01.

Forward commitments:A forward is an agreement between two parties to buy or sell an asset at a pre-determined future time for a certain price.

- Forward price for a forward contract is defined as the delivery price, which make the value of the contract at initiation be zero.
- The buyer of a forward contract has a "long position" in the asset/commodity.

2. PRINCIPLES OF ARBITRAGE-FREE PRICING AND VALUATION OF FORWARD COMMITMENTS

Forward commitment pricing:Forward commitment pricing involves determining the appropriate forward commitment price or rate at which the forward commitment contract is initiated.
Forward commitment valuation: Forward commitment valuation involves determining the appropriate value of the forward commitment once it has been initiated.Forward value refers to the monetary value of an existing forward or futures contract.

Key assumptions made in pricing and valuation of contracts:

i. Replicating instruments are identifiable and investable;
ii. There are no market frictions;
iii. Short selling is allowed with full use of proceeds;
iv. Borrowing and lending are available at a known risk-free rate.

Note:Cash inflows to the arbitrageur have a positive sign and outflows are negative.

Carry arbitrage models used for forward commitment pricing and valuation are based on the no-arbitrage approach.

- Arbitrage occurs when equivalent assets or combination of assets sell for two different prices.
- The law of one price states that two identical goods must sell for the same current price in the absence of transaction costs. According to law of one price, arbitrage will drive prices of equivalent assets to a single price so that no riskless profits can be earned. The law of one price is based on the value ***additivity principle***, according to which the value of a portfolio is simply the sum of the values of each instrument held in the portfolio.
- Arbitrage opportunities should disappear quickly in an efficient and frictionless market.

3. PRICING AND VALUING FORWARD AND FUTURES CONTRACTS

3.1 Our Notation

Notation:

- 0 = today, T = expiration, underlying asset = $S_{0(\text{or t or } T)}$, forward = F(0,T)
- S_0 denotes the underlying price at the time of forward contract initiation
- S_T denotes the underlying price when the forward contract expires.
- $F_0(T)$ denote the forward price established at the initiation date, 0, and expiring at date T, where T represents a period of time later.
- Uppercase "F" denotes the forward price, whereas lowercase "f" denotes the futures price. Similarly, uppercase "V" denotes the forward value, whereas lowercase "v" denotes the futures value.

- Forward contracts are traded over-the-counter, no money changes hand initially and during the life time of the contract. Hence, the contract value at the initiation of the contract is ZERO. The forward contract value when initiated is expressed as $V_0(T) = v_0(T) = 0$.
- The contract price is set such that the value of the contract is Zero, that is,
  Present value of contract price = Prevailing spot price of the underlying
- Subsequent to the initiation date, the value can be significantly positive or negative.

At Market Contract: The forward contracts having value of zero at contract initiation are referred to as at market.

Property of Convergence: According to property of convergence, at Time T (expiration), both the forward price and the futures price are equivalent to the spot price, that is,

$$F_T(T) = f_T(T) = S_T$$

Important to Remember:

- The market value of a **long position** in a forward contract value at maturity is $V_T(T) = S_T - F_0(T)$.
- The market value of a **short position** in a forward contract value at maturity is $V_T(T) = F_0(T) - S_T$.
- The market value of a **long position** in a futures contract value *before marking to market* is $vt(T) = f_t(T) - f_{t-}(T)$.
- The market value of a short position in a futures contract value *before marking to market* is $v_t(T) = f_{t-}(T) - f_t(T)$.
- The futures contract value after daily settlement is $v_t(T) = 0$.
- If value of underlying > initial forward price, a long position in a forward contract will have a positive value.
- If value of underlying < initial forward price, a short position in a forward contract will have a positive value at expiration.

Note: The forward value and the futures value will be different because futures contracts are marked to market while forward contracts are not being marked to market.

3.2 No-Arbitrage Forward Contracts

3.2.1.) Carry Arbitrage Model When There Are *No* Underlying Cash

Carry arbitrage model is based on following two rules:

1) Do not use your own money, i.e. borrow money to buy the underlying.
2) Do not take any price risk (here refers to market risk); i.e. invest the proceeds from short selling transactions at risk-free rate or in other words, lend the money by selling the underlying).

Cash Flows related to Carrying the Underlying through Calendar Time:

If an arbitrageur enters a forward contract to sell an underlying instrument for delivery at Time T, then this exposure can be hedged by buying the underlying instrument at Time 0 with borrowed funds and carry it to the forward expiration date so it can be sold under the terms of the forward contract.

The table below shows Cash Flows Related to Carrying the Underlying through Calendar Time.

	Underlying Purchased	Underlying Sold
	0	T
Underlying:	$-S_0$	$+S_T$
Borrow:	$+S_0$	$-FV(S_0)$
Forward:	0	$F_0(T) - S_T$
Net:	0	$F_0(T) - FV(S_0)$

- The above figure shows that arbitrageur borrows the money to buy the asset, so at Time T, he will pay back $FV(S_0)$, based on the risk-free rate.
- When $S_T < FV(S_0)$, the arbitrageur will suffer a loss.
- When $S_T = FV(S_0)$, there will be breakeven.
- If we assume continuous compounding (r_c), then $FV(S_0) = S_0e^{r_cT}$
- If we assume annual compounding (r), then $FV(S_0) = S_0(1 + r)^T$

Carry Arbitrage Model Steps[1]: Assuming $S_0 = 100$, r = 5%, T = 1, and $S_T = 90$

1) Purchase one unit of the underlying at Time 0 and sell at T:
   At Time 0: cash outflow of $-S_0 = -100$.
   At Time T: cash inflow of $+S_T = +90$.
2) Borrow the purchase price at Time 0 and repay with interest at Time T.
   At Time 0:cash inflow of $+S_0 = +100$.
   At Time T: cash outflow of $-FV(S_0) = -100(1 + 0.05)^1 = -105$
   Net Cash Flows for Financed Position in the Underlying Instrument
   - Net Cash flow at Time 0: zero
   - Net Cash flow at Time T: $+S_T - FV(S_0) = 90 - 105 = -15$
3) Sell a forward contract on the underlying. Assuming, the forward price is trading at 105.
   At Time 0: Cash inflow of $+V_0(T)$
   At Time T: $V_0(T) = F_0(T) - S_T = 105 - 90 = 15$
4) Pre-capture your arbitrage profit (or in other words borrow it) by bringing it to the present so as to receive it at Time 0. The amount borrowed is forward price minus the future value of the spot price when compounded at the risk-free rate[2].
   At Time 0: Cash inflow of $+PV[F_0(T) - FV(S_0)]$
   At Time T: $V_0(T) = -[F_0(T) - FV(S_0)] = -[105 - 100(1 + 0.05)] = 0$

[1]Note that all four transactions are done simultaneously not sequentially.
[2]Note that if the forward contract is priced correctly, there will be no arbitrage profit and, hence, no Step 4.
The lending case is not discussed here because it would occur only if a strategy is executed to capture a certain loss.

Net Cash flow:

At Time 0: $+V_0(T) + PV[F_0(T) - FV(S_0)]$

At Time T: $V_0(T) = -[F_0(T) - FV(S_0)] = 0$ (for every underlying value)

The no-arbitrage forward price is simply the future value of the underlying as stated below:

$$F_0(T) = FV(S_0)$$

- If $F_0(1) = 106$, which is higher than that determined by the carry arbitrage model ($F_0(T) = FV(S_0) = 105$). This shows that market forward price is too high and should be sold.
- If the forward price were 106, the value of the forward contract at time 0 would be $V_0(T) = PV[F_0(T) - FV(S_0)] = (106 - 105)/(1 + 0.05) = 0.9524$.
- If the counterparty enters a long position in the forward contract at a forward price of 106, then the forward contract seller has the opportunity to receive the 0.9524 with no liability in the future.

Cash Flows with Forward Contract Market Price Too High Relative to Carry Arbitrage Model

1) Sell forward contract on underlying at $F_0(T) = 106$
   At Time 0: $V_0(T) = 0$
   At Time T: $V_0(T) = V_T(T) = F_0(T) - S_T = 106 - 90 = 16$
2) Purchase underlying at 0 and sell at T:
   At Time 0: $-S_0 = -100$
   At Time T: $-FV(S_0) = +S_T = 90$
3) Borrow funds for underlying purchase
   At Time 0: $+S_0 = 100$
   At Time T: $-FV(S_0) = -100(1 + 0.05) = -105$
4) Borrow arbitrage profit
   At Time 0: $+PV[F_0(T) - FV(S_0)] = (106 - 105)/(1+0.05) = 0.9524$
   At Time T: $-[F_0(T) - FV(S_0)] = -[106 - 100(1+0.05)] = -1$

Net cash flow
At Time 0: 0.9524
At Time T: 16 + 90 – 105 – 1 or –4 + 110 – 105 – 1 = 0

Reverse Carry Arbitrage:
Suppose forward price of $F_0(T) = 104$, which is less than the forward price determined by the carry arbitrage model (105). In this case, the opposite strategy – named "***Reverse Carry Arbitrage***" is followed. It involves the following steps:

1) Buy a forward contract, and the value at T is $S_T - F_0(T)$.
2) Sell short the underlying instrument.
3) Lend the short sale proceeds.
4) Borrow the arbitrage profit.

Important to Remember:

- If $F_0(T) \neq FV(S_0)$, there is an arbitrage opportunity.
- If $F_0(T) > FV(S_0)$, then the forward contract is sold and the underlying is purchased.
- If $F_0(T) < FV(S_0)$, then the forward contract is purchased and the underlying is sold short.
- If the forward contract price is equal to its equilibrium price, there will be no arbitrage profit and thus no Step 4.
- The quoted forward price does not directly reflect expectations of future underlying prices.

Relationship between Forward price and interest rate:
Forward price is directly related to interest rates – i.e., when interest rate falls (rises), forward price decreases (increases). This relationship between forward prices and interest rates will generally hold except for interest rates forward contracts.

Practice:Example 1, Reading 39, Curriculum.

Cash Flows for the Valuation of a Long Forward Position:

Steps	Cash Flow at Time 0	Value at Time t	Cash Flow at Time T
1. Buy forward contract at 0 at $F_0(T)$	0	$V_t(T)$	$V_T(0, T) = S_T - F_0(T)$
2. Sell forward contract at t at $F_t(T)$	NA	0	$V_T(t, T) = F_t(T) - S_T$
Net cash flows/Value	0	$V_t(T)$	$+F_t(T) - F_0(T)$

- "Value at Time t" represents the value of the forward contracts.

$$F_t(T) = FV_{t,T}(St)$$

- The value observed at Time t of the original forward contract initiated at Time 0 and expiring at Time T is simply the present value[3] of the difference in the forward prices, as stated below.

$$V_t(T) = \text{Present value of difference in forward prices} = PV_{t,T}[F_t(T) - F_0(T)]$$

Alternatively,

$$V_t(T) = S_t - PV_{t,T}[F_0(T)]$$

Practice:Example 2, Reading 39, Curriculum.

3.2.2.) Carry Arbitrage Model When Underlying Has Cash Flows

[3] Present value is calculated over the remaining life of the contract.

In Carry arbitrage, we are required to pay the interest cost, whereas in reverse carry arbitrage, we receive the interest benefit.

