vapour pressure ?

Ans. When the liquid confined in a closed vessel, the ejected vapour molecule get accumulated in the space between the free liquid surface and the top of the vessel. This accumulated vapour of the liquid exerts a partial pressure on the liquid surface which is known as vapour pressure.

1.20. What are manometers ?

Ans. Manometers are defined as the devices used for measuring the pressure at a point in a fluid by balancing the column of fluid by the same or another column of fluid.

1.21. Classify manometers.

Ans. Manometers are classified as follows :
1. Simple manometers :
   a. Piezometer,
   b. U-tube manometer, and
   c. Single column manometer.
2. Differential manometer.

1.22. Define piezometer.

Ans. A piezometer is the simplest form of manometer which can be used for measuring moderate pressure of liquids.

1.23. What is centre of pressure ?

Ans. Centre of pressure is defined as the point of application of the total pressure on the surface.

1.24. Define buoyancy.

Ans. The tendency for an immersed body to be lifted up in the fluid due to an upward force opposite to action of gravity is known as buoyancy.

1.25. What do you understand by buoyant force ? **AKTU 2015-16, Marks 02**

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Ans. When a body is either wholly or partially immersed in a fluid, the hydrostatic lift due to the net vertical component of hydrostatic pressure force experienced by the body is called the buoyant force.

1.26. What do you mean by centre of buoyancy ?

Ans. The point of application of the force of buoyancy on the body is known as the centre of buoyancy.

1.27. Explain metacentre and metacentric height.

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Ans. **Metacentre :** The metacentre may be defined as the point at which the line of action of the force of buoyancy will meet the normal axis of the body when the body is given a small angular displacement.

Metacentric Height : The distance between the centre of gravity of a floating body and the metacentre is called metacentric height.

1.28. Calculate the work done in blowing a soap bubble of diameter 100 mm. Assume the surface tension of soap solution = 0.038 N/m.

Ans. **Given :** d = 100 mm or 0.1 m,
σ = 0.038 N/m.

The soap bubble has two interfaces.

$\therefore$ Work done = surface tension × total surface area

$$= 0.038 \times 4\pi \times \left(\frac{0.1}{2}\right)^2 \times 2$$

$$= 0.002388 \text{ N-m}$$

Evaluation Based Questions

1.29. Discuss the effect of temperature and pressure on the viscosity of fluid.

Ans. **Effect of Temperature :** The viscosity of liquids decreases but that of gases increases with increase in temperature.

Effect of Pressure : Under ordinary conditions, effect of pressure on viscosity is negligible. However, the viscosity of some oils has been found to increase with increase in pressure.

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2 UNIT Types of Fluid and Continuity Equation (2 Marks Questions)

Memory Based Questions

2.1. Classify the fluid flow.

Ans. Fluid flow may be classified as follows :

1. Steady and unsteady flows.
2. Uniform and non-uniform flows.
3. One, two and three dimensional flows.
4. Rotational and irrotational flows.
5. Laminar and turbulent flows.
6. Compressible and incompressible flows.

2.2. Differentiate between steady and unsteady flow.

AKTU 2015-16, Marks 02

Ans. **Steady Flow :** The type of flow in which the fluid properties like velocity, pressure, density, etc. at a point do not change with time is called steady flow. Mathematically,

$$\left(\frac{\partial u}{\partial t}\right)_{x,y,z} = 0, \left(\frac{\partial \rho}{\partial t}\right)_{x,y,z} = 0, \text{ etc.}$$

Unsteady Flow : Unsteady flow is that type of flow in which the velocity, pressure or density at a point change with respect to time. Mathematically,

$$\left(\frac{\partial u}{\partial t}\right)_{x,y,z} \neq 0, \left(\frac{\partial v}{\partial t}\right)_{x,y,z} \neq 0, \left(\frac{\partial w}{\partial t}\right)_{x,y,z} \neq 0, \text{ etc.}$$

2.3. Explain uniform flow.

Ans. The type of flow, in which the velocity at any given time does not change with respect to space, is called uniform flow. Mathematically,

$$\left(\frac{\partial v}{\partial s}\right)_{t = \text{constant}} = 0$$

2.4. Discuss non-uniform flow.

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Ans. Non-uniform flow is that type of flow in which the velocity at any given time changes with respect to space.
Mathematically,

$$\left(\frac{\partial v}{\partial s}\right)_{t = \text{constant}} \neq 0$$

2.5. Define laminar and turbulent flow.

Ans.
Laminar Flow : A flow in which paths taken by individual particle do not cross one another and move along well defined paths is known as laminar flow.
Turbulent Flow : A turbulent flow is that flow in which fluid particles move in a zig-zag way.

2.6. Define rotational and irrotational flow ?

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Ans. **Rotational Flow :** A flow is said to be rotational if the fluid particles while moving in the direction of flow rotate about their mass centers.
Irrotational Flow : A flow is said to be irrotational if the fluid particles while moving in the direction of flow do not rotate about their mass centers.

2.7. Discuss compressible flow.

Ans. Compressible flow is that type of flow in which the density of the fluid changes from point to point.

2.8. Explain incompressible flow.

Ans. In fluid mechanics incompressible flow refers to a flow in which the material density is constant within a fluid parcel.

2.9. Define sub-sonic, sonic and supersonic flow ?

Ans.
1. When Mach number is less than 1 (M < 1), flow is called subsonic flow.
2. When Mach number is equal to 1 (M = 1), flow is called sonic flow.
3. When Mach number is greater than 1 (M > 1), flow is called supersonic flow

2.10. Define sub-critical, critical and supercritical flow :

Ans.
1. When Froude number is less than one (Fe < 1), the flow is subcritical flow.
2. When Froude number is equal to one (Fe = 1), the flow is critical flow.
3. When Froude number is greater than one (Fe > 1), the flow is supercritical flow.

2.11. Define one-dimensional flow.

Ans. One-dimensional flow is that type of flow in which the flow parameter such as velocity is a function of time and one space coordinate only. Mathematically,

$$u = f(x),\ v = 0 \text{ and } w = 0$$

where, u = Velocity in X-direction.
v = Velocity in Y-direction.
w = Velocity in Z-direction.

2.12. What is two-dimensional flow ?

Ans. The flow in which the velocity is a function of time and two rectangular space coordinates is called two-dimensional flow.

2.13. Define three-dimensional flow.

Ans. The flow in which the velocity is a function of time and three mutually perpendicular directions is called three-dimensional flow.

2.14. Define streamline.

Ans. A streamline may be defined as an imaginary line within the flow so that the tangent at any point on it indicates the velocity at that point.

2.15. Explain stream tube.

Ans. A stream tube is a fluid mass bounded by a group of streamlines. The contents of a stream tube are known as current filament.

2.16. Explain the continuity equation.

Ans. The equation based on the principle of conservation of mass is called continuity equation.
Rate of flow at pipe section 1-1 = Rate of flow at pipe section 2-2

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2 \quad \text{...(2.6.1)}$$

eq. (2.6.1) is called continuity equation.

2.17. What do you understand by circulation ?

AKTU 2015-16, Marks 02

Ans. Circulation is defined as the line integral of the tangential velocity about a closed path (contour). Circulation around regular curves can be obtained by integration.

2.18. Write down the definition of stream function ?

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Ans. Stream function is defined as a function of space and time, such that its partial derivative with respect to any direction gives the velocity component at right angles to this direction. It is denoted by ψ.

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$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}$$

2.19. Discuss velocity potential function.

Ans. Velocity potential function is defined as a scalar function of space and time such that its negative derivative with respect to any direction gives the fluid velocity in that direction. It is denoted by ϕ.

$$u = -\frac{\partial \phi}{\partial x}, v = -\frac{\partial \phi}{\partial y}, w = -\frac{\partial \phi}{\partial z}$$

Application Based Questions

2.20. Verify whether the following function is valid potential function or not :
$\phi = A(x^2 - y^2)$.

Ans.

$$\phi = A(x^2 - y^2)$$

$$\frac{\partial \phi}{\partial x} = 2Ax; \frac{\partial \phi}{\partial y} = -2Ay$$

$$\frac{\partial^2 \phi}{\partial x^2} = 2A; \frac{\partial^2 \phi}{\partial y^2} = -2A$$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 2A + (-2A) = 0$$

Hence $\phi = A(x^2 - y^2)$ is a valid potential function.

2.21. For the $\psi = 3xy$, stream function, calculate velocity at a point (1, 2).

Ans. Stream function,

$$\psi = 3xy$$

$$u = \frac{\partial \psi}{\partial y} = 3x$$

$$v = -\frac{\partial \psi}{\partial y} = -3y$$

At (1, 2),

$$u = 3 \times 1 = 3$$

$$v = -3 \times 2 = -6$$

$$V = \sqrt{u^2 + v^2} = \sqrt{(3)^2 + (-6)^2} = \sqrt{45}$$

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3 UNIT Potential Flow, Bernoulli's Equation and Its Applications (2 Marks Questions)

Memory Based Questions

3.1. Define source flow.

Ans. The source flow is the flow coming from a point and moving out radially in all directions of a plane at uniform rate.

3.2. Explain sink flow.

Ans. The sink flow is the flow in which fluid moves radially inwards towards where it disappears at a constant rate. This flow is just opposite to the source flow.

3.3. What is doublet ?

Ans. Doublet is a special case of source and sink pair when the two approach each other in such a way that the distance ($2a$) between them approaches zero and the product ($2a.q$) remains constant where q is the discharge.

3.4. What is half body ?

Ans. It is formed due to a particular combination of a uniform flow and a plane source. The body is known as half body because it has only the leading point, it trails to infinity at the downstream end.

3.5. Describe the assumptions of Bernoulli's equation.

Ans. Assumptions are as follows :

1. Fluid is ideal *i.e.*, viscosity is zero.
2. Flow is steady.
3. Flow is incompressible.
4. Flow is irrotational.

3.6. State the Bernoulli's theorem.

Ans. The theorem states that in a steady, ideal flow of an incompressible fluid, the total energy at any point of the fluid is constant.

3.7. What is venturimeter ?

Ans. A venturimeter is a device used for measuring the rate of flow of a fluid flowing through a pipe.

3.8. Discuss orifice meter.

Ans. Orifice meter is a device used for measuring the rate of flow of a fluid through a pipe. It is a cheaper device as compared to venturimeter.

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3.9. Define Pitot tube.

Ans. Pitot tube is a device used for measuring the velocity of flow at any point or a channel.

3.10. What is stagnation point ?

Ans. Stagnation point is a point where the velocity of the fluid becomes zero.

3.11. Differentiate between notch and weir.

Ans.

S. No.	Notch	Weir
1.	It is a device used for measuring the rate of flow of a liquid through a small channel or a tank.	1. A weir is a concrete or masonry structure, placed in an open channel over which the flow occurs.

3.12. Write the advantages of triangular notch or weir over rectangular notch or weir.

Ans. Following are the advantages :

1. Ventilation of a triangular notch is not necessary.
2. For measuring low discharge, a triangular notch or weir is preferred.
3. In case of triangular notch, only one reading (H), is required for the computation of discharge, etc.

3.13. What is coefficient of discharge ?

Ans. Coefficient of discharge is defined as the ratio of the actual discharge to the theoretical discharge of flow. Thus,

$$C_d = \frac{Q_{act.}}{Q_{the.}}$$

3.14. Write down minor energy losses in pipes.

Ans. The minor energy losses in pipes due to following ways :

1. Sudden expansion of pipe
2. Sudden contraction of pipe
3. Bend in pipe
4. Pipe fitting
5. An obstruction in pipe

3.15. What do you understand by TEL and HGL ? **AKTU 2015-16, 2016-17, Marks 02**

Ans. **TEL :** Total energy line (TEL) is defined as the line which gives the sum of pressure head, datum head and kinetic head of a flowing fluid in a pipe with respect to some reference line.

HGL : Hydraulic gradient line is defined as the line which gives the sum of pressure head $\left(\frac{p}{\omega}\right)$ and datum head (z) of a flowing fluid with respect to some reference line.

3.16. What is syphon ?

Ans. Syphon is a long bent pipe which is used to transfer liquid from a reservoir at a higher elevation to another reservoir at a lower level when the two reservoirs are separated by a hill or high level ground.

3.17. Discuss compound pipes.

Ans. Compound pipes are defined as the pipes of different lengths and different diameters connected end to end (in series) to form a pipe line.

3.18. In which cases syphon is used ?

Ans. Syphon is used in the following cases :

1. To carry water from one reservoir to another reservoir separated by a hill or ridge, and
2. To empty a channel not provided with any outlet sluice.

3.19. Define water hammer in pipes.

Ans. The wave of high pressure has the effect of hammering action on the walls of the pipe. This phenomenon is known as water hammer in pipes.

3.20. On what factors, does water hammer pressure depend ?

Ans. Water hammer pressure depends upon the following factor :

1. Velocity of flow,
2. Length of pipe,
3. Time taken to close the valve, and
4. Elastic properties of the material of pipe.

3.21. What is branching pipe system ?

Ans. When three or more reservoirs are connected by means of pipes, having one or more junctions, the system is called a branching pipe system.

3.22. What are the necessary conditions for a pipe network ?

Ans. Followings are the necessary conditions for any network of pipes :

1. Flow into each junction must be equal to the flow out of the junction.
2. The algebraic sum of head losses round each loop must be zero.

3.23. Explain Darcy-Weisbach equation with notations.

Ans. Darcy-Weisbach gave the following equation for the major frictional losses :

$$h_f = \frac{fLV^2}{d \times 2g}$$

Where, f = Friction factor, L = Length of pipe, V = Velocity of flow, d = Diameter of pipe, and h_f = Head loss.

3.24. What does Hagen-Poiseuille equation refer to ? What is Hagen-Poiseuille's formula ? **AKTU 2016-17, Marks 02**

Ans. Hagen-Poiseuille equation refers to head loss. Hagen-Poiseuille's formulla is given by :

For circular plate = $\frac{32\,\mu\bar{u}L}{\rho\, g d^2}$, For parallel plate = $\frac{12\,\mu\bar{u}L}{\rho\, g b^2}$

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4 UNIT

Flow Through Pipes (2 Marks Questions)

Memory Based Questions

4.1. Write the characteristics of laminar flow.

Ans. Characteristics of laminar flow are as follows :

1. Flow is irrotational.
2. No slip will occur at the boundary.
3. Each fluid layer flows separately.

4.2. Discuss Stoke's law.

Ans. According to Stoke's law, a drag force (F) acting on the sphere moving with a constant velocity in a viscous fluid of viscosity (μ) is given by,

$$F = 3\pi\mu Ud$$

Where, F = Drag force.
U = Velocity of sphere.
d = Diameter of sphere.
μ = Viscosity.

4.3. What do you understand by kinetic energy correction factor ? **AKTU 2015-16, 2016-17, Marks 02**

Ans. Kinetic energy correction factor is defined as the ratio of the kinetic energy of the flow per second based on actual velocity across a section to the kinetic energy of the flow per second based on average velocity across the same section.

4.4. When will a laminar flow change to turbulent flow ?

Ans. A laminar flow may change to turbulent flow when :

1. There is an increased velocity of flow.
2. There is an increased diameter of pipe.
3. The viscosity of fluid is decreased.

4.5. Give the classification of turbulent flow.

Ans. The turbulent flow can be classified as follows :

1. Wall turbulence, 2. Free turbulence, and
3. Convective turbulence.

4.6. Describe the characteristics of turbulent flow.

Ans.

1. The velocity distribution in turbulent flow is more uniform than in laminar flow.
2. The velocity distribution which is parabola in laminar flow tends to follow power law and logarithmic law in turbulent flow.
3. Random orientation of fluid particle in a turbulent flow gives rise to additional stress called Reynolds stresses.

4.7. Give some example of turbulence flow.

Ans.

1. Smoke rising from a cigarette
2. Flow over a golf ball.
3. The mixing of warm and cold air in the atmosphere.

4.8. Define isotropic turbulence.

Ans. **Isotropic Turbulence :** Turbulence whose properties, especially statistical, correlations do not depend on direction is known as isotropic turbulence.

4.9. Define homogeneous turbulence.

Ans. If turbulence has some structure quantitatively in all parts of flow fluid, the turbulence is said to be homogeneous.

4.10. Define Eddy viscosity.

Ans. The viscosity which accounts for momentum transport by turbulent eddies is known as eddy viscosity.

4.11. Explain kinematic eddy viscosity.

Ans. The ratio of η (eddy viscosity) and ρ (mass density) is known as kinematic eddy viscosity and is denoted by ε.
Mathematically,

$$\varepsilon = \frac{\eta}{\rho}$$

4.12. Discuss mixing length in turbulent flow.

Ans. According to Prandtl, the mixing length is that distance between two layers in the transverse direction such that the lumps of fluid particles from one layer could reach the other layer and the particles are mixed in the other layer in such a way that the momentum of the particles in the direction of X-axis is same.

