# Do LDA algorithms learn LDA models?

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#### Abstract

The Latent Dirichlet Allocation topic model has received enormous attention as a method for extracting insight from documents and a wide variety of other data sources. Two interesting aspects in its application are the effectiveness of learning algorithms, and the applicability to real data. In this paper, we primarily study the former but also apply some of what we learn to the latter.

For a common view, we define a natural (for LDA) prediction task and generate data according to the LDA model. We then evaluate a range of algorithms, including one of our invention, on this generated data. We characterize features of the generated data to help predict the performance of these algorithms. We also run the algorithms on some real world datasets.

We find that the standard algorithms for LDA are quite poor compared with principal components methods (latent semantic indexing or LSI) and our new method with data generated from their model. In contrast, the LDA methods do perform better than principal components methods on real world data, though, fall far short of nearest neighbor methods. While the goal of topic modelling is not nessarily prediction, we believe this study provides useful information about their use.

We note that it is ironic that the algorithms for LDA fail to beat LSI based methods on their "LDA" data but do on real world data.

## 1 Introduction

With the vast production of data comes a great desire to understand and make use of its structure. One avenue has been to use generative models to provide insight. Topic models, in particular, have been quite popular to investigate documents and an increasing number of other domains (See [7] for a survey of this field.)

A relevant, prehistorical, viewpoint is the concept of latent semantic indexing which views a document as being about a subset of a larger set of topics. Topics are in turn are about (positive numbers) or not about (negative numbers) certain words [16]. They used this view to provide an algorithm based on linear algebra that learned the structure of their model. They also argued that this model provided some insight into the wide applicability of principal components analysis in data analysis. The algorithmic tool at the core of their work (and principal components) was the singular value decomposition.

Shortly thereafter, a probabilistic framework was provided by Puzicha and Hofmann [10]. They termed their method probabilistic latent semantic indexing (pLSI), and modelled topics as probability distributions over words (no negative numbers in this description). Documents were then presumed to be generated as a mixture of these topics. The large number of parameters in the PLSI model motivated Blei, Ng and Jordan to provide a generative model for these parameters as well; they provided the enormously influential Latent Dirichlet Allocation(LDA) topic model [9].

Both pLSI and LDA have had enormous impact, but remain challenging both in terms of the appropriateness of the model and in terms of algorithms fitting the model. The evaluation of efficacy departed from tradition in machine learning in that one evaluated the learned parameters in terms of predicted probability of the corpus ("perplexity") rather than on the performance of some task. Indeed, this perspective remains compelling. For example, in [15] the evaluation of new algorithms still proceeds very much along these internal

measures. This has merits in that one goal of topic modelling is to find something "interesting", for which performance is difficult to define. Still, the form of the measures suggested in the literature can be very complex. See, for example, the definition of "free energy" in [15].

This complexity can make it very difficult to see when such models are appropriate or whether the algorithms are effectively learning the model. We repeat for emphasis that there are *two issues here*: the applicability of the model, and the effectiveness of the algorithm. Neither is easy to evaluate in engineering the methods for data analytics.

We seek to provide insight into the performance of algorithms and to some extent understand the application to data. In contrast to the tendencies in topic modelling [15], we wish to provide simple methods for evaluation. In this paper, we primarily study the effectiveness of algorithms for discerning LDA models when we know they apply; for data generated by the model.

An effective model should lead to good performance on prediction. We thus return to a simple prediction task as a primary figure of merit. Topic models, particularly LDA, generate documents word by word. Thus, a fair evaluation is to use a training set of documents and a subset of a document to predict an unseen word. This task, as we show, entails both learning the topics and the distribution of topics in a particular document. The "one-more" (or "k-more") word task is central to our study. This task also has natural applications which we touch upon later.

A feature of this approach is that one can compare to methods not based on modelling. Thus, we can compare performance based on modelling topics to standard learning methods. In particular, we include nearest neighbor methods which have been found to be highly effective in the literature and in our experiments.

We found that for generated LDA data that the algorithms widely used for learning LDA appear to be reasonably effective, but traditional algorithms based on LSI are, in fact, more effective. We further engineered a new method combining LSI, k-means, and the inference part of LDA. This method is more robust than LSI and has the best performance of any method we tested.

