FinQuiz Notes 2 0 1 9

2. Spot Rates and Forward Rates

The price of risk-free single-unit payment at time T is referred to as '**Discount Factor**', denoted as P (T).

$$P(T) = \frac{1}{[1+spot\ rate]^T} = \frac{1}{[1+r(T)]^T}$$

Discount Function is the discount factor for a range of maturities in years (T) greater than zero while the **spot yield curve** represents the term structure of interest rates at any point in time. The shape and level of spot yield curve changes over time because the spot rate represents the annualized return on an option-free and default risk-free zero-coupon bond with a single payment of principal at maturity. Under the spot yield curve, there is no reinvestment risk and the stated yield is equal to the actual realized return if the zero-coupon bond is held till maturity.

The yields to maturity on coupon paying government bonds, priced at par, over a range of maturities is called **par curve**. Typically, recently issued ("*on the run*") bonds are used to build the par curve because on the run issues are generally priced at or close to par. The one-year zero-coupon rate is equal to the one-year par rate.

Forward rate is an interest rate for a loan initiated T* years from today with maturity of T years. It is denoted by f (T*, T). The term structure of forward rates for a loan made on a specific initiation date is called the **forward curve**. In a forward contract, the parties to the contract do not exchange money at contract initiation; rather, the buyer of forward contract pays the seller the contracted forward price value at time T* and receives from the seller the principal payment of bond at time T* + T.

The forward pricing model is stated as below:

$$\mathbf{P(T^* + T) = P(T^*) \times F(T^*,T)}$$

- P (T*+ T) is the cost of a zero-coupon bond, having maturity of T* + T years.
- The right hand side of the equation reflects a forward contract where, P (T*) × F(T*,T) is the present value of a zero-coupon bond with maturity T at time T*.
- The equation implies that initial costs of the two investments must be the same because both investments have same payoffs at time T* + T. If the initial cost is not same, an investor can earn risk-free profits with zero net investment by selling the overvalued instrument and buying the undervalued investment.

Practice: Example 1, Reading 34.

2.1 The Forward Rate Model

Forward rate: The forward rate f (T*,T) is the discount rate for a risk-free unit-principal payment T* + T years from today, valued at time T*, such that the present value equals the forward contract price, F(T*,T). E.g. f (5, 1) is the rate agreed on today for a one-year loan to be made five years from today. Forward rate can be viewed as a rate that can be locked in by extending maturity by one year. Forward rate can also be viewed as a break-even interest rate because it is the rate at which an investor is indifferent between buying a six-year zero-coupon bond or in vesting in a five-year zero-coupon bond and at maturity reinvesting the proceeds for one year.

$$\mathbf{F(T^*, T) = \frac{1}{[1+f(T^*,T)]^T}}$$

Forward rate model:

$$[1 + r(T^* + T)]^{(T^* + T)} = [1 + r(T^*)]^{T^*} \times [1 + f(T^*, T)]^{T}$$

- Forward rate model reflects how we can extrapolate forward rates from spot rates.
- Spot rate for T* + T is r(T* + T).
- Spot rate for T* is r (T*).

Practice: Example 2, Reading 34.

Spot rate for a security, having maturity of T > 1 can be estimated by calculating geometric mean of spot rate for a security with a maturity of T = 1 and a series of T – 1 forward rates as shown below:

$$r(T) = \{[1 + r(1)]\,[1 + f(1,1)]\,[1 + f(2,1)]\,[1 + f(3,1)] \ldots [1 + f(T-1,1)]\}^{(1/T)}$$

$$\left\{\frac{[1+r(T^*+T)]}{[1+r(T^*)]}\right\}^{\frac{T^*}{T}}[1+r(T^*+T)] = [1+f(T^*,T)]$$

E.g. suppose T* = 1, T = 5, r (1) = 2%, and r (6) = 4%;

$$\left(\frac{1.04}{1.02}\right)^{\frac{1}{5}}(1.04) = 1.044 \rightarrow f(1,5) = 4.405\%$$

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- When the spot curve is ***upward (downward) sloping***, the forward curve will lie ***above (below) the spot curve***. This implies that when the yield curve is upward sloping, r(T* + T) > r(T*) and the forward rate rises as T* increases; which means that the forward rate from T* to T is greater than the long-term (T* + T) spot rate: f(T*,T) > r(T* + T). Opposite occurs when yield curve is downward-sloping. In the above example, 4.405% > 4%.
- When the yield curve is flat, all one-period forward rates = spot rate.

Practice: Example 3 & 4, Reading 34.

Bootstrapping:
It is the process of sequentially calculating spot rates from securities with different maturities using the yields on Treasury bonds from the yield curve.

Example:

6-month U.S. Treasury bill has an annualized yield of 5% and 1-year Treasury STRIP has an annualized yield of 4.5%. The yields are spot rates since these are discount securities. Assume that 1.5 year Treasury is priced at $98 and its coupon rate is 5% i.e. $2.5 every six months.
1.5-year spot rate is calculated as follows:
Price = $2.5 / (1 + [6-month spot/2])^1 + $2.5 / (1 + [12-month spot/2]])^2 + $102.5 / (1 + [18-month spot/2])^3
$98 = $ 2.5/ (1 + [5% ÷2])^1 + $2.5 / (1 + [4.5% ÷2])^2 + $ 102.5/ (1 + [18-month spot ÷2])^3
18-month spot rate = 6.464%

Shapes of Yield Curves and their implications:

- **Upward sloping Yield Curve:** Generally, in developed markets, yield curves are upward sloping; and for longer maturities, yield curve tends to flatten, reflecting diminishing marginal increase in yield for identical changes in maturity. An upward sloping yield curve is associated expectations of higher future inflation resulting due to strong future economic growth. Upward sloping curve also indicates higher risk premium for assuming greater interest rate risk associated with longer-maturity bonds.
- **Downward sloping Yield Curve:** Downward sloping curve indicates expectations of declining future inflation due to recession or slow economic activity.
- **Flat yield curve:** A flat yield curve is unusual and typically indicates a transition to either an upward or downward slope. E.g. in order to restrain rapidly growing economy, a central bank may raise interest rates that results in rise in short-term yields to reflect hike in rates, while long-term rates fall in anticipation of inflation moderate.

2.2 Yield to Maturity in Relation to Spot Rates and Expected and Realized Returns on Bonds

Under no arbitrage principle, the yield-to-maturity of the bond should be weighted average of spot rates, so that sum of present values of bond's payments discounted by their corresponding spot rates is equal to the value of a bond.

Yield-to-maturity (YTM) is the *expected rate of return* for a bond that is held until its maturity, assuming that all coupon and principal payments are made in full when due and that coupons are reinvested at the original YTM. In contrast, **realized rate of return** is the *actual holding period return of the bond.*

The YTM provides a poor estimate of expected return if:

1) Interest rates are volatile, which implies that coupons would not be reinvested at the YTM.
2) Yield curve is steeply sloped (either upward or downward), which implies that coupons would not be reinvested at the YTM.
3) There is significant risk of default, implying that actual cash flows may not be the same as calculated using YTM.
4) The bond is not option-free (e.g. has put, call, or conversion option), implying that a holding period may be shorter than the bond's original maturity.

Practice: Example 5, Reading 34.

Example: Suppose a five-year annual coupon bond with a coupon rate of 10%. Spot rates are *r*(1) = 5%, *r*(2) = 6%, *r*(3) = 7%, *r*(4) = 8%, and *r*(5) = 9%.

The forward rates extrapolated from the spot rates (as explained in section 2.1) are calculated as below:

$$f(1,1) = \left(\frac{1.06}{1.05}\right)^{\frac{1}{1}}(1.06) = 1.07 \rightarrow 7\%$$

$$f(2,1) = \left(\frac{1.07}{1.06}\right)^{\frac{2}{1}}(1.07) = 1.09 \rightarrow 9\%$$

$$f(3,1) = \left(\frac{1.08}{1.07}\right)^{\frac{3}{1}}(1.08) = 1.1105 \rightarrow 11.1\%$$

$$f\,(4,1) = \left(\frac{1.09}{1.08}\right)^{\frac{4}{1}}(1.09) = 1.1309 \rightarrow 13.1\%$$

The price, determined as a percentage of par, is $105.43.

Expected cash flow at the end of Year 5, using the forward rates as the expected reinvestment rates, is calculated as below:

10(1 + 0.07) × (1 + 0.09) × (1 + 0.111) × (1 + 0.131) + 10(1 + 0.09) × (1 + 0.011) × (1 + 0.131) + 10(1 + 0.111) × (1 + 0.131) + 10(1 + 0.131) + 110 ≈ $162.22

Expected bond return = ($162.22 – $105.43)/$105.43 = 53.87%

Expected annualized rate of return = (1 + 53.87%) [5] = 9.00%

Practice: Example 6, Reading 34.

2.3 Yield Curve Movement and the Forward Curve

If the future spot rate is expected to be lower than the prevailing forward rate, the forward contract value is expected to increase and accordingly, demand for forward contract would increase. In contrast, if the future spot rate is expected to be higher than the prevailing forward rate, the forward contract value is expected to decrease and accordingly, demand for forward contract tends to decrease. This implies that any change in the forward price results from deviation of the spot curve from that predicted by today' forward curve.

Forward contract price that delivers a T-year-maturity bond at time T* is estimated as below:

$$F\,(T^*, T) = \frac{P\,(T* + T)}{P\,(T*)}$$

$$\textbf{Discount factor} = \textbf{P* (T)} = \frac{P\,(t+T)}{P\,(t)}$$

Forward contract price at time t is F*(t, T*, T) = $\frac{P*\,(T* + T - t)}{P*\,(T* - t)}$

$$\textbf{F*(t, T*, T)} = \frac{\frac{P\,(t+T*+T-t)}{P\,(t)}}{\frac{P\,(t+T*-t)}{P\,(t)}} = \frac{P*\,(T* + T)}{P\,(T*)} = F\,(T^*, T)$$

Example: Suppose a flat yield curve with 4% interest rate. The discount factors for the one-year, two-year, and three-year terms are calculated as follows:

$$P\,(1) = \frac{1}{(1.04)^1} = 0.9615$$

$$P\,(2) = \frac{1}{(1.04)^2} = 0.9246$$

$$P\,(3) = \frac{1}{(1.04)^3} = 0.8890$$

The forward contract price that delivers a one-year bond at Year 2 is estimated as follows:

$$F\,(2,1) = \frac{P(2+1)}{P\,(2)} = \frac{0.8890}{0.9246} = 0.9615$$

The discount factors for the one-year and two-year terms one year from today are calculated as below:

$$P^*\,(1) = \frac{P(1+1)}{P\,(1)} = \frac{0.9246}{0.9615} = 0.9616$$

$$P^*\,(2) = \frac{P(1+2)}{P\,(1)} = \frac{0.8890}{0.9615} = 0.9246$$

The price of the forward contract one year from today =

$$F^*\,(1, 2, 1) = \frac{P*\,(2+1-1)}{P*\,(2-1)} = \frac{P*(2)}{P*\,(1)} = \frac{0.9246}{0.9616} = 0.9615$$

It can be observed that due to flat yield curve price of forward contract is not changed. When the spot rate curve is constant, then each bond earns the forward rate.

2.4 Active Bond Portfolio Management

If the spot curve one year from today reflects the current forward curve, then the total return of the bond over a one-year period, irrespective of its maturity, is always equal to the risk-free rate over one-year period. But if the spot curve one year from today differs from today's forward curve, then the return of a bond for the one-year holding period will not all be equal to risk-free rate over one-year period.

$$\frac{[1 + r(T+1)]^{T+1}}{[1 + f\,(1,T)]^T} = [1 + r(1)]$$

Example: Suppose a one-year zero-coupon bond, with a price of $91.74 and face value of $100. r (1) is 9%. Its return over the one-year holding period is estimated as follows:

$$\left(100 \div \frac{100}{1+r(1)}\right) - 1 = \left(100 \div \frac{100}{1+0.09}\right) - 1 = \frac{100}{91.74} - 1 = 9\%$$

Similarly, assuming r (2) of 10%, then the return of the two-year zero-coupon bond over the one-year holding period is estimated as:

$$\left(\frac{100}{1+f\,(1,1)} \div \frac{100}{[1+r\,(2)]^2}\right) - 1 = \left(\frac{100}{1+0.1101} \div \frac{100}{(1+0.10)^2}\right) - 1 = \frac{90.08}{82.64} - 1 = 9\%$$

In other way,

Return of the two-year zero-coupon bond over the one-year holding period =

$$\frac{\text{Price of a two-year zero-coupon bond one year from today}}{\text{Purchase price of the bond}}$$

Where,

Price of a two-year zero-coupon bond one year from today = $\frac{\text{Par value of bond}}{(1+\text{Forward rate for one year bond one year from today})}$

Similarly, Price of a three-year zero-coupon bond one year from today =

$$\frac{\text{Par value of bond}}{(1+\text{Forward rate for two year bond one year from today})} = \frac{\text{Par value of bond}}{1+f\,(1,2)}$$

Hence, return on a three-year zero-coupon bond over one-year holding period = $\left(\frac{100}{(1+0.10)^2} \div \frac{100}{(1+0.11)^3}\right) - 1 = 13.03\%$

This equation, $\frac{[1+r(T+1)]^{T+1}}{[1+f(1,T)]^{T}} = [1 + r(1)]$, can be used to evaluate the cheapness or expensiveness of a bond of a certain maturity.

- All else being constant, if expected future spot rate < (>) quoted forward rate for the same maturity, then bond is considered to be undervalued (overvalued) because the bond's payments are being discounted at a higher (lower) interest rate.
- All else being equal, if the projected spot curve is above (below) the forward curve, the return on a bond will be less (more) than the one-period risk-free interest rate.
- The greater the difference between the projected future spot rate and forward rate, the greater the difference between the trader's expected return and original yield to maturity.
- The longer the bond's maturity, the greater the sensitivity of the bond's return to the changes in spread between the forward rate and the spot rate.

Riding the yield curve or rolling down the yield curve:

- When the yield curve is upward sloping → the forward curve is above the current spot curve → total return on bonds with a maturity longer than the investment horizon would be greater than the return on a maturity matching strategy.
- When the yield curve is downward sloping → the forward curve is below the current spot curve → total return on bonds with maturity longer than the investment horizon would be lower than the return on a maturity matching strategy.

Practice: Example 7, Reading 34.

3. The Swap Rate Curve

Swap contract is a type of derivative contracts in which an investor can exchange or swap fixed-rate interest payments for floating-rate interest payments. Swap contracts are used to speculate or modify risk.

- A fixed-rate leg of an interest rate swap is referred to as **swap rate**.
- The floating rate is based on short-term reference interest rate i.e. 3-month LIBOR.
- Libor can be used for short-maturity yields; whereas, swap rates can be used for yields with a maturity of more than one year.
- A swap contract has *zero value* at the start of the contract (the present values of the fixed-rate is equal to the benchmark floating-rate leg) i.e. when a contract is initiated, neither party pays any amount to the other.

The yield curve of swap rates is called the **swap rate curve**. The swap curve is a type of par curve because it is based on par swaps.

The advantages of the Swap Curve over a government bond yield curve are:

1) There is almost no government regulation of the swap market making swap rates across different markets more comparable.
2) The supply of swaps depends only on the number of counterparties that are seeking or are willing to enter into a swap transaction at any given time. Swap curve is not affected by technical market factors that can affect government bonds.
3) The swap market is more liquid than bonds because a swap market has counterparties who exchange cash flows, allowing investors flexibility and customization; whereas, in bonds market, there are multiple borrowers or lenders.
4) Swap curves across countries are more comparable as they reflect similar levels of credit risk. While comparisons across countries of government yield curves are difficult because of the differences in sovereign credit risk.
5) Swap rate more appropriately reflects the default risk of private entities, having rating of A1/A+.
6) There are more maturity points available to construct a swap curve than a government bond yield curve i.e. swap rates for 2, 3, 4, 5, 6, 7, 8, 9, 10, 15, and 30 year maturities are available.
7) Swap contracts can be used to hedge interest rate risk.
8) Swap curve is considered to be a better benchmark for interest rates in the countries where private sector market is bigger than the public sector market.

3.2 Why Do Market Participants Use Swap Rates When Valuing Bonds?

The choice of benchmark for interest rates between government spot curves and swap rate curves depends on many factors, including:

- Relative liquidity, i.e. if a swap market is relatively less active than Treasury security market, then government spot rate would be preferred as benchmark interest rates.
- Business operations of the institution using the benchmark; e.g. wholesale banks tend to use swap curve to value assets and liabilities as they typically use swaps to hedge their balance sheet.