4.13. Define boundary layer thickness.

Ans. Boundary layer thickness is defined as the distance from the boundary of the solid body measured in the Y-direction to the point, where the velocity of the fluid is approximately equal to 0.99 times the free stream velocity of the fluid.

4.14. What do you understand by displacement thickness ? **AKTU 2015-16, Marks 02**

Ans. Displacement thickness is defined as the distance, measured perpendicular to the boundary of the solid body, by which the

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boundary should be displaced to compensate for the reduction in flow on account of boundary layer formation.

4.15. Explain momentum thickness.

Ans. Momentum thickness is defined as the distance, measured perpendicular to the boundary of the solid body, by which the boundary should be displaced to compensate for the reduction in momentum of the flowing fluid on account of boundary layer formation.

4.16. What is energy thickness ?

Ans. Energy thickness is defined as the distance, measured perpendicular to the boundary of the solid body, by which the boundary should be displaced to compensate for the reduction in kinetic energy of the flowing fluid on account of boundary layer formation.

4.17. Define turbulent boundary layer and laminar sub layer.

Ans. **Turbulent boundary :** Downstream to the transition zone, the boundary layer is turbulent and continues to grow in thickness. This layer of boundary is called turbulent boundary layer.

Laminar sub layer : This is the region in boundary layer zone, adjacent to solid surface of the plate.

4.18. Enumerate the methods of preventing the separation of boundary layer.

Ans. Following are the methods for preventing the separation of boundary layer :

1. Suction of the slow moving fluid by a suction slot.
2. Supplying additional energy from a blower.
3. Providing guide blades in a bend.
4. Acceleration of fluid in the boundary layer.
5. Using stream lined body shape.

4.19. Describe on what factors separation depends and the cases in which separation occurs.

Ans. **Factors on which separation depends :**

1. Reynold number of the flow.
2. Curvature of the surface.
3. Roughness of the surface.

Separation may occur in following cases :

1. In pumps
2. In aerofoils
3. In turbine blades
4. In diffusers
5. In fans etc.

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5 UNIT — Drag and Lift, Similarities Law and Dimensional Analysis (2 Marks Questions)

Memory Based Questions

5.1. What do you mean by drag ?

Ans. The component of the total force (F_R) in the direction of motion is called drag. This component is denoted by F_D.

5.2. Define lift.

Ans. The component of the total force (F_R) in the direction perpendicular to the direction of motion is known as lift. This is denoted by F_L.

5.3. What is average coefficient of drag ?

Ans. Average coefficient of drag is defined as the ratio of the total drag force to the quantity $\frac{1}{2}\rho AU^2$. It is also called coefficient of drag and is denoted by C_D.

$$C_D = \frac{F_D}{\frac{1}{2}\rho AU^2}$$

5.4. Define local coefficient of drag.

Ans. Local coefficient of drag is defined as the ratio of the shear stress (τ_0) to the quantity $\frac{1}{2}\rho U^2$. It is denoted by (C_D*).

Mathematically,

$$C_D^* = \frac{\tau_0}{\frac{1}{2}\rho U^2}$$

5.5. Define Aerofoil.

Ans. An Aerofoil is a streamlined body which may be either symmetrical or unsymmetrical.

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5.6. Discuss Magnus effect.

Ans. When a cylinder is rotated in a uniform flow, a lift force is produced on the cylinder. This phenomenon is known as Magnus effect.

5.7. Define model and prototype.

Ans. The model is a small scale replica of the actual structure or machine. The actual structure or machine is called prototype.

5.8. Differentiate between model and prototype

Ans.

S. No.	Model	Prototype
1.	It is not necessarily functional.	It is fully functional.
2.	Used for visual demonstration of product.	Used for performance evaluation and further improvement of product.

5.9. What is the meaning of geometric similarity between model and prototype ? **AKTU 2015-16, Marks 02**

Ans. The geometric similarity is said to exist between the model and the prototype if the ratio of all corresponding linear dimension in the model and prototype are equal.

5.10. Discuss kinematic similarity.

Ans. Kinematic similarity means the similarity of motion between model and prototype. Thus, it is said to exist between the model and the prototype if the ratios of the velocity and acceleration at the corresponding points in the model and at the corresponding points in the prototype are same.

5.11. What is dynamic similarity ?

Ans. Dynamic similarity is the similarity of forces. The flows in the model and in prototype are dynamically similar if at all the corresponding points, identical types of forces are parallel and bear the same ratio.

5.12. Enumerate the different model laws or similarity laws.

Ans. Following are the model laws :

1. Reynolds model law,
2. Froude model law,
3. Euler model law,
4. Weber model law, and
5. Mach model law.

5.13. Discuss undistorted models.

Ans. Undistorted models are those models which are geometrically similar to their prototypes. In other words, if the scale ratio for linear dimensions of model and its prototype is same, model is called undistorted model.

5.14. What are distorted models ?

Ans. A model is said to be distorted if it is not geometrically similar to its prototype. For this type of model different scale ratios are adopted for the linear dimensions.

5.15. State the Buckingham's π-theorem.

Ans. The theorem states that if there are n variables (independent and dependent variables) in a physical phenomenon and if these variables contain m fundamental dimensions (M, L, T), then the variables are arranged into $(n-m)$ dimensionless terms.

5.16. Discuss Reynolds number.

Ans. Reynolds number is defined as the ratio of inertia force of a flowing fluid to the viscous force of the fluid. Thus,

$$Re = \frac{v \times d}{\nu} \text{ or } \frac{\rho v d}{\mu}$$

5.17. Explain Froude number.

Ans. Froude number is defined as the square root of the ratio of inertia force of a flowing fluid to the gravity force.
Mathematically,

$$Fe = \sqrt{\frac{F_i}{F_g}} = \frac{v}{\sqrt{Lg}}$$

5.18. What is Euler number ?

Ans. Euler number is defined as the square root of the ratio of the inertia force of a flowing fluid to the pressure force.
Mathematically,

$$Eu = \sqrt{\frac{F_i}{F_p}} = \frac{v}{\sqrt{P/\rho}}$$

5.19. Define Weber number.

Ans. Weber number is defined as the square root of the ratio of the inertia force of a flowing fluid to the surface tension force. It is expressed as,

$$We = \sqrt{\frac{F_i}{F_p}} = \frac{v}{\sqrt{\sigma/\rho L}}$$

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5.20. Where is Mach model law applicable ?

Ans. The Mach model law finds application in the following :

1. Aerodynamic testing,
2. Phenomena involving velocities exceeding the speed of sound,
3. Hydraulic model testing for the cases of unsteady flow, especially water hammer problems, and
4. Underwater testing of torpedoes.

5.21. What are the uses of Weber model law ?

Ans. Weber model law is applied in the following flow situations :

1. Flow over weirs involving very low heads,
2. Very thin sheet of liquid flowing over a surface,
3. Capillary waves in channels,
4. Capillary rise in narrow passages,
5. Capillary movement of water in soil.

5.22. Write down the application of Reynolds model law ?

Ans.

1. Motion of aeroplanes,
2. Flow of incompressible fluid in closed pipes,
3. Motion of submarines completely under water, and
4. Flow around structures and other bodies immersed completely under moving fluids.

5.23. In which conditions Euler model law is suitable ?

Ans. This law is applied in the following flow problems :

1. Enclosed fluid system where the turbulence is fully developed so that viscous forces are negligible and also the forces of gravity and surface tension are entirely absent, and
2. Where the phenomenon of cavitation occurs.

5.24. What is the necessity of Froude model law ?

Ans.

1. Free surface flows such as flow over spillways, sluices etc.,
2. Flow of jet from an orifice or nozzle,
3. Where waves are likely to be formed on the surface, and
4. Where fluids of different mass densities flow over one another.

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B.Tech.
(SEM. III) ODD SEMESTER THEORY EXAMINATION, 2012-13
FLUID MECHANICS

Time : 3 Hours **Total Marks : 100**

Note :
a. Attempt all questions.
b. Each question carries equal marks.
c. Assume any missing data suitably.

1. Attempt any **four** parts of the following : **(5 × 4 = 20)**

a. Explain the concept of fluid continuum. A rectangular plate of 0.50 m × 0.50 m weighing 500 N slides down an inclined plane making 30° angle with horizontal at a velocity of 1.75 m/s. If the 2 mm gap between the plate and the inclined surface is filled with lubricating oil, find its dynamic viscosity in poise.

Ans. Refer Q. 1.10, Page 1–10A, Unit-1.

b. Determine the reading of the pressure gauge 'B' shown in Fig. 1.

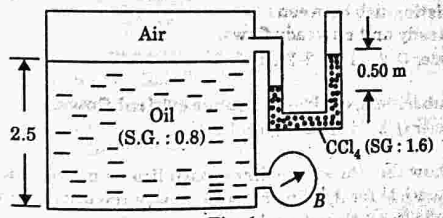

Fig. 1.

Ans. Refer Q. 1.21, Page 1–23A, Unit-1.

c. An annular circular plate of 2 m external diameter and 1 m internal diameter is immersed vertically in water so that its lowest edge is 5 m below the free water surface. Determine the total force and position of centre of pressure.

Ans. Refer Q. 1.26, Page 1–34A, Unit-1.

d. A sphere of diameter 0.50 m and S.G. 7.0 when submerged completely in a liquid causes a tension of 3.465 kN in string from which it is suspended. What is the specific gravity of the liquid ?

Ans. Refer Q. 1.8, Page 1–8A, Unit-1.

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e. What is the significance of metacentric height ? For rotational stability is it enough that the floating body has as large metacentric height as possible ? Explain.

Ans. 1. The metacentric height is a measurement of the initial static stability of a floating body.

2. It is calculated as the distance between the center of gravity of a ship and its metacenter.

3. A larger metacentric height implies greater initial stability against overturning.

4. Metacentric height also has implication on the natural period of rolling of a hull, with very large metacentric heights being associated with shorter periods of roll which are uncomfortable for passengers.

5. Hence, a sufficiently high but not excessively high metacentric height is considered ideal for passenger ships.

f. A tank 3 m in length, 2 m wide and 2 m deep containing 1.5 m depth of water is given a constant horizontal acceleration of 3 m/s². Determine the water surface slope ? Will the water spill out ?

Ans. Refer Q. 1.39, Page 1–51A, Unit-1.

2. Attempt any four parts of the following : (5 × 4 = 20)

a. Distinguish between :

1. Steady and unsteady flows.

Ans. Refer Q. 2.1, Page 2–2A, Unit-2.

2. Subcritical, critical and super critical flows.

Ans. Refer Q. 2.2, Page 2–4A, Unit-2.

b. Show that the stream lines, path lines and streak lines are identical for a 2-dimensional steady irrotational flow.

Ans. Refer Q. 2.3, Page 2–5A, Unit-2.

c. Derive the continuity equation for steady irrotational flow in cartesian coordinates for incompressible fluids.

Ans. Refer Q. 2.5, Page 2–6A, Unit-2.

d. Find the circulation around a closed curve defined by :

$$y = 1, x = 2, y = 4 \text{ and } x = 4$$

when the velocity field is given by

$$u = 16y - 8x$$
$$v = 8y - 7x.$$

Ans. Refer Q. 2.17, Page 2–19A, Unit-2.

e. If $\phi = -\frac{A}{2\pi} \ln r$; where A is a positive constant, determine ψ and plot typical equipotential lines and stream lines. Identify the flow pattern.

Ans. Refer Q. 2.26, Page 2–28A, Unit-2.

f. List the assumptions in the flownet study. Under what condition a solid boundary will act as extreme streamline ? In a converging section of open channel why it is difficult to draw a flow net for flows ?

Ans.

A. Assumptions in the Flownet Study : Refer Q. 2.25, Page 2–28A, Unit-2.

B. Condition at which Solid Boundary will act as a Streamline :

1. At stagnation point the solid boundary will act as extreme streamline.

2. At this point $u = v = 0$ *i.e.*, $\frac{\partial \psi}{\partial x} = \frac{\partial \psi}{\partial y} = 0$.

C. Reason why it is Difficult to Draw a Flownet in Converging Section :

1. It is difficult to draw a flow net for flows through a converging section of open channel because in this situation the streamlines are no longer remains straight so that it is not possible to draw flownet for flow.

3. Attempt any two parts of the following : (10 × 2 = 20)

a. If the velocity distribution in an open channel with depth 'D_0' follows a triangular profile (zero velocity at the bed and maximum free stream velocity u_m at top free surface) : what will be the value of kinetic energy correction factor ?

Ans.

Given : Depth = D_0, Maximum free stream velocity = u_m
To Find : The value of kinetic energy correction factor.

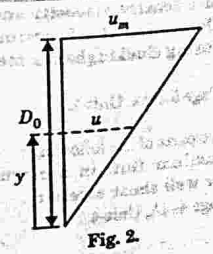

Fig. 2.

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1. Since, $\alpha = \dfrac{\int_0^A u^3\, dA}{\bar{u}^3 A}$ ...(1)

2. From Fig. 2 $u = \dfrac{u_m}{D_0} \times y,\ A_0 = D_0 \times 1,\ A = y \times 1,\ dA = dy$

$\dfrac{u_{max}}{\bar{u}} = 2$ (for circular flow)

$$\bar{u} = \frac{u_{max}}{2} = \frac{u_m}{2}$$

3. Putting these values in eq. (1),

$$\alpha = \frac{\int_0^{D_0} \left(\dfrac{u_m}{D_0} \times y\right)^3 dy}{\left(\dfrac{u_m}{2}\right)^3 D_0}$$

$$= \frac{\dfrac{u_m^3}{D_0^3}\int_0^{D_0} y^3\, dy}{\dfrac{u_m^3}{8} \times D_0} = \frac{\dfrac{u_m^3}{D_0^3} \times \dfrac{D_0^4}{4}}{\dfrac{u_m^3}{8} \times D_0}$$

$\alpha = 2.$

b. In an experiment of flow through orifices the following data were recorded :
Diameter of sharp orifice : 100 mm
Diameter of jet at vena contracta : 78.42 mm
Height of water tank maintained : constant : 3.60 m
Discharge measured (Tank-method) : 0.0385 cumec
Determine C_c, C_v and C_d.

Ans: Refer Q. 3.13, Page 3–17A, Unit-3.

c. For the flow over a triangular notch the discharge Q depends on head of water 'H', angle of notch 'θ', acceleration due to gravity 'g' : mass density, viscosity and surface tension *i.e.*, ρ, μ and σ. Determine the functional relationship for discharge Q; using Buckingham's method of dimensional analysis.

Ans: Refer Q. 5.18, Page 5–20A, Unit-2.

4. Attempt any two parts of the following : (10 × 2 = 20)

a. For steady laminar flow in a circular pipe, obtain an expression for wall shear stress τ_w.

Ans: Refer Q. 4.2, Page 4–4A, Unit-4.

b. Explain the Prandtl mixing length concept to describe the turbulence during the fluid flows at high Reynold's number. Describe hydrodynamically smooth and rough boundary surface.

Ans:
A. Prandtl Mixing Length Concept for Turbulence : Refer Q. 4.16, Page 4–22A, Unit-4.
B. Hydrodynamically Smooth and Rough Boundaries : Refer Q. 4.18, Page 4–24A, Unit-4.

c. If 300 m length of 200 mm diameter pipe with friction factor 0.018 is to be replaced by 150 mm diameter pipe with friction factor 0.02 to carry the same discharge, what length will have to be provided ?

Ans: Refer Q. 3.26, Page, 3–34A, Unit-3.

5. Attempt any two parts of the following : (10 × 2 = 20)

a. A flat plate of 2 m width and 4 m length is kept parallel to air flow at 5.0 m/s velocity at 15 °C. Determine the length of plate over which the boundary layer is laminar, shear at the location where boundary layer ceases to be laminar, and total force on both sides on that portion of plate where the boundary layer is laminar.

Ans: Refer Q. 4.32, Page 4–44A, Unit-4.

b. If velocity distribution in laminar boundary layer is given by :

$\dfrac{u}{v} = \eta$ **where** $\eta = \dfrac{y}{\delta}$**; obtain values for** $\dfrac{\theta}{\delta}, \dfrac{\delta}{x}$ **and** $\dfrac{\delta^*}{\theta}$ **where the symbols appear for their usual meanings.**

Ans: Refer Q. 4.28, Page 4–38A, Unit-4.

c. How will you control the separation of flow ? Explain the Magnus effect with suitable example.

Ans:
A. Control of the Separation of Flow : Refer Q. 4.35, Page 4–47A, Unit-4.
B. Magnus Effect : Refer Q. 5.9, Page, 5–9A, Unit-5.

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B.Tech.
(SEM. III) ODD SEMESTER THEORY EXAMINATION, 2013-14
FLUID MECHANICS

Time : 3 Hours **Total Marks : 100**

Note : Attempt all questions. All questions carry equal marks. Assume any suitable data, if missing. Use sketches and diagrams to illustrate your answers.