We characterize the instances that we can learn, the instances where knowing the parameters give power (even when we fail to learn them), and finally where the model fails to give any predictive advantage at all even cheating. The resulting characterization is unsuprising: simply measure the independence or lack thereof of the selection of words in a particular document is key. On the other hand, taking advantage of such simple measures has been key to giving provable algorithms for numerous mixture problems ranging from learning mixtures of Gaussians [18], to hidden markov problems [14], to recent provable method for learning topic models [5] and even the LDA model [4]. Indeed, we must first recognize order before distinguishing it from chaos

Finally, we test the methods on some real data sets. Here, we find that topic models do distinguish from chaos. Indeed, we find that the LDA algorithms do better than both LSI and our engineered methods which dominate on the simulated data. This is quite interesting, and gives gives testament to the notion that topic modellers have worked extensively with real data; model or not. Still, the most effective algorithms in our experiments by far for real datasets is the nearest neighbor algorithms.

To reiterate, the main study here is a simulation study to see whether the modelling process produces better algorithms. What appears to be the case is that decent, though not champion, algorithms for real data sets were developed for LDA despite them not being particularly good for data generated by the actual model. This suggests that the algorithms may be more experimentally developed than formally motivated. It remains quite interesting to characterize what model is most appropriate for real world data, but understanding the relationship between algorithms and models should be an important aspect of this. We proceed in the sequel by describing the LDA model in section 3. We decribe the algorithms in 6. We give some experiments for a range of parameters and for real datasets in section 7. We explore the border

<sup>&</sup>lt;sup>1</sup>We primarily report on a variational algorithm for optimizing LDA. We experimented with Gibbs Sampling implementations as well as pLSI implementations. The former was slow but performed similarly to the variational approach, the latter performed poorly.

of where the algorithms and than even the topic parameterization fail to provide performance in section 8. We conclude in section 9.

## 2 Related work

The most closely related work in theory are the aforementioned algorithms provided by Arora et.al. [5] which learn pLSI under certain assumptions, and the breakthrough by Anandkumar[4] for LDA. There is a long literature in learning mixtures of distributions in statistics and more recently in theoretical computer science. See [11] for a recent breakthrough on learning mixtures of Gaussians and a discussion of this field. There is ample work describing methods for optimizing LDA [7]. Again, recent examples is described in [15]. There is also significant work on extending LDA and topic models to better model data as well as to augment them with other types of information. These are discussed in [7]. The hierarchical LDA model [8] is perhaps the most relevant. This model is based on the chinese restuarant process where new customers arrive and chose to join a table or create a new one. This process can be used to create a hierarchical structure of documents and topics. The authors argue that this is a better model for real data and we tend to agree. Still, we began by understanding the simpler model which continues to have wide application. While we have included the hLDA generator is included in our framework, the implementations distributed by the authors are not yet stable within our system.

To some extent, we seek to provide an infrastructure to evaluate topic modelling. We should point out that the machine learning community has been active at producing infrastructure for evaluation. See for example, MLComp [12]. A particularly related java based infrastructure for topic modelling is [13]. We plan to test their implementations in the final version of this paper.

## 3 LDA model

LDA is introduced in [9] as a generative process. As a model it is widely applied in various domains such as information retrieval, collaborative filtering, vision, and bioinformatics. In this work we will adopt the language of text collections, and denote entities as 'word', 'document', and 'corpus', since they give intuition in discussing topic models.

The basic idea is that there exist k underlying latent topics. Each document is a mixture of the latent topics, where the topic mixture is drawn from a dirichlet distribution. More precisely, there are n documents, m words, and k topics. The model has a  $m \times k$  word-topic matrix A, where the i-th column  $A_i$  specifies a multinomial distribution on the m words for topic i. For a document w, we first choose  $\vec{\theta}$ , its distribution over topics, which can take values in the (k-1)-simplex, and has the following Dirichlet distribution

$$p(\theta|\vec{\alpha}) = \frac{\Gamma(\sum_{i=1}^{k} \alpha_i)}{\prod_{i=1}^{k} \Gamma(\alpha_i)} \theta_i^{\alpha_1 - 1} \cdots \theta_k^{\alpha_k - 1}$$