- Let γ denote the carry benefits (for example, dividends, foreign interest, and bond coupon payments that would arise from certain underlyings).

Future value of underlying carry benefits = $\gamma_T = FV_{0,T}(\gamma_0)$

Present value of underlying carry benefits = $\gamma_0 = PV_{0,T}(\gamma_T)$

- Let θ denote the carry costs. These refer to additional costs to hold the commodities, like storage, insurance, deterioration, etc. These can be considered as negative dividends. Carry costs are zero for financial instruments but holding these assets does involve opportunity cost of interest.

Future value of underlying costs = $\theta_T = FV_{0,T}(\theta_0)$

Present value of underlying costs = $\theta_0 = PV_{0,T}(\theta_T)$

Forward price is the future value of the underlying adjusted for carry cash flows. **Forward pricing equation** is stated as below:

$$F_0(T) = \text{Future value of underlying adjusted for carry cash flow}$$
$$= FV_{0,T}(S_0 + \theta_0 - \gamma_0)$$

- Carry costs (e.g. interest rate) are added to forward price because they increase the cost of carrying the underlying instrument through time.
- Carry benefits are subtracted from forward price because they decrease the cost of carrying the underlying instrument through time.

Example: Suppose, $S_0 = 100$, r = 5%, T = 1, and $S_T = 90$.Assuming the underlying will distribute 2.9277 at Time t = 0.5: γt = 2.9277. The time until the distribution of 2.9277 is t, and hence, the present value is

$$\gamma_0 = 2.9277/(1 + 0.05)^{0.5} = 2.8571$$

The time between the distribution and the forward expiration is T – t = 0.5, and thus, the

$$\text{Future value} = \gamma_T = 2.9277(1 + 0.05)^{0.5} = 3$$

Cash Flows for Financed Position in the Underlying with Forward: The steps involved in this strategy are as below:

1) Purchase the underlying at Time 0, receive the dividend at Time t = 0.5 and sell the underlying at Time T.
2) Reinvest the dividend received at Time t = 0.5 at the risk-free interest rate until Time T.
3) Borrow the initial cost of the underlying.
4) Sell a forward contract at Time 0 and the underlying will be delivered at Time T.
5) Borrow the arbitrage profit.

Cash flows are reflected in the table:

Steps	Cash Flow at Time 0	Cash Flow at Time t	Cash Flow at Time T
1. Purchase underlying at 0, sell at T	$-S_0 = -100$	$+\gamma_t = 2.9277$	$+S_T = 90$ or $+S_T = 110$
2. Reinvest distribution		$-\gamma_t = -2.9277$	$+\gamma_T = 2.9277(1 + 0.05)^{0.5} = 3$
3. Borrow funds	$+S_0 = 100$		$-FV(S_0) = -100(1 + 0.05)^1 = -105$
4. Sell forward contract	$V_0(T)$		$V_T(T)=F_0(T)-S_T = 102 - 90 = 12$ or $= 102 - 110 = -8$
5. Borrow arbitrage profit	$+PV[F_0(T) + \gamma_T - FV(S_0)]$		$-[F_0(T) + \gamma_T - FV(S_0)]$
Net cash flows	$V_0(T) + PV[F_0(T) + \gamma_T - FV(S_0)]$	0	$+S_T + \gamma_T - FV(S_0) + F_0(T) - S_T - [F_0(T) + \gamma_T - FV(S_0)] = 0$

The value of the cash flow at Time 0 is zero, or

$$V_0(T) + PV[F_0(T) + \gamma_T - FV(S_0)] = 0$$

and

$$V_0(T) = -PV[F_0(T) + \gamma T - FV(S_0)].$$

If theForward contract has zero value, then

Forward Price = $F_0(T)$ = Future value of underlying – Future value of carry benefits= $FV(S_0) - \gamma_T$

Initial forward price = Future value of the underlying - Value of any ownership benefits at expiration

or

$$F_0(T) = FV_{0,T}(S_0 - \gamma_0)$$

Forward value for a long position is estimated using the following:

$$V_t(T) = \text{Present value of difference in forward prices}$$
$$= PV_{t,T}[F_t(T) - F_0(T)]$$

Where,$F_t(T) = FV_{t,T}(S_t + \theta_t - \gamma_t)$

Annual compounding and continuous compounding:
The equivalence between annual compounding and continuouscompounding can be expressed as follows:

$$(1 + r)^T = e^{rct}$$

or

$$r_c = \ln[(1 + r)^T]/T = \ln(1 + r);$$

If the quoted interest rate is 5% based on annual compounding, then the implied interest rate based on continuous compounding is

$$r_c = \ln(1 + r) = \ln(1 + 0.05) = 0.0488, \text{ or } 4.88\%$$

- This implies that a cash flow compounded at 5% annually is equivalent to being compounded at

4.88% continuously.
- Continuous compounding results in a lower quoted rate.

Carry arbitrage model with continuous compounding:
The carry arbitrage model with continuous compounding is expressed as

$F_0(T) = S_0 e^{(r_c+\theta-\gamma)T}$ (Future value of the underlying adjusted for carry)

The future value of the underlying adjusted for carry, i.e., the dividend payments, is $F_0(T) = S_0 e^{(r_c-\gamma)T}$

- If a dividend payment is announced between the forward's valuation and expiration dates, assuming the news announcement does not change the current underlying price, the forward value will most likely decrease.
- If a new dividend is imposed, the new forward price will decrease and consequently, the value of the old forward contract will be lower.

3.3 Equity Forward and Futures Contracts

Since, futures contracts are marked to market daily, the equity futures value is zero each day after settlement has occurred.

Practice:Example 3, 4 & 5 Reading 39, Curriculum.

3.4 Interest Rate Forward and Futures Contracts

Libor, which stands for London Interbank Offered Rate, is a widely used interest rate that serves as the underlying for many derivative instruments. It represents the rate at which London banks can borrow from other London banks.

- When these loans are in dollars, they are known as ***Eurodollar*** time deposits, with the rate referred to as dollar Libor.
- Average Libor rates are derived and posted each day at 11:30 a.m. London time.
- Libor is stated on an actual over 360-day count basis (often denoted ACT/360) with interest paid on an add-on basis.

Let,
$L_i(m)$ = Libor on an m-day deposit observed on day i
NA = notional amount, quantity of funds initially deposited
NTD = number of total days in a year, used for interest calculations (always 360 in the Libor market)
t_m = accrual period, fraction of year for m-day deposit—tm = m/NTD
TA = terminal amount, quantity of funds repaid when the Libor deposit is withdrawn

Example: Suppose day i is designated as Time 0, and we are considering a 90-day Eurodollar deposit (m = 90). Dollar Libor is quoted at 2%; thus, $L_i(m) = L_0(90) = 0.02$. $50,000 is initially deposited, i.e. NA = $50,000. Hence,

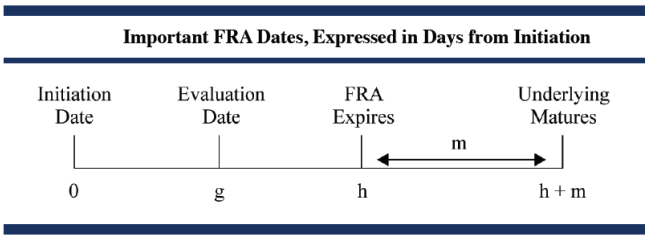
$t_m = 90/360 = 0.25$
$TA = NA\,[1 + L_0(m)tm] = \$50{,}000[1 + 0.02(90/360)] = \$50{,}250$
Interest paid = TA – NA = $50,250 – $50,000 = $250

Forward market for Libor: A forward rate agreement (FRA) is an over-the-counter (OTC) forward contract in which the underlying is an interest rate on a deposit. An FRA involves two counterparties: the fixed receiver (short) and the floating receiver (long).

- Being long the FRA means that we gain when Libor rises.
- The fixed receiver counterparty receives an interest payment based on a fixed rate and makes an interest payment based on a floating rate.
- The floating receiver counterparty receives an interest payment based on a floating rate and makes an interest payment based on a fixed rate.
- FRA price is the fixed interest rate such that the FRA value is zero on the initiation date.
- The underlying of an FRA is an interest payment.
- It is also important to understand that the parties to an FRA do not necessarily engage in a Libor deposit in the spot market. Rather, a Libor spot market is simply the benchmark from which the payoff of the FRA is determined.

A 3 × 9 FRA is pronounced as "3 by 9." It implies that FRA expires in ***three*** months and the payoff of the FRA is 6months Libor (i.e. 9 -3) when the FRA expires in 3 months.

- A short (long) FRA will effectively replicate going short (long) a nine-month Libor deposit and long (short) a three-month FRA deposit.
- FRA value is the market value on the evaluation date and reflects the fair value of the original position.



Example: A 30-day FRA on 90-day Libor would have h = 30, m = 90, and h + m = 120. If we want to value the FRA prior to expiration, g could be any day between 0 and 30.

- FRA (0, h, m) denotes the fixed forward rate set at Time 0 that expires at Time h wherein the

underlying Libor deposit has m days to maturity at expiration of the FRA.
- Thus, the rate set at initiation of a contract expiring in 30 days in which the underlying is 90-day Libor is denoted FRA (0, 30, 90).
- Like all standard forward contracts, at initiation, no money changes hands, implying value is zero.
- We can estimate price of FRA by determining the fixed rate [FRA(0,30,90)] such that the value is zero on the initiation date.

How to settle interest rate derivative at expiration: There are two ways to settle an interest rate derivative when it expires:

1) **Advanced set, settled in arrears:**Advanced set implies that the reference interest rate is set at the time the money is deposited. The term settled in arrears means that the interest payment is made at Time h + m, (i.e. at the maturity of the underlying instrument). Swaps and interest rate options are normally based on advanced set, settled in arrears.
2) **Advanced set, advanced settled:** FRAs are typically settled based on advanced set, advanced settled. In an FRA, the term "advanced" refers to the fact that the interest rate is set at Time h, the FRA expiration date, which is the time when the underlying deposit starts. Here, advanced settled means the settlement is made at Time h. Libor spot deposits are settled in arrears, whereas FRA payoffs are settled in advance.

The **settlement amounts for advanced set, advanced settled** are determined in the following manner:
- Settlement amount at h for receive-floating: $NA\{[(m)L_h - FRA(0,h,m)]t_m\}/[1 + D_h(m)t_m]$
- Settlement amount at h for receive-fixed: $NA\{[FRA(0,h,m) - L_h(m)]t_m\}/[1 + D_h(m)t_m]$

Where, $1 + D_h(m)t_m$ is a discount factor applied to the FRA payoff.It reflects that the rate on which the payoff is determined, $L_h(m)$, is obtained on day h from the Libor spot market, which uses settled in arrears, that is, interest to be paid on day h + m.

Example: In 30 days, a UK company expects to make a bank deposit of £10,000,000 for a period of 90 days at 90-day Libor set 30 days from today. The company is concerned about a possible decrease in interest rates. The company enters into a £10,000,000 notional amount 1 × 4 receive-fixed FRA that is advanced set, advanced settled. This implies that an instrument that expires in 30 days and is based on 90-day (4 – 1) Libor. The discount rate for the FRA settlement cash flows is 0.40%. After 30 days, 90-day Libor in British pounds is 0.55%.