1. Attempt any **four** parts of the following : (5 × 4 = 20)

a. What is the difference between dynamic viscosity and kinematic viscosity ? State their units of measurements. Determine the bulk modulus of elasticity of a fluid that has a density increase of 0.002 percent for a pressure increase of 45 kN/m².

Ans. Refer Q. 1.4 Page 1–4A, Unit-1.

b. State and prove the Pascal's law. Also explain the 'metacentre' and 'metacentric height'.

Ans.

A. Pascal's Law :

1. It states that the pressure at a point in a static fluid is equal in all directions.
2. The fluid element is of very small dimensions *i.e.*, dx, dy and ds.
3. Consider an arbitrary fluid element of wedge shape in a fluid mass at rest as shown in Fig. 1.

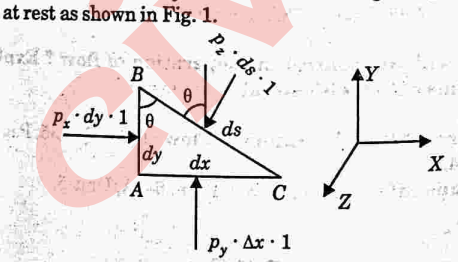

Fig. 1.

4. Let the width of the element perpendicular to the plane of paper is unity and p_x, p_y and p_z are the pressures acting on the face AB, AC and BC respectively.
5. Let $\angle ABC = \theta$.
6. Then the forces acting on the element are :

a. Pressure forces normal to the surfaces.
b. Weight of element in the vertical direction.

7. Force on the face $AB = p_x \times$ Area of face AB
$= p_x \times dy \times 1$
Similarly force on the face $AC = p_y \times dx \times 1$
Force on the face $BC = p_z \times ds \times 1$
Weight of element = Mass of element × g

8.
$$= \left(\frac{AB \times AC}{2} \times 1\right) \times \rho \times g,$$
where ρ = density of fluid.

9. Resolving the forces in x-direction,
$$p_x \times dy \times 1 - p_z\,(ds \times 1)\sin(90° - \theta) = 0$$
$$p_x \times dy \times 1 - p_z\, ds \times 1 \cos\theta = 0.$$

10. But from Fig. 1, $ds \cos\theta = AB = dy$
$$\therefore p_x \times dy \times 1 - p_z\, dy \times 1 = 0$$
or $$p_x = p_z \quad ...(1)$$

11. Similarly, resolving the forces in y-direction,
$$p_y \times dx \times 1 - p_z \times ds \times 1 \cos(90° - \theta) - \frac{dx \times dy}{2} \times 1 \times \rho \times g = 0$$
or $$p_y \times dx - p_z\, ds \sin\theta - \frac{dxdy}{2} \times \rho \times g = 0$$

12. But $ds \sin\theta = dx$ and also the element is very small and hence weight is negligible.
$$p_y dx - p_z \times dx = 0$$
or $$p_y = p_z \quad ...(2)$$

13. From eq. (1) and eq. (2), ...(3)
$$p_x = p_y = p_z$$

14. The eq. (3) shows that the pressure at any point in x, y and z directions is equal.
15. Since the choice of fluid element was completely arbitrary, which means the pressure at any point is the same in all directions.

B. Metacentre and Metacentric Height : Refer Q. 1.32, Page 1–40A, Unit-1.

c. A piece of wood of specific gravity 0.6 and size 10 cm square in cross-section and 2.5 m long floats in water. How much lead (of specific gravity 12) needs to be fastened at the lower end of the stick so that it flows up right with 0.5 m length out of water ?

Ans. Refer Q. 1.28, Page 1–36A, Unit-1.

d. Derive an expression for the resultant pressure force on a

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curved surface immersed in a liquid.

Ans. Refer Q. 1.24, Page 1–27A, Unit-1.

e. **A Newtonian fluid is filled in the clearance between a shaft and a concentric sleeve. The sleeve attains a speed of 50 cm/s, when a force of 40 N is applied to the sleeve parallel to the shaft. Determine the speed if a force of 200 N is applied.**

Ans. Refer Q. 1.6, Page 1–6A, Unit-1.

2. Attempt any four parts of the following : (5 × 4 = 20)

a. **Explain the following terms :**

1. **Pathline,** 2. **Streakline,**
3. **Streamline, and** 4. **Streamtube.**

Ans. 1. **Pathline :** A pathline is the actual path travelled by an individual fluid particle over some time period.

2. **Streakline :** It is the locus of particles that have earlier passed through a prescribed point.

3. **Streamline :** Refer Q. 2.14, 2 Marks Question, Page SQ–7A, Unit-2.

4. **Streamtube :** Refer Q. 2.15, 2 Marks Question, Page SQ–7A, Unit-2.

b. **Explain whether the velocity components given by $u = 2x^3$ and $v = -6x^2y$ represent a possible steady two dimensional incompressible flow. State whether the flow is irrotational and if so, work out the velocity potential function. If rotational, determine the vorticity, shear strain and dilatency.**

Ans. Refer Q. 2.23, Page 2–25A, Unit-2.

c. **If the velocity field is given by $u = x + y$ and $v = x^3 - y$, find the circulation around a closed contour defined by $x = 1, y = 0, y = 1$ and $x = 0$.**

Ans. Same as Q. 2.17, Page 2–19A, Unit-2.
Ans. – 1 unit.

d. **Explain the uniform flow with source and sink. Obtain the expressions for stream function and velocity potential functions.**

Ans. Refer Q. 2.15, Page 2–16A, Unit-2.

e. **A flow is described by the stream function $\psi = 3\sqrt{2}\,xy$. Locate the point at which the velocity vector has a magnitude of 6 units and makes an angle of 145° with the X-axis.**

Ans. Refer Q. 2.24, Page 2–27A, Unit-2.

f. **Differentiate between the Eulerian and Lagrangian methods of representing fluid flow. Also derive the 3-D continuity equation.**

Ans. Refer Q. 2.8, Page 2–10A, Unit-2.

3. Attempt any two parts of the following : (10 × 2 = 20)

a. **What is Euler's equation of motion and how will you obtain the Bernoulli's equation from it ? The rate of water flow through a vertical conical draft tube of a Kaplan turbine is 17.5 m³/s. The diameter of the draft tube on the side connected to the outlet of the turbine runner is 2.5 m and the average velocity at exit is 1.5 m/s. If the pressure at the inlet of the tube is not to be less than – 0.7 bar, how far the tube should extend above the tail race ? Neglect the frictional effect and presume that exit of the draft tube lies 1.2 m below the water level.**

Ans. Refer Q. 3.8, Page 3–8A, Unit-3.

b. **Explain the principle of venturimeter with a neat sketch. Derive the expression for rate of flow of fluid through it. 250 litres per second of water is flowing in a pipe having a diameter of 30 cm. If the pipe is bent by 135°, find the magnitude and direction of resultant force on the bend. The pressure of the water flowing in the pipe is 400 kPa.**

Ans. Refer Q. 3.23, Page 3–28A, Unit-3.

c. **Use Buckingham's π-theorem to show that boundary shear stress τ_0, on turbulent flow through rough pipe is given by :**

$$\tau_0 = \rho . v^2 . \varphi \left(\frac{\rho v D}{\mu}, \frac{K}{D} \right).$$

Ans. Refer Q. 5.21, Page 5–24A, Unit-5.

4. Attempt any two parts of the following : (10 × 2 = 20)

a. **Velocity distribution for laminar flow of real fluid in a pipe is given as $v' = v_{max}\left[1 - \left(\frac{r}{R}\right)^2\right]$**

Determine the momentum correction factor. Also explain the mixing-length hypothesis and determine the total shear stress in terms of mixing length.

Ans.

A. **Momentum Correction Factor :**

1. Consider an elementary area dA in the form of a ring at a radius r and of thickness dr, then $dA = 2\pi r dr$

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2. Flow rate through the ring = dQ = Elemental area × local velocity
$= 2\pi r dr \times u$

3. Total flow
$$Q = \int_0^R 2\pi u r dr$$
$$= \int_0^R 2\pi U_m \left(1 - \frac{r^2}{R^2}\right) r\, dr$$
i.e.
$$Q = 2\pi U_m \left(\frac{R^2}{4}\right) \quad ...(i)$$

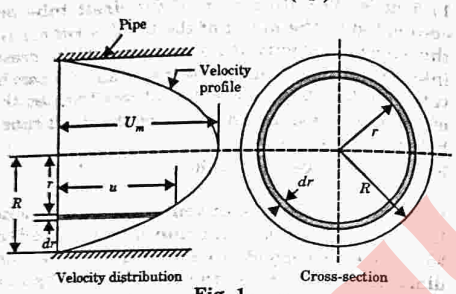

Fig. 1.

4. If $\bar{u}$ is the average flow velocity, then
$$Q = A\bar{u} = \pi R^2 \bar{u} \quad ...(ii)$$

5. From eq. (*i*) and eq. (*ii*), we get
$$\pi R^2 \bar{u} = 2\pi U_m \left(\frac{R^2}{4}\right)$$
$$\therefore \quad \bar{u} = \frac{2\pi U_m \left(\frac{R^2}{4}\right)}{\pi R^2} = \frac{U_m}{2}$$

6. Momentum correction factor :
$$\beta = \frac{1}{A\bar{u}^2}\int u^2 dA = \frac{1}{A\bar{u}^2}\int_0^R U_m^2 \left[1 - \left(\frac{r}{R}\right)^2\right]^2 2\pi r dr$$
$$= \frac{2\pi U_m^2}{A\bar{u}^2}\int_0^R \left(1 - 2\times\frac{r^2}{R^2} + \frac{r^4}{R^4}\right) r\, dr$$
$$= \frac{2\pi U_m^2}{A\bar{u}^2}\int_0^R \left(r - 2\times\frac{r^3}{R^2} + \frac{r^5}{R^4}\right) dr = \frac{2\pi U_m^2}{A\bar{u}^2}\left(\frac{R^2}{6}\right)$$

7. Substituting the values $A = \pi R^2$ and $\bar{u} = \frac{U_m}{2}$,

$$\beta = \frac{2\pi U_m^2}{\pi R^2 \times \left(\frac{U_m}{2}\right)^2}\left(\frac{R^2}{6}\right) = 1.33$$

B. Mixing Length Hypothesis : Refer Q. 4.16, Page 4–22A, Unit-4.

b. In a 80 mm diameter pipe line, an oil of specific gravity 0.8 is flowing with the rate of 0.0125 m³/s. A sudden expansion take place in to a second pipe line of such diameter that maximum rise in pressure is obtained. Find :

i. Loss of energy in sudden expansion.

ii. Differential gauge length indicated by oil mercury manometer connected between two pipes.

Ans.

Given : Diameter of the smaller pipe, D_1 = 80 mm = 0.08m
Specific gravity of oil, S = 0.8, Discharge, Q = 0.0125 m³/s.
To Find : i. Loss of energy in sudden expansion.
ii. Differential gauge length indicated by oil mercury manometer connected between two pipes.

A. Loss of energy in sudden expansion :

1. Velocity of flow, $v_1 = \frac{Q}{\text{Area}} = \frac{0.0125}{\frac{\pi}{4}\times 0.08^2} = 2.49$ m/s

2. The pressure rise will be maximum when
$$\frac{D_1}{D_2} = \frac{1}{\sqrt{2}} \quad \text{(where, } D_2 = \text{ diameter of the larger pipe)}$$
or,
$$D_2 = \sqrt{2}\, D_1 = \sqrt{2} \times 0.08 = 0.1131 \text{ m}$$

3. $\therefore$
$$v_2 = \frac{Q}{\frac{\pi}{4}D_2^2} = \frac{0.0125}{\frac{\pi}{4}\times(0.1131)^2} = 1.244 \text{ m/s}$$

4. Loss of energy (or head) in sudden expansion,
$$h_e = \frac{(v_1 - v_2)^2}{2g} = \frac{(2.49 - 1.244)^2}{2\times 9.81} = 0.079 \text{ m of oil}$$

B. Reading of the manometer :

1. The energy equation is given as,
$$\frac{p_1}{w} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{v_2^2}{2g} + z_2 + h_e$$
($z_1 = z_2$, the pipe being horizontal)

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$$\left(\frac{p_2 - p_1}{w}\right) = \frac{v_1^2}{2g} - \frac{v_2^2}{2g} - hw$$

$$= \frac{2.49^2}{2 \times 9.81} - \frac{1.244^2}{2 \times 9.81} - 0.079 = 0.158 \text{ m of oil}$$

2. Let, h = Reading of the U-tube oil-mercury manometer where limbs are connected across the expanded transition.

3. Then, $\frac{p_2 - p_1}{w} = h\left(\frac{S_m}{S_0} - 1\right)$

[where, S_m = specific gravity of mercury = 13.6]

$$0.158 = h\left(\frac{13.6}{0.8} - 1\right) = 16\text{ h}$$

or, $$h = \frac{0.158}{16} = 0.009875 \text{ m or } 9.875 \text{ mm}$$

c. Prove that the difference of local velocity and average velocity for turbulent flow through rough or smooth pipe is given by :

$$\frac{u - V}{u_*} = 5.75 \log_{10}\left(\frac{y}{R}\right) + 3.75 .$$

Ans. Refer Q. 4.21, Page 4–27A, Unit-4.

5. Attempt any two parts of the following : (10 × 2 = 20)

a. Describe the concept of separation of boundary layer with neat sketches. Also discuss the method to control the separation.

Find the energy thickness for velocity distribution on the boundary layer is given by :

$$\frac{u}{v} = \frac{3}{2}\eta - \eta^2 .$$

Ans. **Separation of Boundary Layer :** Refer Q. 4.33, Page 4–45A, Unit-4.

Method to Control the Separation : Refer Q. 4.35, Page 4–47A, Unit-4.

Numerical : Refer Q. 4.29, Page 4–40A, Unit-4.

b. Explain the boundary layer thickness, displacement thickness, momentum thickness and energy thickness. The velocity distribution in boundary layer is given by

$$\frac{u}{v} = \frac{3}{2}\eta - \frac{\eta^2}{2}; \text{ find } \left(\frac{\delta^*}{\delta}\right) \text{ and } \left(\frac{\theta}{\delta}\right).$$

Ans. Refer Q. 4.27, Page 4–37A, Unit-4.

c. Draw a graph for C_D at various values of R_e and explain the Stoke's flow.

A flat plate of 400 m long, 120 wide is covered through sea water. Determine the power required at speed of 3.0 m/s. If $\nu = 1.35 \times 10^{-6}$ m²/s and $\rho = 1020$ kg/m³. Justify the use of turbulent boundary layer formula for this case.

Ans. **Graph between C_D and R_E :**

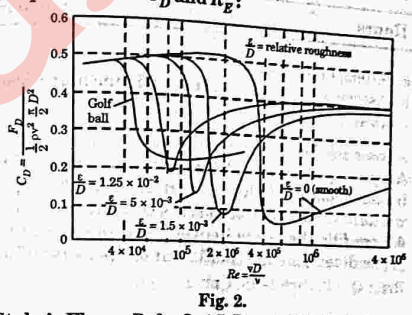

Fig. 2.

Stoke's Flow : Refer Q. 4.7, Page 4–13A, Unit-4.

Numerical : Refer Q. 5.7, Page 5–7A, Unit-5.

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B.Tech.

(SEM. III) ODD SEMESTER THEORY EXAMINATION, 2014-15

FLUID MECHANICS

Time : 3 Hours **Total Marks : 100**

Note : a. Attempt all questions.
b. Assume suitable data if not given in the numerical question.
c. Draw neat sketches wherever required.

1. Answer any four parts of the following : (5 × 4 = 20)

a. **A square plate 50 cm × 50 cm weighing 200 N slides down an inclined plane of slope 1 vertical : 2.5 horizontal with a uniform velocity of 0.40 m/s. If a thin layer of oil of thickness 0.5 cm fills the space between the plate and the inclined plane determine the coefficient of viscosity of oil.**

Ans. Refer Q. 1.9, Page 1–9A, Unit-1.

b. **Draw and explain Rheological diagram.**

Ans. Refer Q. 1.15, Page 1–14A, Unit-1.

c. **An oil of viscosity 5 poise is used for lubrication between a shaft and sleeve. The diameter of the shaft is 0.5 m and it rotates at 200 rpm. Calculate the power lost in oil for the sleeve length of 100 mm the thickness of the oil film is 1 mm.**

Ans. Refer Q. 1.7, Page 1–7A, Unit-1.

d. **With the neat sketches, explain the conditions of equilibrium for floating and submerged bodies.**

Ans. Refer Q. 1.34, Page 1–43A, Unit-1.

e. **A tank contains water upto a height of 0.5 m above the base. An immiscible liquid of specific gravity 0.8 is filled on the top of water upto 1 m height. Calculate total pressure on one side of tank and the position of centre of pressure for one side of the tank which is 2 m wide.**

Ans. Refer Q. 1.25, Page 1–32A, Unit-1.

f. **Fig. 1 shows a conical vessel having its outlet at *A* to which a *U*-tube manometer is connected. The reading of the manometer given in the Fig. 1 shows when the vessel is empty. Find the reading of the manometer when vessel is completely filled with water.**

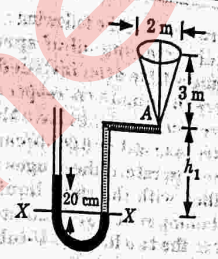

Fig. 1.