where  $\vec{\alpha}$  is parameter of the model. The number of words in document w is sampled from Poisson(l). For each word  $w_i$ , a topic  $z_i \sim Multinomial(\vec{\theta})$  is chosen, then the actual word  $w_i \sim Multinomial(A_{z_i})$  is generated. Equivalently, in matrix form, there are the  $m \times k$  word-topic matrix A, and  $k \times n$  topic-document matrix W whose columns are drawn i.i.d from  $Dirichlet(\vec{\alpha})$ . The product M = AW is the  $m \times n$  term-document matrix where column  $M_i$  is document i's distribution on words. Document i is generated by sampling words i.i.d from  $Multinomial(M_i)$ . We are interested in the case where A is of full rank, since if the columns of A are not independent, intuitively it means there exists some document which is covered completely by a set of topics I, but at the same time also completely covered by another set of topics I which is disjoint from I. In our experiments, the randomly generated A matrices are almost always of full rank.

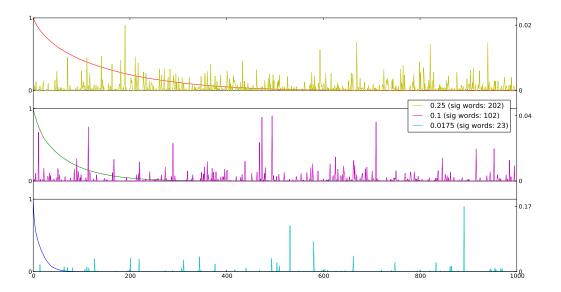


Figure 1: Plot of distributions on words for various  $\beta$ . m = 1000, each distribution is plotted along with its cdf after sorting the words by popularity. Refer to the y-axis on the right for the scaling of the distributions. In general, larger  $\beta$  values yield flatter distributions.

## 4 Data

Since our focus is on how effectively the algorithms learn the model, we use synthetic datasets generated from the LDA model for a range of parameters. Our data generator takes in parameters

- $\bullet$  n, m, k, number of documents, words, and topics respectively
- $\alpha$ , the Dirichlet parameter for generating documents' distributions over topics as in the LDA model. In our experiments we work with symmetric Dirichlet distributions, where  $\alpha_i = \ldots = \alpha_k = \alpha$
- $\beta$ , we generate the columns of word-topic matrix A from a m dimensional Dirichlet distribution with parameter  $\vec{\beta}$ . Again we work with symmetric Dirichlet where  $\beta_1 = \ldots = \beta_m = \beta$ .
- l, the Poisson parameter controlling the expected number of words in a document.

Intuitively the Dirichlet parameter  $\alpha$  is a crude measure of the sparsity of the sampled distribution over topics. When  $\alpha=1$ , all points in the k-1 simplex have the same probability density. When  $\alpha<1$ , distributions that are more concentrated on a few topics are prefered by the Dirichlet. The same applies to  $\beta$  and the topic's distribution on words. See figure 1 for typical word distributions sampled from the Dirichlet distribution with various  $\beta$ 's as parameter. To help understand the dataset, we compute the values  $sig\_topic$  and  $sig\_word$ . For a document with distribution  $\vec{\theta}$  over topics,  $sig\_topic$  is the smallest t such that the union of the t heaviest topics in  $\vec{\theta}$  has an aggregate probability of at least 0.8. Intuitively,  $sig\_topic$  is the smallest number of significant topics of a document. Analogously, for a topic's distribution over words,  $sig\_word$  is the smallest number of most popular words with an aggregate probability of at least 0.8.Instead of using  $\alpha$  and  $\beta$ , we use the average  $sig\_topic$  and average  $sig\_word$  to characterize our datasets.

We also have collected some real world datasets. We used the Classic-3 datasets [3], a corpus from the Associated Press [1] pruning stop words and infrequent words, and a bag of words dataset from UC Irvine[2].

## 5 Experiments

### 5.1 Prediction task

For a corpus of documents, we randomly divide the documents into the training set and the testing set, where each document is put into the training set with probability  $p_t$  independently. For each document in the testing set, we hold out a certain percentage H of the distinct words in the document uniformly at random. The training set is given to the algorithms. After training, each algorithm gets the testing documents, and for each document predicts s terms not in the observed part of the testing document. We use the precision of the s predicted terms as the score of an algorithm on a specific testing document. The score of an algorithm on the corpus is its average score over all testing documents. In our experiments, we use  $p_t = 0.9, H = 30\%, s = 3$  as our parameters.