3.3 How Do Market Participants Use the Swap Curve in Valuation?

Swap contracts are ***non-standardized, customized*** contracts between two parties in the ***over-the-counter market***.

3.4 The Swap Spread

Swap Spread = Fixed-rate payer of an interest rate swap – Interest rate on "on-the-run" government security
Suppose a fixed rate of a five-year fixed-for-float Libor swap is 3.00% and the five-year Treasury rate is 1.50%, the swap spread = 3.00% – 1.50% = 0.50%, or 50 bps.

Uses of Swap Spread: The swap spread can be used to determine the time value, credit, and liquidity components of a bond's yield to maturity. That is, the higher the swap spread, the higher the return required by investors for assuming credit and/or liquidity risks.

Zero-Spread or Z-spread: The Zero-volatility spread / Z-spread or the Static spread is the spread that when added to all of the spot rates on the yield curve will make the present value of the bond's cash flow equal to the bond's market price. Therefore, it is a spread over the entire spot rate curve. The zero-volatility spread is a spread relative to the Treasury spot rate curve. Z-spread is a more accurate measure of credit and liquidity risk.

$$\text{Discount factor for one year} = \frac{\text{Current price of security}}{\text{Expected payment in one year}}$$

Interest Rate associated with discount factor = 1/ Discount Factor

The swap rates can be determined from the spot rates and the spot rates can be determined from the swap rates.

Value of a floating-rate leg of swap is always 1 at contract initiation; whereas, the swap rate is determined using the following equation:

$$\underbrace{\sum_{t=1}^{T}\frac{s\,(T)}{[1+r(t)]^{t}}+\frac{1}{[1+r(T)^{T}]}}_{\textit{Value of fixed rate leg}} \quad \underset{\textit{value of floating rate leg}}{\overset{=1}{\downarrow}}$$

Practice: Example 8, Reading 34.

Interpolated Spread or I-spread = Yield to maturity of the bond - Linearly interpolated yield to the same maturity on an appropriate reference curve

Example: Suppose, a bond with a coupon rate of 1.625% (semi-annual) and face value of $1 million, maturing on 2 July 2015. The evaluation date is 12 July 2012, so the remaining maturity is 2.97 years [= 2 + (350/360)]. The swap rates for two-year and three-year maturities are 0.525% and 0.588%, respectively. And the swap spread for 2.97 years is 0.918%.

Swap rate for 2.97 years = 0.525% + [(350/360)(0.588% – 0.525%)] = 0.586%

Yield to maturity on bond = 0.918% + 0.586% = 1.504%

Invoice price (price including accrued interest) for US$1 million face value = $\frac{1{,}000{,}000\left(\frac{0.01625}{2}\right)}{\left(1+\frac{0.01504}{2}\right)^{\left(1-\frac{10}{180}\right)}}+$

$$\frac{1{,}000{,}000\left(\frac{0.01625}{2}\right)}{\left(1+\frac{0.01504}{2}\right)^{\left(2-\frac{10}{180}\right)}}+\ldots+\frac{1{,}000{,}000\left(\frac{0.01625}{2}\right)}{\left(1+\frac{0.01504}{2}\right)^{\left(6-\frac{10}{180}\right)}}+\frac{1{,}000{,}000}{\left(1+\frac{0.01504}{2}\right)^{\left(6-\frac{10}{180}\right)}}=$$

$1,003,954.12

Accrued interest rate amount = 1,000,000 × [(0.01625/2) (10/180)] = $451.39

Clean price = 1,003,954.12 – 451.39 = US$1,003,502.73

3.5 Spreads as a Price Quotation Convention

Price quote convention refers to quoting the price of a bond using the bond yield net of either a benchmark Treasury yield or swap rate.

Cash flows from swap contracts are subject to higher default risk compared with treasury bonds; hence, the swap rate is usually greater than the corresponding Treasury note rate and the swap spread is usually, but not always, positive. Similarly, for some maturities, swap contracts have more actively traded market than that of treasury bonds. Therefore, it is not possible to perfectly execute arbitrage between these two markets.

TED spread = LIBOR - T-bill rate

- TED spread is a measure of the credit risk in the general economy as well as counter party risk in the swap market. If TED increases (decreases), it indicates increase (decrease) in the risk of default on interbank loans.
- TED spread is a more accurate measure of credit risk in the banking system; whereas, swap spread more accurately reflects varying demand and supply conditions.

Libor–OIS spread = Libor - Overnight indexed swap (OIS) rate

- An OIS in an interest rate swap in which the periodic floating rate of the swap is equal to the geometric average of an overnight rate (or overnight index rate) over every day of the payment period.
- The index rate is typically the rate for overnight unsecured lending between banks— for example, the federal funds rate for US dollars.
- The Libor–OIS spread is a measure of the risk and liquidity of money market securities.

4. Traditional Theories of the Term Structure of Interest Rates

Unbiased expectations theory or pure expectations theory:
According to the pure expectations theory, forward rates exclusively represent expected future spot rates. Thus, the entire term structure at a given time reflects the market's current expectations of the short-term rates. In other words, ***long term interest rates are equal to the mean of future expected short-term rates.***

Forward rate can be viewed as a **"break-even rate"** i.e. an investor would be indifferent between investing for two years at 6% or investing at 4% for the first year and reinvesting in one year at 8% breakeven rate.

Forward rate can also be interpreted as a rate that allows the investor to lock in a rate for some future period e.g. an investor can invest in the 2-year bond at 6% instead of 1-year bond and essentially lock in an 8% rate for the 1-year period starting in one year.

The pure expectations theory predicts that the expected spot rate in one year is equal to the implied 1-year forward rate of 8%. Thus, ***Expectations are Unbiased***. The pure expectations theory is consistent with the assumption of risk neutrality, where the investors are unaffected by uncertainty and there are no risk premiums.

Local expectations theory: According to the local expectations theory, interest rate and reinvestment risks are important in the long term only. This theory states that the expected return for every bond over ***short time periods*** is the risk-free rate and thus, there is no risk premium. In the short term, these risks are ignored and investors are assumed to be indifferent between different instruments. This theory is consistent with the assumption of no-arbitrage opportunity.

$$\frac{1}{P(t,T)} = [1 + r(1)][1 + f(1,1)][1 + f(2,1)][1 + f(3,1)] \ldots [1 + f(T-1,1)]$$

Where, P (t, T) is the discount factor for a T-period security at time t.

If the forward rates are realized, the one-period return of a long-term risky bond is the one-period risk-free rate.

Typically, both the yields and actual return for a short-term security is lower than that of long-term security because investors tend to prefer short-term securities to long-term securities to meet liquidity needs and to hedge risk.

4.2 Liquidity Preference Theory

The liquidity theory states that forward rates reflect investors' expectations of future spot rates plus a liquidity premium positively related to maturity to compensate them for exposure to interest rate risk i.e. 20-year bond has a larger liquidity premium than a 5-year bond. This theory states that investors will hold longer-term maturities if they are offered a long-term rate higher than the average of expected future rates by a risk premium.

According to the liquidity preference theory, forward rates will not be an unbiased estimate of the market's expectations of future interest rates because they contain a liquidity premium. Thus, an upward-sloping yield curve may reflect expectations that future interest rates either:

1) Will rise or
2) Will be unchanged or even fall but with a liquidity premium increasing faster

A downward-sloping yield curve may reflect expected decline in interest rates being greater than the effect of the liquidity premiums. Typically, yield curve tends to upward sloping in presence of liquidity premiums.

The size of the liquidity premiums depends on risk aversion among investors i.e. the greater the risk aversion, the higher would be the liquidity premium. It is important to note that liquidity premium is not the same as the yield premium demanded by investors for lack of liquidity.

Practice: Example 9, Reading 34.

4.3 Segmented Markets Theory

According to Segmented markets theory, yields of securities of a particular maturity depend on the supply and demand for funds of that particular maturity (i.e. a segmented market). For example, investors with long-term liabilities (like pension funds) tend to prefer to invest in long-term securities. In contrast, money market funds tend to prefer to invest in short-term securities.

4.4 Preferred Habitat Theory

According to preferred habitat theory, the term structure reflects the expectation of the future path of interest rates as well as a risk premium. The yield premium need not reflect a liquidity risk but instead reflects imbalance between the demand and supply of funds in a given maturity range. Usually lenders prefer to invest for a short term and borrowers prefer to raise long term capital. Investors will shift out of their preferred maturity sectors if they are given a sufficient high risk premium. For example, borrowers require cost savings (lower yields) and lenders require a yield premium (higher yields) to move out of their preferred habitats.

Under this theory, a yield curve may take any shape.

5. Modern Term Structure Models

5.1 Equilibrium Term Structure Models

Equilibrium term structure models are based on fundamental economic variables.

Characteristics of Equilibrium Term Structure Models:

- Equilibrium term structure models can be based on single factor (referred to as state variable, e.g. short-term interest rate) or multiple factors.
- Equilibrium term structure models make assumption about the factors e.g. mean reversion of short-term rates.
- Equilibrium term structure models tend to be more cautious the number of parameters that must be estimated compared with arbitrage-free term structure models.

Types of Equilibrium models (section 5.1 – 5.2):

1) Cox–Ingersoll–Ross (CIR) Model: The CIR model is based on single factor (i.e. short-term interest rate). The CIR model assumes that the short-term interest rates in an economy converges to constant long-run interest rate because of two reasons:

a) Unlike stock prices, interest rates cannot rise indefinitely as higher interest rates lead to slow down the economic activity and ultimately, interest rates need to be decreased.

b) Nominal interest rates cannot be negative.

$$dr = a\,(b - r)\,dt + \sigma\sqrt{r dz}$$

Where,

dt = drift term. It is known as the deterministic part of the model.

dz = stochastic or random part of the model, i.e. infinitely small changes in "random walk". It is used to model risk.

r = short-term rate

b = long-run rate

a = speed of adjustment of interest rate

$\sigma\sqrt{r dz}$ = volatility term. It follows random normal distribution with mean of zero and standard deviation of 1.

$\sigma\sqrt{r}$ = Standard deviation factor. This implies that the higher the interest rates, the greater the volatility.

Under this model, interest rate is assumed to revert to mean toward a long-run value "b", with the speed of adjustment governed by the strictly positive parameter "a", implying that the higher (lower) the value of "a", the more (less) quicker the mean reversion towards the long-run rate "b".

2) Vasicek Model: The Vasicek model is also based on single factor (i.e. short-term interest rate). Like CIR mode, it assumes that the short-term interest rate in an economy converges to constant long-run interest rate.

$$dr = a(b - r)dt + \sigma dz$$

Unlike the CIR Model, interest rates in Vasicek model are calculated assuming constant volatility over the period of analysis.

Disadvantage of the Vasicek model: Under this model, it is theoretically possible for the interest rate to become negative.

5.2 Arbitrage-Free Models: The Ho–Lee Model

As the name implies, prices estimated using arbitrage-free models are equal to the market prices. Unlike Vasicek and CIR models, which have only a finite number of free parameters, arbitrage-free model is based on dynamic parameters which can be used to value derivatives and bonds with embedded options as well.

The **Ho-Lee model** is a type of arbitrage-free model. Under this model it is assumed that movement in yield curve is consistent with no-arbitrage condition. In the Ho–Lee model, the short rate follows a normal process, as reflected in the equation below:

$$dr_t = \theta_t dt + \sigma dz_t$$

The model generates a symmetrical ("bell-shaped" or normal) distribution of future rates; hence, interest rates can be negative.

Example: Suppose current short-term rate is 4%. Drift terms are θ_1 = 1% in the first month and θ_2 = 0.80% in the second month. Time period is monthly and annual volatility is 2%. A two period binomial lattice-based model for the short-term rate is as below.

$$\textbf{Monthly volatility} = \sigma\sqrt{\frac{1}{t}} = 2\%\sqrt{\frac{1}{12}} = 0.5774\%$$

$$\textbf{Time step} = 1/12 = 0.0833$$

$$dr_t = \theta_t dt + \sigma dz_t = \theta_t(0.0833) + (0.5774)dz_t$$

- If the rate increases in the first month, r = 4% + (1%)(0.0833) + 0.5774% = 4.6607%
- If the rate increases in the first month and in the second month, r = 4.6607% + (0.80%)(0.0833) + 0.5774% = 5.3047%
- If the rate increases in the first month but decreases in the second month, r = 4.6607% + (0.80%)(0.0833) – 0.5774% = 4.1499%
- If the rate decreases in in the first month, r = 4% + (1%)(0.0833) – 0.5774% = 3.5059%
- If the rate decreases in the first month and increases in the second month, r = 3.5059% + (0.80%)(0.0833) + 0.5774% = 4.1499
- If the rate decreases in the first month and in the second month, r = 3.5059% + (0.80%)(0.0833) – 0.5774% = 2.9951%

Practice: Example 10, Reading 34.

6. Yield Curve Factor Models

6.1 A Bond's Exposure to Yield Curve Movement

Shaping risk: The sensitivity of a bond's price to the *changing shape* of the yield curve is known as shaping risk. Shaping risk also affects the value of many options.

6.2 Factors Affecting the Shape of the Yield Curve

A yield curve factor model is a model that is used to describe the yield curve movements. According to the **three-factor model of Litterman and Scheinkman**, a type of yield curve factor model, yield curve movements can be explained by three independent movements, i.e. level, steepness, and curvature.

- ***Level movement*** refers to an upward or downward shift in the yield curve.
- ***Steepness movement*** refers to a non-parallel shift in the yield curve e.g. when either change in short-term rates is more than that of long-term rates or change in long-term rates is more than that of short-term rates.
- The curvature movement refers to movement in three segments of the yield curve, i.e. change in the short-term and long term rates is more than that of middle-term rates or vice versa.

Another yield curve factor model is **Principal component analysis** (PCA). PCA involves reducing the observed variables into a smaller number of principal components (artificial variables) that best explain the variance in the observed variables.

6.3 The Maturity Structure of Yield Curve Volatilities

For fixed-income management, it is highly important to quantify the volatility in interest rates because option values, and consequently, the values of the fixed income securities, crucially depend on the level of interest rate volatilities. In addition, it is important to measure interest rate volatility for managing interest rate risk. Interest rate volatility can be measured using the following equation:

Interest rate volatility for a security with maturity T at time

$$t = \sigma(t, T) = \frac{\sigma\left[\frac{\Delta r\,(t,T)}{r\,(t,T)}\right]}{\sqrt{\Delta t}}$$

- ➢ If the time interval is a one-month period, then the specified time interval = 1/12 years.

Term structure of interest rate volatilities is a graphical representation of the yield volatility of a zero-coupon bond for every maturity of security. The yield curve risk can be measured by using term structure of interest rate volatility. Typically, short-term rates are more volatile than long-term rates because short-term volatility is most strongly associated with uncertainty regarding monetary policy whereas long-term volatility is most strongly associated with uncertainty regarding the real economy and inflation. Moreover, the co-movement between short-term and long-term volatilities tends to depend on dynamic correlations between three factors i.e. monetary policy, the real economy, and inflation.

6.4 Managing Yield Curve Risks

Yield curve risk refers to sensitivity of value of portfolio to unanticipated changes in the yield curve. Yield curve risk can be measured by using following two measures:

1) **Effective duration**: It measures the sensitivity of a bond's price to a ***small*** **parallel shift** in a benchmark yield curve. The portfolio's effective duration is the weighted sum of the effective duration of each bond position. For zero-coupon bonds, the effective duration of each bond is the same as the maturity of the bond.
2) **Key rate duration**: It measures a bond's sensitivity to a ***small*** change in a benchmark yield curve at a **specific maturity segment**. The portfolio key rate duration for a specific maturity is the weighted value of the key rate durations of the individual issues for that maturity.

Example:

Key Rate Durations											
Issue	**Value ($1,000's)**	**weight**	**3 m**	**2 yr**	**5 yr**	**10 yr**	**15 yr**	**20 yr**	**25 yr**	**30 yr**	**Effective Duration**
Bond 1	100	0.10	0.03	0.14	0.49	1.35	1.71	1.59	1.47	4.62	11.4
Bond 2	200	0.20	0.02	0.13	1.47	0.00	0.00	0.00	0.00	0.00	1.62
Bond 3	150	0.15	0.03	0.14	0.51	1.40	1.78	1.64	2.34	2.83	10.67
Bond 4	250	0.25	0.06	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06
Bond 5	300	0.30	0.00	0.88	0.00	0.00	1.83	0.00	0.00	0.00	2.71
Total Portfolio	1000	1.00	0.0265	0.325	0.4195	0.345	0.987	0.405	0.498	0.8865	3.8925

NOTE:

- The duration of a zero-coupon security is approximately the number of years to maturity.
- Rate Duration = Weight × Duration

- **Effective Duration:** For Bond 2, the effective duration is:

  0.02 + 0.13 + 1.47 = 1.62

- **Portfolio key rate duration:** The 10-year key rate duration for the portfolio is:

  (0.10)(1.35) + (0.20)(0.00) + (0.15)(1.40) + (0.25)(0.00) + (0.30)(0.00) = 0.345

- **Value of the portfolio when the entire yield curve undergoes a parallel shift i.e. the rate at all key maturities increases by 50 basis points:**

Since the yield curve underwent a parallel shift, the impact on portfolio value can be computed directly using the portfolio's effective duration.