Ans. Refer Q. 1.23, Page 1–25A, Unit-1.

2. Answer any four parts of the following : (5 × 4 = 20)

a. **For steady incompressible flows derive the continuity equation using 3-D rectangular coordinate system.**

Ans. Refer Q. 2.5, Page 2–6A, Unit-2.

b. **Sketch the stream lines represented by $\psi = x^2 + y^2$. Also find out the velocity and its direction at point (1, 2).**

Ans. Refer Q. 2.21, Page 2–23A, Unit-2.

c. **What is flownet ? Describe any one method of drawing flownet.**

Ans. Refer Q. 2.25, Page 2–28A, Unit-2.

d. **If for a 2-D potential flow, the velocity potential is given by $\phi = x(2y - 1)$. Determine the velocity at the point *P*(4, 5). Determine also the value of stream function at the point *P*.**

Ans. Refer Q. 2.22, Page 2–24A, Unit-2.

e. **A jet of water from a 25 mm diameter nozzle is directed vertically upwards. Assuming that the jet remains circular and neglecting any loss of energy, what will be the diameter at a point 4.5 m above the nozzle, if the velocity with which the jet leaves the nozzle is 12 m/s ?**

Ans. Refer Q. 2.6, Page 2–8A, Unit-2.

f. **Write short note on :**
1. **Subcritical, critical and supercritical flow.**
2. **Subsonic, sonic and supersonic flow.**
3. **Laminar and turbulent flow.**

Ans. 1. Refer Q. 2.2, Page 2–4A, Unit-2.
2. Refer Q. 2.2, Page 2–4A, Unit-2.
3. Refer Q. 2.1, Page 2–2A, Unit-2.

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3. Answer any two parts of the following : (10 × 2 = 20)

a. In a vertical conveying oil of specific gravity 0.8, two pressure gauges have been installed at A and B where the diameters are 16 cm and 8 cm respectively. A is 2 m above B. The pressure gauge readings have shown that the pressure at B is greater than at A by 0.981 N/cm^2. Neglecting all losses, calculate the flow rate. If the gauges at A and B are replaced by tubes filled with the same liquid and connected to a U-tube containing mercury. Calculate the difference of level of mercury in the two limbs of U-tube.

Ans. Refer Q. 3.14, Page 3–18A, Unit-3.

b. In a 45° bend a rectangular air duct of 1 m^2 cross-sectional area is gradually reduces to 0.5 m^2 area. Find the magnitude and direction of the force required to hold the duct in position if the velocity of flow at the 1 m^2 section is 10 m/s and pressure is 2.943 N/cm^2. Take density of air as 1.16 kg/m^3.

Ans. Refer Q. 3.24, Page 3–30A, Unit-3.

c. Using Buckingham's π theorem, show that the discharge, Q consumed by an oil ring is given by

$$Q = (Nd^3)\, f[\mu/(\rho N d^2), \sigma/(\rho N^2 d^3), \omega/(\rho N^2 d)]$$

Where, d is internal diameter of ring, N is rotational speed, ρ is density, μ is viscosity, σ is surface tension and ω is the specific weight of oil.

Ans. Refer Q. 5.17, Page 5–18A, Unit-5.

4. Answer any two parts of the following : (10 × 2 = 20)

a. Derive the equation of motion for laminar flow through pipes. Also derive the expression for velocity and shear stress distribution across the pipe with neat sketches.

Ans. Refer Q. 4.2, Page 4–4A, Unit-4.

b. Prove that the difference of local velocity u and average velocity U for turbulent flow through smooth or rough pipe is given by :

$$(u - U)/u_f = 5.75 \log_{10} (y/R) + 3.75$$

Ans. Refer Q. 4.21, Page 4–27A, Unit-4.

c. i. A pipe of diameter 300 mm and length 3500 m is used for the transmission of power by water. The total head at the inlet of the pipe is 500 m. Find the maximum power available at the outlet, if the value of $f = 0.006$.

Ans.

Given : Diameter, $d = 300$ mm, length, $l = 3500$ m
Total head, $H_1 = 500$, $f = 0.006$
To Find : Maximum power available at the outlet.

1. By the condition of maximum power transmitted,

$$h_f = \frac{H_1}{3} = \frac{500}{3}$$

$$H_2 = 500 - \frac{500}{3} = \frac{1000}{3}$$

2. We know that $h_f = \dfrac{flv^2}{2dg}$

$$\frac{500}{3} = \frac{0.006 \times 3500 \times v^2}{2 \times 0.3 \times 9.81}$$

$$v^2 = \frac{500}{3} \times \frac{2 \times 0.3 \times 9.81}{0.006 \times 3500}$$

$$v^2 = 46.714$$

$$v = 6.83 \text{ m/s}$$

3. $$Q = v \times A = 6.83 \times \frac{\pi}{4} \times 0.3^2 = 0.483 \text{ m}^3/\text{s}$$

4. Maximum power transmitted $= \rho g Q H_2$

$$= 1000 \times 9.81 \times 0.483 \times \frac{1000}{3} = 1.58 \text{ MW}$$

ii. What do you understand by water hammer ? Derive an expression for sudden closure of the valve considering pipe material is elastic.

Ans. Refer Q. 3.31, Page 3–39A, Unit-3.

5. Answer any two parts of the following : (10 × 2 = 20)

a. i. What do you understand by momentum thickness, displacement thickness and energy thickness ?

Ans. Momentum Thickness : Refer Q. 4.25, Page 4–33A, Unit-4.
Energy Thickness : Refer Q. 4.26, Page 4–36A, Unit-4.

ii. An oil with density 900 kg/m^3 and kinematic viscosity 10^{-5} m^2/s is flowing over a plate of 3 m long and 2 m wide with a velocity of 3 m/s parallel to 3 m side. Find the boundary layer thickness at the point of transition and at the end of the plate.

Ans. Refer Q. 4.31, Page 4–43A, Unit-4.

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b. i. A kite 60 cm × 60 cm in size weighing 3 N makes an angle of 10° with horizontal. The thread attached to it makes an angle of 45° to the horizontal and pull on the starting is 25 N. The wind is flowing over the kite at 15 m/s. Find C_L and C_D for kite.

Ans. Same as Q. 5.8, Page 5–8A, Unit-5.
Using changed value.

$$C_D = 0.349$$
$$C_L = 0.408$$

ii. What do you understand by coefficient of lift, coefficient of drag and aerofoil ?

Ans. **Coefficient of Lift :** It is defined as the ratio of the total lift force to the quantity $\frac{1}{2}\rho AU^2$.

Mathematically, $C_L = \dfrac{F_L}{\frac{1}{2}\rho AU^2}$

Coefficient of Drag : Refer Q. 5.3, 2 Marks Questions, Page SQ–15A, Unit-5.
Aerofoil : Refer Q. 5.5, Page 5–5A, Unit-5.

c. i. Explain the phenomenon of drag on a sphere and define Stoke's law. Draw a graph between C_D and various values of Reynold's number.

Ans. **Drag on a Sphere and Stoke's Law :** Refer Q. 4.7, Page 4–13A, Unit-4.
Graph : Refer Q. 5(c), Page SP–13A, Paper 2013-14.

ii. Discuss the effect of pressure gradient on boundary layer separation with neat sketches.

Ans. Refer Q. 4.34, Page 4–46A, Unit-4.

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B.Tech.
(SEM. III) ODD SEMESTER THEORY EXAMINATION, 2015-16
FLUID MECHANICS

Time : 3 Hours **Total Marks : 100**

SECTION – A

1. Attempt all parts. All parts carry equal marks. Write answer of each part in short : **(2 × 10 = 20)**

a. Write down the definitions of gauge pressure and absolute pressure ?
Ans. Refer Q. 1.18, 2 Marks Questions, Page SQ–3A, Unit-1.

b. Explain metacentre and metacentric height.
Ans. Refer Q. 1.27, 2 Marks Questions, Page SQ–4A, Unit-1.

c. What do you understand by buoyant force ?
Ans. Refer Q. 1.25, 2 Marks Questions, Page SQ–3A, Unit-1.

d. Differentiate between steady and unsteady flow.
Ans. Refer Q. 2.2, 2 Marks Questions, Page SQ–5A, Unit-2.

e. Write down the definition of stream function.
Ans. Refer Q. 2.18, 2 Marks Questions, Page SQ–7A, Unit-2.

f. What do you understand by circulation ?
Ans. Refer Q. 2.17, 2 Marks Questions, Page SQ–7A, Unit-2.

g. What do you understand by kinetic energy correction factor ?
Ans. Refer Q. 4.3, 2 Marks Questions, Page SQ–12A, Unit-4.

h. What is the meaning of geometric similarity between model and prototype ?
Ans. Refer Q. 5.9, 2 Marks Questions, Page SQ–16A, Unit-5.

i. What do you understand by TEL and HGL ?
Ans. Refer Q. 3.15, 2 Marks Questions, Page SQ–10A, Unit-3.

j. What do you understand by displacement thickness ?
Ans. Refer Q. 4.14, 2 Marks Questions, Page SQ–13A, Unit-4.

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SECTION – B

Note : Attempt any five questions from this section. **(10 × 5 = 50)**

2. What should be the diameter of a droplet of water, if the pressure inside is to be 0.0018 kg(f)/cm² greater than the outside ? Given the value of surface tension of water in contact with air at 20 °C as 0.0075 kg(f)/m.

Ans. Refer Q. 1.14, Page 1–14A, Unit-1.

3. A 3.6 m by 1.5 m wide rectangular gate is vertical and is hinged at point 0.5 m below the centre of gravity of the gate. The total depth of water is 6 m. What horizontal force must be applied at the bottom of the gate to keep the gate closed ?

Ans. Refer Q. 1.29, Page 1–37A, Unit-1.

4. A *U*-tube having its vertical legs 0.6 m apart is partially filled with carbon tetrachloride (specific gravity 1.6) and rotated about a vertical axis 0.15 m, from one leg. What will be the difference in elevation of the two free surfaces with the angular velocity 100 revolutions per minute ?

Ans.

Given : Difference in the vertical legs = 0.6 m
Specific gravity of CCl_4 = 1.6
Distance of vertical axis from one leg = 0.15 m
Angular velocity = 100 rpm
To Find : Difference in elevation of the two free surfaces.

1. Let us consider an element at x distance from axis of rotation with length dx and area dA,

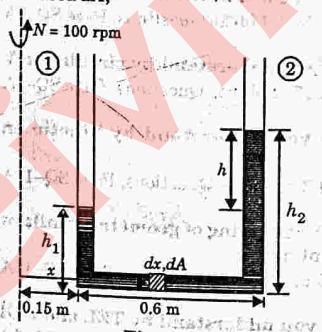

Fig. 1.

2. Free body diagram of element,

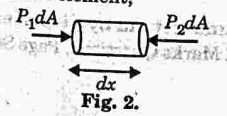

Fig. 2.

3. From Newton's law of motion,

$$P_2\,dA - P_1\,dA = m\omega^2 x \qquad (\because m = \rho dA dx)$$

$$(P_2 - P_1)\,dA = \rho dA.\ dx\ \omega^2 x$$

$$P_2 - P_1 = \rho\omega^2\ x dx$$

$$\rho g h_2 - \rho g h_1 = \rho\omega^2\ x dx$$

$$(h_2 - h_1) = \frac{\rho\omega^2}{\rho g}\ x dx$$

4. $$\int_0^h dh = \int_{0.15}^{0.75} \frac{\omega^2}{g}\ x\ dx$$

$$h = \frac{\omega^2}{g}\left[\frac{x^2}{2}\right]_{0.15}^{0.75} = 3.018 \text{ m}$$

5. Water flows through a 0.9 m diameter pipe at the end of which there is a reducer connecting to a 0.6 m diameter pipe. If the gauge pressure at the entrance to the reducer is 412.02 kN/m² and the velocity is 2 m/s, determine the resultant thrust on the reducer, assuming that the frictional loss of head in the reducer is 1.5 m.

Ans. Refer Q. 3.9, Page 3–9A, Unit-3.

6. Derive continuity equation for a three dimensional steady or unsteady flows in a cartesian coordinate system.

Ans. Refer Q. 2.5, Page 2–6A, Unit-2.

7. With the help of a diagram explain streamlines, equipotential lines and flownet. Prove that equipotential lines and streamlines intersect each other orthogonally.

Ans. **Streamlines and Equipotential Lines :** Curves of constant values of ψ define streamlines of the flow, while curves of constant values of ϕ define equipotential lines.

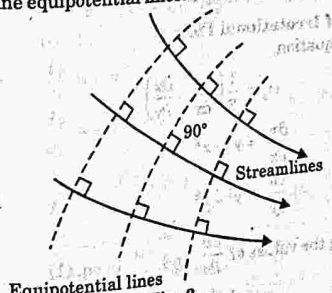

Fig. 3.

Flownet : Refer Q. 2.25, Page 2–28A, Unit-2.
Relationship between Equipotential lines and Streamlines : Refer Q. 2.12, Page 2–14A, Unit-2.

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8. Two velocity components are given in the following equations, find the third component such that they satisfy the continuity equation :

$$u = x^3 + y^2 + 2z^2,\ v = -x^2 y - yz - xy$$

Ans. Refer Q. 2.7, Page 2–9A, Unit-2.

9. For laminar flow of an oil having dynamic viscosity μ = 1.766 Pa.s in a 0.3 m diameter pipe, the velocity distribution is parabolic with a maximum point velocity of 3 m/s at the centre of the pipe. Calculate the shear stresses at the pipe wall and within the fluid 50 mm from the pipe wall.

Ans. Refer Q. 4.9, Page 4–15A, Unit-4.

SECTION – C

Note : Attempt any **two** questions from this section. **(15 × 2 = 30)**

10. The velocity components in a two-dimensional flow field for an incompressible fluid are expressed as

$$u = \frac{y^3}{3} + 2x - x^2 y\ ;\ v = xy^2 - 2y - \frac{x^3}{3}$$

a. Show that these functions represent a possible case of an irrotational flow.

b. Obtain an expression for stream function ψ.

c. Obtain an expression for velocity potential ϕ.

Ans.

Given : $u = y^3/3 + 2x - x^2 y,\ v = xy^2 - 2y - x^3/3.$

To Find : i. Show that these functions represent a possible case of an irrotational flow.

ii. Expression for stream function ψ.

iii. Expression for velocity potential ϕ.

a. Case of Irrotational Flow :

1. From equation,

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \quad ...(1)$$

2. Now,

$$\frac{\partial v}{\partial x} = y^2 - x^2$$

$$\frac{\partial u}{\partial y} = y^2 - x^2$$

3. Putting the values of $\frac{\partial v}{\partial x}$ and $\frac{\partial u}{\partial y}$ in eq. (1),

$$\omega_z = \frac{1}{2}\,[y^2 - x^2 - (y^2 - x^2)] = 0$$

4. Since, ω_z is zero, therefore these functions represent a possible case of an irrotational flow.

b. Expression for ψ :

1. The velocity components in terms of stream function are

$$\frac{\partial \psi}{\partial x} = v = xy^2 - 2y - x^3/3 \quad ...(2)$$

$$\frac{\partial \psi}{\partial y} = -u = -y^3/3 - 2x + x^2 y \quad ...(3)$$

2. Integrating eq. (2) w.r.t. x, we get

$$\psi = \int (xy^2 - 2y - x^3/3)\, dx$$

or

$$\psi = \frac{x^2 y^2}{2} - 2xy - \frac{x^4}{4 \times 3} + k, \quad ...(4)$$

where k is a constant of integration which is independent of x but can be a function of y.

3. Differentiating eq. (4) w.r.t. to y, we get

$$\frac{\partial \psi}{\partial y} = \frac{2x^2 y}{2} - 2x + \frac{\partial k}{\partial y} = x^2 y - 2x + \frac{\partial k}{\partial y}$$

4. On comparing the value of $\frac{\partial \psi}{\partial y}$ with eq. (3),

$$x^2 y - 2x + \frac{\partial k}{\partial y} = -y^3/3 - 2x + x^2 y$$

$$\therefore \quad \frac{\partial k}{\partial y} = -y^3/3$$

On integrating,

$$k = \int (-y^3/3)\, dy = \frac{-y^4}{4 \times 3} = \frac{-y^4}{12}$$

5. Substituting this value in eq. (4), we get

$$\psi = \frac{x^2 y^2}{2} - 2xy - \frac{x^4}{12} - \frac{y^4}{12}$$

c. Expression for ϕ :

1. We know

$$\frac{d\phi}{dx} = -u$$

$$\frac{d\phi}{dy} = -v \quad ...(5)$$

2. Therefore,

$$\frac{d\phi}{dx} = -\frac{y^3}{3} - 2x + x^2 y \quad ...(6)$$

$$\frac{\partial \phi}{\partial y} = -xy^2 + 2y + \frac{x^3}{3} \quad ...(7)$$

3. Integrating eq. (5) w.r.t. x, we get

$$\phi = -\frac{y^3 x}{3} - x^2 + \frac{x^3 y}{3} + k$$

where, k is a constant of integration which is independent of x but can be function of y.