This prediction task is widely applicable in practice, especially in online settings with a large amount of user-generated data. A familiar example is the collaborative filtering system by which Netflix leverages the preferences of a large user population to recommend new films to a customer after observing his or her viewing history. For our purpose, the prediction task provides a more straightforward algorithmic measure than statistical measures such as perplexity and likelihood, which are commonly adopted in machine learning, but not applicable to algorithms that don't reconstruct a statistical model.

## 5.2 Recovery task

For the algorithms that reconstruct a topic matrix  $\hat{A}$  in the process, we also measure how close  $\hat{A}$  and A are. For each learned topic  $\hat{A}_i$  and real topic  $A_j$ , we compute  $cos(\hat{A}_i, A_j)$ , then find a maximum matching between the learned topics and real topics. We evaluate the average cosine similarity between the matched real and learned topics. We also carry out the above computation using total variation distance between distributions, and get same qualitative results between algorithms.

# 6 Algorithms

### 6.1 Baseline.

For each testing document, the baseline algorithm predicts s unseen words that are most frequent in the training documents.

## 6.2 K Nearest Neighbors (KNN)

For parameter k, KNN finds the k most similar training documents to a testing document, where similarity is defined as the cosine between the two documents as vectors in the word space. For a testing document, KNN predicts the s unseen words that are most frequent in its k closest training documents. Notice baseline is just KNN with k equal to the number of training documents.

## 6.3 LDA algorithm

It is NP-hard to find the maximum likelihood fit to the LDA model, so in practice, the prevailing approach to learn LDA model is local search. We adopted the algorithm implemented by Blei [6] based on variational

EM (see [9]). We also used an implementation [17] based on Gibbs sampling. The two algorithms give close results on the prediction task for representative datasets. We didn't use the Gibbs sampling implementation on all datasets since it is much slower. In our discussion, LDA algorithm refers to the first implementation based on variational EM.

The LDA algorithm has two procedures, "estimation" and "inference". "estimation" takes a collection of documents as input, and outputs estimated model parameters for the corpus, in particular the estimated term-topic matrix. "inference' takes a collection of documents as well as a LDA model, and outputs an inferred topic mixture for each document. For our prediction task, we use the estimated term-topic matrix of the training set and the topic mixture for each testing document to estimate the most likely unseen words.

#### 6.4 LDAT and LDAC

For generated LDA datasets, we have two "cheating" algorithms as benchmarks. LDAT knows the real term-topic matrix A of the model. We skip "estimation", and use the real A at "inference". LDAC knows the real term-document matrix M, and for each testing document, just predicts the most likely unseen word according to the real distribution on words of the specific testing document. LDAC is supposedly the best we can do given sampling noises.

### 6.5 LSI

In the LDA model, we have M = AW, where A and W are the word-topic matrix and topic-document matrix respectively. Each column of M is a probability distribution on words. The ith document we observe is a set of i.i.d samples from the distribution  $M_i$ , and we have the observed word distribution  $\hat{M}_i$ . In the word space  $\mathbb{R}^m$ , all columns of M lie in the k-dimensional subspace spanned by the k topics A. The sampled document  $\hat{M}_i$  will be a noisy version of  $M_i$ , so in the word space  $\mathbb{R}^m$ , the points in  $\hat{M}$  will be scattered close to the subspace of A.

LSI works by computing the best rank k approximation to  $\hat{M}$  of the training documents

$$min_{U,\Sigma,V} \| \hat{M} - U\Sigma V \|_F$$
  
such that  $U \in \mathbb{R}^{m \times k}, V \in \mathbb{R}^{k \times n}$  orthonormal,  $\Sigma \in \mathbb{R}^{k \times k}$  diagonal.

The optimal U, S, V are computed using singular value decomposition (SVD) of  $\hat{M}$ . LSI is not a statistical model in that the U and V matrices contain negative entries. The subspace spanned by columns of U serve as an approximation of the subspace of A. Notice if we carry out SVD on M, the subspace of U will be exactly the same as subspace of U. For a testing document U, we find its projection  $\hat{U} = UU^T U$  on the subspace of U, and predict U0 unseen words with largest entries in  $\hat{U}$ 0.