TA = 10,000,000[1 + 0.0055(0.25)] = £10,013,750.
Interest paid at maturity = TA – NA = £10,013,750 - £10,000,000 = £13,750.

- If the FRA was initially priced at 0.60%, the payment received to settle it will be closest to:

m = 90 (number of days in the deposit)
tm = 90/360
h = 30 (number of days initially in the FRA)
The settlement amount of the 1 × 4 FRA at h for receive-fixed = [10,000,000(0.0060 – 0.0055)(0.25)]/[1 + 0.0040(0.25)] = £1,248.75

- If the FRA was initially priced at 0.50%, the payment received to settle it will be closest to as follows:

Settlement amount of the 1 × 4 FRA at h for receive-fixed = [10,000,000(0.0050 – 0.0055)(0.25)]/[1 + 0.0040(0.25)] = –£1,248.75

- In pay floating FRA, the long benefits when interest rate declines.

Practice:Example 6, Reading 39, Curriculum.

FRA pricing: Steps are as follows:
Step 1: Deposit funds for h + m days:
- **At Time 0:**deposit an amount = $1/[1 + L_0(h)t_h]$, the present value of 1 maturing in h days, in a bank for h+ m days at an agreed upon rate of $L_0(h + m)$.
- **After h + m days**,withdraw an amount = $[1 + L_0(h + m)t_{h+m}]/[1 + L_0(h)t_h]$

Step 2: Borrow funds for h days:
- **At Time 0:** Borrow $\{1/[1 + L_0(h)t_h]\}$, for h days so that the net cash flow at Time 0 is zero.
- In h days, this borrowing will be worth 1.

Step 3: At Time h, roll over the maturing loan in Step 2 by borrowing funds for m days at the rate $L_h(m)$. At the end of m days, we will owe $[1 + L_h(m)t_m]$.
In order to mitigate the risk of increase in interest rate, we would enter into a receive-floating FRA on m-day Libor that expires at Time h and has the rate set at FRA(0,h,m) as defined in step 4.
Step 4: Enter a receive-floating FRA and roll the payoff at h to h + m at the rate $L_h(m)$. The payoff at Time h will be $([L_h(m) – FRA(0,h,m)]t_m)/(1 + L_h(m)t_m)$. There will be no cash flow from this FRA at Time h because this amount will be rolled forward at the rate $L_h(m)t_m$. Therefore, the value realized at Time h + m will be $[L_h(m) – FRA(0,h,m)]t_m$.

Cash Flow Table for Deposit and Lending Strategy with FRA

Steps	Cash Flow at Time 0	Cash Flow at Time h	Cash Flow at Time h + m
1. Make deposit for h + m days	$-1/[1 + L_0(h)t_h] = -0.996264$	0	$+[1 + L_0(h + m)t_{h+m}]/[1 + L_0(h)t_h] = 1.006227$
2. Borrow funds for h days	$+1/[1 + L_0(h)t_h] = +0.996264$	−1	
3. Borrow funds for m days initiated at h		+1	$-[1 + L_h(m)t_m] = -1.0075$
4. Receive-floating FRA and roll payoff at $L_h(m)$ rate from h to h + m	0	0	$+[L_h(m) - FRA(0, h, m)]t_m = [0.03 - FRA(0,h,m)](90/360)$
Net cash flows	0	0	$+[1 + L_0(h + m)t_{h+m}]/[1 + L_0(h)t_h] - [1 + L_h(m)t_m] + [L_h(m) - FRA(0,h,m)]t_m$

The terminal cash flows as expressed in the table can be used to solve for the FRA fixed rate. Because the transaction starts off with no initial investment or receipt of cash, the net cash flows at Time h + m should equal zero; thus,

$$+[1 + L_0(h+m)t_{h+m}]/[1 + L_0(h)t_h] - [1 + L_h(m)t_m] + [L_h(m) - FRA(0,h,m)]t_m = 0$$

FRA fixed rate:

$$FRA(0,h,m) = \{[1 + L_0(h+m)t_{h+m}]/[1 + L_0(h)t_h] - 1\}/t_m$$

E.g. $FRA(0,90,90) = \{[1 + L_0(180)t_{180}]/[1 + L_0(90)t_{90}] - 1\}/t_{90}$

Practice:Example 7, Reading 39, Curriculum.

Valuing an existing FRA:If we are long the old FRA, we will receive the rate $L_h(m)$ at h. We will go short a new FRA that will force us to pay $L_h(m)$ at h. Suppose that we initiate an FRA that expires in 90 days and is based on 90-day Libor. The fixed rate at initiation is 2.49%. Thus, tm = 90/360, and FRA (0,h,m) = FRA(0,90,90) = 2.49%.

- When the FRA expires and makes its payoff, assume that we roll it forward by lending it (if a gain) or borrowing it (if a loss) from period h to period h + m at the rate $L_h(m)$. We then collect or pay the rolled forward value at h + m. Thus, there is no cash realized at Time h.
- Assume 30 days later, the rate on an FRA based on 90-day Libor that expires in 60 days is 2.59%. Thus, FRA (g, h – g, m) = FRA(30,60,90) = 2.59%. We go short this FRA, and as with the long FRA, we roll forward its payoff from Time h to h + m. Therefore, there is no cash realized from this FRA at Time h.

Value of the offset position = (2.59% – 2.49%) = 10 bps times 90/360 paid at Time h + m

- To determine the fair value of the original FRA at Time g, we need the present value of this Time h + m cash flow at Time g.

Value of the old FRA = Present value of the difference in the new FRA rate and the old FRA rate

Hence, the value is

$$V_g(0,h,m) = \{[FRA(g,h-g,m) - FRA(0,h,m)]t_m\}/[1 + D_g(h+m-g)t_{h+m-g}]$$

$$FRA(g,h-g,m) = \{[1 + L_g(h+m-g)t_{h+m-g}]/[1 + L_g(h-g)t_{h-g}] - 1\}/t_m$$

Where, $V_g(0,h,m)$ is the value of the FRA at Time g that was initiated at Time 0, expires at Time h, and is based on m-day Libor. $D_g(h + m – g)$ is the discount rate. Traditionally, it is assumed that the discount rate, $D_g(h + m – g)$, is equal to the underlying floating rate, $L_g(h + m – g)$, but that is not necessary.

Example: Suppose a 60- day rate of 3% on day g. Thus, $L_g(h – g) = L_{30}(60) = 3\%$. Then the value of the FRA would be

$V_g(0,h,m) = V_{60}(0,90,90) = 0.00025/[1 + 0.03(60/360)] = 0.000249$

Cash Flows for FRA Valuation are as following:

Steps	Flow at Time g	Time h	Cash Flow at Time h + m
1. Receive-floating FRA (settled in arrears) at Time 0; roll forward at Rate $L_h(m)$ from h to h + m		0	$+\{[L_h(m) - FRA(0,h,m)]t_m\} = +(L_h(m) - 0.0249)(90/360)$
2. Receive-fixed FRA (settled in arrears) at Time g; roll forward at Rate $L_h(m)$ from h to h + m	0	0	$+[FRA(g,h-g,m) - L_h(m)]t_m = +[0.0259 - L_h(m)](90/360)$
Net cash flows	0	0	$+[FRA(g,h-g,m) - FRA(0,h,m)]t_m = +(0.0259 - 0.0249)(90/360) = 0.00025$

Practice:Example 8, Reading 39, Curriculum.

3.5 Fixed-Income Forward and Futures Contracts

Accrued interest = Accrual period × Periodic coupon amount

or

$$AI = (NAD/NTD) \times (C/n)$$

Where NAD denotes the number of accrued days since the last coupon payment, NTD denotes the number of total days during the coupon payment period, n

denotes the number of coupon payments per year, and C is the stated annual coupon amount.

Example: After two months (60 days), a 3% semi-annual coupon bond with par of 1,000 would have accrued interest of AI = (60/180) × (30/2) = 5.

Important to remember:

- The accrued interest is expressed in currency (not percent) and the number of total days (NTD) depends on the coupon payment frequency (semi-annual on 30/360 day count convention would be 180).

We know that Forward price is equal to Future value of underlying adjusted for carry cash flows, as stated below:

$$= FV_{0,T}(S_0 + \Theta_0 - \gamma_0)$$

- For the fixed-income bond, let T + Y denote the underlying instrument's current time to maturity. Therefore, Y is the time to maturity of the underlying bond at Time T, when the contract expires.
- Let $B_0(T + Y)$ denote the quoted price observed at Time 0 of a fixed-rate bond that matures at Time T + Y and pays a fixed coupon rate.
- For bonds quoted without accrued interest, let AI_0 denote the accrued interest at Time 0.
- The carry benefits are the bond's fixed coupon payments, γ_0 = present value of all coupon interest paid over the forward contract horizon from Time 0 to Time T = $PVCI_{0,T}$.
- Future value of these coupons is $\gamma_T = FVCI_{0,T}$.
- Assuming no carry costs, $\Theta_0 = 0$.

S_0 = Quoted bond price + Accrued interest = $B_0(T + Y) + AI_0$ (1)

Fixed-income futures contracts: Fixed-income futures contracts often have more than one bond that can be delivered by the seller. These bonds are usually traded at different prices based on maturity and stated coupon, therefore, an adjustment known as the ***conversion factor*** is used to make prices of all deliverable bonds equal (roughly, not exactly).

In Fixed-incomefutures contracts markets, the futures price, $F_0(T)$, is defined as

Quoted futures price × conversion factor= $QF_0(T) \times CF(T)$

In general, the futures contract are settled against the quoted bond price without accrued interest. Thus, the total profit or loss on a long futures position = $B_T(T + Y) - F_0(T)$. Based on above equation (1), this profit or loss can be expressed as follows:

$$(S_T - AI_T) - F_0(T)$$

Adjusted Price of fixed-income forward or futures price including the conversion factor can be expressed as

$F_0(T) = QF_0(T)\, CF(T)$ =Future value of underlying adjusted for carry cash flows = $FV_{0,T}[S_0 - PVCI_{0,T}]$ = Future value (Quoted bond price + accrued interest - coupon payments made during the life of the contract) = $FV_{0,T}[B_0(T+Y) + AI_0 - PVCI_{0,T}]$

Steps of Carry arbitrage in the bond market:

Step 1: Buy the underlying bond, requiring S_0 cash flow.
Step 2: Borrow an amount equivalent to the cost of the underlying bond, S_0.
Step 3: Sell the futures contract at $F_0(T)$.
Step 4: Borrow the arbitrage profit.