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4. Differentiating eq. (7) w.r.t. y, we get

$$\frac{\partial \phi}{\partial y} = -y^2 x + \frac{x^3}{3} + \frac{\partial k}{\partial y} \quad ...(8)$$

5. Comparing the values of $\frac{\partial \phi}{\partial y}$ from eq. (6) and eq. (8),

$$\frac{\partial k}{\partial y} = 2y$$

$$k = y^2$$

6. Substituting this value in eq. (7),

$$\phi = \frac{x^3 y}{3} - \frac{xy^3}{3} - x^2 + y^2$$

11. A rectangular door covering an opening 3 m wide and 2 m high in a vertical wall is hinged about its vertical edge by two pivots placed symmetrically 0.25 m from either end. The door is locked by a clamp placed at the centre of the vertical edge. Determine the reactions at the two hinges and the clamp, when the height of water is 1.5 m above the top edge of the opening.

Ans. Refer Q. 1.30, Page 1–38A, Unit-1.

12. a. For the velocity distribution $\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$, find the energy thickness δ^{**}.

Ans. Same as Q. 4.29, Page 4–40A, Unit-4.
Using changed values,

$$\delta^{**} = 22\delta/105$$

b. A compound piping system consists of 1800 m of 0.50 m, 1200 m of 0.40 m and 600 m of 0.30 m new cast iron pipes connected in series. Convert the system to (*a*) an equivalent length of 0.40 m pipe, and (*b*) equivalent size pipe 3600 m long.

Ans. Refer Q. 3.29, Page 3–38A, Unit-3.

c. The pressure drop 'Δp' in a pipe of diameter 'D' and length 'L' due to viscous flow depends on the velocity 'v', dynamic viscosity 'μ', average height 'k' and mass density 'ρ' using Buckingham's theorem obtain expression for 'Δp'.

Ans. Same as Q. 5.21, Page 5–24A, Unit-5.
Using changed values,

$$\Delta p = \rho v^2 \phi\left[\frac{l}{D}, \frac{\mu}{\rho v D^3}, \frac{k}{D}\right]$$

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B.Tech.
(SEM. III) ODD SEMESTER THEORY EXAMINATION, 2016-17
FLUID MECHANICS

Time : 3 Hours **Total Marks : 100**

Section-A

1. Answer all parts. All parts carry equal marks. Write answer of each part in short. (2 × 10 = 20)

a. Define surface tension.

Ans. Refer Q. 1.12, 2 Marks Questions, Page SQ–2A, Unit-1.

b. What is vapour pressure ?

Ans. Refer Q. 1.19, 2 Marks Questions, Page SQ–3A, Unit-1.

c. Define control volume.

Ans. Refer Q. 1.5, 2 Marks Questions, Page SQ–1A, Unit-1

d. Define rotational and irrotational flow ?

Ans. Refer Q. 2.6, 2 Marks Questions, Page SQ–6A, Unit-2.

e. Define impulse momentum equation.

Ans. It states that the impulse of force F acting on a fluid mass m in a short interval of time dt is equal to the change of momentum $d(mv)$ in direction of force.
Mathematically,

$$Fdt = d(mv)$$

f. What does Haigen-Poiseulle equation refer to ? What is Haigen-Poiseulle's formula ?

Ans. Refer Q. 3.24, 2 Marks Questions, Page SQ–11A, Unit-3.

g. What is kinetic energy correction factor ?

Ans. Refer Q. 4.3, 2 Marks Questions, Page SQ–12A, Unit-4.

h. Define hydraulic gradient line and total energy line.

Ans. Refer Q. 3.15, 2 Marks Questions, Page SQ–10A, Unit-3.

i. Differentiate between model and prototype.

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Ans. Refer Q. 5.8, 2 Marks Questions, Page SQ–16A, Unit-5.

j. What is displacement thickness ?

Ans. Refer Q. 4.14, 2 Marks Questions, Page SQ–13A, Unit-4.

Section-B

Note : Attempt any five question from this section. **(10 × 5 = 50)**

2. What is boundary layer separation ? Explain with neat sketches, the necessary conditions for boundary layer separation. What are common methods to control boundary layer separation ?

Ans. **Boundary Layer Separation :** Refer Q. 4.33, Page 4–45A, Unit-4.

Necessary Conditions for Boundary Layer Separation : For boundary layer separation, pressure gradient should be positive in the direction of flow $\left(\frac{\partial p}{\partial x} > 0\right)$ *i.e.*, the pressure should be in increasing manner in the direction of flow. (Refer Fig. 4.34.1, Page 4–46A, Unit-4).

Methods to Control Boundary Layer Separation : Refer Q. 4.35, Page 4–47A, Unit-4.

3. Classify different types of fluid flow with example. Derive the continuity equation in cartesian coordinates.

Ans. **Types of Fluid Flow :** Refer Q. 2.1, Page 2–2A and Q. 2.2, Page 2–4A, Unit-2.

Continuity Equation in Cartesian Coordinates : Refer Q. 2.5, Page 2–6A, Unit-2.

4. Define the terms : major energy losses and minor energy losses in pipe. Derive expression for calculating loss of head due to sudden enlargement and sudden contraction.

Ans. Refer Q. 3.25, Page 3–31A, Unit-3.

5. What are differential manometers ? A "*U*" tube manometer containing mercury of density 13600 kg/m³ is used to measure the pressure drop along a horizontal pipe. If the fluid in the pipe has a relative density of 0.8 and the manometer reading is 0.6 m, what is the pressure difference measured by the manometer ?

Ans. **Differential Manometers :** Refer Q. 1.20, Page 1–21A, Unit-1.

Numerical :

Give : $\rho_m = 13600$ kg/m³, $s = 0.8$, $h = 0.6$ m

To Find : Pressure difference measured by the manometer.

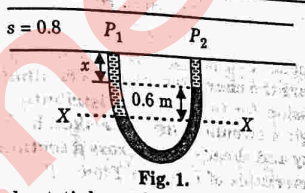

Fig. 1.

Apply hydrostatic law at datum line X–X

$P_1 + (x + 0.6) \times 0.8 \times \rho_w \times g = P_2 + x \times 0.8 \times \rho_w \times g + 0.6 \times 13600 \times g$

$P_1 - P_2 = -0.6 \times 0.8 \times 1000 \times 10 + 0.6 \times 13600 \times 10$

$P_1 - P_2 = 76.8$ m of water

6. Assuming the drag force, *F*, exerted on a body is a function of the following :

Fluid density ρ, Fluid viscosity μ, Diameter *d*

Velocity *u*

Show that the drag force can be expressed as,

$$F = d^2 u^2 \rho \phi (Re)$$

Where ϕ is some unknown function and *Re* is Reynold's number.

Ans. Refer Q. 5.20, Page 5.23A, Unit-5.

7. A horizontal venturimeter with a discharge coefficient of 0.98 is being used to measure the flow rate of a liquid of density 1030 kg/m³. The pipe diameter at entry to the venturi is 75 mm and the venturi throat has an area of 1000 mm². If the flow rate is 0.011 m³/s. Determine the height difference recorded on a *U*-tube manometer connecting the throat to the upstream pipe. Take the relative density of mercury to be 13.6.

Ans. Refer Q. 3.11, Page 3–15A, Unit-3.

8. A circular plate 3 meters in diameter is submerged in water in such a way that the greatest and least depths of the surface, (below water surface) are 2 m, and 1 m respectively, calculate :

i. The total pressure on front face of the plate, and

ii. The position of centre of pressure.

Ans. Refer Q. 1.31, Page 1–39A, Unit-1.

9. In a two-dimensional flow field for an incompressible fluid the velocity components are :

$$u = y^3/3 + 2x - x^2 y$$

$$v = xy^2 - 2y - x^3/3$$

Find an expression for the stream function ψ.

Ans. Refer Q. 10, Page SP–22A, Solved Paper 2015-16.

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Section-C

Note : Attempt any two questions from this section. **(15 × 2 = 30)**

10. What are the characteristics of a laminar flow ? Derive the expression for the velocity distribution for viscous flow through a circular pipe. Also sketch the distribution of velocity and shear stress across a section of pipe.

Ans. **Characteristics of Laminar Flow :** Refer Q. 4.1, Page 4–3A, Unit-4.

Expression for the Velocity Distribution for Viscous Flow : Refer Q. 4.2, Page 4–4A, Unit-4.

11. Water is flowing in a 300 mm pipeline fitted with a 45° bend in the vertical plane. The diameter at the outlet of the bend is 150 mm. The pipe axis at the inlet is horizontal and the outlet is 1.5 m above the inlet. If the flow through the bend is 0.4 m³/s and a head-loss of 0.5 m occurs in the bend, calculate the magnitude and direction of the resultant force the bend support must withstand. The volume of the bend is 0.075 m³ and the pressure at the inlet is 300 kN/m².

Ans. Refer Q. 3.22, Page, 3–27A, Unit-3.

12. Using continuity and the Bernoulli's equation derive the following expression that can be used to measure flow rate with a venturimeter.

$$Q_{actual} = C_d A_1 A_2 \sqrt{\frac{2g\left[\frac{p_1 - p_2}{\rho g} + z_1 - z_2\right]}{A_1^2 - A_2^2}}$$

Ans. Refer Q. 3.10, Page 3–10A, Unit-3.

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B.Tech.
(SEM. III) ODD SEMESTER THEORY EXAMINATION, 2017-18
FLUID MECHANICS

Time : 3 Hours **Max. Marks : 70**

Note : Attempt all sections. Assume missing data, if any.

Section-A

1. Attempt all question in brief. **(2 × 7 = 14)**

a. Define the term cohesion and adhesion.

Ans. Refer Q. 1.11, Page 1–10A, Unit-1.

b. Explain gauge pressure, vacuum pressure and absolute pressure with suitable sketch.

Ans. Refer Q. 1.16, Page 1–16A, Unit-1.

c. Write the difference between Eulerian and Lagrangian approach.

Ans. Refer Q. 2.8, Page 2–10A, Unit-2.

d. Explain the rotational and irrotational flow.

Ans. Refer Q. 2.6, 2 Marks Questions, Page SQ–6A, Unit-2.

e. Write short note on Pitot static tube.

Ans. Refer Q. 3.9, 2 Marks Questions, Page SQ–10A, Unit-3.

f. What do you understand by shape factor ?

Ans. The ratio of momentum thickness (θ) to displacement thickness (δ*) is known as shape factor.

$$\text{Shape factor} = \frac{\theta}{\delta^*}$$

g. Explain the drag and lift.

Ans. **Drag :** Refer Q. 5.1, 2 Marks Questions, Page SQ–15A, Unit-5.
Lift : Refer Q. 5.2, 2 Marks Questions, Page SQ–15A, Unit-5.

Section-B

2. Attempt any three parts of the following : **(7 × 3 = 21)**

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a. Liquid of specific gravity 1.0 flows through pipes A and B at positive pressure of 0.5 bar and 0.25 bar respectively. Pipe A is 1.6 m higher than B, what would be the difference in the level of U-tube manometer connected to A and B, having manometer liquid of specific gravity 13.6 ? Liquid level in the limb attached to A is lower than that in the order.

Ans.

Given : Specific gravity of liquid in pipe, $\rho_1 = 1.0$,
Specific gravity of liquid in manometer, $\rho_2 = 13.6$,
$P_A = 0.5$ bar, $P_B = 0.75$ bar
To Find : Difference in the level of U-tube manometer.

1. Equating the pressure above datum line X-X, we have,

$$P_A + (1.6 + y + x)\,\rho_1 = P_B + (y)\,\rho_1 + (x)\,\rho_2$$
$$P_A - P_B + (1.6 + y + x)\,1.0 = (y)\,1.0 + (x)\,13.6$$
$$0.5 - 0.25 + 1.6 + y + x = y + 13.6x$$
$$1.85 = 12.6x$$
$$x = 0.147 \text{ m}$$
$$x = 14.7 \text{ cm}$$

Fig. 1.

b. Derive the continuity equation for steady irrotational flows in cartesian coordinate for incompressible fluids.

Ans. Refer Q. 2.5, Page 2–6A, Unit-2.

c. What are the minor losses and major losses in a pipe flow ?

Ans. Refer Q. 3.25, Page 3–31A, Unit-3.

d. The velocity distribution in the boundary layer is given by,

$$\frac{u}{U} = \sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)$$

Find displacement thickness and momentum thickness.

Ans.

Given : $\frac{u}{U} = \sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)$
To Find : i. Displacement thickness.
ii. Momentum thickness.

1. Displacement thickness, δ^* :

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy$$

$$\delta^* = \int_0^\delta \left[1 - \sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)\right] dy$$

$$\delta^* = \left[y + \frac{\cos\left(\frac{\pi}{2}\frac{y}{\delta}\right)}{\frac{\pi}{2} \times \frac{1}{\delta}}\right]_0^\delta$$

$$\delta^* = \left[\delta - \frac{2\delta}{\pi}\right]$$

$$\delta^* = \left[\frac{\pi - 2}{\pi}\right]\delta$$

2. Momentum thickness, θ :

$$\theta = \int_0^\delta \frac{u}{U}\left[1 - \frac{u}{U}\right] dy$$

$$\theta = \int_0^\delta \sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)\left[1 - \sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)\right] dy$$

$$= \int_0^\delta \left[\sin\left(\frac{\pi}{2}\frac{y}{\delta}\right) - \sin^2\left(\frac{\pi}{2}\frac{y}{\delta}\right)\right] dy$$

$$= \int_0^\delta \left\{\sin\left(\frac{\pi}{2}\frac{y}{\delta}\right) - \left[\frac{1}{2} - \frac{\cos\{\pi(y/\delta)\}}{2}\right]\right\} dy$$

$$= \left[\frac{-\cos\left(\frac{\pi}{2}\frac{y}{\delta}\right)}{\frac{\pi}{2\delta}} - \frac{1}{2}y + \frac{\sin\left(\pi\frac{y}{\delta}\right)}{\frac{2\pi}{\delta}}\right]_0^\delta$$

$$= \left[\frac{-\cos\left(\frac{\pi}{2}\right)}{\frac{\pi}{2\delta}} - \frac{1}{2}\delta + \frac{\sin(\pi)}{\frac{2\pi}{\delta}}\right] - \left[\frac{-\cos(0)}{\frac{\pi}{2\delta}} - 0 + \frac{\sin(0)}{\frac{2 \times \pi}{\delta}}\right]$$

$$= \left[\frac{1}{2}\delta\right] - \left[\frac{-2\delta}{\pi}\right]$$

$$= \frac{2\delta}{\pi} - \frac{\delta}{2}$$

$$= \left[\frac{2}{\pi} - \frac{1}{2}\right]\delta$$

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e. Explain the Magnus effect with an example.

ANS.

A. Magnus Effect : Refer Q. 5.9, Page 5–9A, Unit-5.

B. Example :

1. The Magnus effect can be seen in advanced external ballistics.
2. First, a spinning bullet in flight is often subject to a crosswind.
3. In addition to this, a bullet experiences a small sideways wind component due to its yawing motion.
4. This yawing motion along the bullet's flight path means that the nose of the bullet is pointing in a slightly different direction from the direction in which the bullet is travelling.
5. The combined sideways wind component of these two effects causes a Magnus force to act on the bullet, which is perpendicular both to the direction the bullet is pointing and the combined sideways wind.
6. In a very simple case where we ignore various complicating factors, the Magnus force from the crosswind would cause an upward or downward force to act on the spinning bullet, causing an observable deflection in the bullet's flight path up or down, thus changing the point of impact.
7. Overall, the effect of the Magnus force on a bullet's flight path itself is usually insignificant compared to other forces such as aerodynamic drag.
8. The stability of the bullet is affected because the Magnus effect acts on the bullet's centre of pressure instead of its centre of gravity. This means that it affects the yaw angle of the bullet i.e., it tends to twist the bullet along its flight path, either towards the axis of flight or away from the axis of flight.

Section-C

3. Attempt any one part of the following : (7 × 1 = 7)

a. Explain the condition of stability for floating body and immersed body with neat sketch.

ANS. Refer Q. 1.34, Page 1–43A, Unit-1.

b. A circular plate 6 m diameter is immersed in water in such a way that its greatest and least depth below the free surface of water is 4 m and 2 m respectively. Determine the total pressure on one face of the plate and position of the centre of pressure.