### 6.6 Projector

Projector is our new algorithm that builds upon LSI, and reconstructs a term-topic probability matrix  $\hat{A}$ . The motivation is that SVD is computationally more efficient than the LDA algorithm, and has a clear geometric interpretation, but doesn't recover the topics as distributions of words. We aim to start from the subspace computed by SVD, and use some straightforward operation to construct the topics. Our algorithm is based on geometric intuition of the documents as points in the high dimensional word space. The algorithm is as follows

**Input** M: observed distributions of training documents, k: number of topics,  $\delta$ : algorithm parameter

<sup>&</sup>lt;sup>2</sup>This assumes that k is the number of topics used in the LDA model which generated the data. This k is provided to all algorithms. For generated data, this parameter is easy to recover in any case.

Shift Shift the training documents to be centered at the origin.

$$center = \frac{1}{n} \sum_{i=1}^{n} \hat{M}_{i}$$

$$\hat{M}_{i} = \hat{M}_{i} - center \quad \forall i = 1, \dots, n$$

**SVD** Compute the U, the best rank (k-1)-dimension approximation to the column space of  $\hat{M}$  Project all  $\hat{M}_i$ 's to the subspace U, denote  $V_i$  as the projections.

Clustering Use k-means to cluster the  $V_i$ 's into k clusters, where in the k-means algorithm the distance between two points x, y is defined as  $1 - \cos(x, y)$ .

Let  $C_1, \ldots, C_k$  be the centers of the k clusters (center in the sense as in euclidean distance).

Scale Scale  $C_1, \ldots, C_k$  by the smallest common scalar so that  $\delta n$  of  $V_1, \ldots, V_n$  are contained in the hull with  $C_1, \ldots, C_k$  as vertices.

Whitening Make all  $C_i$  distribution over words:  $C_i = C_i + center$ , truncate the negative entries in  $C_i$  to be 0, normalize  $C_i$  so the sum of entries is 1.

Return  $A_i = C_i$  be the recovered topics.

We illustrate in figure 2 how our algorithm works using the a visualization on two datasets with k = 3 topics. Notice after the *Shift* step, we want to find the best (k - 1)-dimensional subspace since the columns from the topic-document matrix are from the (k - 1)-simplex.

We use estimated  $\hat{A}$  and the inference procedure of the LDA algorithm to predict words for testing documents. We use the inference procedure of LDA since LDAT also uses it, and then we can attribute the performance difference between LDAT and Projector to the quality of  $\hat{A}$  compared to the real topics.

## 7 Experiment Results

We generated datasets from the LDA model for a range of parameters, and tested the algorithms discussed in previous section on the prediction task. For the LDA algorithm and our Projector algorithm, we also have results for the recovery task. We experimented with k from 3 to 30, and in each case, a set of  $\alpha$  and  $\beta$  to cover a wide range of  $sig\_word$  and  $sig\_topic$ . See figure 3 for the prediction results of the algorithms on a representative set of datasets.

	Baseline	GibbsLda	LSI	PLSI	knn-25	lda	projector
AP	0.14	0.21	0.24	0.15	0.25	0.2	0.21
Cisi	0.09	0.12	0.09	0.1	0.14	0.12	0.1
Cran	0.17	0.21	0.19	0.17	0.25	0.23	0.21
Kos	0.21	0.38	0.26	0.19	0.38	0.38	0.3
Med	0.08	0.11	0.16	0.08	0.17	0.11	0.16
NIPS	0.64	0.77	0.47	0.64	0.75	0.73	0.57

Table 1: Experiment results on real datasets. We pick the result of the best among a few parameters for each algorithm.

Also see table 1 for prediction results on real datasets. There we see that LDA is no longer dominated by LSI and Projector. Still, nearest neighbor's performance dominates. In the appendix, in table 2 we show the most popular words in a sampling of topic vectors recovered by Projector in the AP dataset.

Typically, we saw the topic matching to correlate with prediction performance. See figure 5 to see this result.

## 8 Analysis

In the previous section, we defined the notion of typical number of words in the support of a topic, or sig\_words, and the notion of a typical number of topics in a document. Clearly, if both get very large one gets to a trivial topic model where each document is generated by choosing words independently from a single probability distribution.