Exhibit 11. Cash Flows for Fixed Rate Coupon Bond Futures Pricing

Steps	Cash Flow at Time 0	Cash Flow at Time T
1. Buy bond	$-S_0 = -[B_0(T + Y) + AI_0]$ $= -[107 + 0.07]$ $= -107.07$	$S_T + FVCI_{0,T}$ $= 110.20 + 0.0$ $= 110.20$
2. Borrow	$+S_0 = 107.07$	$-FV_{0,T}(S_0)$ $= -(1+0.002)^{0.25}(107.07)$ $= -107.12$
3. Sell futures	0	$F_0(T) - B_T(T + Y)$ $= 108 - 110$ $= -2$
4. Borrow arbitrage profit	$+PV_{0,T}[F_0(T) - FV_{0,T}(S_0) + AI_T + FVCI_{0,T}]$ $= (1 + 0.002)^{-0.25}[108 - 107.12 + 0.20 + 0.0]$ $= 1.0795$	$-[F_0(T) - FV_{0,T}(S_0) + AI_T + FVCI_{0,T}]$ $= -[108 - 107.12 + 0.20 + 0.0]$ $= -1.08$
Net cash flows	$+PV_{0,T}[F_0(T) - FV_{0,T}(S_0) + AI_T + FVCI_{0,T}]$ $=1.0795$	0

- The value of the Time 0 cash flows ***should be zero*** or else there is an arbitrage opportunity.
- If the value in the Time 0 column for net cash flows is positive, then we buy bond, borrow, and sell futures.
- If the Time 0 column is negative, then we conduct the reverse carry arbitrage strategy, i.e. short sell bond, lend, and buy futures.

In equilibrium, to eliminate an arbitrage opportunity, $PV_{0,T}[F_0(T) - FV_{0,T}(S_0) + AI_T + FVCI_{0,T}] = 0$ or $F_0(T) = FV_{0,T}(S_0) - AI_T - FVCI_{0,T}$

$QF_0(T)$ = Conversion factor adjusted future value of underlying adjusted for carry = $[1/CF(T)]\{FV_{0,T}[B_0(T + Y) + AI_0] - AI_T - FVCI_{0,T}\}$

Practice:Example 9, Reading 39, Curriculum.

Cash Flows for Offsetting a Long Forward Position:

Steps	Cash Flow at Time 0	Cash Flow at Time t	Cash Flow at Time T
1. Buy bond forward contract at 0	0	$V_t(T)$	$V_T(0,T) = B_T(T+Y) - F_0(T)$ = 108 – 107.12 = 0.88
2. Sell bond forward	NA	0	
contract at t			$V_T(t,T) = F_t(T) - B_T(T+Y)$ = 107.92 – 108 = –0.08
Net cash flows	0	$V_t(T)$	$F_t(T) - F_0(T)$ = 107.92 – 107.12 = 0.8

Practice:Example 10, Reading 39, Curriculum.

3.6 Currency Forward and Futures Contracts

The carry arbitrage model with foreign exchange presented here is also known as covered interest rate parity and sometimes just ***interest rate parity***. We will discuss two strategies here.

Strategy #1: Invest one currency unit in a domestic risk-free bond. Thus, at Time T, we have the original investment grossed up at the domestic interest rate or the future value of 1DC, denoted FV(1DC). Future value at Time T of this strategy is expressed as $FV_{£,T}(1)$, given British pounds as the domestic currency.

Strategy #2:

1) Firstly, the domestic currency is converted at the current spot exchange rate, S_0(FC/DC), into the foreign currency (FC), that is, S_0(DC/FC) = 1/S_0(FC/DC).
2) Then, FC is invested at the foreign risk-free rate until Time T. For example, the future value at Time T of this strategy can be expressed as $FV_{€,T}(1)$ - denoting the future value of one euro, given that the euro is the foreign currency.
3) And then, we enter into a forward foreign exchange contract to sell the foreign currency at Time T in exchange for domestic currency with the forward rate denoted F_0(DC/FC,T). So, for example, F_0(£/€,T) is the rate on a forward commitment at Time 0 to sell one euro for British pounds at Time T. This transaction is equivalent to taking short position in the euro in pound terms or being long the pound in euro terms for delivery at Time T.

Based on the two strategies, the value at Time T follows:

Strategy 1: Future value at Time T of investing £1: $FV_{£,T}(1)$

Strategy 2: Future value at Time T of investing £1: F_0(£/€,T)$FV_{€,T}(1)S_0$(€/£)

Solving for the forward foreign exchange rate, the forward rate can be expressed as

F_0(£/€,T) = Future value of spot exchange rate adjusted for foreign rate

$$= FV_{£,T}(1)/\left[FV_{€,T}(1)S_0(€/£)\right] = S_0(£/€)FV_{£,T}(1)/FV_{€,T}(1)$$

- The higher the foreign interest rate, the greater the benefit, and hence, the lower the forward or futures price.

Practice:Example 11, Reading 39, Curriculum.

Assumingannual compounding and denoting the risk-free rates r£ and r€, respectively, we have

F_0(£/€,T) = S_0(£/€)(1 + r£)^T/(1 + r€)^T

Assuming continuous compounding and denoting these risk-free rates in domestic (UK) and eurozone as $r_{£,c}$ and $r_{€,c}$, respectively, we have

$$F_0(£/€,T) = S_0(£/€)\, e^{(r_{£,c} - r_{€,c})T} \text{ (Continuously compounded version)}$$

Carry arbitrage model: The carry arbitrage model based on S_0(FC/DC) = 1/S_0(DC/FC) and F_0(FC/DC) =1/F_0(DC/FC) can be expressed as follows:

$$F_0(DC/FC,T) = S_0(DC/FC)\frac{(1+r_{DC})^T}{(1+r_{FC})^T} \text{ or } F_0(FC/DC,T) = S_0(FC/DC)\frac{(1+r_{FC})^T}{(1+r_{DC})^T}$$

For, continuous compounding:

F_0(DC/FC,T) = S_0(DC/FC)$e^{(rDC,c - rFC,c)T}$

F_0(FC/DC,T) = S_0(FC/DC)$e^{(rFC,c - rDC,c)T}$

- The interest rate in the numerator should be the rate for the country whose currency is specified in the spot rate quote. The interest rate in the denominator is the rate in the other country.
- Similarly, in continuous compounding formula, the first interest rate in the exponential will be the rate for the country whose currency is specified in the spot rate quote.

In equilibrium,

F_0(£/€,T) = S_0(£/€)$FV_£(1)/FV_€(1)$

Please refer to following table for cash flows for offsetting a long forward position:

Cash Flows for Offsetting a Long Forward Position

Steps	Cash Flow at Time 0	Cash Flow at Time t	Cash Flow at Time T
1. Buy forward contract at 0	0	$V_t(T)$	$V_T(0,T) = S_T(£/€) - F_0(£/€,T)$ $= 1.2 - 0.804 = 0.396$
2. Sell forward contract at t	NA	0	$V_T(t,T) = F_t(£/€,T) - S_T(£/€)$ $= 0.901 - 1.2 = -0.299$
Net cash flows	0	$V_t(T)$	$+F_t(£/€,T) - F_0(£/€,T)$ $= 0.901 - 0.804 = 0.097$

The forward value observed at t of a T maturity forward contract = Present value of the difference in foreign exchange forward prices. That is,

$$V_t(T) = \text{Present value of the difference in forward prices}$$
$$= PV_{£,t,T}[F_t(£/€,T) - F_0(£/€,T)]$$

Practice:Example 12, Reading 39, Curriculum.

3.7 Comparing Forward and Futures Contracts

Forward pricing: $F_0(T) = FV_{0,T}(S_0 + \Theta_0 - \gamma_0)$

Note that the price of a forward commitment is a function of the price of the underlying instrument, financing costs, and other carry costs and benefits.

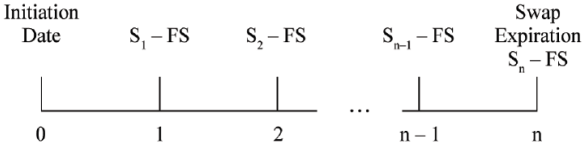
Forward valuation: $V_t(T) = PV_{t,T}[F_t(T) - F_0(T)]$

Futures prices are generally found using the same model, but unlike forwards, futures values are zero at the end of each day because daily market to market settlement.

4. PRICING AND VALUING SWAP CONTRACTS

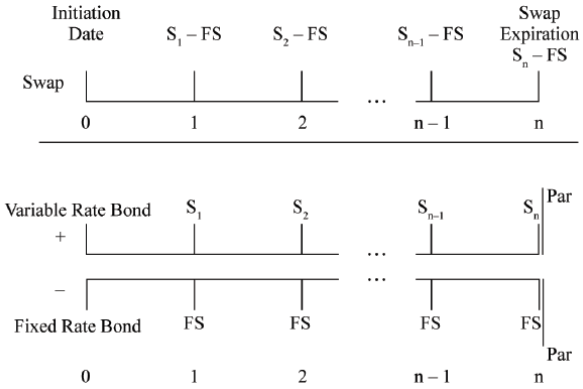
Swap contracts can be synthetically created by either a portfolio of underlying instruments or a portfolio of forward contracts. Thus, swaps can be viewed as a portfolio of futures contracts. A swap can also be viewed as a portfolio of option because a single forward contract can be viewed as a portfolio of a call and a put option.

Generic Swap Cash Flows: Receive-Floating, Pay-Fixed



- A ***receive-floating, pay-fixed swap*** is equivalent to being long a floating-rate bond and short a fixed-rate bond. If both bonds are purchased at par, the initial cash flows are zero and the par payments at the end offset each other. Also, note that the coupon dates on the bonds match the settlement dates on the swap and the maturity date matches the expiration date of the swap.

Receive-Floating, Pay-Fixed as a Portfolio of Bonds



Uses of Swaps: Swaps can be used to manage interest rate risk. E.g. we can create a synthetic floating-rate bond by entering a receive-fixed, pay-floating interest rate swap. This swap can be used to hedge exposure to fixed rate loan. The two fixed rate payments (i.e. on loan and swap) cancel each other, leaving on net the floating-rate payments.

There are also currency swaps and equity swaps. Currency swaps can be used to manage both interest rate and currency exposures. Equity swaps can be used to manage equity exposure.

Like OTC products, swaps can be designed with an infinite number of variations. A swap can have both

semi-annual payments and quarterly payments, as well as actual day counts and day counts based on 30 days per month. Also, the notional amount can vary across the maturities. Due to differences in payment frequency and day count methods as well as identifying the appropriate discount rate to apply to the future cash flows, the pricing and valuation of swaps is a bit tricky.

4.1 Interest Rate Swap Contracts

Interest rate swaps have two legs, typically a floating leg (FLT) and a fixed leg (FIX). The floating leg cash flow (denoted Si) can be expressed as follows:

$$S_i = CF_{FLT,i} = AP_{FLT,i} r_{FLT,i} = \left(\frac{NAD_{FLT,i}}{NTD_{FLT,i}}\right) r_{FLT,i}$$

The fixed leg cash flow (denoted FS) can be expressed as follows:

$$FS = CF_{FIX,i} = AP_{FIX,i} r_{FIX} = \left(\frac{NAD_{FIX,i}}{NTD_{FIX,i}}\right) r_{FIX}$$

Where,

- CF_i represents cash flows
- APi denotes the accrual period
- r denotes the observed floating rate appropriate for Time i
- NADi denotes the number of accrued days during the payment period
- NTDi denotes the total number of days during the year applicable to cash flow i
- rFIX denotes the fixed swap rate.

Types of day count methods: The two most popular day count methods are known as 30/360 and ACT/ACT.