Method 1: Effective duration of the portfolio is the sum of the weighted averages of the key rate durations for each issue. The 3-month key rate durations for the portfolio can be calculated as follows:

(0.10)(0.03) + (0.20)(0.02) + (0.15)(0.03) + (0.25)(0.06) + (0.30)(0) = 0.0265

Method 2: Effective duration of the portfolio is the weighted average of the effective durations for each issue. Using this approach, the effective duration of the portfolio can be computed as:

(0.10)(11.4) + (0.20)(1.62) + (0.15)(10.67) + (0.25)(0.06) + (0.30)(2.71) = 3.8925

Using an effective duration of 3.8925, the value of the portfolio following a parallel 50 basis point shift in the yield curve is computed as follows:

Percentage change = (50 basis points) (3.8925) = 1.9463% decrease

Value of the portfolio when only 3 rates change while the others remain constant i.e.

- The 3-month rate increases by 20 basis points.
- The 5-year rate increases by 90 basis points.
- The 30-year rate decreases by 150 basis points.

<u>Change in Portfolio Value:</u>

Change from 3-month key rate increase:	(20 bp)(0.0265)	= 0.0053% decrease
Change from 5-year key rate increase:	(90 bp)(0.4195)	= 0.3776% decrease

Change from 30-year key rate decrease:	(150 bp)(0.8865)	= 1.3298% increase
Net change		**0.9469% increase**

This means that the portfolio value after the yield curve shift = 1,000,000(1 + 0.009469) = $1,009,469.00

Practice: Example 11, Reading 34 and end of chapter problems.

FinQuiz Notes 2 0 1 9

2. The Meaning of Arbitrage-Free Valuation

Arbitrage-free valuation is a security valuation approach that calculates that value of security, which does not provide any arbitrage opportunities.

A fundamental principle of valuation: A fundamental principle of valuation states that the value of any financial asset (e.g. zero-coupon bonds, interest rate swaps etc.) is equal to the present value of its expected future cash flows. If the yield curve is flat, value of a bond is estimated by discounting all cash flows with the same discount rate.

- ***Yield Curve*** shows the relationship between yield and maturity for coupon bonds.
- ***Spot Rate Curve*** shows the relationship between spot rates and maturity. Note that the Spot rates are the appropriate rates to use to discount cash flows.

2.1 The Law of One Price

The law of one price states that two identical goods must sell for the same current price in the absence of transaction costs. Otherwise, in the absence of transaction costs, an investor may simultaneously buy the good at the lower price and sell at the higher price, resulting in riskless profit. This transaction is repeated without limit until the two prices converge.

2.2 Arbitrage Opportunity

Arbitrage opportunity refers to opportunity to earn riskless profits without any net investment of money.
There are two types of arbitrage opportunities.

1) **Value Additivity:** It implies that the value of the whole equals the sum of the values of the parts. In other words, if the bond's value is less than the sum of the values of its cash flows individually, there is an opportunity to earn riskless profit by buying the bond while selling claims to the individual cash flows. E.g. suppose Asset A risk-free zero-coupon bond with a face value of one dollar is priced today at 0.952381 (\$1/1.05). Asset B is a portfolio of 105 units of Asset A with face value of \$105 and price of \$95 at time period 0. An investor can earn riskless profit by selling 105 units of Asset A at \$100 (0.952381 × 105) while simultaneously buying one portfolio Asset B for \$95.
2) **Dominance:** A security is said to be dominant over another when both can be purchased at the same price at t = 0, but the dominant security will yield higher return in every state of the world. In case of arbitrage, an investor can make riskless profit by buying the dominant security and selling the dominated security, if they are traded at the same price.

Practice: Example 1, Reading 35.

2.3 Implications of Arbitrage-Free Valuation for Fixed-Income Securities

Under the arbitrage-free approach, any fixed-income security can be considered as a portfolio of zero-coupon bonds. E.g. a ten-year 2% coupon Treasury issue should be viewed as a package of twenty one zero-coupon instruments (20 semiannual coupon payments, one of which is made at maturity, and one principal value payment at maturity). Under the arbitrage-free valuation approach, an investor cannot earn an arbitrage profit through stripping and reconstitution.

Where,

Stripping: The process of separating the bond's individual cash flows and trading them as zero-coupon securities is known as stripping.

Reconstitution: The process of recombining the individual zero-coupon securities and reproducing the underlying coupon Treasury is known as reconstitution.

3. Interest Rate Trees and Arbitrage-Free Valuation

For option-free bonds, the arbitrage-free value is calculated as the sum of the present values of expected future values using the benchmark spot rates.

Interest-rate tree: A set of possible interest rate paths is referred to as an Interest Rate Tree. It is generated based on some assumed interest-rate model and interest-rate volatility. Interest rates on the tree are used to generate

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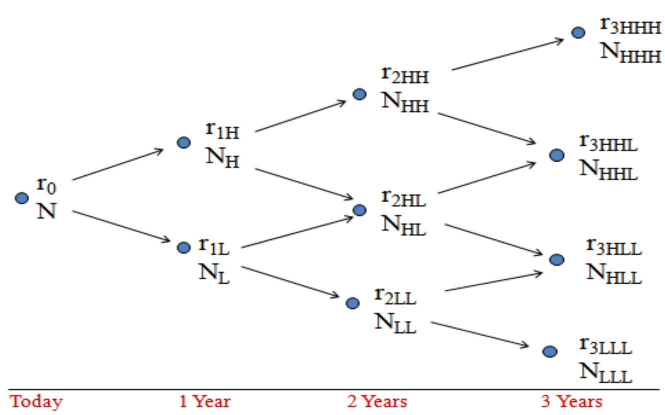
future cash flows of the bonds with embedded options and to compute the present value of the cash flows.

3.1 The Binomial Interest Rate Tree

Binomial interest-rate model: Under a binomial interest-rate model, it is assumed that interest rates have an *equal probability* of taking on *one of two possible rates* in the next period. The graphical representation of the Binomial Model is called binomial interest-rate tree.

Three-Year Binomial Interest-Rate Tree



- The dot in the figure is referred to as a "***node***". A node is a point in time when interest rates can take one of two possible paths i.e. an upper path H (or U) or a lower path 'L'.
- The first node is called the root of the tree and is simply the current one-year rate at Time 0.
- At each node, a random event (e.g. change in interest rates) or a decision takes place (e.g. decision to exercise the option).
- At each node of the tree there are interest rates and these rates are effectively ***forward rates i.e. one-period rates starting in period t***.
- rL or iL is denoted as the rate lower than the implied forward rate and rH or iH is denoted as the higher forward rate.
- For each year, there is a unique forward rate, implying that for each year, there is a *set of forward rates*.

Methodology for constructing an arbitrage-free interest rate tree:

Step 1: Given the coupon rate & maturity, use the yield on the current 1-year on-the-run U.S. Treasury security issue for r_0.

Step 2: Assume the level of interest rates volatility i.e. σ. The changes in the assumed interest rate volatility will affect the rates at every node in the tree.

Step 3: Given the coupon rate and market value of the 2-year on-the-run issue, select a value of lower rate i.e. $r_{1,L}$ (the lower one-year forward rate one year from now). Then compute the upper rate i.e. $r_{1,U}$.

$$\mathbf{r_{1,U} = r_{1,L} \times e^{2\sigma}}$$

Where,

- r1,L = the lower one-year forward rate one year from now at Time 1, and
- r1,H = the higher one-year forward rate one year from now at Time 1.
- σ = assumed volatility of the 1-period rate
- e = natural antilogarithm, 2.71828

As the standard deviation (i.e., volatility) increases, the multiplier increases and the difference between two rates increases.

For example, suppose that i1,L is 1.194% and σ is 15% per year, then i1,H = 1.194%($e^{2\times0.15}$) = 1.612%.

At Time 2, there are three possible values for the one-year rate, which we will denote as follows:

- i2,LL = one-year forward rate at Time 2 assuming the lower rate at Time 1 and the lower rate at Time 2
- i 2,HH = one-year forward rate at Time 2 assuming the higher rate at Time 1 and the higher rate at Time 2
- i2,HL = one-year forward rate at Time 2 assuming the higher rate at Time 1 and the lower rate at Time 2, or equivalently, the lower rate at Time 1 and the higher rate at Time 2.

This type of tree is called a ***recombining tree*** because there are two paths to get to the middle rate. The relationship between r2, LL and the other two one-year rates is as follows:

$$r_{2,HH} = r_{2,LL}\,(e^{4\sigma}) \text{ and } r_{2,HL} = r_{2,LL}\,(e^{2\sigma})$$

Similarly, there are four possible values for the one-year forward rate at Time 3. These are represented as follows: $r_{3,HHH}$, $r_{3,HHL}$, $r_{3,LLH}$ and $r_{3,LLL}$.

The lowest possible forward rate at Time 3 is $r_{3,LLL}$ and is related to the other three as given below:

- $r_{3,HHH} = (e^{6\sigma})r_{3,LLL}$
- $r_{3,HHL} = (e^{4\sigma})r_{3,LLL}$
- $r_{3,LLH} = (e^{2\sigma})r_{3,LLL}$

Step 4 (section 3.3): Compute the bond's value one year from now using the interest rate tree i.e.

a) Compute the bond's value two years from now.
b) Calculate the PV of the bond's value determined in part "a" using higher discount rate. This value is denoted as V_H.
c) Calculate the PV of the bond's value determined

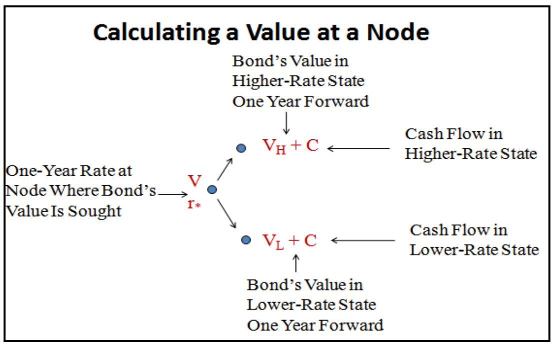
in part "a" using lower discount rate. This value is denoted as V_L.

d) Add the coupon payments to V_H and V_L and calculate PV of (V_H + C) and (V_L + C) using the one-year forward rate.

e) Calculate the ***average*** of present values determined in part "d" i.e. $\frac{1}{2}\left[\frac{V_H + C}{1 + r_*} + \frac{V_L + C}{1 + r_*}\right]$

Step 5: If the value calculated using the model is > market price, use higher rate of $r_{1,L}$, re-compute $r_{1,U}$ and then calculate the new value of the on-the-run issue using new interest rates. If the value is too low, decrease the interest rates in the tree.

Step 6: The five steps are repeated with a different value for *lower interest rate i.e.* $r_{1,L}$ until the value estimated by the model is equal to the market price.



Practice: Example 2, Reading 35.

3.2 What Is Volatility and How Is It Estimated?

Volatility is measured by using a standard deviation, which is the square root of the variance. In a lognormal distribution, the changes in interest rates are proportional to the level of the one-period interest rates each period. This implies that interest rate changes by a greater amount when interest rates are high and by a smaller amount when interest rates are low. In addition, under lognormal model, interest rates cannot be negative.

For a lognormal distribution the standard deviation of the one-year rate is equal to $r_0\sigma$. For example, if σ is 10% and the one-year rate (r_0) is 3%, then the standard deviation of the one-year rate is 3% × 10% = 0.3% or 30 bps.

Following two methods are commonly used to estimate interest rate volatility.

1) ***By estimating historical interest rate volatility:*** In this method, volatility is calculated by using historical data with the assumption that history will be repeated.

2) ***Implied Volatility:*** In this method, interest rate volatility is calculated based on observed market prices of interest rate derivatives (e.g., swaptions, caps, floors).

Practice: Example 3, Reading 35.

Example: Consider a bond with a 5% *semi-annual* coupon, maturing in two years at par value. The current one-year spot rate is 6.20%. For the second year, the yield volatility model forecasts that the one-year rate will be either 5.90% or 7.30%. Using a binomial interest rate tree, the value of an option free bond is calculated as follows:

- The prices at node A is calculated as follows:

  Price $_A$ = [Probability × (P_{up} + (coupon / 2))] + [Probability × (P_{down} + (coupon / 2))] / [1 + (rate / 2)]

  = [0.5 × (100 + 2.5)] + [0.5 × (100 + 2.5)] / [1 + (0.0730 / 2)] = 98.89

- The prices at node B is calculated as follows:

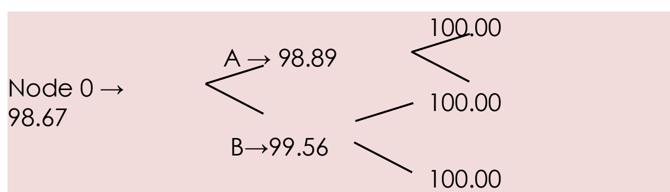
  Price $_B$ = [Probability × (P_{up} + (coupon / 2))] + [Probability × (P_{down} + (coupon / 2))] / [1 + (rate / 2)]

  = [0.5 × (100 + 2.5)] + [0.5 × (100 + 2.5)] / [1 + (0.0590 / 2)] = 99.56

- The price at node 0 is calculated as follows:

Price at node 0 = 0.5[(98.89 + 2.5) / (1+ {0.062 / 2}) + (99.56 + 2.5) / (1 + {0.062 / 2})] = 0.5(98.3414 + 98.9913) = 98.6663

The option-free bond price tree is as follows:



The value of a bond estimated using a binomial interest rate tree should be equal to the value of bond estimated by discounting the cash flows with the spot rates. It is explained in the example below.

Example: Suppose, the one-year par rate is 2.0%, the two-year par rate is 3.0%, and the three-year par rate is 4.0%. Consequently, the spot rates are S_1 = 2.0%, S_2 = 3.015% and S_3 = 4.055%. Zero-coupon bond prices are P_1 = 1/1.020 = 0.9804, P_2 = 1/(1.03015)^2 = 0.9423, and P_3 = 1/(1.04055)^3 = 0.8876. Interest volatility is 15% for all years.

Time 0: The par, spot, and forward rates are all the same for the first period in a binomial tree. Consequently, Y_0 = S_0 = F_0 = 2.0%.

Time 1: The two-year spot rate is the geometric average of the one-year forward rate at Time 0 and the one-year forward rate at Time 1; so, we can infer the average forward rate for Time 2 as follows.

$$1.030152 = (1.02) \times (1+F_{1,2})$$

Under lognormal model on interest rate changes are estimated as follows:

$$F_{1,2U} = (F_{1,2d})(e^{2\sigma})$$

$\mathbf{F_{1,2d}} =$ (4.040%)($e^{-0.15}$) = 3.477% and $\mathbf{F_{1,2u}} =$ (4.040%)($e^{0.15}$) = 4.694%

Price for the two-year zero-coupon bond = [(0.5)(1/1.03477) + (0.5)(1/1.04694)]/1.02 = 0.9419.

Time 2: The average forward rate for Time 2 is estimated as follows:

$$F_{2,3} = (1.04055^3/1.030152^2) - 1 = 6.167\%$$

$$\text{Lower value} = (6.167\%) \times (e^{-0.3}) = 4.569\%$$

$$\text{Upper value} = (6.167\%) \times (e^{0.3}) = 8.325\%$$

Using these interest rates, a price for a three-year zero-coupon bond is estimated to be 0.8866. This price is close to correct price of Bond of 0.8876. Using Excel's Solver, the three correct one-year forwards are estimated as 4.482%, 6.051%, and 8.167%.

Working backward through the tree, values at Time 2 is calculated as 1/1.08167 = 0.9245, 1/1.06051 = 0.9429, and 1/1.04482 = 0.9571.

At Time 1, the tree values are calculated as (0.5)(0.9245)/1.04602 + (0.5)(0.9429)/1.04602 = 0.8923 and (05)(0.9429)/1.03409 + (0.5)(0.9571)/1.03409 = 0.9188.