On the other hand if sig\_words and sig\_topics are small each document should have relatively small support compared with the corpus. Thus, in such cases we should easily distinguish the data from the trivial topic model.

We will proceed by showing that the probability of cooccurrence in documents of two words i and j differs significantly from in the trivial topic model. This can be represented as a matrix which refer to as the co-occurrence matrix.

The expected co-occurrence matrix can, of course, be calculated precisely from the topic and document distributions. But we give simple, even trivial, calculations that provide insight based on a simplified topic model.

## 8.1 A Uniform Topic Model.

We proceed by calculating the difference in co-occurrence matrices of a corpus generated by a nontrivial topic model from the trivial topic model.

Let k be the total number of topics. Let m be the vocabulary size. Let t be the number of topics in a document and assume that each word is chosen uniformly from these t topics. Each topic is a uniform distribution over w words. Let l be the number of words in the document.

We examine word cooccurrences for  $w_i$  and  $w_j$ : that is, the probability that two words generated in a document are  $w_i$  and  $w_j$ . Note that in a document of length l, there are  $\binom{l}{2}$  possible cooccurrences between  $w_i$  and  $w_j$ .

We compute the probability of a word cooccurrence for data from a topic distribution versus the trivial model with the working vocabulary: the union of significant words in all topics which roughly has size  $v = \min(m, kw)$ .

For two words from the same topic, a document contains their common topic with probability t/k, and the probability that both words are chosen is  $(1/tw)^2$ . Thus the probability of cooccurrence from being in the same topic is  $\frac{t}{k}(1/tw)^2$ . We assume that the background probability that the two words cooccur in the other case (or in the case that there are no topics) is  $1/v^2$ .

When the background co-occurrence is much smaller than the topic cooccurrence the topic model should be easy to learn. For example, when  $\frac{t/k}{1/(tw)^2} >> 1/v^2$  or when  $1 >> (\frac{kw}{v})^2 \frac{t}{k}$ , then we should have good performance. In figure 4, we plot this ratio against the performance of the projector algorithm and see that things degrade as this value increases.

### 8.2 Dependence on document size and length.

When the ratio is close to 1, the topic cooccurrence should be viewed as an additive advantage over the "background" probability of two words co-occurring, e.g.,  $1/v^2$  where v is the effective vocabulary size as defined before.

That is, for a pair of words not in the same topic the probability of co-occurrence is  $1/v^2$  and for a pair of words in the same topic the probability of cooccurrence is  $1/v^2(1-(t/k))+(t/k)(1/tw)^2$ . The latter

<sup>&</sup>lt;sup>3</sup>We note that it is possible to set up topics so that the additional correlations one recieves in one topic are exactly cancelled out by other topics. This setup corresponds to coding up a parity problem in the set of topics, but it seems unlikely to arise in any reasonable topic model. Still, the "rough" calculations here fail. Indeed, we emphasize that the arguments are heuristic.

summand being the contribution from when the common topic is chosen, the former being an approximation to the background.

Indeed with enough data the distribution will be distinguishable from a random one whenever tw < v. The question then is "How much data does one need?"

We define the density,  $\Delta$  to be the ratio total number of co-occurrences in the documents to the total number of word, word pairs. For our model, this is roughly  $\frac{n\binom{l}{2}}{\binom{v}{2}}$ .

We consider a corpus with additive advantage  $\epsilon = (t/k)((1/tw)^2 - (1/v)^2)$ .

The total signal on a topic of size w is  $\epsilon \Delta w^2$ . The noise (standard deviation) of this process is roughly  $\sqrt{\Delta w^2}$ . As there are  $\binom{v}{w}$  subsets the ratio of signal to noise needs to exceed  $\sqrt{\log\binom{v}{w}} \approx \sqrt{w \log v}$  to properly distinguish a topic from one that may arise "on accident" (assuming a Gaussian distribution on the deviation). Thus, we would roughly need

$$\frac{\epsilon \Delta w^2}{\sqrt{\Delta w^2}} \ge \sqrt{w \log v}.$$

That is, the density  $\Delta$  should be larger than roughly,  $\frac{\log v}{\epsilon^2 w}$ . Recalling that  $\Delta$  is  $(nl^2)/v^2$  where n is the number of documents and l is the number of words in a document. So, as the number of words in each document and number of documents increase so does the distinguishability. Moreover, longer documents are more advantageous than more documents.