- As the name suggests, 30/360 treats each month as having 30 days, and thus a year has 360 days.
- ACT/ACT treats the accrual period as having the actual number of days divided by the actual number of days in the year (365 or 366).

In swap market, the floating interest rate is assumed to be advanced set and settled in arrears; thus, $r_{FLT,i}$ is set at the beginning of period and paid at the end. If we assume constant accrual periods, the receive-fixed, pay-floating net cash flow can be expressed as follows:

$$FS - S_i = AP(r_{FIX} - r_{FLT,i})$$

And the receive-floating, pay-fixed net cash flow can be expressed as follows:

$$S_i - FS = AP(r_{FLT,i} - r_{FIX})$$

Example: Suppose, a fixed rate is 5%, the floating rate is 5.2%, and the accrual period is 30 days based on a 360 day year, the payment of a receive-fixed, pay-floating swap is calculated as (30/360)(0.05 – 0.052) = –0.000167 per notional of 1. Because the floating rate > fixed rate, the party that pays floating (and receives fixed)would pay this amount to the party that receives floating (and pays fixed).

Swap pricing: Swap pricing involves determining the equilibrium fixed swap rate. The fixed swap rate is simply one minus the final present value term divided by the sum of present values (as discussed in detail below). Suppose the arbitrageur enters a receive-fixed, pay-floating interest rate swap with some initial value V. Please see the cash flows for receive-fixed swap hedge with bonds as stated below:

Cash Flows for Receive-Fixed Swap Hedge with Bonds

Steps	Time 0	Time 1	Time 2	...	Time n
1. Receive fixed swap	–V	+FS – S_1	+FS – S_2	...	+FS – S_n
2. Buy floating-rate bond	–VB	+S_1	+S_2	…	+S_n + Par
3. Short sell fixed-rate bond	+FB	–FS	–FS	...	–(FS + Par)
Net cash flows	–V – VB + FB	0	0	0	0

In equilibrium, we must have –V – VB + FB = 0 or else there is an arbitrage opportunity.
For a receive fixed and pay floating swap, the value of the swap is

V = Value of fixed bond – Value of floating bond = FB – VB

- The value of a receive-fixed, pay-floating interest rate swap is simply the value of buying a fixed-rate bond and issuing a floating-rate bond.
- The value of a floating-rate bond, assuming we are on a reset date and the interest payment matches the discount rate, is par, assumed to be 1 here.
- The value of a fixed bond is as follows:

$$\text{Fixed bond rate: } FB = C\sum_{i=1}^{n} PV_{0,t_i}(1) + PV_{0,t_n}(1)$$

Where, C denotes the coupon amount for the fixed-rate bond and $PV_{0,ti}$ (1) is the appropriate present value factor for the i^{th} fixed cash flow.

$$\text{Swap pricing equation: } r_{FIX} = \frac{1 - PV_{0,t_n}(1)}{\sum_{i=1}^{n} PV_{0,t_i}(1)}$$

- The fixed swap leg cash flow for a unit of notional amount is simply the fixed swap rate adjusted for the accrual period, i.e. $FS_i = AP_{FIX,i} r_{FIX}$.
- The annualized fixed swap rate = fixed swap leg cash flow / fixed rate accrual period, or
  $r_{FIX,i} = FS/AP_{FIX}$
- The fixed swap payment will vary if the accrual period varies across the swap payments.

Practice:Example 13, Reading 39, Curriculum.

Interest rate swap valuation:
The value of a fixed rate swap at some future point in Time t is simply the sum of the present value of the difference in fixed swap rates times the stated notional amount (denoted NA), or

$$V = NA(FS_0 - FS_t)\sum_{i=1}^{n'} PV_{t,t_i}$$

- Positive (negative) value of FS_0 represents value to the party receiving (paying) fixed.

Please refer below to the Cash Flows for Receive-fixed Swap Valued at Time t:

Steps	Time t	Time 1	Time 2	...	Time n′
1. Receive fixed swap (Time 0)	–V	$+FS_0 - S_1$	$+FS_0 - S_2$	...	$+FS_0 - S_{n'}$
2. Receive floating swap (Time t)	0	$S_1 - FS_t$	$S_2 - FS_t$	...	$S_{n'} - FS_t$
Net cash flows	–V	$FS_0 - FS_t$	$FS_0 - FS_t$	...	$FS_0 - FS_t$

Practice:Example 14, Reading 39, Curriculum.

4.2 Currency Swap Contracts

A currency swap is a contract in which two counterparties agree to exchange future interest payments in different currencies. There are four major types of currency swaps:

1. Fixed-for-fixed
2. Floating for- fixed
3. Fixed-for-floating
4. Floating-for-floating

Important to Remember:

- Currency swaps often (but do not always) involve an exchange of notional amounts at both the initiation of the swap and at the expiration of the swap.
- The payment on each leg of the swap is in a different currency unit, such as euros and dollars, and the payments are not netted.
- Each leg of the swap can be either fixed or floating.

Currency swap pricing has three key variables: These include two fixed interest rates and one notional amount.

The value of a fixed-rate bond in Currency k can be expressed as

$$FB_k = C_k\sum_{i=1}^{n} PV_{0,t_i,k}(1) + PV_{0,t_n,k}(Par_k)$$

Where k = a or b, C_k denotes the periodic fixed coupon amount in Currency k,

$\sum_{i=1}^{n} PV_{0,t_i,k}(1)$ denotes the present value from Time 0 to Time t_i discounting at the Currency k risk-free rate, and Par_k denotes the k currency unit par value.

Here, par is not assumed to be equal to 1 because the notional amounts are typically different in each currency within the currency swap. Please refer to table below for cash flows for currency swaps hedged with Bonds.

Cash Flows for Currency Swap Hedged with Bonds

Steps	Time 0	Time 1	Time 2	...	Time n
1. Enter currency swap	$-V_a$	$+FS_a - S_1FS_b$	$+FS_a - S_2FS_b$	...	$+FS_a + NA_a - S_n(FS_b + NA_b)$
2. Short sell bond in Currency a	$+FB_a(C_a = FS_a)$	$-FS_a$	$-FS_a$	...	$-(FS_a + Par_a)$
3. Buy bond in Currency b	$-S_0FB_b(C_b = FS_b)$	$+S_1FS_b$	$+S_2FS_b$	...	$+S_n(FS_b + Par_b)$
Net cash flows	$-V_a + FB_a - S_0FB_b$	0	0	0	0

Based on this table, in equilibrium we must have $-V_a + FB_a - S_0FB_b = 0$

Fixed-for-fixed currency swap value is $Va = FB_a - S_0FB_b$ or else there is an arbitrage opportunity.
Note that the exchange rate S_0 is the number of Currency a units for one unit of Currency b at Time 0; thus, S_0FB_b is expressed in Currency a units.

Swap value after initiation = $V_a = FB_a - S_0FB_b$

In equilibrium, the notional amounts of the two legs of the currency swap are $NA_b = Par_b$ and $NA_a = Par_a = S_0Par_b$.

In order to determine the fixed rates of the swap such that the current swap value is zero, we have

$FB_a(C_{0,a}, Par_a) = S_0FB_b(C_{0,b}, Par_b)$

The equilibrium fixed swap rate equations for each currency:

$$r_{FIX,a} = \frac{1 - PV_{0,t_n,a}(1)}{\sum_{i=1}^{n} PV_{0,t_i,a}(1)}$$

and

$$r_{FIX,b} = \frac{1 - PV_{0,t_n,b}(1)}{\sum_{i=1}^{n} PV_{0,t_i,b}(1)}$$

The fixed swap rate in each currency is simply one minus the final present value term divided by the sum of present values.

Numerical Example of Currency Swap Hedged with Bonds

Steps	Time 0	Time 1	Time 2	...	Time 10
1. Enter currency swap	0	+$10 – ($1.5/€)€9 = –$3.5	+$10 – ($1.1/€)€9 = $0.1	...	+$10 + $1,300 –($1.2/€)(€9 + €1,000) = $99.2
2. Short sell US dollar bond	+$1,300	–$10	–$10	…	–($10 + $1,300)
3. Buy euro bond	– ($1.3/€) €1,000	+($1.5/€)€9	+($1.1/€)€9	...	+($1.2/€)(€9 + €1,000)
Net cash flows	0	0	0	0	0

- If the initial swap value is positive, then we would follow the set of transactions stated in the table above.
- If the initial swap value is negative, then the opposite set of transactions would be implemented, that is, we would enter into a pay-US dollar, receive-euro swap, buy Currency a bonds, and short sell Currency b bonds.

Practice:Example 15, Reading 39, Curriculum.

Fixed-for-floating currency swap: A fixed-for-floating currency swap is simply a fixed-for-fixed currency swap paired with a floating-for-fixed interest rate swap.

Cash Flows for Currency Swap Hedged with Bonds

Steps	Time t	Time 1	Time 2	...	Time n′
1. Currency swap	$-V_a$	$+FS_{a,0} - S_1FS_{b,0}$	$+FS_{a,0} - S_2FS_{b,0}$	...	$+FS_{a,0} + NA_{a,0} - S_{n'}(FS_{b,0} + NA_{b,0})$
2. Short sell bond (a)	$+FB_a$	$-FS_{a,0}$	$-FS_{a,0}$	...	$-(FS_{a,0} + NA_{a,0})$
3. Buy bond (b)	$-S_tFB_b$	$+S_1FS_{b,0}$	$+S_2FS_{b,0}$	...	$-S_{n'}(FS_{b,0} + NA_{b,0})$
Net cash flows	0	0	0		0

Value of a fixed-for-fixed currency swap = $V_a = FBa - S_0FB_b$

$$= FS_{a,0}\sum_{i=1}^{n'} PV_{t,t_i,a} + NA_{a,0}PV_{t,t_{n'},a} - S_t\left(FS_{b,0}\sum_{i=1}^{n'} PV_{t,t_i,b} + NA_{b,0}PV_{t,t_{n'},b}\right)$$

The currency swap valuation equation can be expressed as

$$V_a = NA_{a,0}\left(r_{FIX,a,0}\sum_{i=1}^{n'} PV_{t,t_i,a} + PV_{t,t_{n'},a}\right) - S_tNA_{b,0}\left(r_{FIX,b,0}\sum_{i=1}^{n'} PV_{t,t_i,b} + PV_{t,t_{n'},b}\right)$$

Practice:Example 16, Reading 39, Curriculum.

4.3 Equity Swap Contracts

An equity swap is an OTC derivative contract in which two parties agree to exchange a series of cash flows whereby one party pays variable cash flows based on an equity and the other party pays either (1) a variable cash flows based on a different equity or rate or (2) a fixed cash flow. Equity swaps are widely used in equity portfolio investment management to modify returns and risks.

Three common types of equity swaps are

i. Receive-equity return
ii. Pay-fixed; receive equity return
ii. Pay-floating; and receive-equity return, pay-another equity return. It can be viewed simply as a receive-equity a, pay-fixed swap combined with a pay-equity b, receive-fixed swap. The fixed payments cancel out each other – remaining with equity portion.

Important to Remember:

- The underlying reference instrument for the equity leg of an equity swap can be an individual stock, a

published stock index, or a custom portfolio.
- The equity leg cash flow can include or exclude dividends.
- Like interest rate swaps, equity swaps have a fixed or floating interest rate leg.