At Time 0, the tree values are calculated as (0.5)(0.8923)/1.02 + (0.5)(0.9188)/1.02 = 0.88778876.

When the tree gives correct prices for zero-coupon bonds maturing in one, two, and three years, the tree is calibrated to be arbitrage free.

3.6 Pathwise Valuation

Pathwise valuation calculates the present value of a bond for each possible interest rate path and takes the average of these values across paths. Pathwise valuation involves the following steps:

1) Specify a list of all potential paths through the tree
2) Determine the present value of a bond along each potential path, and
3) Calculate the average across all possible paths

The total number of paths for each period/year can be easily determined by using ***Pascal's Triangle***. For a three-year zero-coupon bond, there are four possible paths to arrive at Year 3 using a Pascal's Triangle, i.e. HH, HT, TH, TT.

Example: Suppose a three-year, annual-pay, 5 % coupon bond. For a three-year tree, there are eight paths, four of which are unique. The cash flows along each of the eight paths are discounted and then average is taken as follows.

Cashflows:

Path	Time 0	Time 1	Time 2	Time 3
1	0	5	5	105
2	0	5	5	105
3	0	5	5	105
4	0	5	5	105
5	0	5	5	105
6	0	5	5	105
7	0	5	5	105
8	0	5	5	105

Discount Rates:

Path	Time 0	Time 1	Time 2	Time 3
1	2%	4.602%	8.167%	
2	2%	4.602%	8.167%	
3	2%	4.602%	8.167%	
4	2%	4.602%	8.167%	
5	2%	4.602%	8.167%	
6	2%	4.602%	8.167%	
7	2%	4.602%	8.167%	
8	2%	4.602%	8.167%	

Present Values:

Path	Time 0
1	100.5296
2	100.5296
3	102.3449
4	102.3449
5	103.4792
6	103.4792
7	104.8876
8	104.8876
Average	102.8103

Practice: Example 5 & 6, Reading 35.

4. Monte Carlo Method

Monte Carlo method involves randomly selecting paths in order to approximate the results of a complete path-wise valuation. Monte Carlo method is preferred to use when a security's cash flows are path dependent, implying that when cash flow to be received in a particular period depends on the path followed to reach its current level as well as the current level itself.

Monte Carlo Method involves the following steps:

1) Simulate different number of paths of interest rates under some volatility assumption and probability distribution;
2) Generate spot rates from the simulated future interest rates;
3) Determine the cash flow along each interest rate path;
4) Calculate the present value for each path;
5) Calculate the average present value across all interest rate paths;

At all interest rate paths, a constant (referred to as drift) should be added to all interest rates so that the average present value for each benchmark bond equals its market value. This method is known as drift adjustment.

End of Reading Practice Problems:

2. Overview of Embedded Options

The term "embedded bond options" or "embedded options" refers to contingency provisions associated with a bond.

2.1 Simple Embedded Options

2.1.1) Call Options

The call option is the right to redeem the bond issue prior to maturity. The call option is typically exercised when interest rates have declined or when the issuer's credit quality has improved.

- The initial call price (exercise price) is typically set at a premium above par.
- The call price gradually declines to par as the bond reaches towards maturity.
- Make-whole call provision: In a make-whole call provision, the bond's price is set such that the bondholders are more than compensated for prepayment risk.
- Lock-out period: It is the period during which the issuer cannot call the bond. E.g. a 15-year callable bond, having a lock-out period of 5 years mean that the first potential call date is five years after the bond's issue date.

Exercising styles associated with Options:

- ***European-style exercise:*** European-style options can be exercised only on its expiration day.
    - However, in some cases, options can be exercised during that day before expiration.
- ***American-style exercise:*** American-style options can be exercised any time before expiration.
- A **Bermudan-style call** option can be exercised only on a predetermined schedule of dates after the end of the lockout period. These dates are specified in the bond's indenture or offering circular.

Examples:

- Tax-exempt municipal bonds (often called "munis") are almost always callable at 100% of par any time after the end of the 10th year. In tax-exempt municipal bonds, advance refunding may also be allowed.
- Except for few, bonds issued by government-sponsored enterprises in the United States (e.g., Fannie Mae, Freddie Mac, Federal Home Loan Banks, and Federal Farm Credit Banks) are callable, typically at 100% of par. These bonds tend to have relatively short maturities (5–10 years) and very short lockout periods (three months to one year) and the call option is often Bermudan style.

2.1.2) Put Options and Extension Options

A putable bond is one in which the bondholder has the right to force the issuer to repurchase the security at specified dates before maturity. The repurchase price is set at the time of issue, and is usually par value. The put option is usually exercised when interest rates increase. Like callable bonds, most putable bonds have lock-out periods. Putable bond option can be European style, rarely, Bermudan style, but not American-style.

Extension Option:

Extension option is an option that grants the bondholder the right to extend the expiration date. This implies that if extension option is exercised, the bondholder can keep the bond for a number of years after maturity, possibly with a different coupon. In case of exercise of extension option, the terms of the bond's indenture or offering circular are modified.

- The value of a putable bond, say a three-year bond putable in Year 2, should be the same as that of a two-year bond extendible by one year, otherwise, arbitrage opportunities exist; however, their underlying option-free bonds are different.
- If one-year forward rate at the end of Year 2 is higher than the coupon rate, the putable bond will be put because the bondholder can reinvest the proceeds of the retired bond at a higher yield, and the extendible bond will not be extended for the same reason.

2.2 Complex Embedded Options

The conversion option is an option that grants the bondholders to convert their bonds into the issuer's common stock. Usually, convertible bonds are callable by the issuer.

Colloquially or Death-put Bonds: Death-put bond is the bond having estate put or survivor's put option, which grants the heirs of a deceased bondholder to redeem the bond at par value.

- The value of a bond with an estate put depends not only on interest rate movements, like any bond with an embedded option, but also on the bondholder's life expectancy. That is, the shorter (longer) the life expectancy, the greater (smaller) the value of the estate put.
- Death-put bonds should be put only if they sell at a discount. Otherwise, they should be sold in the

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market at a premium.
- Typically, there is a limit on the principal amount of the bond the issuer is required to accept in a given year, such as 1% of the original principal amount.
- If the amount requested to be redeemed is greater than the limit, it is sent into a queue in chronological order.
- Typically, estate put option bonds are also callable, usually at par and within five years of the issue date. If the bond is called early, the estate put option is extinguished.

Sinking Fund Bonds:
In a sinking fund bond, the issuer is required to set aside funds over time to retire the bond issue. Hence, bondholders in sinking fund bonds are exposed to relatively less credit risk. Sinking fund bonds may be callable and may also include unique options, such as an acceleration provision and a delivery option.

Acceleration provision:
An acceleration provision allows the issuer to repurchase the bond at par three times the mandatory amount on any scheduled sinking fund date.

Delivery option:
A delivery option allows the issuer to satisfy a sinking fund payment by delivering bonds to the bond's trustee rather than paying cash.

- If the bonds are currently trading at discount, it is more cost effective for the issuer to buy back bonds from investors to meet the sinking fund requirements than to pay par.
- In contrast, if the bonds are currently trading at premium (i.e. when interest rates rise), it is more cost effective for the issuer to pay at par or exercising the delivery option rather than buying back bonds from investors to meet the sinking fund requirements.

From the issuer's perspective, the combination of the call option and the delivery option is effectively a "**long straddle**", that is, buying a call and buying a put, both with the same strike price and expiration. This implies that a sinking fund bond benefits the issuer both when interest rates decline and rise.

Practice: Example 1, Reading 36.

3. Valuation and Analysis of Callable and Putable Bonds

Callable Bond:
In a callable bond, the investor is long the bond but short the call option. Hence, from the investor's perspective, the value of the call option decreases the value of the callable bond relative to the value of the straight bond.

Value of callable bond =
Value of straight bond – Value of issuer call option

Value of issuer call option =
Value of straight bond – Value of callable bond

Putable Bond:
In a putable bond, the investor has a long position in both the bond and the put option. Hence, the value of the put option increases the value of the putable bond relative to the value of the straight bond.

Value of putable bond =
Value of straight bond + Value of investor put option

Value of investor put option =
Value of putable bond – Value of straight bond

3.3 Valuation of Default-Free Callable and Putable Bonds in the Absence of Interest Rate Volatility

3.3.1) Valuation of a Callable Bond at Zero Volatility
The call option is exercised by the issuer when the value of the bond's future cash flows is higher than the call price (exercise price).

Example: A Bermudan-style three-year 4.25% annual coupon bond that is callable at par one year and two years from now. One-year forward rate two years from now is 4.564% and one-year forward rate one year from now is 3.518%.

Present value at Year 2 of the bond's future cash flows =

$$\frac{104.250}{1.04564} = 99.70$$

- This value < call price of 100, so a rational borrower will not call the bond at that point in time.

Present value at Year 1 of the bond's future cash flows =

$$\frac{99.70+4.250}{1.03518} = 100.417$$

- This value > call price of 100, so a rational borrower will call the bond at that point in time.

Present value at Year 0 of the bond's future cash flows = $\frac{100.417+4.250}{1.02500}$ = **101.707**

Value of issuer call option = 102.114 – 101.707 = 0.407

Where, value of straight bond is 102.114.

The bondholder exercises the put option when the value of the bond's future cash flows is lower than the put price (exercise price).

Practice: Example 2, Reading 36.

3.4 Effect of Interest Rate Volatility on the Value of Callable and Putable Bonds

- As interest rate volatility increases, the value of call option increases and consequently, the value of the callable bond decreases.
- As interest rate volatility increases, the value of put option increases and consequently, the value of the putable bond increases.

3.4.2) Level and Shape of the Yield Curve

Callable Bond:

- When the yield curve is upward sloping, the one-period forward rates on the interest rate tree are high and hence, the call option of a callable bond **issued at par** is out-of-the money.
- In contrast, when the yield curve is upward sloping, the one-period forward rates on the interest rate tree are high and hence, the call option of a callable bond **issued at premium** is in-the money.
- When the yield curve flattens or inverts, the one-period forward rates on the interest rate tree are low, and hence, the call option of a callable bond issued at par is in-the-money.

Putable Bond:

- When the yield curve is upward sloping, the one-period forward rates in the interest rate tree are high and hence, the put option of a putable bond issued at par is in-the-money.
- When the yield curve flattens or inverts, the one-period forward rates on the interest rate tree are low, and hence, the put option of a putable bond issued at par is out-of-the-money.

3.5 Valuation of Default-Free Callable and Putable Bonds in the Presence of Interest Rate Volatility

When bond is both putable and callable:

At each node two decisions must be made about exercising of an option i.e.

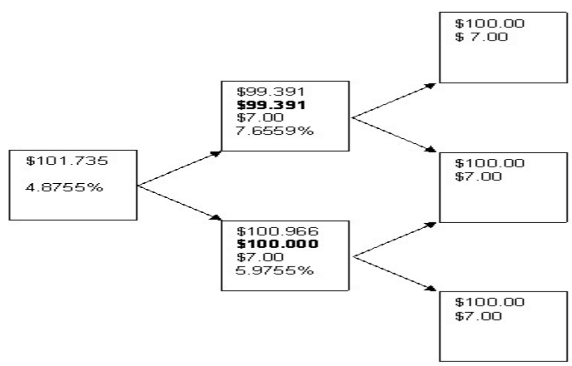
i. If the call option is exercised, the value at the node is replaced by the call price. The call price is then used in subsequent calculations.
ii. If the put option is exercised, then the put price is substituted at that node and is used in subsequent calculations.

Example of Valuing a Callable Bond Using the Binomial Model:

A callable bond pays interest annually over a two-year life, has a 7% coupon payment, and has a par value of \$100. The bond is callable in one year at par (\$100). The current one-year spot rate is 4.8755%. For the second year, the yield volatility model forecasts that the one-year rate will be either 7.6559% or 5.9755%. Suppose that the value of the option-free counterpart is \$102.196. This price came from discounting cash flows at on-the-run rates for the issuer.

Using the binomial tree model, the value of the callable bond is estimated as follows:

Calculations:



- ➢ The price of the callable bond is \$101.735.
- ➢ The value of the call option is \$102.196 – \$101.735 = \$0.461.

Note: The rate in the up state (Ru) is calculated as

$$R_u = R_d \times e^{2\sigma\sqrt{t}}$$

Where,

Rd = Rate in the down state

σ = Interest rate volatility

t = Time in years between "time slices"

Practice: Example 3, Reading 36.

3.5.2) Valuation of a Putable Bond with Interest Rate Volatility

Example of Valuing a Putable Bond Using the Binomial Model:
A putable bond with a 6.4% annual coupon will mature in two years at par value. The current one-year spot rate is 7.6%. For the second year, the yield volatility model forecasts that the one-year rate will be either 6.8% or 7.6%. The bond is putable in one year at 99.

Using a binomial interest rate tree, the value of putable bond is estimated as follows:

Calculations:
The tree will have three nodal periods: 0, 1, and 2. The goal is to find the value at node 0. We know the value at all nodes in nodal period 2: V_2=100. In nodal period 1, there will be two possible prices:

$V_{i,U}$ = [(100 + 6.4) / 1.076 + (100+6.4) / 1.076] / 2 = 98.885

$V_{i,L}$ = [(100 + 6.4) / 1.068 + (100 + 6.4) / 1.068] / 2 = 99.625

Since 98.885 is less than the put price, $V_{i,U}$ = 99

V_0 = [(99 + 6.4) / 1.076) + (99.625 + 6.4) / 1.076)] / 2 = 98.246.

Practice: Example 4 & 5, Reading 36.

3.6 Valuation of Risky Callable and Putable Bonds

There are two different methods used to estimate value of bonds that are subject to default risk.

1) **Increase the discount rates above the default-free rates to incorporate the default risk**: Discounting the risky bond using higher discount rate will result in the value of a risky bond being less than that of an otherwise identical default-free bond.
2) **By assigning the default probabilities to each time period and by specifying the recovery value given default:** The probability of default in Year 1 may be 1%; the probability of default in Year 2, conditional on surviving in Year 1, may be 1.30%; and so on.

3.6.1) Option-Adjusted Spread
There are two standard approaches to construct a suitable yield curve for a risky bond.

1) Using an issuer-specific curve, which represents the issuer's borrowing rates over the relevant range of maturities.
2) Uniformly raising the one-year forward rates derived from the default-free benchmark yield curve by a fixed spread (i.e. the zero-volatility spread, or Z-spread). The Z-spread for an option-free bond is simply its option-adjusted spread (OAS) at zero volatility.

Option-adjusted Spread (OAS): OAS is the constant spread that, when added to all the one period forward rates on the interest rate tree, makes the arbitrage-free value of the bond equal to its market price.

- When an OAS for a bond is lower than that for a bond with similar characteristics and credit quality, it indicates that the bond is overpriced (rich) and should be sold.
- When an OAS for a bond is greater than that for a bond with similar characteristics and credit quality, it indicates that the bond is underpriced (cheap) and should be purchased.
- When the OAS for a bond is equal to that of a bond with similar characteristics and credit quality, the bond is said to be fairly priced.

3.6.2) Effect of Interest Rate Volatility on Option-Adjusted Spread:
As interest rate volatility increases, the OAS for the callable bond decreases.

Practice: Example 6, Reading 36.

4. Interest Rate Risk of Bonds with Embedded Options

Two key measures of interest rate risk include:

1) **Duration:** Duration is the approximate percentage change in the value of a security for a 100 bps change in interest rates (assuming a parallel shift in the yield curve).

$$\boldsymbol{Duration = \frac{V_- - V_+}{2 \times V_0 \times (\Delta Y)}}$$

2) **Convexity:** Convexity is a measure of the sensitivity of the duration of a bond to changes in interest rates. In general, the higher the convexity, the more sensitive the bond price is to decreasing interest rates and the less sensitive the bond price is to increasing rates.

$$Convexity = \frac{V_+ + V_- - (2 \times V_0)}{2 \times V_0 \times (\Delta Y)^2}$$

Where,
Δy = change in rate used to calculate new values
V_+ = estimated value if yield is increased by Δy
V_- = estimated value if yield is decreased by Δy
V_0 = initial price (per $100 of par value)

Example:
Δ y = 0.0025
V_+ = 101.621
V_- = 102.765
V_0 = 102.218

$$Duration = \frac{102.765 - 101.621}{2 \times 102.218 \times (0.0025)} = 2.24$$

$$Convexity = \frac{101.621 + 102.765 - (2 \times 102.218)}{2 \times 102.218 \times (0.0025)^2} = -39.1321$$

Note: This callable bond exhibits negative convexity.