In figure 5, we see that algorithms become increasing effective with either an increase in document size or number as suggested by these calculations. Also, we see from the first few points that doubling the document size was far more effecting than incresing the number of documents.

Finally, we note this is statistically distinguishable but to date not necessarily algorithmically distinguishable. For example, principal components analysis (e.g., LSI) has greater difficulty distinguishing distributions when there are many small topics. In particular, LSI finds "clusters" real or imagined that have the greatest excess of common pairs. Thus the algorithm must contend with the noise on a large fake topic, i.e., the noise is roughly  $\sqrt{\Delta m^2}$  versus just the noise on a small topic  $\sqrt{\Delta w^2}$ . Thus, one needs  $\Delta$  to be a factor of  $(m/w)^2$ greater for LSI to learn the topics than would be required information theoretically.

#### 8.3 An external measure.

For real world data, there is no access, of course, to the parameters used in the discussion above. We instead use the chi squared measure on the cooccurrence matrix, W. The Chi Squared measure is defined as

$$\sum_{i,j} \frac{(W_{i,j} - E_{i,j})^2}{E_{i,j}},$$

where  $E_{i,j}$  is the expected number of cooccurrences if documents are generated from a trivial topic model with a word distribution equal to the marginal distribution of the data.

We remove all words for the corpus that occur infrequently and compute the Chi-squared measure on the corresponding co-occurrence matrix. In figure 6, we see that this measure gives us reasonable insight on the generated data into when we get good performance on the prediction task.

#### 9 Conclusion

In this paper, we made progress toward understanding the performance of algorithms on data generated by the LDA model. Using our prediction task, we were able to find an improved algorithm for this task. This prediction task itself may be a reasonable candidate for practitioners to use to evaluate algorithms that are trying to find interesting topics. We will provide our framework either in MALLET [13] or separately for this purpose.

We provided some rules of thumb for when algorithms can make use of topic structure in generated data. It is important to see if these rules of thumb extend to real world datasets. We are working on this aspect currently.

We also have begun to extend our framework to work with hierarchical LDA, and will explore that in future work.

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## 10 Appendix

Civil Rights	Economics	Politics	Stock	Legal
years	percent	states	stock	court
west	rate	government	exchange	wednesday
black	year	united	points	federal
american	prices	union	closed	judge
war	reported	war	tuesday	trial
lived	increase	minister	average	convicted
jr	compared	countries	tokyo	state
chicago	rose	military	nikkei	death
story	report	meeting	close	attorney
social	higher	soviet	share	police
martin	highest	leader	financial	prison
progress	time	world	shares	district
series	index	president	index	charges
blacks	march	administration	volume	accused
side	mortgages	political	timesstock	man
neighborhood	billion	forces	times	charged
dream	figures	nations	million	filed
king	inflation	foreign	percent	case
influence	retail	prime	prices	years
remains	august	capital	investors	ordered

Table 2: The top 20 words (ranked by probability) for 5 sparse topics recovered by Projector on the AP dataset. Topic names derived from top words.

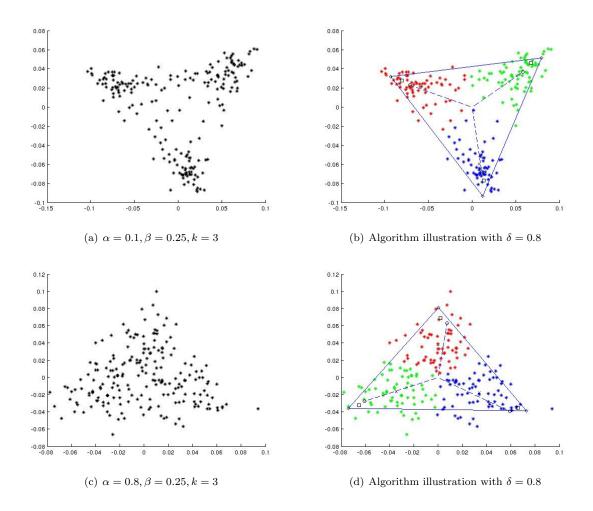


Figure 2: Illustration of Projector. The left figures are the  $V_i$ 's after the SVD step. In the right figures, the black 'o's at the ends of dotted lines are the real topic, black  $\times$  are the  $C_i$ 's before scaling, black  $\diamond$  are  $C_i$ 's after scaling, and black  $\square$  are the recovered  $\hat{A}_i$ 's. All points in the plot are after shifting and projected on the SVD subspace.