The equity swap cash flows can be expressed as follows:

For receive-equity, pay-fixed = NA (Equity return – Fixed rate)

For receive-equity, pay-floating = NA(Equity return – Floating rate)

For receive-equity, pay-equity = NA(Equity return$_a$ – Equity return$_b$)

Where, a and b denote different equities.

- When the equity leg of the swap is negative, then the receive-equity counterparty must pay both the equity return as well as the fixed rate.
- Equity swaps may cause liquidity problems because if the equity return is negative, then the receive-equity return, pay-floating or pay-fixed swap may result in a large negative cash flow.

Practice:Example 17, Reading 39, Curriculum.

The ***cash flows for the equity leg of an equity swap*** can be expressed as

$$S_i = NA_E R_{Ei}$$

Where, NA_E denotes the notional amount and R_{Ei} denotes the periodic return of the equity either with or without dividends as specified in the swap contract.

The ***cash flows for the fixed interest rate leg of the equity swap*** can be expressed as

$$FS = NA_E AP_{FIX} r_{FIX}$$

Where AP_{FIX} denotes the accrual period for the fixed leg for which we assume the accrual period is constant and r_{FIX}denotes the fixed rate on the equity swap.

Please refer to the table below for Cash Flows for Receive-Fixed Equity Swap Hedged with Equity and Bond.

Steps	Time 0	Time 1	Time 2	...	Time n
1. Enter equity swap	–V	+FS – S_1	+FS – S_2	...	+FS – S_n
2. Buy NA_E equity	$-NA_E$	$+S_1$	$+S_2$	…	$+S_n + NA_E$
3. Short sell fixed-rate bond	+FB(C = FS)	–FS	–FS	...	–(FS + Par)
4. Borrow arbitrage profit	–PV(Par – NA_E)				Par – NA_E
Net cash flows	$-V - NA_E$ + FB – PV(Par – NA_E)	0	0	0	0

Equity swap value is $V = -NA_E + FB - PV(Par - NA_E)$

The fixed swap rate can be expressed as

$$r_{FIX} = \frac{1 - PV_{0,t_n}(1)}{\sum_{i=1}^{n} PV_{0,t_i}(1)}$$

- In a pay-floating swap, there is no need to calculate price of the swap because the floating side effectively prices itself at par automatically at the start.
- If the swap involves paying one equity return against another, there would be no need to price the swap because this arrangement can be viewed as paying equity a and receiving a fixed rate as specified above and receiving equity b and paying the same fixed rate. The fixed rates would cancel each other.
- Valuing an equity swap after the swap is initiated (Vt) is similar to valuing an interest rate swap except that rather than adjust the floating-rate bond for the last floating rate observed (remember, advanced set), the value of the notional amount of equity is adjusted as below.

$$V_t = FB_t(C_0) - (S_t/S_{t-})NA_E - PV(Par - NA_E)$$

Where,
- $FB_t(C_0)$ denotes the Time t value of a fixed-rate bond initiated with coupon C_0 at Time 0,
- S_t denotes the current equity price,
- S_{t-} denotes the equity price observed at the last reset date, and
- PV() denotes the present value function from Time t to the swap maturity time.

Practice: Example 18, Reading 39, Curriculum.

1. INTRODUCTION

A contingent claim is a derivative instrument whose payoff depends on occurrence of a future event. In a contingent claim (unlike forward and futures contracts), one party to the contract receives the right – not the obligation – to buy or sell an underlying asset from another party. The purchase price is fixed over a specific period of time and will eventually expire. Contingent claims include options.

- Options derive their value from an underlying asset, which has value. E.g. the payoff on a call (put) option occurs only if the value of the underlying asset is greater (lesser) than an exercise price that is specified at the time the option is created. If this contingency does not occur, the option is worthless.
- Like forward, futures, and swaps contracts, option valuation models are based on the principle of no arbitrage. Option valuation models typically use two approaches.

1) **Binomial model** – based on discrete time. The binomial model is used to value path-dependent options, which are options whose values depend both on the value of the underlying at expiration and how it got there. Such as American options, which can be exercised prior to expiration. (*Discussed in detail in Section 3*).
2) **Black–Scholes–Merton (BSM) model**, which is based on continuous time. The BSM model is based on the key assumption that the value of the underlying instrument follows a statistical process called ***geometric Brownian motion***. Geometric Brownian motion implies a lognormal distribution of the return, which implies that the continuously compounded return on the underlying is normally distributed. The BSM model values only ***path-independent*** options (i.e. European options), which depend on only the values of their respective underlyings at expiration.

2. PRINCIPLES OF A NO-ARBITRAGE APPROACH TO VALUATION

As discussed in Reading 40, Arbitrage is based on following two fundamental rules as well as law of one price.
Rule #1: Do not use your own money.
Rule #2: Do not take any price risk.

Key assumptions in Option Valuation[1]: In this reading, we will make following key assumptions in estimating values of Options:

1) Replicating instruments are identifiable and investable.
2) There are no market frictions, i.e. transaction costs and taxes.
3) Short selling is allowed with full use of proceeds.
4) The underlying instrument follows a known statistical distribution.
5) Borrowing and lending at a risk-free interest rate is available.

The option payoffs can be replicated with a dynamic portfolio of the underlying instrument and financing.

- **Dynamic Portfolio:** A dynamic portfolio is one whose composition changes over time.

[1] Throughout this reading, cash outflows are treated as negative and inflows as positive.

3. BINOMIAL OPTION VALUATION MODEL

The binomial option valuation model is based on the no-arbitrage approach to valuation.

Value of Call Option at expiration: $c_T = Max(0, S_T - X)$

Value of Put Option at expiration: $p_T = Max(0, X - S_T)$

Where,

- S_t denote the underlying instrument price observed at Time t, where t is expressed as a fraction of a year. E.g. a call option had 60 days to expiration when purchased (T = 60/365), but now only has 35 days to expiration (t = 25/365).
- S_T denotes the underlying instrument price observed at the option expiration date, T.
- c_t denote a European-style call price at Time t and with expiration on Date t = T, where both t and T are expressed in years.
- C_t denote an American-style call price.
- X denote the exercise price.

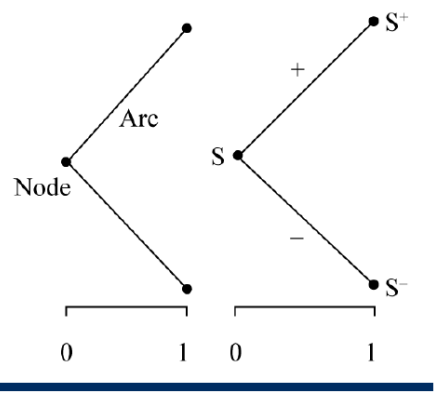
If the option values deviate from these expressions, then there will be arbitrage profits available.

Since, European options cannot be exercised until expiration, they do not technically have exercise values prior to expiration.

Time Value of Options: The time value is always nonnegative for options because of the asymmetry of option payoffs at expiration. For example, for a call option, the upside is unlimited, whereas the downside is limited to zero. At expiration, time value is zero.

3.1 One-Period Binomial Model

The following figure represents a tree for possible outcomes in one-period Binomial Model.



- Each dot represents a particular outcome at a particular point in time in the binomial lattice. These dots are termed ***nodes.***
- At the Time 0 node, there are only two possible future paths in the binomial process, an ***up move*** and a ***down move***, termed as ***arcs***.
- At Time 1, there are only two possible outcomes: S^+ denotes the outcome when the underlying goes up, and S^- denotes the outcome when the underlying goes down.

$$u = \frac{S^+}{S} \text{ (up factor)}$$

$$d = \frac{S^-}{S} \text{ (down factor)}$$

- The up factors and down factors are the total returns.
- The magnitudes of the up and down factors are based on the volatility of the underlying. In general, the higher the volatility, the higher will be the up values and the lower will be the down values.

At expiration, Option value is either

$$c^{++} = Max(0, S^{++} - X) = Max(0, u^2S - X)$$

or

$$c^{+-} = Max(0, S^{+-} - X) = Max(0, udS - X)$$

- The value of a call option is positively related to the value of the underlying. That is, if the underlying goes up (down), value of call option increases (decreases). This implies that in order to ***hedge position***, a trader to be a ***long position*** in the underlying. Specifically, the trader buys a certain number of units, '**h**', of the underlying. The symbol h represents a **hedge ratio** – as estimated below.

$$h = \frac{c^+ - c^-}{S^+ - S^-} \geq 0$$

  - The above formula states that Hedge ratio is the value of the call if the underlying goes up minus the value of the call if the underlying goes down divided by the value of the underlying if it goes up minus the value of the underlying if it goes down.
  - Hedge ratio is non-negative because call prices are positively related to changes in the underlying price.

Writing One Call Hedge with h Units of the Underlying and Finance: The following table shows payoffs of writing one call hedge with h units of the underlying and finance.

Strategy	Time Step 0	Time Step 1 Down Occurs	Time Step 1 Up Occurs
1) Write one call option	+c	−c−	−c+
2) Buy h underlying units	−hS	+hS−	+hS+
3) Borrow or lend	−PV(−hS− + c−) = −PV(−hS+ + c+)	−hS− + c−	−hS+ + c+
Net Cash Flow	+c − hS −PV(−hS− + c−)	0	0

- At Time 0, the value of the net portfolio should always be zero, else there will be an arbitrage opportunity.
- If the net portfolio has positive value, then arbitrageurs will write call option, long the "h" underlying units, and then finance his transaction through borrowing.
- If the net portfolio has negative value, then arbitrageurs will buy call option, short sell the "h" underlying units, and then lend (or invest the proceeds) – pushing the call price up and the underlying price down until the net cash flow at Time 0 is no longer positive.

Long a call option = Owning 'h' shares of stock partially financed

Where,

$$\text{Financed amount} = PV(-hS^- + c^-)$$

or using the per period rate,

$$(-hS^- + c^-) / (1 + r)$$

Single-period call option valuation using no-arbitrage approach:

$$c = hS + PV(-hS^- + c^-)$$

or, equivalently,

$$c = hS + PV(-hS^+ + c^+)$$

Replicating a Call option: A call option can be replicated with the underlying and financing. Specifically, the call option is equivalent to a leveraged position in the underlying. The trading strategy that will generate the payoffs of taking a ***long position*** in a call option within a single-period binomial framework is as follows:

Buy h = (c+ – c–)/(S+ – S–) units of the underlying and financing of – PV(–hS⁻ + c⁻)

Please refer to table below:

Strategy	Time Step 0	Time Step 1 Down Occurs	Time Step 1 Up Occurs
Buy 1 call option	−c	+c−	+c+
OR A REPLICATING PORTFOLIO			
Buy h underlying units	−hS	+hS−	+hS+
Borrow or lend	−PV(−hS− + c−) = −PV(−hS+ + c+)	−hS− + c−	−hS+ + c+
Net	−hS − PV(−hS− + c−)	+c−	+c+

Practice: Example 1, Reading 40, Curriculum.