ANS. Same as Q. 1.31, Page 1–39A, Unit-1.
[Ans. Total pressure = 831.986 kN, Centre of pressure = 3.083 m]

4. Attempt any one part of the following : (7 × 1 = 7)

a. Velocity field in fluid medium is given by :

$$\mathbf{v} = 10x^2y\,\hat{i} + 15xy\,\hat{j} + (25t - 3xy)\,\hat{k}$$

Find acceleration at (1, 2, – 1) m and t = 0.5 sec.

ANS.

Given : $\mathbf{v} = 10x^2y\,\hat{i} + 15xy\,\hat{j} + (25t - 3xy)\,\hat{k}$
To Find : Acceleration at (1, 2, – 1) m and t = 0.5 sec.

1. The velocity components u, v, and w are,
$u = 10x^2y$, $v = 15xy$, $w = 25t - 3xy$
2. Acceleration is given by,

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t} \quad ...(1)$$

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t} \quad ...(2)$$

$$a_z = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t} \quad ...(3)$$

3. Now from velocity component, we have

$$\frac{\partial u}{\partial x} = 20xy,\ \frac{\partial u}{\partial y} = 10x^2,\ \frac{\partial u}{\partial z} = 0 \text{ and } \frac{\partial u}{\partial t} = 0$$

$$\frac{\partial v}{\partial x} = 15y,\ \frac{\partial v}{\partial y} = 15x,\ \frac{\partial v}{\partial z} = 0 \text{ and } \frac{\partial v}{\partial t} = 0$$

$$\frac{\partial w}{\partial x} = -3y,\ \frac{\partial w}{\partial y} = -3x,\ \frac{\partial w}{\partial z} = 0 \text{ and } \frac{\partial w}{\partial t} = 25$$

4. Substituting these values in eq. (1), eq. (2) and eq. (3), we have

$$a_x = 10x^2y\,(20xy) + 15xy\,(10x^2) + (25t - 3xy)\,(0) + 0$$
$$= 200x^3y^2 + 150x^3y$$
$$a_y = 10x^2y\,(15y) + 15xy\,(15x) + (25t - 3xy)\,(0) + 0$$
$$= 150x^2y^2 + 225\,x^2y$$
$$a_z = 10x^2y(-3y) + 15xy\,(-3x) + (25t - 3xy)\,(0) + 25$$
$$= -30x^2y^2 - 45x^2y + 25$$

5. Acceleration component at (1, 2, – 1) m and t = 0.5 sec,

$$a_x = 200\,(1)^3\,(2)^2 + 150\,(1)^3\,(2)$$
$$= 800 + 300 = 1100$$
$$a_y = 150\,(1)^2\,(2)^2 + 225\,(1)^2\,(2)$$
$$= 600 + 450 = 1050$$
$$a_z = -30\,(1)^2\,(2)^2 - 45\,(1)^2\,(2) + 25$$
$$= -120 - 90 + 25 = -185$$

6. Acceleration is given as,

$$A = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$
$$A = 1100\,\hat{i} + 1050\,\hat{j} - 185\,\hat{k}$$

7. Resultant, $A = \sqrt{(1100)^2 + (1050)^2 + (-185)^2}$
$A = 1531.9$ m/sec^2

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b. A 500 mm diameter pipe carrying water at rate 0.5 m³/sec branches into two pipes of 200 mm and 400 mm diameters. If the rate of flow of water through small diameter pipe is 0.2 m³/sec. Determine velocity of flow in each pipe.

Ans.

Given : Diameter of main pipe, $d = 500$ mm $= 0.5$ m
Rate of flow of main pipe, $Q = 0.5$ m³/sec
Diameter of small pipe ①, $d_1 = 200$ mm $= 0.2$ m
Diameter of small pipe ②, $d_2 = 400$ mm $= 0.4$ m
Rate of flow of small pipe ①, $Q_1 = 0.2$ m³/sec
To Find : Velocity of flow in each pipe.

1. We know, $Q = Q_1 + Q_2$
$Q_2 = Q - Q_1$
$= 0.5 - 0.2$
$Q_2 = 0.3$ m³/sec.

2. Now, Q = Area of main pipe × Velocity
$$Q = \frac{\pi}{4} d^2 v$$
$$0.5 = \frac{\pi}{4} \times (0.5)^2 \times v$$
$$v = 2.5 \text{ m/sec}$$

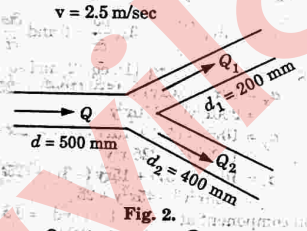

Fig. 2.

3. Similarly, Q_1 = Area of pipe ① × Velocity
$$Q_1 = \frac{\pi}{4} d_1^2 v_1$$
$$0.2 = \frac{\pi}{4} \times (0.2)^2 \times v_1$$
$$v_1 = 6.36 \text{ m/sec}$$

4. Similarly, Q_2 = Area of pipe ② × Velocity
$$Q_2 = \frac{\pi}{4} d_2^2 v_2$$
$$0.3 = \frac{\pi}{4} \times (0.4)^2 \times v_2$$
$$v_2 = 2.38 \text{ m/sec}$$

5. Attempt any one part of the following : (7 × 1 = 7)

a. Write about venturimeter. Derive the expression for rate of flow of fluid through venturimeter.

Ans. Refer Q. 3.10, Page 3–10A, Unit-3.

b. Find the discharge through a trapezoidal notch which is 1 m wide at the top and 0.4 m at the bottom and is 30 cm in height. The head of water on the notch is 20 cm. Assume C_d for rectangular portion = 0.62 while for triangular portion = 0.60.

Ans.

Given : Top width, $AE = 1$ m, Base width, $CD = L = 0.4$ m, Height, $h = 0.3$ m, Head of water, $H = 0.20$ m, For rectangular portion, $C_{d1} = 0.62$, For triangular portion, $C_{d2} = 0.60$
To Find : Discharge through trapezoidal notch.

1. From ΔABC, we have
$$\tan\frac{\theta}{2} = \frac{AB}{BC} = \frac{(AE - CD)/2}{BC} = \frac{(1.0 - 0.4)/2}{0.3} = \frac{0.6/2}{0.3} = \frac{0.3}{0.3} = 1$$

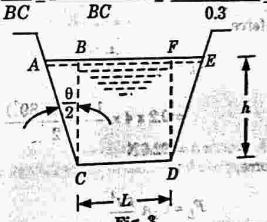

Fig. 3.

2. Discharge through trapezoidal notch is given as,
$$Q = \frac{2}{3} C_{d1} L \sqrt{2g}\, H^{3/2} + \frac{8}{15} C_{d2} \tan\frac{\theta}{2} \sqrt{2g}\, H^{5/2}$$
$$= \frac{2}{3} \times 0.62 \times 0.4 \times \sqrt{2 \times 9.81} \times (0.2)^{3/2} + \frac{8}{15} \times 0.60 \times 1 \times \sqrt{2 \times 9.81} \times (0.2)^{5/2}$$
$$= 0.0655 + 0.02535 = 0.09085 \text{ m}^3/\text{s}$$
$$= 90.85 \text{ litres/s}$$

6. Attempt any one part of the following : (7 × 1 = 7)

a. Oil with density 900 kg/m³ and kinematic viscosity 10^{-5} m²/sec is flowing over a plate of 3 m long and 2 m wide with a velocity of 3 m/sec parallel to 3 m side. Find the boundary layer thickness at the point of transition and at the end of plate.

Ans. Refer Q. 4.31, Page 4–43A, Unit-4.

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b. What do you mean by separation of boundary layer ? Define with neat sketch.

Ans. Refer Q. 4.33, Page 4–45A, Unit-4. (Refer Fig. 4.34.1, Page 4–46A)

7. Attempt any one part of the following : (7 × 1 = 7)

a. A square plate of side 2 m is moved in a stationary air of density 1.2 kg/m³ with a velocity of 50 km/hr. If the coefficient of drag and lift are 0.2 and 0.8 respectively, determine the drag force, lift force and resultant force.

Ans.

Given : Area of plate, $A = 2 \times 2 = 4\ m^2$
Velocity of the plate, $v = 50\ km/hr = 13.89\ m/s$
Density of air, $\rho = 1.2\ kg/m^3$, Coefficient of drag, $C_D = 0.2$
Coefficient of lift, $C_L = 0.8$

To Find : i. Drag force.
ii. Lift force.
iii. Resultant force.

1. Drag force,

$$F_D = C_D A \frac{\rho v^2}{2}$$

$$= 0.2 \times 4 \times \frac{1.2 \times (13.89^2)}{2}$$

$$= 92.6\ N$$

2. Lift force,

$$F_L = C_L A \frac{\rho v^2}{2}$$

$$= 0.8 \times 4 \times \frac{1.2 \times (13.89^2)}{2}$$

$$= 370.43\ N$$

3. Resultant force,

$$F_R = \sqrt{F_D^2 + F_L^2}$$

$$F_R = \sqrt{(92.6)^2 + (370.43)^2}$$

$$F_R = 381.83\ N$$

b. Find the form of equation for discharge Q through a sharp edged triangular notch; assuming Q depends upon the central angle α of the notch, head H, gravitational acceleration g, and on the mass density ρ, viscosity μ, and surface tension σ of the fluid.

Ans. Refer Q. 5.18, Page 5–20A, Unit-5.

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B.Tech.
(SEM. III) ODD SEMESTER THEORY EXAMINATION, 2018-19
FLUID MECHANICS

Time : 3 Hours **Max. Marks : 70**

Note : Attempt all sections. Assume missing data, if any.

Section-A

1. Attempt all question in brief. (2 × 7 = 14)

a. Draw the figure of shear stress v/s rate of deformation.

Ans. Refer Q. 1.3, Page 1–3A, Unit-1.

b. Define perfect gas.

Ans. Perfect gas is defined as an ideal gas which strictly obeys all the gas laws under all conditions of pressure and temperature.

c. What do you understand by stable equilibrium ?

Ans. Refer Q. 1.34, Page 1–43A, Unit-1.

d. The velocity distribution between two parallel plate is given by $u = (a^2 - y^2)$, where u is the velocity at a distance y from the middle of the two plates. Find the expression for stream function.

Ans.

Given : $u = (a^2 - y^2)$
To Find : Expression for stream function.

$$\frac{\partial \psi}{\partial y} = -u$$

$$\partial \psi = -(a^2 - y^2)\, dy$$

$$\partial \psi = (y^2 - a^2)\, dy$$

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$$\psi = \frac{y^3}{3} - ya^2 + c$$

e. Define surface loss.

Ans. Surface loss is loss of pressure or head that occurs in pipe flow due to effect of the fluid's viscosity near the surface of pipe.

f. What do you understand by dimensional homogeneity ?

Ans. Dimensional homogeneity means the dimensions of each terms in an equation on both sides are equal.

g. Find the frequency of oscillation when a 72 km/hr wind blows across a telephone wire of 3 mm diameter. Take $\nu = 1.5 \times 10^{-5}$ m²/s

Ans.

Given : $v = 72$ km/hr $= 20$ m/sec, $d = 3$ mm $= 3 \times 10^{-3}$
$A/t = 1.5 \times 10^{-5}$ m²/s
To Find : Frequency of oscillation.

We know, $\frac{1}{4}\pi d^2 v = \frac{A}{t}$

$$\frac{1}{4}\pi \times (3 \times 10^{-3})^2 \times v = 1.5 \times 10^{-5}$$

$$v = 2.12 \text{ Hz}$$

Section-B

2. Attempt any three parts of the following : (7 × 3 = 21)

a. Explain the procedure of finding hydrostatic forces on curved surfaces.

Ans. Refer Q. 1.24, Page 1–28A, Unit-1.

b. What are the different laws on which models are designed for dynamic similarity ?

Ans. Refer Q. 5.10, Page 5–10A, Unit-5.

c. What are the different laws on which models are designed for dynamic similarity ?

Ans. Refer Q. 5.10, Page 5–10A, Unit-5.

d. Draw the pressure distribution, theoretical as well as experimental on an airfoil in the fluid flow.

Ans.

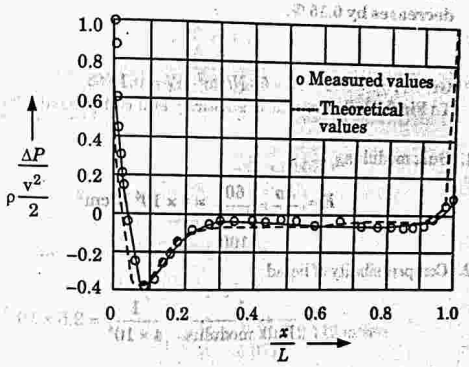

Fig. 1. Pressure distribution around an airfoil.

e. What is the difference between Eulerian and Lagrangian approach ?

Ans. Refer Q. 2.8, Page 2–10A, Unit-2.

Section-C

3. Attempt any one part of the following : (7 × 1 = 7)

a. What is the importance of model testing ?

Ans. The following are the importance of model analysis :

1. The model tests are quite economical and convenient (because the design, construction and operation of a model may be changed several times if necessary, without increasing much expenditure, till most suitable design is obtained).

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2. With the use of models the performance of hydraulic structures/ hydraulic machines can be predicted in advance.

3. While designing a particular portion of the structure if clear cut analytical and reliable method is not available then in such cases it becomes absolutely necessary to know about the safety and reliability of such parts which is possible by means of model testing.

4. Model testing can be used to detect and rectify the defects of an existing structure which is not functioning properly.

b. Determine the bulk modulus of elasticity and compressibility of a liquid. If the pressure of liquid is increased from 70 N/cm² to 130 N/cm². The volume of liquid decreases by 0.15 %.

Ans.

Given : $dp = 130 - 70 = 60$ N/cm², $dV = 0.15$ %
To Find : Bulk modulus of elasticity and compressibility of liquid.

1. Bulk modulus,

$$K = \frac{dp}{dV} = \frac{60}{\frac{0.15}{100}} = 4 \times 10^4 \text{ N/cm}^2$$

2. Compressibility of liquid

$$= \frac{1}{\text{Bulk modulus}} = \frac{1}{4 \times 10^4} = 2.5 \times 10^{-5}$$

4. Attempt any one part of the following : (7 × 1 = 7)

a. A model boat, 1/50 of its prototype experienced 0.2 N when simulating a speed of 5 m/s. Find the corresponding resistance of the prototype considering resistance at free surface only. Water is used for model as well as prototype also.

Ans. Refer Q. 5.14, Page 5–15A, Unit-5.

b. Mention the important dimensionless number used in fluid mechanics and their significance.

Ans. Refer Q. 5.22, Page 5–26A, Unit-5.

5. Attempt any one part of the following : (7 × 1 = 7)

a. A 30 cm diameter horizontal pipe terminates in a nozzle with the exit diameter of 7.5 cm. If the water flows through the pipe at a rate of 0.15 m³/sec what force will be exerted by the fluid on the nozzle ?

Ans.

Given : $d_1 = 30$ cm $= 0.3$ m, $d_2 = 7.5$ cm $= 0.075$ m,
$Q = 0.15$ m³/sec
To Find : Force exerted by the fluid on the nozzle.

1. Area of pipe, $A_1 = \frac{\pi}{4} d_1^2$

$$= \frac{\pi}{4} \times (0.3)^2$$
$$= 0.071 \text{ m}^2$$

2. Area of nozzle,

$$A_2 = \frac{\pi}{4} d_2^2 = \frac{\pi}{4} \times (0.075)^2$$
$$= 4.418 \times 10^{-3} \text{ m}^2$$

3. Applying continuity equation,

$$A_1 v_1 = A_2 v_2 = Q$$

$$v_1 = \frac{Q}{A_1} = \frac{0.15}{0.071} = 2.112 \text{ m/sec}$$

$$v_2 = \frac{Q}{A_2} = \frac{0.15}{4.418 \times 10^{-3}}$$
$$= 33.95 \text{ m/sec}$$

4. We know that,

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} = \frac{v_2^2}{2g} \quad \left[\because \frac{p_2}{\rho g} = \text{Atmospheric pressure} = 0, z_1 = z_2\right]$$

$$\frac{p_1}{\rho g} = \frac{v_2^2 - v_1^2}{2g}$$

$$p_1 = \left(\frac{33.95^2 - 2.112^2}{2}\right) \times 1000$$

$$= 574070.978 \text{ N/m}^2$$

5. Net force in direction of x = Rate of change of momentum in direction x.

$$p_1 A_1 - p_2 A_2 + F_n = \rho Q (v_2 - v_1)$$

Where, F_n = Force exerted by fluid on nozzle, and

p_2 = Atmospheric pressure = 0

$$574070.978 \times 0.071 - 0 + F_n = 1000 \times 0.15 \times (33.95 - 2.112)$$

$$40759.03 + F_n = 477.57$$

$$F_n = -40281.46 \text{ N}$$

Here negative sign indicates that the force exerted by the nozzle on water is acting from right to left.

b. Find the discharge from an 80 mm diameter external mouth piece fitted to a side of a large vessel if the head over the mouthpiece is 6 m.