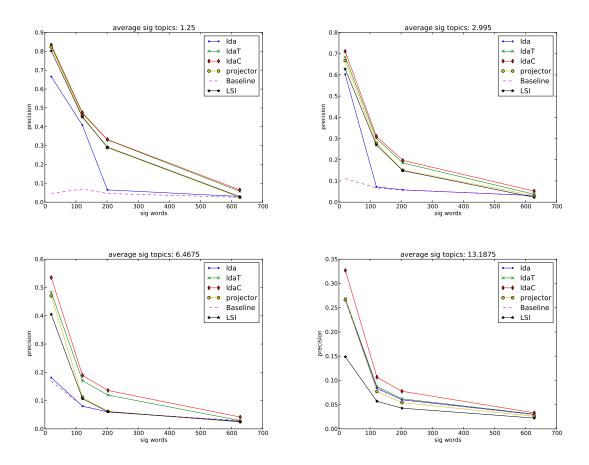


Figure 3: Results of various algorithms on 16 generated datasets, with k=20, n=1000, m=1000, l=75. Each subfigure has a fixed  $sig\_topic$ , the plots are result of the prediction task versus  $sig\_word$ 

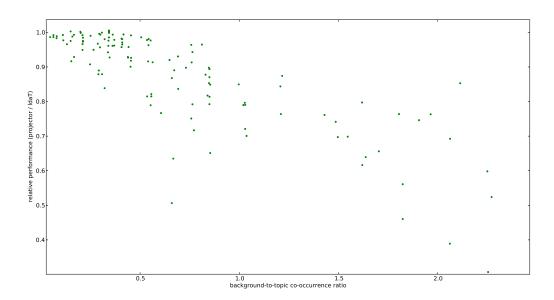


Figure 4: The performance of projector relative to LDAT falls off as the ratio of the background to topic cooccurrences increases.

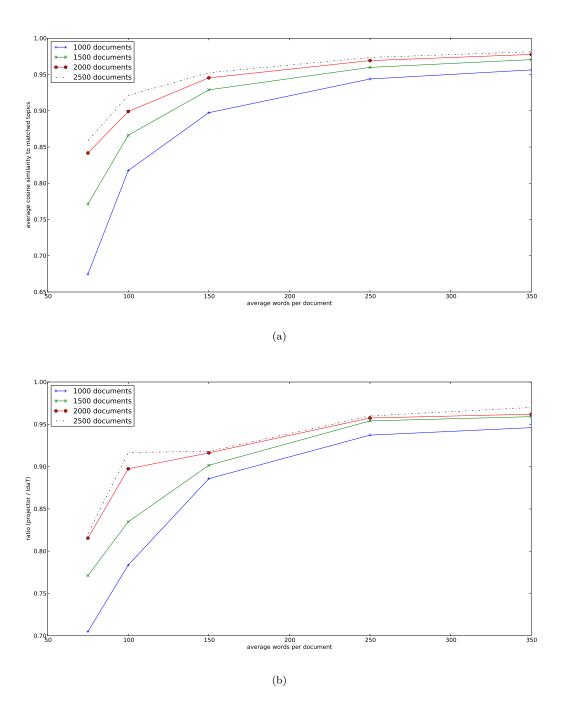


Figure 5: Figure 5(a) above shows that as documents grow and document lengths grow, the projector algorithm learns the topics better. Figure 5(b) a corresponding increase in prediction performance comparing the LdaT algorithm that has perfect knowledge of the topics. The different lines correspond to different numbers of documents, and the x-axis corresponds to the number of words in the document.

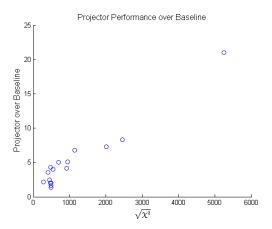


Figure 6: The x-axis is the log of the Chi Squared measure on the co-occurrence matrix for the words that occur with more than average frequency. Performance relative to baseline increases with increasing Chi squared value.