No-arbitrage single-period put option valuation equation is as follows:

$$p = hS + PV(-hS^- + p^-)$$

or, equivalently,

$$p = hS + PV(-hS^+ + p^+)$$

Where,

$$h = \frac{p^+ - p^-}{S^+ - S^-} \leq 0$$

- For put options, the hedge ratio is negative because p^+ is less than p^-.
- In order to hedge a ***long put*** position, the arbitrageur will ***short sell*** the underlying (–h = –(p+ – p–)/(S+ – S–) units of the underlying) and lend a portion of the proceeds.
- Note that since –h is positive, the value –hS results in a positive cash flow at Time Step 0.

Please refer to table below.

Strategy	Time Step 0	Time Step 1 Down Occurs	Time Step 1 Up Occurs
Buy 1 Put Option	−p	+p−	+p+
OR A REPLICATING PORTFOLIO			
Short sell –h Underlying Units	−hS	+hS−	+hS+
Borrow or Lend	−PV(−hS− + p−) = −PV(−hS+ + p+)	−hS− + p−	−hS+ + p+
Net	−hS − PV(−hS− + p−)	+p−	+p+

Practice: Example 2, Reading 40, Curriculum.

Expectations Approach: The expectations approach results in an identical value as the no-arbitrage approach, but it is usually easier to compute. The formulas are given as follows:

$$c = PV[\pi c^{+} + (1 - \pi)c^{-}]$$

and

$$p = PV[\pi p^{+} + (1 - \pi)p^{-}]$$

Where,

$$\text{Probability of an up move} = \pi = [FV(1) - d]/(u - d)$$

Expected terminal option payoffs: The option values are present value of the expected terminal option payoffs. The expected terminal option payoffs can be expressed as follows:

$$E(c_1) = \pi c^{+} + (1 - \pi)c^{-}$$

and

$$E(p_1) = \pi p^{+} + (1 - \pi)p^{-}$$

Where c1 and p1 are the values of the options at Time 1.

The option values based on the expectations approach can be expressed as follows:

$$c = PV_f[E(c_1)]$$

and

$$p = PV_f[E(p_1)]$$

Difference between Expectations approach and discounted cash flow approach to securities valuation: The expectations approach is often regarded as superior method to the discounted cash flow approach because it is based on objective measures as follows.

i. The expectation is not based on the investor's beliefs regarding the future course of the underlying – implying that the probability, **π**, is ***objectively*** determined and not based on the investor's personal view. This probability is referred to as **risk-neutral (RN) probability** – reason being the expectations approach is not based on assumption regarding risk preferences.
ii. In expectations approach, the discount rate is ***not risk adjusted***, rather it is based on the estimated risk-free interest rate.

Note: The expectations approach can be applied to European-style options. The no-arbitrage approach can be applied to either European-style or American style options because it provides the intuition for the fair value of options.

Practice: Example 3, Reading 40, Curriculum.

Put–call parity:

$$S + p = PV(X) + c$$

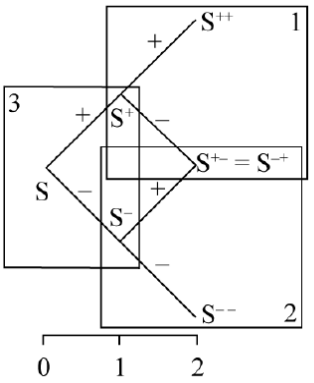
- The value of a put or call option can be found based on put–call parity.
- E.g. Call option can be expressed as a position in a stock, financing, and a put, i.e.

$$c = S - PV(X) + p$$

Practice: Example 4, Reading 40, Curriculum.

3.2 Two-Period Binomial Model

Following figure reflects Two-Period Binomial Lattice as Three One-Period Binomial Lattices.



- For simplicity, it is assumed that the ***up and down factors are constant throughout the lattice***, that is $S^{+-} = S^{-+}$. For example, assume u = 1.25, d = 0.8, and $S_0 = 100$. Note that $S^{+-} = 1.25(0.8)100 = 100$ and $S^{-+} = 0.8(1.25)100 = 100$. So the middle node at Time 2 is 100 and can be reached from either of two paths.
- It is important to remember that Option valuation relies on self-financing, dynamic replication. Dynamic replication is obtained by using a portfolio of stock and the financing. The strategy is self-financing because the funds borrowed at Time 1 grew to 'x' amount.

Call Option Payoffs at Time 2:

$$c^{++} = \text{Max}(0, S^{++} - X) = \text{Max}(0, u^2S - X),$$

$$c^{+-} = \text{Max}(0, S^{+-} - X) = \text{Max}(0, udS - X), \text{ and}$$

$$c^{--} = \text{Max}(0, S^{--} - X) = \text{Max}(0, d^2S - X)$$

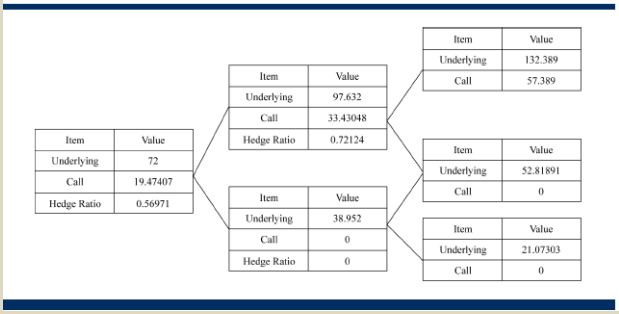
Put Option Payoffs at Time 2:

$$p^{++} = \text{Max}\ (0,\ X - S^{++}) = \text{Max}\ (0, X - u^2S),$$

$$p^{+-} = \text{Max}\ (0,\ X - S^{+-}) = \text{Max}\ (0, X - udS), \text{ and}$$

$$p^{--} = \text{Max}\ (0, X - S^{--}) = \text{Max}\ (0,\ X - d^2S)$$

Example: Following lattice shows the no-arbitrage approach for solving the two-period binomial call value. Suppose the annual interest rate is 3%, the underlying stock is S = 72, u = 1.356, d = 0.541, and the exercise price is X = 75. The stock does not pay dividends.



$$c = PV\ [\pi^2 c^{++} + 2\pi\ (1 - \pi)\ c^{+-} + (1 - \pi)^2 c^{--}]$$

$$p = PV\ [\pi^2 p^{++} + 2\pi\ (1 - \pi)\ p^{+-} + (1 - \pi)^2 p^{--}]$$

The expected terminal option payoffs are expressed as below:

$$E(c_2) = \pi^2 c^{++} + 2\pi\ (1 - \pi)\ c^{+-} + (1 - \pi)^2 c^{--}$$

and

$$E(p_2) = \pi^2 p^{++} + 2\pi\ (1 - \pi) p^{+-} + (1 - \pi)^2 p^{--}$$

The two-period binomial option values based on the expectations approach are expressed as:

$$= PV_r[E\pi(c_2)]$$

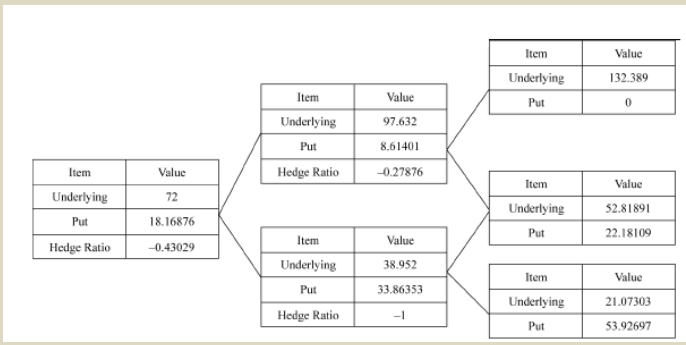
and

$$p = PV_r[E\pi(p_2)]$$

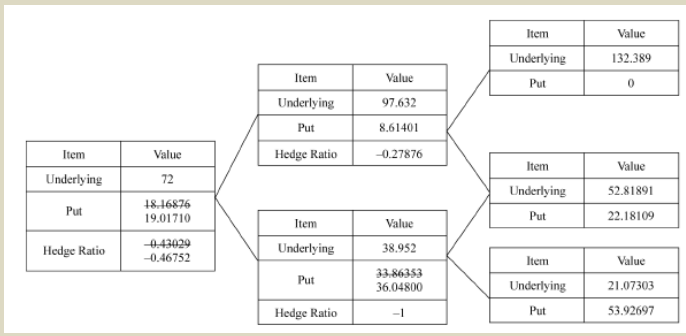
Practice: Example 5, Reading 40, Curriculum.

American-style options: American options are options which can be exercised prior to expiration. A non-dividend paying call options on stock will not be exercised early because the minimum price of the option exceeds its exercise value. However, this is not true for put options (particularly a deep in the- money put) because the sale proceeds can be invested at the risk-free rate and earn interest worth more than the time value of the put.

Example: Suppose the periodically compounded interest rate is 3%, the non-dividend-paying underlying stock is currently trading at 72, the exercise price is 75, u = 1.356, d = 0.541, and the put option expires in two years. Following lattice reflects **"Two-Period Binomial Model for a European-Style Put Option":**



Following lattice reflects **"Two-Period Binomial Model for an American-Style Put Option":**



Put value = $p = PV[\pi p^{+} + (1 - \pi)p^{-}]$

Practice: Example 6, Reading 40, Curriculum.

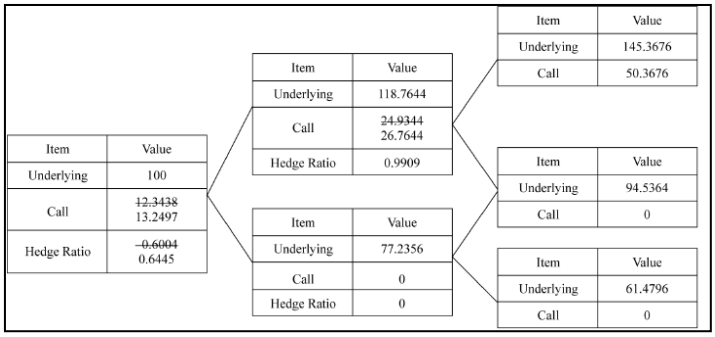
Escrow method: Dividends negatively affect the value of a call option because dividends lower the value of the stock. Most option contracts do not provide protection against dividends. Assuming dividends are perfectly predictable, we can split the underlying instrument into two components: the underlying instrument without the known dividends and the known dividends. For example, the current value of the underlying instrument ***without dividends*** can be expressed as follows:

$$\hat{S} = S - \gamma$$

Where,

γ denotes the present value of dividend payments. ^ symbol is used to denote the underlying instrument without dividends. At expiration, the underlying instrument value is the same, $\hat{S}_T = S_T$. because it is assumed that any dividends have already been paid. The value of an investment in the stock, however, would be $S_T + \gamma T$, which assumes the dividend payments are reinvested at the risk-free rate.

Following lattice reflects **"Two-Period Binomial Model for an American-Style Call Option with Dividends"**



- At Time 0, the present value of the US$3 dividend payment is US$2.970297 (= 3/1.01). Therefore, 118.7644 = (100 – 2.970297)1.224 is the stock value without dividends at Time 1, assuming an up move occurs.
- The exercise value for this call option, including dividends, is 26.7644 [= Max (0,118.7644 + 3 – 95)], whereas the value of the call option per the binomial model is 24.9344.
- The stock price just before it goes ex-dividend is 118.7644 + 3 = 121.7644, so the option can be exercised for 121.7644 – 95 = 26.7644.
- If not exercised, the stock drops as it goes ex-dividend and the option becomes worth 24.9344 at the ex-dividend price.