Yield Duration Measure:
The sensitivity of the bond's full price (including accrued interest) to changes in the bond's yield to maturity is referred to as yield duration measure. Yield duration measure is appropriate to use only for option-free bonds.

Curve Duration Measure:
The sensitivity of the bond's full price (including accrued interest) to changes in benchmark interest rates is referred to as curve duration measure. Curve duration measure (also known as effective duration) is appropriate to use for bonds with embedded options.

4.1.1) Effective Duration
Effective duration indicates the sensitivity of the bond's price to a 100 bps parallel shift of the benchmark yield curve (particularly, the government par curve) assuming no change in the bond's credit spread. The formula for calculating a bond's effective duration is

$$\text{Effective duration} = \frac{(PV_-) - (PV_+)}{2 \times (\Delta\text{Curve}) \times (PV_0)}$$

Where
ΔCurve = Magnitude of the parallel shift in the benchmark yield curve (in decimal);
PV– = the full price of the bond when the benchmark yield curve is shifted down by ΔCurve;
PV+ = the full price of the bond when the benchmark yield curve is shifted up by ΔCurve; and
PV0 = the current full price of the bond (i.e., with no shift).

The effective duration of a callable bond as well as putable bond cannot be greater than that of the straight bond.

- When interest rates are high relative to bond's coupon rate, the call option is out-of-the-money and hence, callable bond's effective duration is equal to that of straight bond.
- When interest rates are low relative to bond's coupon rate, the call option is in-the-money which limits the price appreciation, and hence, effective duration of the callable bond is less than that of a straight bond.
- When interest rates are low relative to the bond's coupon, the put option is out of the money, and hence, the effective duration of the putable bond is equal to that of identical option-free bond.
- When interest rates are high relative to the bond's coupon, the put option is in-the-money which limits the price depreciation, and hence, shortens its effective duration compared to that of identical option-free bond.

Type of Bond	Effective Duration
Cash	0
Zero-coupon bond	≈Maturity
Fixed-rate bond	< Maturity
Callable bond	≤ Duration of straight bond
Putable bond	≤ Duration of straight bond
Floater (Libor flat)	≈ Time (in years) to next reset

Note: Generally, a bond's effective duration cannot be greater than its maturity, except for tax-exempt bonds (on an after tax basis).

4.1.2) One-Sided Durations
The price sensitivity of bonds with embedded options is not symmetrical to positive and negative changes in interest rates of the same magnitude.

One-sided durations:
One-sided duration refers to effective durations when interest rates go up or down. It is more appropriate to use for callable or putable bond than the (two-sided) effective duration, particularly when the embedded option is near the money.

- For a callable bond, the one-sided **up-duration** is higher than the one-sided **down-duration** because the callable bond is more sensitive to interest rate rises than to interest rate declines.
- For a putable bond, the one-sided **down-duration** is higher than the one-sided **up-duration** because putable bond is more sensitive to interest rate declines than to interest rate rises.

4.1.3) Key Rate Durations
Key Rate Duration:
It is the approximate percentage change in the value of a bond or bond portfolio in response to a 100 basis point

change in the corresponding key rate, holding all other rates constant. The impact of any type of yield curve shift can be measured using key rate durations. Every security or portfolio has a set of key rate durations, one for each key rate. Key rate duration can be used to identify "**shaping risk**" for bonds i.e. the bond's sensitivity to changes in the shape/slope of the yield curve (e.g., steepening and flattening).

- For an option-free bond trading at par, the maturity-matched par rate is the only rate that affects the bond's value because the largest cash flow of a fixed-rate bond (principal plus final coupon payment) occurs at maturity.
- For zero-coupon bonds or bonds with very low coupon, the maturity points are shorter than the maturity of bond and hence, the key rate durations are negative.
- The key rate durations of bonds with embedded options depend not only on the time to maturity but also on the time to exercise.

4.2 Effective Convexity

Positive convexity: When price of a bond rises slightly more when interest rates move down than it declines when interest rates move up by the same amount is referred to as positive convexity. The option-free bond exhibits positive convexity.

- The effective convexity of the callable bond turns negative when the call option is near the money because when interest rates decline, the price of the callable bond is capped by the price of the call option if it is near the exercise date.
- In contrast, putable bonds always have positive convexity because the price of a putable bond is floored by the price of the put option if it is near the exercise date.
- Callable bonds have more upside potential than otherwise identical putable bonds when interest rates increase.
- Putable bonds have more upside potential than otherwise identical callable bonds when interest rates decline.

Practice: Example 7, Reading 36.

5. Valuation and Analysis of Capped and Floored Floating-Rate Bonds

5.1 Valuation of a Capped Floater

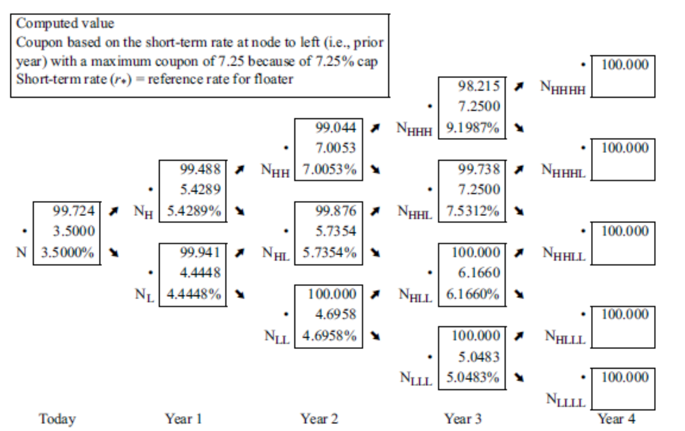
The cap provision in a floater is a provision that prevents the coupon rate from increasing above a specified maximum rate. Hence, a capped floater protects the issuer against rising interest rates. From investor's perspective, the value of the cap decreases the value of the capped floater relative to the value of the straight bond:

Value of capped floater = Value of straight bond – Value of embedded cap

To estimate the value of a floating rate note with a cap i.e. capped floater using a binomial model, the coupon rate needs to be adjusted based on the 1-year rate which is assumed to be the reference rate.

- The coupon rate is set at the beginning of the period but paid at the end of the period i.e. the coupon interest is paid in arrears.
- The adjustment is made to the backward induction method by discounting the coupon payment to be made in the next period for a floater based on the beginning of the period reference rate.
- When a coupon rate exceeds the cap, the coupon rate is replaced by the capped rate.

Valuing a Floating Rate Note with a 7.25% Cap (10% Volatility Assumed)



At each node where the 1-year rate exceeds 7.25%, a coupon of $7.25 is substituted. The value of this capped floater is 99.724. Thus, the cost of the cap is the difference between par and 99.724.

NOTE: The higher the cap, the closer the capped floater will trade to par.

Ratchet bonds:
In a ratchet bonds, the coupon can only decline at the time of reset; that is, it cannot be greater than the existing interest rate. However, in order to compensate investors for the potential loss of interest income over time, the initial coupon rate in ratchet bonds is set well above the issuer's long-term option-free borrowing rate. Hence, a ratchet bond is similar to a conventional callable bond. The only difference is that in ratchet bonds, calling back the bond entails no transaction cost and the call decision is on autopilot.

In ratchet bonds, the investor also has the option to put the bonds back to the issuer at par if the coupon is reset. Therefore, this option is called "contingent put" option. The contingent put protects investors against an adverse credit event.

Practice: Example 8, Reading 36.

In a ratchet bond, the coupon is reset based on a formula such that market price at the time of reset is above par, provided that there is no deterioration in the issuer's credit quality.

5.2 Valuation of a Floored Floater

The floor provision in a floater is a provision that prevents the coupon rate from decreasing below a specified minimum rate. Hence, a floored floater protects the investor against declining interest rates. From investor's perspective, the value of the floor increases the value of the floored floater relative to the value of the straight bond:

Value of floored floater = Value of straight bond + Value of embedded floor

6. Valuation and Analysis of Convertible Bonds

6.1 Defining Features of a Convertible Bond

A convertible bond is a security in which the investor has the right to convert the security into pre-determined (fixed) number of shares of common stock of the issue during a pre-determined period (known as the conversion period) at a pre-determined price (known as the conversion price).

Advantages:

- Convertible bonds tend to offer lower coupon rate than for otherwise identical non-convertible bonds because they allow investors to convert their bonds into shares at a cost lower than market value.
- Convertible bonds offer two benefits to issuers i.e. a lower coupon rate and conversion of their debt into equity.

Disadvantages:

- In case of conversion, the existing shareholders face dilution in shareholding.
- If the underlying share price remains below the conversion price and the bond is not converted, the issuer has to repay the debt or refinance it, potentially at a higher cost.
- If the underlying share price remains below the conversion price, the bondholder will earn lower interest income relative to otherwise identical nonconvertible bond.

Corporate actions, i.e. stock splits, bonus share issuances, and rights or warrants issuances, affect a company's share price and may reduce the benefit of conversion for the convertible bondholders.
In addition, to compensate investors for dividend payments, a threshold dividend is defined in the terms of issuance. The conversion price is adjusted downward for annual dividend payments above the threshold dividend to offer compensation to convertible bondholders.

- **Change-of-control events**: These are events (e.g. merger or acquisition) when occur allow the investors to exercise a put option during a specified period after the change-of-control event or the conversion price is adjusted downward to facilitate investors to convert their bonds into shares earlier and at more advantageous terms.

- **Some convertible bonds are putable.** Put options can be classified as
  1) ***Hard Puts:*** A hard put is one in which the convertible security must be redeemed by the issuer for cash.
  2) ***Soft Puts:*** In soft puts, the issuer has the option to select the mode of payment e.g. an issuer may redeem the convertible security for cash, common stock, subordinated notes or a combination of the three.
- **Almost all convertible issues are callable.** There are some issues that have a provisional call feature that allows the issuer to call the issue during the non-call period if interest rates are falling, if the share price increases above the conversion price, or if its credit rating is revised upward, thus enabling the issuance of debt at a lower cost. Such an event is called forced conversion because it forces bondholders to convert their bonds into shares. The forced conversion strengthens the issuer's capital structure.
- To protect investors against early repayment, convertible bonds usually have a lockout period.

6.2 Analysis of a Convertible Bond

1. **Conversion Ratio:** The number of shares of common stock that the security holder will receive from exercising the call option of a convertible security is called the conversion ratio.
   - Conversion ratio may change over time.
   - Conversion ratio is adjusted proportionately for stock splits and stock dividends.

2. **Conversion Price (or stated conversion price) = Par value of convertible bond ÷ Conversion ratio**
   For a bond that is not issued at par i.e. Zero-coupon bond,

   Conversion Price (or stated conversion price) = Issue price per $1,000 of par value of convertible bond ÷ Conversion ratio

3. **Conversion Value or Parity Value:** The conversion value or a parity value of a convertible security is the value of the security if it is converted immediately i.e.

   Conversion Value = Market price of common stock × Conversion ratio

NOTE:
When reverse stock split occurs, conversion value remains same i.e. if price rises, conversion ratio will fall or if price falls, conversion ratio will rise. Hence, conversion value remains constant.

4. **Straight Value or Investment Value:** It is a value of a security without the conversion option. It is estimated as the PV of bond's cash flows discounted at the required return on a comparable option-free issue.
   - The straight value acts as a floor for the convertible security's price. However, it is a moving floor as the straight value will change with changes in interest rates or if the issuer's credit quality changes.

5. **Minimum Value of a Convertible Security:** It is the greater of:
   1. Its conversion value, or
   2. Its straight value or investment value.
   - The minimum value acts as a **floor** for the convertible security's price. However, it is a moving floor as the straight value will change with changes in interest rates or if the issuer's credit quality changes. If interest rates rise, the value of the straight bond falls, making the floor fall. Similarly, if the issuer's credit spread decreases (e.g. due to upgrade of its credit rating), the floor value will increase.

6. **Market Conversion Price or Conversion Parity Price:** It is the price that an investor effectively pays for the common stock if the convertible bond is purchased and then converted into common stock.

   $$\textbf{Market Conversion Price} = \frac{\text{Market Price of Convertible Security}}{\text{Conversion Ratio}}$$

   - Market conversion price can be viewed as a break-even price i.e. it is the price at which an investor is indifferent between selling the bond and converting it.

7. **Market Conversion Premium:** The market conversion premium per share allows investors to identify the premium or discount payable when buying the convertible bond rather than the underlying common stock.

   Market Conversion Premium per share = Market Conversion Price – Current Market Price

   - Like call option premium, the premium paid when buying a convertible bond allows investors to limit their downside risk to the straight value.
   - However, in call option, the bondholders know exactly the amount of the downside risk; whereas, in convertible bonds, the bondholders know only that the most they can lose is the difference between the convertible bond price and the straight value because the straight value is not fixed.

8. **Market Conversion Premium Ratio:** The market conversion premium per share is usually expressed as a percentage of the current market price i.e.

Market Conversion Premium Ratio = $\frac{\textbf{Market Conversion premium per share}}{\textbf{Market price of common stock}}$

9. **Premium Payback Period:** Premium Payback Period tells the time it takes to recover the premium per share. It is also known as Break-even time.

Premium Payback Period = $\frac{\textbf{Market Conversion premium per share}}{\textbf{Favorable Income Differential per share}}$

NOTE: The premium payback period does not take into account the time value of money or changes in the dividend over the period.

10. **Favorable Income Differential per share =** $\frac{\textbf{Coupon interest} - (\textbf{Conversion Ratio} \times \textbf{Common stock dividend per share})}{\textbf{Conversion Ratio}}$

6.2.4) Downside risk with a Convertible Bond: The downside risk is measured as a percentage of the straight value i.e.

Premium over straight value = $\frac{\textbf{Market Price of Convertible Bond}}{\textbf{Straight Value}}$ **- 1**

- The higher the premium over straight value, all other factors constant, the less attractive the convertible bond.
- Downside risk measure changes as interest rates change because the straight value (floor) changes as interest rates change.

Example:
DATA:

- *The straight value of the bond is $98.19 per $100 of par value.*
- *Market price per share of common stock is $33.*
- *Conversion ratio is 25 shares.*
- *Market price of convertible security = $1065 or $106.50 per $100 of par value*
- *Coupon interest from bond = 0.0575 × $1,000 = $57.50*
- *Dividend per share = $0.90*

1. **Conversion Value** = $33 × 25 = $825 or $82.50 per $100 of par value. Since the straight value is $98.19 and conversion value is $82.50,
2. **The minimum value** for the ABC convertible bond = $98.19
3. **Market conversion price** = $\frac{\$1,065}{25}$ = $42.60
4. **Market Conversion Premium per share** = $42.60 – $33.00 = $9.60
5. **Market Conversion Premium Ratio** = $\frac{\$9.60}{\$33}$ = 0.291 or 29.1%
   - This means that investor is paying 29.1% above the market price of $33 by buying the convertible.
6. **Favorable Income Differential per share** = $\frac{\$57.50-(25\times\$0.90)}{25}$ = $1.40
7. **Premium Payback Period** = $\frac{\$9.60}{\$1.40}$ = 6.857 years
   - This means that (without considering the time value of money) the investor would recover the market conversion premium per share assuming unchanged dividends in about 6.857 years.
8. **Premium over straight value** = $\frac{\$106.50}{\$98.19} - 1$ = 0.085 = 8.5%

6.3 Valuation of a Convertible Bond

Typically, the convertible bonds tend to have lighter covenants than otherwise similar non-convertible bonds and are frequently issued as subordinated securities. Therefore, the valuation and analysis of convertible bonds is complex. Convertible bond price is affected by underlying share price and factors that affect issuer's common stock, including dividend payments and the issuer's actions (e.g., acquisitions or disposals, rights issues.

- For a convertible bond with a fixed coupon, all else being equal, price and value of a convertible bond increases (decreases) as the interest rates decline (increase) significantly.
- For a convertible bond, all else being equal, price and value of a convertible bond increases (decreases) as the issuer's credit quality improves (deteriorates).

An Option-Based Valuation Approach: Buying a non-callable/non-putable convertible bond is equivalent to:

- Buying a Noncallable/Nonputable straight security.
- Buying a call option on the stock where the number of shares that can be purchased with the call option is equal to the conversion ratio.

Non-callable/Non-putable Convertible security value = Straight value + Value of the Call option on the stock

NOTE:

- The theoretical value of call option can be estimated using the Black-scholes option pricing model.
- Fair value of Call option on the stock depends on Expected price volatility i.e. the higher the expected price volatility, the greater the value of the call option.

Callable Convertible bond value = Straight value + Value of the call option on the stock – Value of the call option on the bond

Valuing a convertible bond involves two factors:

1) **Stock price movement** that affects the value of the embedded call option on the common stock.
2) **Interest rates movement** that affects the value of the embedded call option on the bond.