Ans:

Given : $d = 80$ mm $= 0.08$ m, $H = 6$ m
To Find : Discharge.

1. Area of mouth piece $= \frac{\pi}{4} d^2$

$$= \frac{\pi}{4}(0.08)^2 = 5.026 \times 10^{-3} \text{ m}^2$$

2. Discharge, $Q = C_d \times$ Area $\times$ Velocity

$$Q = 0.855 \times 5.026 \times 10^{-3} \times \sqrt{2 \times 9.81 \times 6}$$

$$\left[\begin{array}{l} \because C_d \text{ for mouth piece} = 0.855 \\ \text{Velocity} = \sqrt{2gH} \end{array}\right]$$

$$Q = 0.0466 \text{ m}^3/\text{s}$$

6. Attempt any one part of the following : (7 × 1 = 7)

a. A kite 60 cm × 60 cm in size weighing 3 N makes an angle of 10° with the horizontal. The thread attached to it makes an angle of 45° to the horizontal and pull on the string is 25 N. The wind is flowing over the kite at 15 m/s. Find C_L and C_D for the kite.

Ans: Refer Q. 5(b) i, Page SP–18A, Solved Paper 2014-15.

b. Explain the displacement thickness, momentum thickness related to boundary layer.

Ans: Refer Q. 4.25, Page 4–33A, Unit-4.

7. Attempt any one part of the following : (7 × 1 = 7)

a. A pipe tapers from 250 mm to 125 mm, the rate of flow of the liquid in the pipe is 24000 lit/min. Calculate average velocity of flow at two sections.

Ans:

Given : $Q = 24000$ l/min $= 0.4$ m³/sec $d_1 = 250$ mm $= 0.25$ m, $d_2 = 125$ mm $= 0.125$ m
To Find : Average velocity of flow at two sections.

1. Area at section 1,

$$A_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4} \times (0.25)^2$$

$$= 0.049 \text{ m}^2$$

2. Area at section 2,

$$A_2 = \frac{\pi}{4} d_2^2 = \frac{\pi}{4} \times (0.125)^2 = 0.0123 \text{ m}^2$$

3. Average velocity at section 1,

$$v_1 = \frac{Q}{A_1} = \frac{0.4}{0.049} = 8.16 \text{ m/sec}$$

4. Average velocity at section 2,

$$v_2 = \frac{Q}{A_2} = \frac{0.4}{0.0123} = 32.52 \text{ m/sec}$$

b. Find the displacement thickness for velocity distribution in the boundary layer given by

$$\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$$

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Ans.

Given : $\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$

To Find : Displacement thickness.

1. Displacement thickness δ^* is given by

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy$$

$$\delta^* = \int_0^\delta \left\{1 - \left[2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2\right]\right\} dy$$

$$= \int_0^\delta \left\{1 - 2\left(\frac{y}{\delta}\right) + \left(\frac{y}{\delta}\right)^2\right\} dy = \left[y - \frac{2y^2}{2\delta} + \frac{y^3}{3\delta^2}\right]_0^\delta$$

$$= \delta - \frac{\delta^2}{\delta} + \frac{\delta^3}{3\delta^2} = \delta - \delta + \frac{\delta}{3} = \frac{\delta}{3}$$

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B.Tech.
(SEM. III) ODD SEMESTER THEORY EXAMINATION, 2019-20
FLUID MECHANICS

Time : 3 Hours **Max. Marks : 100**

Note : 1. Attempt all section. If require any missing data; then choose suitably.

Section-A

1. Attempt all questions in brief. (2 × 10 = 20)

a. Define weight density.

Ans. Refer Q. 1.2, Page 1–2A, Unit-1.

b. Define piezometer with neat sketch.

Ans. **Piezometer :** Refer Q. 1.22, 2 Marks Questions, Page SQ–3A, Unit-1.

Figure : Refer Q. 1.14 (Fig. 1.14.1), Page 1–18A, Unit-1.

c. What are stream tube ?

Ans. Refer Q. 2.15, 2 Marks Questions, Page SQ–7A, Unit-2.

d. What do you understand by rate of flow ?

Ans. Rate of flow is defined as the quantity of a liquid flowing per second through a section of pipe or a channel. It is expressed as,

$$Q = Av.$$

where, Q = Discharge.
A = Cross-sectional area of pipe.
v = Velocity.

e. Write practical application of Bernoulli's equation.

Ans. Bernoulli's equation is applied in all problems of incompressible fluid flow where energy considerations are involved. But we shall consider its application to the following measuring devices :

i. Venturimeter. ii. Orifice meter. iii. Pitot-tube.

f. Draw the neat sketch of venturimeter.

Ans. Refer Q. 3.10, Fig. 3.10.1, Page 3–10A, Unit-3.

g. Define Stokes's Law.

Ans. Refer Q. 4.2, 2 Marks Questions, Page SQ–12A, Unit-4.

h. Distinguish between rotational and irrotational flow.

Ans. Refer Q. 2.6, 2 Marks Questions, Page SQ–6A, Unit-2.

i. What are the Magnus effects ?

Ans. Refer Q. 5.6, 2 Marks Questions, Page SQ–16A, Unit-5.

j. What is flow separation ?

Ans. Flow separation or boundary layer separation is the detachment of a boundary layer from a surface into a wake. Separation occurs

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in flow that is slowing down, with pressure increasing, after passing the thickest part of a stream line body or passing through a widening passage.

Section-B

2. Attempt any three of the following : (3 × 10 = 30)

a. Given that : Barometer reading = 740 mm of mercury, specific gravity of mercury =13.6, intensity of pressure = 40 kPa. Express the intensity of pressure in SI units, both gauge and absolute.

Ans.

Given : Barometer reading = 740 mm of mercury, S_{Hg} = 13.6, p = 40 kPa

To Find : Gauge pressure and absolute pressure in SI units.

1. Gauge Pressure :

i. Intensity of pressure,

$$p = 40 \text{ kPa} = 40 \text{ kN/m}^2 = 0.4 \times 10^5 \text{ N/m}^2 = 0.4 \text{ bar} \quad (\because \ 1 \text{ bar} = 10^5 \text{ N/m}^2)$$

Specific weight of water, $w = 9.81$ kN/m³.

Specific weight of mercury, $w = 9.81 \times 13.6$ kN/m³.

ii. Height of pressure in term of water,

$$h = \frac{p}{w} = \frac{0.4 \times 10^5}{9.81 \times 10^3} = 4.077 \text{ m of water}$$

iii. Height of pressure in term of mercury,

$$h = \frac{p}{w} = \frac{0.4 \times 10^5}{9.81 \times 10^3 \times 13.6} = 0.299 \text{ m of mercury}$$

2. Absolute Pressure :

i. Barometer reading (atmospheric pressure)

$$= 740 \text{ mm of mercury} = 740 \times 13.6 \text{ mm of water}$$

$$= \frac{740 \times 13.6}{1000} = 10.06 \text{ m of water}$$

ii. Absolute pressure (p_{abs}) = Atmospheric pressure (p_{atm}) + Gauge pressure (p_{gauge}).

$$\therefore \ p_{abs} = 10.06 + 4.077 = 14.137 \text{ m of water}$$

$$= 14.137 \times (9.81 \times 10^3) = 1.38 \times 10^5 \text{ N/m}^2 \quad (\because \ p = wh)$$

$$= 1.38 \text{ bar} \quad (\because \ 1 \text{ bar} = 10^5 \text{ N/m}^2)$$

$$= \frac{14.137}{13.6} = 1.039 \text{ m of mercury}$$

b. Sketch the velocity distribution for uniform irrotational flow.

Ans. Refer Q. 4.2, Page 4–4A, Unit-4.

c. Find an expression for the discharge over a rectangular notch in terms of head of water over the crest of the notch.

Ans. Refer Q. 3.17, Page 3–21A, Unit-3.

d. Prove that viscous flow through a circular pipe the kinetic energy correction factor equal to 2.

Ans. Energy Correction Factor (α) :

1. We know that, the velocity distribution through a circular pipe for laminar flow at any radius (r) is given by,

$$u = \frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right)(R^2 - r^2) \quad \text{...(1)}$$

2. Consider an elementary area dA in the form of a ring at a radius r and of width dr, then, $dA = 2\pi r \, dr$

Fig. 1.

3. Rate of fluid flowing through the ring

$$= dQ = \text{Velocity} \times \text{Area of ring element}$$

$$= u \times 2\pi r \, dr$$

4. Kinetic energy of the fluid flowing through the elementary ring of radius 'r' and of width 'dr' per sec

$$= \frac{1}{2} \times \text{Mass} \times u^2 = \frac{1}{2} \times \rho(dQ) \times u^2$$

$$= \frac{1}{2} \times \rho \times (u \times 2\pi r \, dr) \times u^2 = \frac{1}{2}\rho \times 2\pi r u^3 \, dr = \pi \rho r u^3 \, dr$$

5. Total actual kinetic energy of flow per second

$$= \int_0^R \pi \rho r u^3 \, dr = \int_0^R \pi \rho r \left[\frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right)(R^2 - r^2)\right]^3 dr$$

$$= \pi \rho \times \left[\frac{1}{4\mu}\left(-\frac{\partial p}{\partial \mu}\right)\right]^3 \int_0^R [R^2 - r^2]^3 \, r dr$$

$$= \pi \rho \times \frac{1}{64\mu^3}\left(-\frac{\partial p}{\partial x}\right)^3 \int_0^R (R^6 - r^6 - 3R^4 r^2 + 3R^2 r^4) \, r \, dr$$

$$= \frac{\pi \rho}{64\mu^3}\left(-\frac{\partial p}{\partial x}\right)^3 \int_0^R (R^6 r - r^7 - 3R^4 r^3 + 3R^2 r^5) \, dr$$

$$= \frac{\pi \rho}{64\mu^3}\left(-\frac{\partial p}{\partial x}\right)^3 \left[\frac{R^6 r^2}{2} - \frac{r^8}{8} - \frac{3R^4 r^4}{4} + \frac{3R^2 r^6}{6}\right]_0^R$$

$$= \frac{\pi \rho}{64\mu^3}\left(-\frac{\partial p}{\partial x}\right)^3 \left[\frac{R^8}{2} - \frac{R^8}{8} - \frac{3R^8}{4} + \frac{3R^8}{6}\right]$$

$$= \frac{\pi \rho}{64\mu^3}\left(-\frac{\partial p}{\partial x}\right)^3 R^8 \left[\frac{12 - 3 - 18 + 12}{24}\right] = \frac{\pi \rho}{64\mu^3}\left(-\frac{\partial p}{\partial x}\right)^3 \frac{R^8}{8}$$

6. Kinetic energy of the flow based on average velocity

$$= \frac{1}{2} \times \text{Mass} \times \bar{u}^2 = \frac{1}{2} \times \rho A \bar{u}^3$$

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Substituting the value of $A = (\pi R^2)$ and $u = \frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right) R^2$

7. Kinetic energy of the flow/sec

$$= \frac{1}{2} \times \rho \times \pi R^2 \left[\frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right) R^2\right]^3 = \frac{1}{2} \times \rho \times \pi R^2 \frac{1}{64 \times 8\mu}\left(-\frac{\partial p}{\partial x}\right)^3 \times R^6$$

$$= \frac{\rho\pi}{128 \times 8\mu^3}\left(-\frac{\partial p}{\partial x}\right)^3 \times R^8 \quad ...(2)$$

8. $$\alpha = \frac{\text{KE/ sec based on actual velocity}}{\text{KE/ sec based on average velocity}} = \frac{\text{Eq. (1)}}{\text{Eq. (2)}}$$

$$\alpha = \frac{\frac{\rho\pi}{64\mu^3}\left(-\frac{\partial p}{\partial x}\right)^3 \times \frac{R^8}{8}}{\frac{\rho\pi}{128 \times 8\mu^3}\left(-\frac{\partial p}{\partial x}\right)^3 \times R^8} = \frac{128 \times 8}{64 \times 8} = 2.0$$

e. Give and explain the five errors in CFD and give examples. How can they be determined and reduced ?

Ans. Following are the various types of errors in CFD :

1. Physical Approximation Error :

i. Physical modeling errors are those due to uncertainty in the formulation of the model and deliberate simplifications of the model.

ii. These errors deal with the continuum model only.

iii. Errors in the modeling of the fluids or solids problem are concerned with the choice of the governing equations which are solved and models for the fluid or solid properties.

iv. Further, the issue of providing a well-posed problem can contribute to modeling errors.

v. Often modeling is required for turbulence quantities, transition, and boundary conditions (bleed, time-varying flow, surface roughness).

vi. Even when a physical process is known to a high level of accuracy, a simplified model may be used within the CFD code for the convenience of a more efficient computation

vi. Physical modeling errors are examined by performing validation studies that focus on certain models (*i.e.*, inviscid flow, turbulent boundary layers, real gas flows, etc).

2. Computer Round-Off Error :

i. Computer round-off errors develop with the representation of floating point numbers on the computer and the accuracy at which numbers are stored. With advanced computer resources, numbers are typically stored with 16, 32, or 64 bit

ii. If computer round-off errors are suspected of being significant, one test is to run the code at a higher precision or on a computer known to store floating point numbers at a higher precision.

iii. One can attempt to iterate a coarse grid solution to a residual of machine zero; however, this may not be possible for more complex algorithms.

3. Iterative Convergence Errors :

i. The iterative convergence error exists because the iterative methods used in the simulation must have a stopping point eventually.

ii. The error scales to the variation in the solution at the completion of the simulations.

4. Discretization Errors :

i. Discretization errors are those errors that occur from the representation of the governing flow equations and other physical models as algebraic expressions in a discrete domain of space (finite-difference, finite-volume, finite-element) and time. The discrete spatial domain is known as the grid or mesh. Discretization error is also known as numerical error.

ii. A consistent numerical method will approach t e continuum representation of the equations and zero discretization error as the number of grid points increases and the size of the grid spacing tends to zero.

iii. As the mesh is refined, the solution should become less sensitive to the grid spacing and approach the continuum solution. This is grid convergence.

iv. The grid convergence study is a useful procedure for determining the level of discretization error existing in a CFD solution.

v. The discretization error is of most concern to a CFD code user during an application.

vi. Discretization errors are of major concern because they are dependent on the quality of the grid; however, it is often difficult to precisely indicate the relationship between a quality grid and an accurate solution prior to beginning the simulation.

vii. The level of discretization error is dependent on grid quality. The level of discretization error is dependent on the features of the flow as resolved by the grid.

viii. Error may develop due to representation of discontinuities (shocks, slip surfaces, interfaces) on a grid.

5. Computer Programming Errors :

i. Programming errors are "bugs" and mistakes made in programming or writing the code.

ii. They are the responsibility of the programmers.

iii. These types of errors are discovered by systematically performing verification studies of subprograms of the code and the entire code, reviewing the lines of code, and performing validation studies of the code.

iv. The programming errors should be removed from the code prior to release.

6. Usage Errors :

i. Usage errors are due to the application of the code in a less-than-accurate or improper manner.

ii. The user sets the models, grid, algorithm, and inputs used in a simulation, which then establishes the accuracy of the simulation.

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iii. There may be blatant errors, such as attempting to compute a known turbulent flow with an assumption of inviscid flow.
iv. Usage errors are minimized through proper training.
v. Usage errors should be controllable through proper training and analysis.

Section-C

3. Attempt any one part of the following : (1 × 10 = 10)

a. A crude oil of viscosity 0.97 poise and relative density= 0.9 is flowing through a horizontal circular pipe of diameter 100 mm and length 10 m. Calculate the difference of pressure at two ends of the pipe, if 100 kg of the oil is collected in tank in 30 seconds.

Ans.

Given : Viscosity, $\mu = 0.97$ poise $= \frac{0.97}{10} = 0.097$ Nsec/m^2
Relative density or specific gravity, $S = 0.9$
Diameter of pipe, $D = 100$ mm $= 0.1$ m, Length of pipe, $L = 10$ m
Mass of oil collected, $M = 100$ kg, Time, $t = 30$ sec
To Find : Difference of pressure or $(p_1 - p_2)$.

1. Density of oil, $\rho_0 = 0.9 \times 1000 = 900$ kg/m^3,
2. Mass of oil/sec = 100/30 kg/sec $= \rho_0 \times Q = 900 \times Q$
$\therefore$ $100/30 = 900 \times Q \Rightarrow Q = 0.003704$ m^3/sec
3. Average velocity, $u = \frac{Q}{\text{Area}} = \frac{0.003704}{\frac{\pi}{4} \times D^2} = \frac{0.003704}{\frac{\pi}{4}(0.1)^2} = 0.472$ m/sec
4. For laminar or viscous flow, the Reynolds number (R_e) is less than 2000.