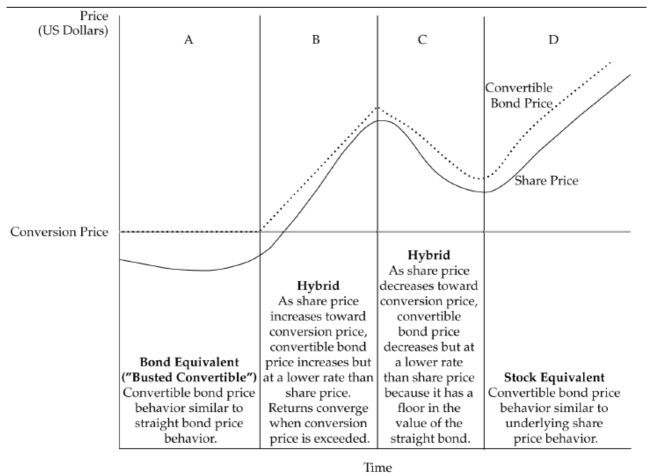
For a Callable Convertible Bond:

- Stock price volatility is positively related to value of call on the stock and value of callable convertible bond.
- Interest rate volatility is positively related to the value of call on the bond but negatively related to value of callable convertible bond.

Callable & Putable Convertible bond value = Straight value + Value of the call option on the stock – Value of the call option on the bond + Value of the Put option on the bond

6.4 Comparison of the Risk–Return Characteristics of a Convertible Bond, the Straight Bond, and the Underlying Common Stock

- When the underlying share price is well below the conversion price, the convertible bond is said to be "***busted convertible***" because the conversion option is not exercised and the call option is out-of-the-money; hence, the risk–return characteristics of the convertible bond is similar to those of the underlying option-free (straight) bond.
- When the underlying share price is above the conversion price, risk-return characteristics of a convertible bond is similar to that the underlying common stock because the call option is in-the-money and thus, price of the call option and the price of the convertible bond is significantly affected by share price movements but unaffected by interest rate movements.
- When the underlying share price is below the conversion price and increases toward it, or when the underlying share price is above the conversion price but decreases toward it, the convertible bond acts like a hybrid instrument.
  - ➢ When the underlying share price is below the conversion price and increases toward it, the value of call option increases as the underlying share price approaches the conversion price.
  - ➢ When the underlying share price is above the conversion price but decreases toward it, the relative change in the convertible bond price is less than the change in the underlying share price because the convertible bond has a floor.



Practice: Example 9, Reading 36.

7. Bond Analytics

For checking the option valuation, investors can use put–call parity. According to put–call parity,

Value of Call Option – Value of Put Option = PV (Forward price of bond on exercise date – Exercise price)

- ➢ Where, European-type call option and put option are on the same underlying bond and have the same exercise date and the same exercise price.

End of Reading Practice Problems:

2. Modelling Credit Risk and the Credit Valuation Adjustment

Credit spread (or z spread):

- Is calculated as the difference between the yield to maturities of a corporate and government bond with the same maturities
- Reflects compensation to the issuer for bearing issuer default risk and for losses incurred in the event of default

Reading assumptions:

1) The terms default risk and credit risk are distinguished in this reading. Default risk is the narrower term and addresses likelihood in the event of default. Credit risk is the broader term and considers 1) default risk and 2) how much will be lost if default occurs.
2) Corporate bonds and default risk-free government bonds have the same liquidity. However, in reality, there is difference between the taxation and liquidity of both types of instruments.

Factors to consider when modelling credit risk:

Factor 1: Expected exposure: The projected amount of money the investor could lose if default occurs before factoring possible recovery. Default events include:

- Nonpayment of obligations
- Failure to meet a different obligation
- Violation of a financial covenant

Factor 2: Assumed recovery rate: %age of loss recovered from a bond in the event of default. The recovery rate varies according to:

- industry,
- degree of seniority in the capital structure,
- the amount of leverage in the capital structure in total, and
- whether a particular security is secured or otherwise collateralized.

Loss given default = Expected exposure to default loss × (1 – Assumed Recovery Rate)
Loss severity = 1 – Assumed Recovery Rate

Factor 3: Probability of default: The probability that a bond issuer will not meet its contractual obligations on schedule.

Why differences between risk neutral and actual probabilities of default exist:

- Risk-neutral probabilities of default are based on the concept of option pricing while actual probabilities of default are based on historical default experience.
- Differences between risk-neutral and actual probabilities of default occur due to the following reasons:
  - Actual probabilities do not include the default risk premium associated with the uncertainty over the timing of possible default losses
  - The observed spread over the yield on a risk-free bond includes liquidity and tax considerations in addition to credit risk

Credit valuation adjustment: is the value of the credit risk in present value terms.

How to Calculate the Credit Valuation Adjustment?

Assumptions:

- Zero interest rate volatility
- Yield curve is assumed to be flat
- Bond is zero-coupon

Step 1: Calculate exposure to default loss:

For each date, calculate: 'Expected Exposure at Maturity/(1 + Risk-free rate)$^{\text{Number of Years Remaining to Maturity}}$'

Expected exposure at maturity is usually 100.

Note: Interest rate volatility is assumed to be zero. To factor in interest rate volatility, the binomial model is used (refer to Section 5).

Step 2: Project assumed recovery if default occurs:

For each date, recovery amount = Exposure (from Step 1) × Recovery rate

Step 3: Calculate Loss Given Default (LGD)

LGD = Exposure for each date (from Step 1) – Assumed recovery (from Step 2)

Step 4: Calculate Risk-Neutral Probability of Default (POD)*

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*POD = Conditional POD which assumes no default in prior years

Initial POD or hazard rate for Date 1 will be given

POD for Date 2 = Probability of surviving (POS) past date 1 and arriving at date 2 × Hazard rate

POD for Date 3 = POS × Hazard rate

POS = $(100\% - \text{Hazard rate})^{\text{Number of years}}$

For Date 1, Number of years = 1

Step 5: Calculate Expected Loss

Expected loss = LGD × POD

Step 6: Calculate Default Risk-Free Discount Factors

Discount factor assumes a flat government bond yield curve.

Discount factor for each date is $1/(1 + \text{risk-free rate})^{\text{Number of Years Remaining}}$

Step 5: Calculate Present Value(PV) of Expected Losses

PV of expected loss = Expected loss × discount factor

Step 6: CVA = ΣPV of expected loss

<u>How to calculate IRR of a bond issue?</u>

Step 1: Calculate the fair value of the bond

Fair value = Current price of bond – CVA

For a default-free bond, price = Par value × Discount factor on maturity date

Step 2: Calculate IRR

IRR depends on when and if default occurs.

On Date 0, bond is worth its fair value

On subsequent dates, IRR is determined using the following expression:

Fair value of bond = $\frac{Recoverable\ amount}{1+IRR}$

If the bond does not default on any date prior to maturity, the investor receives the coupon payment if the bond is coupon paying.

The credit spread is determined by following the steps below:

Step 1: Calculate the yield of the bond

$$\frac{100}{(1+yield)^{Maturity}} = Fair\ value\ of\ bond\ on\ issuance$$

Step 2: Credit spread = Yield on the bond (Step 1) – Yield on an identical maturity government bond

Note:
- POD incorporates likely time of the occurrence of default events and the uncertainty over the timing of these events.
- For a given price and yield, the assumed probability of default and recovery rate are positively correlated

<u>Practice:</u> Example 1 & 2, CFA Curriculum, Volume 5, Reading 37.

3. Credit Scores and Credit Ratings

Credit scores and ratings are used by lenders to decide contract terms and whether to extend credit.

- Credit scores are used in the retail lending market by small businesses & individuals
- Credit ratings are used in the wholesale market for bonds issued by corporations, government entities, and asset-backed securities.

Credit scoring methodologies can vary. Some countries only include negative information while other countries include broader information. A score reflects actual observed factors. Globally, credit scoring agencies are national in scope.

FICO is a commonly used scoring system used by US lenders for retail customers.

<u>Practice:</u> Example 1 & 2, CFA Curriculum, Volume 5, Reading 37.

The three major global credit rating agencies are Moody's Investors Service, Standard and Poor's, and Fitch Ratings. Ratings are ordinal and provide information on default probability as well as ratings for issuers and specific issues.

Corporate debt rated as investment grade at the time of default are unlikely to default while the high-yield sector does experience default.

Credit ratings comprise:

- Expected loss given default by means of *notching*: adjustment to issuer rating to reflect priority of claim and any subordination for specific debt issues
    - Note: Issuer rating is for senior unsecured debt
- Probability of default.

Rating agencies also generate a ratings outlook ('positive', 'negative', or 'stable') as well as when the issuer is under 'watch'.

Rating Transition Matrix

The matrix shows the probabilities of particular rating transitioning to another over a particular time period. Using the matrix and credit spreads, analysts can estimate the periodic return of an issue assuming no default.

For each possible transition in credit rating, Expected price change = - Modified duration × Change in credit spread

Expected overall price change of bond = Σ(Expected price change for each rating transition × Probability of rating transition)

Expected return of the bond (assuming no default) = Expected overall price change of bond – Yield to maturity of bond

Credit spread migration reduces expected return of an issue because:

I. Probabilities are not symmetrically distributed around current rating but are skewed towards a downgrade rather than an upgrade
II. Increase in credit spread is larger for downgrades than decrease in spread for upgrades

Practice: Example 4, CFA Curriculum, Volume 5, Reading 37.

4. Structural Models and Reduced Form Credit Models

Structural models and reduced form credit models are two types of credit analysis models.

Structural Models

- Based on the insight that a company defaults if the value of assets falls below the value of liabilities & probability of default has the features of an option.
- Also known as company-value model because the key variable is the asset value of the company.
- The Black-Scholes-Merton option pricing model assumes that the assets (i.e. company shares) underlying options are actively traded. The assumption is unrealistic for the assets of the company which do not trade.
- Structural models explain why default occurs – value of assets declines below value of liabilities
- Structural model presented as a graph shows the asset value as the vertical axis and time as the horizonal axis
    - A horizontal line represents the default barrier and the area below the line represents the probability of default which is treated as an endogenous variable
    - If the asset value falls below the barrier, the company defaults on the debt
    - Default probability increases with:
        - variance of future asset value
        - greater time
        - greater financial leverage
    - Default probability decreases with less debt
- Structural models interpret debt and equity values in terms of options.
    - The equation, E(T) = max (A(T) – K,0), indicates that equity is a purchased call option on the assets of the company where the strike price is equal to K, the face value of debt
        - Value of equity goes up when asset value goes up and equity does not take on negative values
    - The equation, D(T) = A(T) – Max(A(T) – K,0) ,indicates that debtholders own

assets of the company and have written call options held by shareholders.
  - Debtholders have priority in claims if asset value falls below K.

Reduced-Form Models

- Treats default as an exogenous variable
- Aim to statistically explain when default will occur; this concept is known as default time.
- Also called intensity-based and stochastic default rate models
- The credit risk measure, default intensity, is estimated using regression analysis on company-specific variable and macroeconomic variables.

Advantages and Drawbacks of Structural Models and Reduced-Form Models

Structural Models	Reduced-Form Models
Advantages	
Provide credit risk insight	Inputs are observable variables, historical data
	Credit risk measure directly reflects the business cycle
Disadvantages	
Burdensome to implement	Do not explain the economic reasons for default
Determining default may be difficult in practice due to limited available data e.g. companies may hide debt	Default is assumed to be a surprise event which unrealistic

Practice: Example 5, CFA Curriculum, Volume 5, Reading 37.

Structural models require information best known to the managers of the company, their commercial bankers and credit rating agencies. Uses of models:

- internal risk management,
- bank's internal credit risk measures, and
- publicly available credit ratings.

Reduced-form models only require information available in the market. Uses of models:

- to value risky debt securities
- to credit derivatives.

5. Valuing Risky Bonds in an Arbitrage-Free Framework

The arbitrage-free framework is used to analyze the credit risk of a bond in the context of volatile interest rates. The following steps are followed:

Step 1: Build a binomial interest rate tree based on a no-arbitrage assumption

Step 2: Determine discount rates and spot rates by bootstrapping using cash flows on the underlying benchmark bonds

Step 3: Calculate forward rates as the ratios of discount factors

Step 4: Build a binomial interest rate tree for 1-year forward rates consistent with the pricing of benchmark government bonds and an assumption of future interest rate volatility

Step 5: Use the binomial interest rate tree in combination with a hazard rate, recovery rate, and an interest rate volatility assumption, to determine the value of bond assuming no default (VND)

Step 6: Calculate total expected exposure as: Σ((Bond value at Node_N × Probability of attaining Node N) + Bond value at $\text{Node}_{\text{...Maturity}}$ × Probability of attaining Node...Maturity)) + Coupon payment

Step 7: Calculate Expected Loss

Expected loss = Loss given default (or expected exposure) × probability of default

Step 8: Calculate Contribution of Expected Loss to Total CVA

Present value of expected loss = Contribution to total CVA

Step 9: Determine Total CVA

ΣContribution to total CVA = Total CVA

Step 10: Determine fair value of bond after incorporating default risk

Fair value of bond = VND (Step 5) – Total CVA (Step 9)

YTM of bond is calculated using the coupon rates at each date and fair value of bond derived from Step 10 (above). Credit spread is then equal to the difference between the YTM and the actual yield on the comparable maturity government bond.

Practice: Example 6, CFA Curriculum, Volume 5, Reading 37.

Note:

- Credit spread derived from the binomial interest rate tree analysis above only incorporates credit risk and ignores tax and liquidity differences between government and corporate bonds
- Change in interest rate volatility will only change the fair value of bonds with embedded options
- For option-free bonds, an increase in interest rate volatility:
  - Has a small impact on the fair value
  - Increase the range of forward rates around the implied forward rates for each node asymmetrically
    - Rates at the top of the tree will increase more than rates at bottom
  - Total CVA decreases
  - Loss given default decreases
  - Expected exposure to default loss decreases

Valuation of Risky Floating-Rate Notes

The arbitrage-free framework can also be adapted for a risky floating-rate note. Valuation of risky floater is identical to a risky corporate bond except for:

- A fixed quoted margin is added to each forward rate in the tree
- Interest payment is made in arrears
- A discount margin is estimated as an alternative to a credit spread using trial and error or a software

Refer to pages 32 to 35 of reading for an illustration of the valuation of a floating rate note.

Practice: Example 7, CFA Curriculum, Volume 5, Reading 37.

6. Interpreting Changes in Credit Spread

Corporate bond yield comprises two components:

1) Spread over the benchmark yield: Capture microeconomic factors that pertain to the corporate issuer and specific issue. Spread constitutes:
   a. Expected loss from default
   b. Liquidity
   c. Taxation
   d. Risk aversion – compensation for uncertainty regarding expected loss from default

2) Benchmark yield: Captures macroeconomic factors affecting all debt securities. Spread constitutes:
   a. Expected inflation rate
   b. Expected real rate of return

Liquidity and tax differences between corporate and benchmark bonds are difficult to separate in this analysis. A bond will become less liquid if the recovery rate and probability of default become difficult to assess or when an uncertain tax status on a bond's gains and losses increases the time and cost to estimate value.

Research groups at major banks and consultancies worldwide are working on models to better include counterparty credit risk, funding costs, and liquidity and taxation differences in the valuation of derivatives.

The arbitrage model and credit risk model can also be used to examine connections between default probability, recovery rate, and the credit spread. In reality, these models use Monte Carlo simulations.

Risk-neutral probabilities of default are higher than actual probabilities because:

- Market prices reflect uncertainty over the timing of possible default and actual probabilities ignore this factor
- Credit rating migration risk is captured in risk-neutral probabilities
- Liquidity and tax differences between corporate and benchmark bonds is reflected in risk-neutral probabilities

Reduction in the recovery rate has an impact on the LGD and CVA for each year. This can be determined using the credit risk model and arbitrage-free valuation.

Practice: Example 8, CFA Curriculum, Volume 5, Reading 37.

7. Term Structure of Credit Spreads

Credit curve shows the spread over a benchmark security for an issuer for outstanding fixed-income securities with shorter to longer maturities.

Uses of credit spread term structure:

1. Used by issuers, underwriters and investors in measuring the risk-return tradeoff for a single issuer or set of issuers across ratings and/or sectors across maturities.
2. Determining the terms of a new issuance (example, bid price for a new issuance) or tender for existing debt (example, to inform trading decisions for secondary debt positions).
3. Term structure for a particular rating or sector is used to derived prospective pricing for a new issuance or for determining fair value spreads for outstanding securities
4. High-yield investors gauge the risk/return tradeoffs between different maturities

Drivers of the credit spread term structure:

A. Credit quality

Credit spread term structure for most highly rated securities is either flat or upward sloping. Lower bound on credit spreads means that credit spread migration on investment-grade securities which have the highest credit ratings and lowest credit spreads is possible in only one direction.