Reynolds number, $R_e = \frac{\rho v D}{\mu}$

where, $\rho = \rho_0 = 900$, $v = u = 0.472$ m/sec, $D = 0.1$ m,
$\mu = 0.097$ Nsec/m^2

$$R_e = \frac{900 \times 0.472 \times 0.1}{0.097} = 437.94$$

As Reynolds number is less than 2000, the flow is laminar.
5. The difference of pressure $(p_1 - p_2)$ for viscous or laminar flow is given by,

$$p_1 - p_2 = \frac{32\mu u L}{D^2} = \frac{32 \times 0.097 \times 0.472 \times 10}{(0.1)^2} \text{ N/m}^2 = 1465.09 \text{ N/m}^2$$

b. Explain briefly the following types of equilibrium of floating bodies :
i. Stable equilibrium, ii. Unstable equilibrium, and
iii. Neutral equilibrium.

Ans. Refer Q. 1.34, Page 1–43A, Unit-4.

4. Attempt any one part of the following : (1 × 10 = 10)

a. Write examples of viscous flow and explain the characteristics of Laminar flow.

Ans.

1. **Examples of Viscous Flow :**
i. Flow past tiny bodies.
ii. Underground flow.
iii. Movement of blood in the arteries of a human body.
iv. Flow of oil in measuring instruments.
v. Rise of water in plants through their roots etc.
2. **Characteristics of Laminar Flow or Viscous Flow :** Refer Q. 4.1, Page 4–3A, Unit-4.

b. Find the velocity and acceleration at a point (1, 2, 3) after 1 sec. for a three dimensional flow given by, $u = yz + t$, $v = xz - t$, $w = xy$ m/sec.

Ans.

Given : Three-dimensional flow field,
$u = yz + t$, $v = xz - t$, $w = xy$, m/sec
To Find : Velocity and acceleration at a point (1, 2, 3) after 1 sec.

1. **Velocity at a Point (1, 2, 3), $V_{(1,2,3)}$:**
Velocity at a point (1, 2, 3) after 1 sec is calculated as follows :

$$u = yz + t = 2 \times 3 + 1 = 7 \text{ m/sec},$$
$$v = xz - t = 1 \times 3 - 1 = 2 \text{ m/sec}$$
$$w = xy = 1 \times 2 \text{ m/sec}$$

$$\therefore \quad v_{(1,2,3)} = 7\hat{i} + 2\hat{j} + 2\hat{k} = \sqrt{7^2 + 2^2 + 2^2} = 7.55 \text{ m/sec}$$

Hence, $v_{(1,2,3)} = 7.55$ m/sec

2. **Acceleration, $a_{(1,2,3)}$:**

Now, $v = (yz + t)\hat{i} + (xz - t)\hat{j} + xy\hat{k}$

Acceleration $a = \frac{dv}{dt} = \left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) + \frac{\partial v}{\partial t}$

$$a = (yz + t)(z\hat{j} + y\hat{k}) + (xz - t)(z\hat{i} + x\hat{k}) + xy(y\hat{i} + x\hat{j}) + (1\hat{i} - 1\hat{j})$$

$$\therefore \quad a_{(1,2,3)} = 7(3\hat{j} + 2\hat{k}) + 2(3\hat{i} + 1\hat{k}) + 2(2\hat{i} + 1\hat{j}) + (1\hat{i} - 1\hat{j})$$
$$= (10\hat{i} + 23\hat{j} + 16\hat{k}) + (1\hat{i} - 1\hat{j})$$

3. The convective acceleration components are : (10, 23, 16) m/sec^2
4. The local acceleration components are : (1, – 1) m/sec^2 along x and y directions.
5. The total acceleration of fluid particles at the point (1, 2, 3) is

$$a_{(1,2,3)} = \sqrt{(10+1)^2 + [23 + (-1)]^2 + 16^2}$$
$$= \sqrt{11^2 + 22^2 + 16^2} = 29.343 \text{ m/sec}^2$$

Hence $a_{(1,2,3)} = 29.343$ m/sec^2

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5. Attempt any one part of the following : (1 × 10 = 10)

a. A horizontal pipe of diameter 450 mm is suddenly contracted to a diameter of 200 mm. The pressure intensities in the large and smaller pipe is given as 13.734 N/cm² and 11.774 N/cm² respectively. Find the loss of head due to contraction if C_c = 0.62. Also determine the rate of flow of water.

Ans.

Given : Diameter of large pipe, D_1 = 450 mm = 0.45 m, Diameter of smaller pipe, D_2 = 200 mm = 0.20 m, Pressure in large pipe, p_1 = 13.734 N/cm² = 13.374 × 10⁴ N/m², Pressure in smaller pipe, p_2 = 11.774 N/cm² = 11.774 × 10⁴ N/m², Coefficient of contraction, C_c = 0.62

To Find : The rate of flow of water.

1. Area of larger section, $A_1 = \frac{\pi}{4}(0.45)^2 = 0.159 \text{ m}^2$

2. Area of smaller section, $A_2 = \frac{\pi}{4}(0.20)^2 = 0.0314 \text{ m}^2$

3. Head loss due to contraction,

$$h_c = \frac{v_2^2}{2g}\left[\frac{1}{C_c} - 1.0\right]^2 = \frac{v_2^2}{2g}\left[\frac{1}{0.62} - 1.0\right]^2 = 0.376\frac{v_2^2}{2g}$$

4. From continuity equation, we have $A_1v_1 = A_2v_2$

$$\text{or } v_1 = \frac{A_2v_2}{A_1} = \frac{\frac{\pi}{4}D_2^2 \times v_2}{\frac{\pi}{4}D_1^2} = \left(\frac{D_2}{D_1}\right)^2 \times v_2 = \left(\frac{0.20}{0.45}\right)^2 \times v_2 = 0.1975 v_2$$

5. Applying Bernoulli's equation before and after contraction

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_c$$

But $z_1 = z_2$ (∵ Pipe is horizontal)

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + h_c \quad ...(1)$$

5. But, $h_c = 0.376\frac{v_2^2}{2g}$ and $v_1 = 0.1975 v_2$

Substituting these values in the eq. (1), we get

$$\frac{13.734 \times 10^4}{9.81 \times 1000} + \frac{(0.1975 v_2)^2}{2 \times 9.81} = \frac{11.774 \times 10^4}{1000 \times 9.81} + \frac{v_2^2}{2 \times 9.81} + 0.376\frac{v_2^2}{2 \times 9.81}$$

$$\frac{(0.1975 v_2)^2}{2 \times 9.81} - \frac{v_2^2}{2 \times 9.81} - 0.376\frac{v_2^2}{2 \times 9.81} = \frac{11.774 \times 10^4}{1000 \times 9.81} - \frac{13.734 \times 10^4}{1000 \times 9.81}$$

$$\frac{-1.337 v_2^2}{2 \times 9.81} = \frac{-1.96 \times 10^4}{1000 \times 9.81}$$

$$v_2 = 5.415 \text{ m/sec}$$

6. Loss of head due to contraction,

$$h_c = 0.376\frac{v_2^2}{2g} = \frac{0.376 \times (5.415)^2}{2 \times 9.81} = 0.562 \text{ m}$$

7. Rate of flow of water, $Q = A_2v_2 = 0.0314 \times 5.415 = 0.170031 \text{ m}^3/\text{sec}$ = 170.031 lit/sec

b. Derive an expression for the power transmission through the pipes. Find also the condition of power and corresponding efficiency of transmission.

Ans. **Expression :** Refer Q. 3.30, Page 3–38A, Unit-3.

1. Condition for Maximum Transmission of Power :

i. Power transmission through the pipe is given by,

$$P = \frac{\rho g}{1000} \times \frac{\pi d^2}{4}\left(Hv - \frac{4fLv^3}{2dg}\right) \quad ...(1)$$

ii. The condition for maximum transmission of power is obtained by differentiating eq. (1) with respect to v and equating the same to zero.

Thus $\frac{d}{dv}(P) = 0$

$$\frac{d}{dv}\left[\frac{\rho g}{1000} \times \frac{\pi}{4}d^2\left(Hv - \frac{4fLv^3}{d \times 2g}\right)\right] = 0$$

$$\frac{\rho g}{1000} \times \frac{\pi}{4}d^2\left(H - \frac{4 \times 3 \times f \times L \times v}{d \times 2g}\right) = 0$$

$$\frac{\rho g}{1000} \times \frac{\pi}{4}d^2\left(H - \frac{4 \times 3 \times f \times L \times v^2}{d \times 2g}\right) = 0 \quad \left(\because \frac{4fLv^2}{d \times 2g} = h_f\right)$$

$$\therefore \quad H = 3h_f \text{ or } h_f = H/3 \quad ...(2)$$

2. Condition for Maximum Efficiency of Transmission of Power :

i. Efficiency of power transmission through a pipe is given by,

$$\eta = \frac{H - h_f}{H} \quad ...(3)$$

ii. For maximum power transmission through pipe the condition is given by eq. (2) as :

$$h_f = H/3$$

iii. Substituting the value of h_f in eq. (3), we get

$$\eta_{max} = \frac{H - H/3}{H} = 1 - \frac{1}{3} = \frac{2}{3} \text{ or } 66.7\%$$

6. Attempt any one part of the following : (1 × 10 = 10)

a. If velocity distribution in laminar boundary layer a flat plate is assumed to be given by second order polynomial $u = a + by + cy^2$. Determine its form using the necessary boundary conditions.

Ans.

Given : Velocity distribution : $u = a + by + cy^2$
To Find : Velocity distribution.

1. The following boundary conditions must be satisfied :

i. At $y = 0$, $u = 0$, $u = a + by + cy^2$

$$0 = a + 0 + 0 \quad \Rightarrow a = 0 \qquad ...(1)$$

Put the value of a in eq. (1), we get

$$u = by + cy^2 \qquad ...(2)$$

ii. At $y = \delta$, $u = U$, $U = b\delta + c\delta^2$ ...(3)

iii. At $y = \delta$, $du/dy = 0$

$$\therefore \left(\frac{du}{dy}\right)_{y=\delta} = \frac{d}{dy}(a + by + cy^2) = b + 2cy = b + 2c\delta = 0$$

$$b = -2c\delta \qquad ...(4)$$

2. Substituting the value of b in eq. (3), we get

$$U = (-2c\delta)\delta + c\delta^2 = -2c\delta^2 + c\delta^2 = -c\delta^2$$

$$c = -U/\delta^2$$

3. Put the value of c in eq. (4), we get

$$b = -2c\delta = -2 \times \left(-\frac{U}{\delta^2}\right)\delta = \frac{2U}{\delta}$$

4. Put the values of b and c in eq. (2), we get $u = \frac{2U}{\delta}y - \frac{U}{\delta^2}y^2$

Form of the velocity distribution $\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$

b. Prove that in case of force vortex, the rise of liquid level at the ends is equal to the full liquid level at the axis of rotation.

Ans.

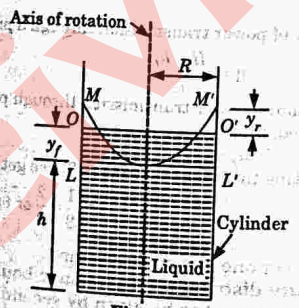

Fig. 2.

1. Let, R = Radius of the cylinder, and OO' = Initial liquid level when the cylinder is stationary.

2. Let the cylinder is rotated at constant angular velocity, ω. The liquid will rise at the ends and will fall at the centre.
Let, y_r = Rise of liquid at the ends (from OO').
y_f = Fall of liquid at the centre (from OO').
3. Now, initial height of liquid = $(h + y_f)$
4. $\therefore$ At initial, volume of liquid in cylinder = $\pi R^2 (h + y_f)$ ...(1)
5. At rise of liquid, volume of liquid = [Volume of cylinder upto level MM' – Volume of paraboloid]

$= (\pi R^2 \times$ Liquid height upto level $MM') - (1/2 \times \pi R^2 \times$ Height of paraboloid)

$$= \pi R^2 \times (h + y_f + y_r) - \frac{\pi R^2}{2}(y_f + y_r)$$

$$= \pi R^2 h + \frac{\pi R^2}{2}(y_f + y_r) \qquad ...(2)$$

6. From eq. (1) and eq. (2), we get

$$\pi R^2(h + y_f) = \pi R^2 h + \frac{\pi R^2}{2}(y_f + y_r)$$

$$\pi R^2 h + \pi R^2 y_f = \pi R^2 h + \frac{\pi R^2}{2}y_f + \frac{\pi R^2}{2}y_r$$

$$\pi R^2 y_f - \frac{\pi R^2}{2}y_f = \frac{\pi R^2}{2}y_r$$

$$\frac{\pi R^2}{2}y_f = \frac{\pi R^2}{2}y_r \Rightarrow y_f = y_r$$

Hence, Fall of liquid at centre = Rise of liquid at the ends.

7. Attempt any one part of the following : **(1 × 10 = 10)**

a. What is meant by geometric, kinematic and dynamic similarities ? Are these similarities truly attainable ? If is not why ?

Ans. Refer Q. 5.11, Page 5–12A, Unit-5.

b. A 1:40 model of an ocean tanker is dragged through fresh water at 2 m/sec with total measured drag of 117.7 N. The skin (factional) drag co-efficient 'f' for model and prototype are 0.3 and 0.02 respectively in the equation $R_f = fAv^2$. The wetted surface area of the model is 25 m². Taking the densities for the prototype and the model as 1030 kg/m³ and 1000 kg/m³ respectively, Determine (i) The total drag on the prototype (ii) Power required to drive the prototype.

Ans.

Given : Linear scale ratio, $L_r = 40$, Velocity of model, $v_m = 2$ m/sec, Total drag of model, $R_m = 117.7$ N, Wetted surface area of model, $A_m = 25$ m², Skin drag friction for prototype $(f_p) = 0.02$ and for model $(f_m) = 0.3$, $\rho_p = 1030$ kg/m³, $\rho_m = 1000$ kg/m³
To Find : i. The total drag on the prototype.
ii. Power required to drive the prototype.

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1. Total Drag on the Prototype, R_p :

i. Frictional drag on model, $(R_f)_m = f_m A_m v^2_m = 0.3 \times 25 \times 2^2 = 30$ N

ii. Wave drag on model, $(R_w)_m = R_m - (R_f)_m = 117.7 - 30 = 87.7$ N

iii. The wave drag for the prototype can be evaluated by satisfying the following condition for dynamic similarity :

$$\left(\frac{R_w}{\rho L^2 v^2}\right)_p = \left(\frac{R_w}{\rho L^2 v^2}\right)_m$$

$$(R_w)_p = \frac{\rho_p}{\rho_m} \times \left(\frac{L_p}{L_m}\right)^2 \times \frac{v_p^2}{v_m^2} \times (R_w)_m$$

But, $$\left(\frac{v_p}{v_m}\right)^2 = \frac{L_p}{L_m}$$

$$(R_w)_p = \frac{\rho_p}{\rho_m} \times \left(\frac{L_p}{L_m}\right)^3 \times (R_w)_m$$

$$= \frac{1030}{1000} \times (40)^3 \times 87.7 \qquad \left[\because \frac{L_p}{L_m} = L_r = 40\right]$$

$$= 5781184 \text{ N}$$

iv. The frictional drag on the prototype is given by,

$$(R_f)_p = f_p A_p v_p^2 \qquad ...(1)$$

where, v_p is velocity of prototype and is obtained from Froude model law as given below :

$$\frac{v_m}{\sqrt{L_m g_m}} = \frac{v_p}{\sqrt{L_p g_p}} \quad or \quad \frac{v_m}{\sqrt{L_m}} = \frac{v_p}{\sqrt{L_p}} \qquad (\because g_m = g_p)$$

$$\therefore \quad v_p = v_m \times \sqrt{\frac{L_p}{L_m}} = 2 \times \sqrt{40} = 12.65 \text{ m/sec}$$

Also, $$\frac{A_p}{A_m} = L_r^2 = 40^2 = 1600$$

$$\therefore \quad A_p = A_m \times 1600 = 25 \times 1600 = 40000 \text{ m}^2$$

v. Substituting the values of $\delta_p A_p$ and v_p in eq. (1), we get

$$(R_f)_p = 0.02 \times 40000 \times (12.65)^2 = 128018 \text{ N}$$

vi. Total drag on the prototype,

$$R_p = (R_w)_m + (R_f)_p = 5781184 + 128018$$

$$= 5909202 \text{ N} \quad or \quad 5909.2 \text{ kN}$$

2. Power Required to Drive the Prototype, P :

Power required = Total drag on prototype × Velocity of prototype

i.e., $$P = (R_f)_p \times v_p$$

$$= 5909.2 \times 12.65 \text{ kW} = 74751.4 \text{ kW}$$

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