Securities with lower credit quality face greater sensitivity to the credit cycle. High-yield issues have a higher likelihood of default resulting in a steeper credit spread curve. A steep curve results in both of the following scenarios:

1. A weaker economy results in credit spread widening
2. An inverted yield curve suggests tighter spreads on longer maturities

B. Financial Conditions

Credit risk of a bond is influenced by expectations for economic growth and inflation. For issues with declining probability of default, a stronger economic climate is associated with higher benchmark yields but lower credit spreads. A countercyclical relationship is commonly observed between spreads and benchmark rates across the business cycle.

C. *Market Supply and Demand Dynamics*

Credit curve is most heavily influenced by most frequently traded securities. The relative liquidity of corporate bonds varies widely and most do not trade on a daily basis.

A flattening of the term structure may occur when market participants anticipate a significant supply in a particular tenor. Infrequently traded bonds with wider bid-ask spreads can also influence the shape of the term structure so it is important to gauge the size and frequency of trades to ensure consistency.

D. Company-value Model Results (in Section 4) from a Microeconomic Perspective

Any microeconomic factor which increases (decreases) implied default probability such as greater (lower) equity volatility will result in a steeper (flatter) yield curve.

Two further considerations when analyzing the term structure of credit spreads:

1. Risk-free/benchmark rates used to determine spreads. An on-the-run government bond with the nearest maturity to an outstanding corporate bond represents a natural benchmark choice as it has the lowest default-risk in developed markets.

However, the maturity and duration of these bonds rarely match those corporate bonds. Hence, interpolation between the yields of government securities with the closest duration is necessary.

2. All-in spread over the benchmark. Term structure analysis should only include bonds with similar credit characteristics which are typically senior, unsecured general obligations of the issuer. Bonds with embedded options, first or second lien provisions, or other unique provisions should be excluded from the analysis.

A change in market expectations of default is a key determinant of the shape of the credit curve term structure.

Credit Curve Term Structure Shape and Interpretation of Slope

Shape	Interpretation of Slope
Flat	Relatively stable expectation of default
Upward-sloping	Greater compensation for assuming issuer default over longer periods
Positively sloped*	Low short-term credit spreads rises with increasing maturity* Slope arises for: • High-quality issuer with a strong competitive position in a stable industry with low leverage, strong cash flow and high profit margin • Investment-grade bond portfolios
Inverted	Slope arises for: • High-yield issuers in cyclical industries because of issuer- or industry-specific reasons • Issuers in a historically cyclical industry are at the bottom of the cycle with expectations of a recovery

*credit spread rises due to macroeconomic environment uncertainty, potential adverse change in competitive landscape, technological change or other factors driving a high probability of default over time.

Bonds with a very high likelihood of default tend to trade on a price basis that converges toward the recovery rate rather than on a spread to benchmark rates. Credit spread structures will not truly reflect the relative risks and rewards of long-term versus short-term bonds from a single issuer.

Interpretation of credit spread term structure is important for investors seeking to capitalize on a market view that differs from that reflected in the credit curve. For example, if an investor disagrees with the market expectation that a high near-term probability of default will decline over time, he should sell short-term protection in the credit default swap market and buy-long term protection.

8. Credit Analysis for Securitized Debt

Credit analysis of structured finance instruments is fundamentally different compared to other risky bonds.

Credit assessment can be based on concepts such as homogeneity and granularity.

Homogeneity refers to the degree to which the underlying debt characteristics in a structured finance instrument are similar across individual obligations.

Granularity is the number of obligations comprising an overall structured finance instrument. For highly granular portfolios, analysts should draw conclusions on creditworthiness based on portfolio summary statistics rather than investigate each borrower because a single portfolio may comprise hundreds of borrowers. For discrete or non-granular portfolio, individual obligations may be analyzed.

Combination of asset type, tenor, granularity and homogeneity drive credit analysis approach:

- Short-term finance vehicles with granular

homogenous assets tend to be evaluated using a statistical approach
- For medium-term granular homogenous obligations, a portfolio-based approach should be used because the portfolio changes over time
- For a discrete or non-granular heterogenous portfolio, a loan-by-loan analysis is more appropriate.

For an asset portfolio whose composition changes over time, creditworthiness of the servicer as well as its track record in meeting servicing obligations should be assessed.

When assessing the credit risk of special purpose entities (SPEs) it is important to consider bankruptcy remoteness – the degree to which bankruptcy of the obligor is related to that of the originator. If transfer of assets from the originator to the SPE is a true sale, risks can be separated between the two parties at a later date.

Additional credit enhancements are a key structural element which needs to be assessed in the context of credit risk.

Covered Bonds

Covered bonds is a senior debt obligation of a financial institution which gives recourse to the originator/issuer as well as the pre-specified collateral pool. The dual recourse principle is of central important in evaluating the credit risk of a covered bond.

Asset pools vary but generally comprise residential mortgages or public sector assets.

For residential mortgages, delinquency rates based on region and asset type using standard criteria will dictate default probability and expected loss.

For public sector assets, performance depends on jurisdiction and asset type.

Practice: End of Chapter Questions, CFA Program Curriculum, Volume 5, Reading 37.

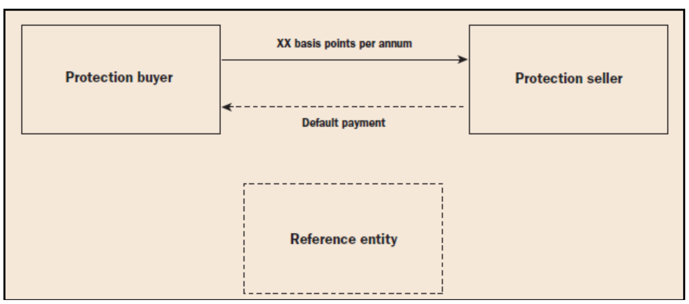
1. INTRODUCTION

Credit derivative: A credit derivative is a derivative instrument in which the underlying is the credit quality of a borrower. Credit derivatives allow investors to transfer, concentrate, dilute, and repackage their credit risk efficiently.

There are four Types of Credit Derivatives:

1) Total return swaps
2) Credit spread options
3) Credit-linked notes
4) Credit default swaps or CDS: A CDS is a contractual agreement to transfer the default risk of one or more reference entities from one party to the other. In a CDS, one party (the ***credit protection buyer***) pays a periodic fee (typically either semiannually or quarterly, called a CDS premium) to the other party (the ***credit protection seller***) during the term of the CDS and in return receives a payoff if an underlying financial instrument defaults or experiences a similar credit event.

- The protection buyer (seller) has a short (long) position in the reference obligation because when the credit quality and market price of the reference obligation declines (improves), the value of the CDS increases.



- CDS are similar to put options on a corporate bond where the protection buyer is protected against the losses resulting from a decline in the value of the bond as a result of a credit event. They are also similar to buying/selling insurance contracts on a corporation or sovereign entity's debt.
- CDS are the most commonly used credit derivatives.

Advantages of CDS:

- By allowing lenders (i.e. banks) to take a short credit position, CDS facilitates lenders to hedge their exposure to credit losses.
- CDS allows lenders to reduce their exposures to certain entities and to attain diversification.
- By allowing lenders to separate interest rate risk from credit risk, credit derivatives facilitate lenders (i.e. banks) to expand their loan business.
- CDS provides protection against changes in market's perception/opinions of a borrower's credit quality i.e. when investors perceive an increase in the likelihood of default, the price of a bond decreases.
- CDS allows protection buyer (seller) to act on a negative (positive) credit view without actually owning the reference entity's debt.
- CDS allows market participants to attain exposure in the form of a long credit position selling.
- CDS allows an investor with a high cost of funding to take on credit exposure without incurring the cost of funding.
- CDS contracts add liquidity to the market and increase the quality of price discovery.
- Credit derivatives facilitate economic growth.

Limitation of CDS:

- CDS contract does not eliminate credit risk because it eliminates the credit risk of one party but substitutes the credit risk of the CDS seller (i.e. counterparty default exposure). However, most active CDS sellers are relatively high-quality borrowers.
- When the reference entity specified in the CDS does not precisely match the hedged asset, the protection buyer faces basis risk.
- Credit derivatives are more effective in bond market where the terms and conditions are standardized compared to bank loan market.

2.1 Types of CDS

There are three types of CDS:

1) Single-name CDS: A single-name CDS is a CDS in which the reference entity is an individual corporation or government. A reference obligation is usually a senior unsecured obligation, referred to as a senior CDS. It is important to understand that any debt obligation issued by the borrower that is pari-passu (ranked equivalently in priority of claims) or higher relative to the reference obligation is covered by CDS.

- The payment received by the CDS holder does not depend on the specific obligation held by him; rather, it will depend on the ***cheapest-to-deliver obligation***, i.e. the debt instrument that can be purchased and delivered at the lowest cost but has the same seniority as the reference obligation.

Practice: Example 1, Volume 6, Reading 38.

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2) Index CDS: In an index CDS, the reference entity is an index of a combination of corporate entities. An index CDS offers protection on all entities in the index, and each entity has an equal share of the notional amount.

- The value of an index CDS depends on credit correlation of the underlying companies i.e. the higher (lower) the correlation between default of companies, the more (less) expensive it is to purchase protection for a combination of companies.

3) Tranche CDS: In a tranche CDS, the underlying is a credit quality of a combination of borrowers but credit protection is provided only on tranches of indices (i.e. up to pre-specified levels of losses). The tranche CDS represent only a small portion of the CDS market.

2.2 Important Features of CDS Markets and Instruments

The CDS market is large, global, and well organized.

The CDS market is governed by the International Swaps and Derivatives Association (ISDA). The terms of the contract are specified in a document called the ISDA Master Agreement and each party to CDS agree to conform to ISDA specifications. If a credit event occurs, the parties settle the CDS obligations according to procedures set forth in the ISDA documentation.

The CDS contracts have become increasingly standardized as market expands.

Each CDS contract is defined by:

a) A ***Reference Entity*** (the underlying entity on which one is buying/selling protection on);
b) A ***Reference Obligation*** (a particular debt instrument issued by the borrower that is being "insured");
c) Term of the contract i.e. maturity or expiration date: It is the date up to which the credit protection is provided.

- Typically, maturity ranges from 1 to 10 years, with 5 years being the most common and actively traded maturity. The two parties can negotiate any maturity, however.
- Typically, maturity dates are the last day of March, June, September, or December, with June and December being the most common.
- As the maturity of that CDS decreases, a new X-year CDS is created, and it begins to be referred to as the X-year CDS.

d) The ***size of the contract***, which is referred to as ***notional principal*** i.e. the amount of protection being purchased. Total amount of CDS notional can be greater than the amount of debt outstanding of the reference entity.

e) Credit events: The credit event is the outcome that triggers a payment from the credit protection seller to the credit protection buyer. Usually, it is difficult to identify credit event or occurrence of default and the extent of damage to the creditor.

Periodic premium paid to the CDS seller: The premium for a credit default swap is commonly known as a CDS spread and is quoted as an annual percentage in basis points of the notional amount.

- The CDS rates are standardized and the most common coupon rates are 1% or 5%. The 1% rate is used for a CDS on an investment-grade company or index. The 5% rate is used for a CDS on a high yield company or index.
- However, not all investment-grade companies have equivalent credit risk and not all high-yield companies have equivalent credit risk.

Upfront premium = Credit spread – Standard rate

- *This upfront premium is converted to a present value basis.*
- When standard rate paid by the protection buyer < credit spread, the protection buyer will have to make a cash upfront payment to compensate the protection seller for assuming credit risk.
- When standard rate paid by the protection buyer > credit spread, the protection seller will have to make a cash upfront payment to the protection buyer.

Impact of changes in the reference entity's credit quality on market value of CDS: The value of the CDS and consequently, the market price of the CDS may change with the changes in the reference entity's credit quality during the life of the contract.

- If the reference entity's credit quality decreases → the risk being covered is greater whereas the coverage and cost of protection are unchanged →leading to increase in the value of the CDS to the credit protection buyer; as a result, the credit protection buyer gains and the credit protection seller losses. These gains and losses are reflected in the market price of the CDS.

In summary:

When credit quality deteriorates, the credit protection buyer benefits.

NOTE:

- If the credit protection buyer holds the debt obligation and the CDS partially covers the debt, then decrease in credit quality of the reference entity will lead to gain on the CDS but loss on the overall position.

When credit quality improves, the credit protection seller benefits.

2.3 Credit and Succession Events

There are three general types of credit events:

1) **Bankruptcy:** It occurs when the reference entity becomes insolvent or is unable to repay its debt obligations and thus, has formally filed a bankruptcy.
2) **Failure to pay:** It occurs when the reference entity fails to make payment of principal or interest after a certain grace period.
3) **Restructuring:** It refers to any changes in the terms of debt obligations that are adverse to creditors e.g. reduction or deferral of principal or interest, change in seniority or priority of an obligation, or change in the currency in which principal or interest is scheduled to be paid. The restructuring must be involuntary i.e. it must be forced on the borrower by the creditors.

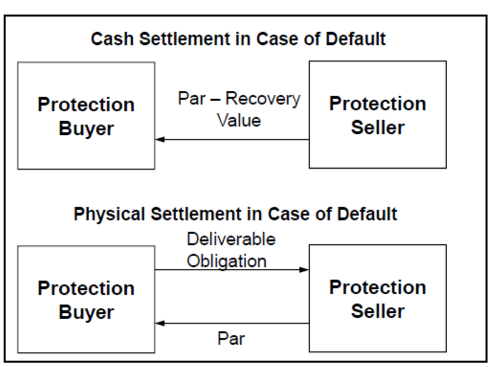
Succession event: Event that results in change in the corporate structure of the reference entity is called a succession event, e.g. merger, divestiture, spinoff etc. Succession events create uncertainty with regard to the obligator for the original debt. Therefore, the CDS contract is modified to reflect entity or entities responsible for the original debt.

2.4 Settlement Protocols

It is important to understand that if credit event occurs, the two parties to a CDS have the *right, but not the obligation*, to settle the contract. Typically, settlement takes place *30 days* after declaration of the credit event.

A CDS can be settled in two ways:

1) **Physical settlement:** In physical settlement, the protection buyer actually delivers the defaulted debt of the reference entity with a face value equal to notional amount specified in the CDS to protection seller and in return the seller pay par value (the notional amount of the contract) to protection buyer. This method is not commonly used.
2) **Cash settlement:** In cash settlement, the protection seller pays (100 – x% of Notional) to the protection buyer.



Recovery rate: It is the % of loss that is recovered. It represents the percentage received by the protection buyer relative to the amount owed.

Expected Credit Loss (%) = Payout ratio
= 1 – Recovery rate (%)

Expected Credit Loss Amount or Payout amount
= Payout ratio × Notional amount

- Actual recovery can be a time-consuming process and can occur much later than the payoff date of the CDS. Thus, an appropriate payout ratio is estimated by using an auction, conducted by the industry, in which bids and offers for the cheapest-to-deliver defaulted debt is submitted by major banks and dealers.
- However, the actual recovery rate can be quite different from the estimated recovery rate and may have material effects on the actual payout amount received by the CDS buyer if the CDS protection buyer also holds the underlying debt.

Example:

Suppose there are two series of senior bonds outstanding issued by the company.

i. Bond A trades at 20% of par. An investor X owns $10 million of Bond A and buys $10 million of CDS protection.
ii. Bond B trades at 35% of par. Investor Y owns $10 million of Bond B and buys $10 million of CDS protection.

- The cheapest-to-deliver obligation is Bond A as it is trading at 20% of par.
  Recovery rate for both CDS contracts = 20%

<u>If investor X uses cash settlement:</u>

Payout amount received by Investor X = [(1 – 20%) × $10 million] = $8 million

Amount received by investor X by selling the bond A = $2 million

Total proceeds received by investor X = $8 million + $2 million = $10 million

<u>If investor X uses physical settlement</u>:

Investor X delivers $10 million face amount of bonds to the counterparty in exchange for $10 million in cash.

- This implies that investor X would be indifferent between using cash settlement and physical settlement.

<u>If investor Y uses cash settlement</u>:

Payout amount received by Investor Y = [(1 – 20%) × $10 million] = $8 million

Amount received by investor Y by selling the bond B = \$3.5 million

Total proceeds received by investor Y = \$8 million + \$3.5 million = \$11.5 million

<u>If investor Y uses physical settlement</u>: Investor Y delivers \$10 million face amount of bonds to the counterparty in exchange for \$10 million in cash.

- This implies that investor Y would prefer a cash settlement because Bond B is worth more than the cheapest-to-deliver obligation.

<u>Practice:</u> Example 2, Volume 6, Reading 38.

2.5 CDS Index Products

CDS indices are provided by a company called Markit. Unlike stock index, a CDS index is not traded in the market. However, traded instruments based on the Markit indices have been developed by the industry. The payoff of these traded instruments depends on default of any entity covered by the index.

- All CDS are equally weighted. E.g. if there are 50 entities, the settlement on one entity is 1/50 of the notional.
- The components of each index are updated on semi-annual basis by Markit.
  - The latest created series is called the ***on-the-run series***.
  - The older series are called ***off-the-run series***.
  - When an investor moves from old series to new series, the move is referred to as a ***roll***.
  - If an entity included in the index defaults, the protection buyer is compensated for the loss and then the CDS notional amount is reduced by the defaulting entity's pro rata share.

Uses of Index CDS:

- Index CDS can be used to take positions on the credit risk of the sectors covered by the indices.
- Index CDS can be used to protect bond portfolios that consist of or are similar to the components of the indices.
- CDS indices on standardized portfolios based on the credit risk of well-identified companies have higher trading volume and are more liquid than that of a single name CDS.

Geographical classification of Markit Indices:

1) **North American Indices,** represented by symbol CDX.
2) **European Indices,** represented by symbol iTraxx.
3) **Asian Indices,** represented by symbol iTraxx.
4) **Australian Indices,** represented by symbol iTraxx.

Within each geographical category there are:

1) **Investment-grade indices:** They are represented by symbol CDX IG and iTraxx Main. Each index comprises 125 investment-grade entities. *Investment-grade index CDS are quoted in terms of spreads.*
2) **High-yield indices:** They are represented by symbol CDX HY (comprises 100 high-yield entities) and iTraxx Crossover (comprising 50 high-yield entities). *High-yield index CDS are quoted in terms of prices.*

Both investment-grade and high-yield indices use standardized coupons.

Example:

Suppose an investor sells \$800 million of protection on the CDX HY index. To hedge a portion of the credit risk, he purchases \$5 million of single-name CDS protection.

Investor is long = \$800 million / 100 = \$8 million notional

Investor is short = \$5 million

Net notional exposure = \$8 million - \$5 million = \$3 million

Exposure hedged by the protection buyer = \$5 million / \$8 million = 62.5%

Remaining notional of the index CDS = \$800 million - \$8 million = \$792

<u>Practice:</u> Example 3, Volume 6, Reading 38.

2.6 Market Characteristics

- CDS transactions are executed in the over-the-counter (OTC) market by phone, instant message, or the Bloomberg message service.
- CDS contracts are typically used by banks and other financial institutions.
- According to new regulations, all CDS must be cleared centrally through clearinghouses that collect and distribute payments; impose margin requirements, and mark positions to market. This process helps reducing the systematic risk.

A bank assumes two primary risks when it lends money:

1. ***Credit risk or default risk*** i.e. the borrower will not be able to repay principal and interest. Credit risk can be managed using:

- *Traditional methods* i.e. analysis of the borrower, its industry, and the macro-economy.
- *Control methods* i.e. credit limits, monitoring, and collateral.
- *Using credit derivatives*: Credit derivatives allow investors to separate interest rate risk from credit risk

by transferring the credit risk from the lender to another party.

- *Insurance contracts*: Insurance products are more consumer-focused than commercially focused. As a result, they are heavily regulated and involve greater costs to expand into new areas with different regulatory authorities.

2. ***Interest rate risk*** i.e. the interest rate will change such that the interest on loans (bank's income) < returns on comparable instruments in the marketplace. Interest rate risk can be managed by using duration-based strategies, gap management, and interest rate derivatives.

3. BASICS OF VALUATION AND PRICING

The price of CDS refers to the CDS spread or upfront payment given a particular coupon rate for a contract. Like corporate bonds, CDS spreads are more informative than their prices.

Unlike conventional derivatives where the underlying (e.g. equities, currencies, interest rates) is traded in active markets, the underlying of a CDS, i.e. credit quality, is not traded explicitly in the marketplace. Therefore, it is difficult to determine the price of credit risk.

3.1 Basic Pricing Concepts

Probability of Default: Probability of default refers to the degree of likelihood that the borrower of a loan or debt will not be able to make the necessary scheduled principal and interest payments. Generally, the probability of default is lower initially (i.e. on the first interest payment) but is greater over a longer period of time (i.e. on the final interest and principal payment).

Hazard Rate: Hazard rate is the probability of default at any point in time (t), given that no default has occurred prior to that time. It basically represents a ***conditional probability*** of default.

- As the hazard rate rises, the credit spread widens, and vice versa.

Example:

A hazard rate of 3% for the first interest payment and 5% for the final interest and principal payment means that there is 5% probability that defaults will occur in Year 2, given that it has not occurred in Year 1. Suppose the recovery rate is 45% and interest payment is $50.

- If default occurs on the first payment, the investor will receive = $50 × 45% = $22.50.
- If default occurs on the final payment, the investor will receive = $1050 × 45% = $472.5

Probability of receiving $50 at year 1 and $1,050 at year 2 =97% × 95% = 92.15%

Probability of receiving $50 at year 1 and $472.5 at year 2 =97% × 5% = 4.85%
Probability of receiving $22.50 at year 1 and $472.5 at year 2 =3%

Probability of default occurring at some time in the life of the loan = 100% - 92.15% = 7.85%

Loss given default: The actual total loss that is experienced by a lender when a borrower defaults on its debt is called loss given default. In the above example,

- If the borrower defaults on the first payment, loss given default = $50 - $22.50 = $27.50.
- If the borrower defaults on the final payment, loss given default = $1,050 - $472.50 = $577.50.

Total loss given default = $27.50 + $577.50 = $605

Total loss given default if the borrower defaults only on the second payment =$577.50
Probability of losing $605 = 3%

Probability of losing $577.50 = 0.97 × 0.05 = 4.85%

Expected loss = Full amount owed – Expected recovery

Or

Expected loss = Loss given default × Probability of default

Expected loss (unadjusted for time value of money) = [(0.03) × ($605)] + [(0.0485) × ($577.50) = $46.15875

The probability that a default will occur at some point during the T years = 1 – Probability of no default in T years

<u>Practice:</u> **Example 4, Volume 6, Reading 38.**

<u>**Two Legs of CDS Contract:**</u> CDS contract consists of two sides or legs, i.e.

1) **Protection leg:** It is the contingent payment that the credit protection seller may have to make to the credit protection buyer.

Value of protection leg = PV of the contingent obligation of the credit protection seller to the credit protection buyer

The value of the protection leg depends on probability of each payment, the timing of each payment, and the discount rate.

Expected payoff of a given payment on the reference entity = ***Present value**** of (Payment adjusted for the expected recovery rate × Probability of survival)

Expected payoff of the bond or loan = Sum of expected payoff of all payments on the reference entity**

*Present value is calculated using an appropriate discount rate. If probability of default is zero, all payments will be discounted using "risk-free" rate.

**it should be equal to the current market price of the bond.

Value of protection leg = Expected payoff of the bond or loan which contains credit risk - Expected payoff of the bond or loan which contains no credit risks

2) Premium leg: It is a series of payments the credit protection buyer promises to make to the credit protection seller.

Value of premium leg = Present value of payments made by the protection buyer to the protection seller

The value of premium leg depends on the coupon rate and the hazard rates.

Upfront payment = PV of protection leg – PV of premium leg

- When PV of protection leg > PV of premium leg, upfront payment is made by the protection buyer to protection seller.
- When PV of premium leg > PV of protection leg, upfront payment is made by the protection seller to protection buyer.

Important to Note:

When the credit spread on reference entity > coupon rate on CDS ➔ PV of protection leg > PV of premium leg ➔ as a result, upfront premium will be paid by the CDS buyer to the CDS seller.

3.2 The Credit Curve

Credit spread of a debt instrument is the rate in excess of LIBOR paid to investors to compensate them for assuming credit risk.

Credit spread ≈ Probability of default × Loss given default (in %)

Credit curve is a curve that graphically exhibits credit spreads for a range of maturities of a company's debt (non-government borrowers). The hazard rate is the primary factor that affects shape of credit curve i.e.

- Credit curve is **flat** when hazard rates are constant. ***Note:*** The credit curve would be completely flat when all hazard rates will be zero.
- Credit curve is **upward-sloping** when hazard rate is greater in later years, reflecting that the reference entity debt is riskier in the long-term.
- Credit curve is **downward-sloping** when hazard rate is greater in earlier years, reflecting that the reference entity debt is riskier in the short-term. Typically, downward-sloping curves are not common.

Practice: Example 5, Volume 6, Reading 38.

3.3 CDS Pricing Conventions

Upfront premium = PV of the credit spread – PV of the fixed coupon

Or

Upfront premium = (Credit spread – Fixed coupon) × Duration of the CDS

PV of credit spread = Upfront premium + PV of fixed coupon

Credit spread ≈ (Upfront premium/Duration) + Fixed coupon

Upfront premium in % = 100 – Price of CDS in currency per 100 par

Price of CDS in currency per 100 par = 100 – Upfront premium %

Practice: Example 6, Volume 6, Reading 38.

3.4 Valuation Changes in CDS during their Lives

Like other financial instruments, value of a CDS changes over time in response to changes in various factors i.e. duration, company's credit quality, probability of default, the expected loss given default, and the shape of the credit curve. The new market value of the CDS reflects gains and losses to the two parties.

Change in value of the CDS for a given change in spread can be estimated as follows:

Profit for the buyer of protection ≈ Change in spread in bps × Duration × Notional amount

% change in CDS price = Change in spread in bps × Duration

Practice: Example 7, Volume 6, Reading 38.

3.5 Monetizing Gains and Losses

Monetizing a gain or loss refers to entering into offsetting CDS contracts (i.e. selling their CDS positions to other parties) in response to changes in the price of a CDS to capture gains or losses.

- If the credit quality of the reference entity improves → credit spread on the reference entity reduces → consequently, the upfront premium on a newly created CDS decreases because of the reduction in risk.
  - In this case, the CDS buyer can offset his original position by entering into a new CDS as a protection seller (either with the same original party or third-party) and would receive the newly calculated upfront premium that is smaller than that of original CDS. Hence, the CDS buyer monetizes a loss.
  - Similarly, the CDS seller can offset his original position by entering into a new CDS as a protection buyer (either with the same original party or third-party) and would pay the newly calculated upfront premium that is less than that of original CDS. Hence, the CDS seller monetizes a gain.
- The protection buyer gains when the company's credit spread increases (i.e. credit quality deteriorates).
- The protection seller gains when the company's credit spread decreases (i.e. credit quality improves).

Methods of realizing a profit or loss on a CDS:

There are two methods to realize a profit or loss on a CDS.

1) By effectively exercising the CDS when credit event occurs.
2) By entering into offsetting CDS in the market.
Similar to a bond which converges toward par as it approaches maturity, when CDS buyer holds the position until expiration and no default occurs, the CDS spread converges to zero.

4. APPLICATIONS OF CDS

The two major uses/ applications of any derivative instrument are:

1. ***To exploit an expected movement in the underlying:*** It is relatively easy, less costly, and more efficient to take a short economic exposure by using derivatives because derivatives market is more liquid than the market for the underlying.

2. ***To earn a return by exploiting mispricing of derivative relative to the underlying:*** If a derivative is undervalued relative to the underlying, the investor can take long position in the derivative and short position in the underlying. However, the return earned by the investor will depend on the efficiency of the market and the quality of the valuation model.

4.1 Managing Credit Exposures

A CDS can be used to increase or decrease credit exposure i.e.

- Rather than selling a bond or loan that involves significant transaction costs, a lender can buy a CDS to reduce its credit exposure to a borrower.
- CDS seller sells CDS to increase credit exposure.

The CDS seller (typically dealers) can manage its credit exposure by either diversifying its credit risks or hedging the risk by entering into an offsetting transaction with another party and investing the funds in a repurchase agreement or repo.

It is important to understand that all CDS sellers are not dealers because a bondholder can assume only the credit risk (not interest risk) by selling a CDS, which involves less capital and low transaction costs.

Naked credit default swap: A naked credit default swap purchase means buying credit protection on the reference entity without actually owning them (i.e. either or both parties have no exposure to the underlying).

- The buyer of naked CDS speculates that reference entity's credit quality will deteriorate.
- The seller of naked CDS speculates that reference entity's credit quality will improve.

Rationale for using Naked CDS:

- It can be used to hedge against adverse economic conditions because even if an investor has no exposure to a borrower, the default of a sovereign entity or municipality imposes costs on investors.
- Naked CDS promotes liquidity and stability in the credit market.

Types of CDS Trading Strategies:

A. **Outright long or short position:** It involves taking an outright long or short position in a CDS contract.

B. **Long/short Trade:** It involves taking a long position in one CDS on one reference entity and short position in another CDS with a different reference entity in anticipation that one entity will improve relative to that of another. This strategy is known as "spread" in options and futures trading. For example,

- If an investor expects a weakening economy, he/she can take long position on investment-grade CDS index and short position on a high-yield CDS index.
- If an investor expects the Asian economy will perform strongly relative to European economy, he/she can take a long position on Asian CDS index and short position in European CDS index.

C. **Curve trade:** It is a type of long/short trade. Curve trade involves buying a CDS of one maturity and selling a CDS on the same reference entity with a *different* maturity.

- ***Credit-curve flattening Trade:*** It involves taking a short position in a short-term CDS and taking a long position in a long-terrm CDS because the investor believes that credit risk is higher in the short-term but will decrease in the long-term.
  - A curve-flattening trade is ***bearish*** in the short-run.
- ***Credit-curve Steepening Trade:*** In a credit curve steepener, the investor takes short position in a long-term CDS and takes long position in a short-terrm CDS because the investor believes that long-term credit risk will increase relative to short-term credit risk.
  - A curve-steepening trade is ***bullish*** in the short-run.
  - Like long-duration bonds, values of longer-term CDS are more sensitive than that of shorter-term CDS. Hence, when all rates are expected to increase, a trader can take short position in long-term CDS and long position in short-term CDS and will adjust the sizes of the positions so that gain in value resulting from one position will be greater than that of other position.

Advantages: Curve trade helps to reduce credit risk and allows investors to partially offset cost of one position.

D. **Basis Trade:** It involves exploiting differences in pricing between the bond issued by the reference entity and the CDS. Such differences may arise due to differences of opinions, differences in models used by participants in the two markets, differences in liquidity in the two markets, and supply and demand conditions in the repo market.

Basis = CDS spread (premium) – Bond's credit spread*

***Bond's Credit spread = Yield on bond - Investor's cost of funding**

NOTE:

Bond yield = Risk-free rate + Funding spread + Credit spread

Where, Risk-free rate + Funding spread = LIBOR

- Ideally, the yield on the bond issued by the reference entity should be equal to the credit spread on a CDS.
- The basis is ***negative*** when the *CDS spread is lower than the bond spread*. This implies that CDS premium is too low relative to the bond credit risk premium, meaning that price of bond is too low (i.e. bond is cheaper than CDS). In this case, the investor should buy the CDS (purchase credit protection) and buy the bond (assuming credit risk). The risk is balanced because the default potential on the bond is protected by the CDS. If convergence occurs, Gained earned by the investor = Yield on bond – Credit spread on CDS
- The basis is ***positive*** when the *CDS spread is higher than the bond spread*. In this case, the investor should long the credit risk (sell protection) and sell the bond short. However, shorting a bond is often not feasible and will highly depend on the liquidity of the underlying bond market.

E. **Credit spread strategy in LBO:** Credit spread increases when a company takes additional debt e.g. undergo a leveraged buyout (LBO). When a company undergoes a LBO, an investor can buy company's stock and purchase CDS protection so that when LBO occurs, stock price increases and CDS price increases (due to widening of spread resulting from increase in probability of default).

F. **Arbitrage Trade:** It involves exploiting perceived mispricing in index i.e. when the cost of the index is not equivalent to the aggregate cost of the index components by buying the cheaper instrument and selling the more expensive instrument.

Synthetic CDO = Portfolio of default-free securities + CDS holdings

- When the price of synthetic CDO< the actual CDO, an investor can capture arbitrage profit by buying the synthetic CDO and selling the actual CDO.

Practice: Example 8-10, Volume 6, Reading 38.