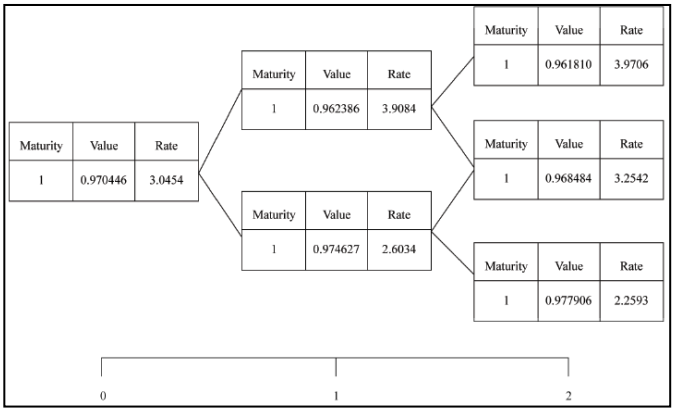
Important to Remember: This example tell us that the American-style call option is worth more than the European-style call option because at Time Step 1 when an up move occurs, the call is exercised early, capturing additional value. For non-dividend paying stocks, the American-style feature has no effect on either the hedge ratio or the option value. American-style put options on non-dividend-paying stock ***may be*** (not necessarily always) worth more than the analogous European style put options.

Practice: Example 7, Reading 40, Curriculum.

3.3 Interest Rate Options

- A call option on interest rates will be in the money when the current spot rate > exercise rate.
- A put option on interest rates will be in the money when the current spot rate < exercise rate.

Example: Following is the Two-Year Binomial Interest Rate Lattice by Year. Assume the notional amount of the options is US$1,000,000 and the call and put exercise rate is 3.25% of par and RN probability is 50%.



- The rates are expressed in annual compounding. Therefore, at Time 0, the spot rate is (1.0/0.970446) – 1 or 3.04540%.
- Note that at Time 1, the value in the column labeled "Maturity" reflects time to maturity not calendar time.

$c^{++} = \text{Max}(0, S^{++} - X) = \text{Max}[0, 0.039706 - 0.0325] = 0.007206$

$c^{+-} = \text{Max}(0, S^{+-} - X) = \text{Max}[0, 0.032542 - 0.0325] = 0.000042$

$c^{--} = \text{Max}(0, S^{--} - X) = \text{Max}[0, 0.022593 - 0.0325] = 0.0$

$p^{++} = \text{Max}(0, X - S^{++}) = \text{Max}[0, 0.0325 - 0.039706] = 0.0$

$p^{+-} = \text{Max}(0, X - S^{+-}) = \text{Max}[0, 0.0325 - 0.032542] = 0.0$

$p^{--} = \text{Max}(0, X - S^{--}) = \text{Max}[0, 0.0325 - 0.022593] = 0.009907$

At Time Step 1, we have

$c^{+} = PV_{1,2}[\pi c^{++} + (1 - \pi)c^{+-}] = 0.962386[0.5(0.007206) + (1 - 0.5)0.000042] = 0.003488$

$c^{-} = PV_{1,2}[\pi c^{+-} + (1 - \pi)c^{--}] = 0.974627[0.5(0.000042) + (1 - 0.5)0.0] = 0.00002$

$p^{+} = PV_{1,2}[\pi p^{++} + (1 - \pi)p^{+-}] = 0.962386[0.5(0.0) + (1 - 0.5)0.0] = 0.0$

$p^{-} = PV_{1,2}[\pi p^{+-} + (1 - \pi)p^{--}] = 0.974627[0.5(0.0) + (1 - 0.5)0.009907] = 0.004828$

At Time Step 0, we have

$c = PV_{rf,0,1}[\pi c^{+} + (1 - \pi)c^{-}] = 0.970446[0.5(0.003488) + (1 - 0.5)0.00002] = 0.00170216$

$p = PV_{rf,0,1}[\pi p^{+} + (1 - \pi)p^{-}] = 0.970446[0.5(0.0) + (1 - 0.5)0.004828] = 0.00234266$

Because the notional amount is US$1,000,000, the call value is = US$1,000,000(0.00170216) = US$1,702.16 and the put value is = US$1,000,000(0.00234266) = US$2,342.66.

3.4 Multiperiod Model

The two-period model divides the expiration into two periods. The three-period model divides expiration into three periods and so forth. Similarly, the multi-period model divides expiration into multiple periods. Each time step is of equal length, i.e., with a maturity of T, if there are n time steps, then each time step is T/n in length.

4. BLACK–SCHOLES–MERTON OPTION VALUATION MODEL

4.2 Assumptions of the BSM model

The stochastic process (wherein value of instrument evolves over time) chosen by Black, Scholes, and Merton is called geometric Brownian motion (GBM).

Assumptions of the BSM model: The standard BSM model assumes a ***constant growth rate*** and ***constant volatility***. The specific assumptions of the BSM model are as follows:

a) The underlying follows a statistical process called ***geometric Brownian motion***, which implies a lognormal distribution of the return – meaning that the continuously compounded return is normally distributed.
b) Geometric Brownian motion implies ***continuous prices***, meaning that the price of underlying instrument does not jump from one value to another; rather, it moves smoothly from value to value.
c) The underlying instrument is ***liquid***, i.e. can be easily bought and sold.
d) ***Continuous trading is available***, i.e. we can trade at every instant.
e) ***Short selling*** of the underlying instrument with full use of the proceeds is allowed.
f) There are ***no market frictions***, i.e. transaction costs, regulatory constraints, or taxes.
g) ***No-arbitrage opportunities are available*** in the marketplace.
h) The ***options are European-style***, meaning that early exercise is not allowed.
i) The continuously compounded ***risk-free interest rate is known and constant***.
j) ***Borrowing and lending*** is allowed at the risk-free rate.
k) The ***volatility of the return*** on the underlying is ***known and constant***.
l) If the underlying instrument pays a yield, it is expressed as ***a continuous known and constant yield at an annualized rate***.

Practice: Example 9, Reading 40, Curriculum.

4.3 BSM model

The BSM model is a ***continuous time version*** of the discrete time binomial model and therefore, continuously compounded interest rate is used in this model. The volatility (σ) is also expressed in annualized percentage terms. The BSM model for stocks can be expressed as follows:

$$c = SN(d_1) - e^{-rT}XN(d_2)$$

and

$$p = e^{-rT}XN(-d_2) - SN(-d_1)$$

Where,

$$d_1 = \frac{\ln(S/X) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

- N(x) reflects the likelihood of observing values less than x from a random sample of observations taken from the ***standard normal distribution***. The ***standard normal distribution*** is a normal distribution with a mean of 0 and a standard deviation of 1.
- The ***normal distribution*** is a symmetric distribution with two parameters, the mean and standard deviation.

BSM model for call option is

$$c = PV_r[E(c_T)]$$

BSM model for put option is

$$p = PV_r[E(p_T)]$$

Where, $E(c_T) = Se^{rT}N(d_1) - XN(d_2)$ and $E(p_T) = XN(-d_2) - Se^{rT}N(-d_1)$. The present value term in this context is simply e^{-rT}.

BSM model can be described as having two components: a stock component and a bond component.

- For call options, the stock component is $SN(d_1)$ and the bond component is $e^{-rT}XN(d_2)$.

BSM model call value = stock component - bond component

- For put options, the stock component is $SN(-d_1)$ and the bond component is $e^{-rT}XN(-d_2)$.

BSM model put value = Bond component - Stock component

- The BSM model can be interpreted as a dynamically managed portfolio of the stock and zero-coupon bonds.
- For both call and put options, we can represent the initial cost of this replicating strategy as follows:

Replicating strategy cost = $n_S S + n_B B$

Where,

- For calls, the equivalent number of underlying shares is $n_S = N(d_1) > 0$ and the equivalent number of bonds is $n_B = -N(d_2) < 0$.
- For puts, the equivalent number of underlying shares is $n_S = -N(-d_1) < 0$ and the equivalent number of bonds $n_B = N(-d_2) > 0$.
- The price of the zero-coupon bond is $B = e^{-rT}X$.

Important to remember: If n is positive, we are buying the underlying and if n is negative we are selling (short selling) the underlying. The cost of the portfolio will exactly equal either the BSM model call value or the BSM model put value.

- A call option can be viewed as a leveraged position in the stock or calls because we are simply buying stock with borrowed money because $n_S > 0$ and $n_B < 0$.
- For call options, $-N(d_2)$ implies borrowing money or short selling $N(d_2)$ shares of a zero-coupon bond trading at $e^{-rT}X$.
- For put options, we are simply buying bonds with the proceeds from short selling the underlying because $n_S < 0$ and $n_B > 0$. A short put can be viewed as an over-leveraged or over-geared position in the stock because the borrowing exceeds 100% of the cost of the underlying. This is because a short position in a put will result in receiving money today and $n_S > 0$ and $n_B < 0$.
- For put options, $N(-d_2)$ implies lending money or buying $N(-d_2)$ shares of a zero-coupon bond trading at $e^{-rT}X$.

Comparison between BSM and Binomial Option Valuation Model: The following table summarized difference between BSM and Binomial Valuation model.

Option Valuation Model Terms	Call Option		Put Option	
	Underlying	Financing	Underlying	Financing
Binomial Model	hS	PV(–hS– + c–)	hS	PV(–hS– + p–)
BSM Model	$N(d_1)S$	$-N(d_2)e^{-rTX}$	$-N(-d_1)S$	$N(-d_2)e^{-rTX}$

- If the value of the underlying, S, increases, then the value of $N(d_1)$ also increases because S has a positive effect on d_1. Thus, the replicating strategy for calls requires continually buying shares in a ***rising market*** and selling shares in a ***falling market***.
- In practical, hedges are imperfect because (i) frequent rebalancing by buying and selling the underlying adds significant costs for the hedger because trading involves transaction costs; (ii) market may move discontinuously (contrary to the BSM model's assumption mentioned above) which requires continuous hedging adjustments, and (iii) volatility cannot be known in advance.

Practice: Example 10, Reading 40, Curriculum.

Probability that the call option expires in the money: Probability that the call option expires in the money is denoted as $N(d_2)$, and correspondingly, $1 - N(d2) = N(-d_2)$ is the probability that the put option expires in the money.

Carry benefits: Carry benefits include dividends for stock options, foreign interest rates for currency options, and coupon payments for bond options. Carry benefits tend to lower the expected future value of the underlying.

Carry costs can be treated as negative carry benefits, i.e. storage and insurance costs for agricultural products. Because the BSM model assumes continuous time, these carry benefits can be modelled as a continuous yield, denoted as γ^c or simply γ.

Carry benefit-adjusted BSM model: The carry benefit-adjusted BSM model is expressed as follows:

$$c = Se^{-\gamma T}N(d_1) - e^{-rT}XN(d_2) \text{ and}$$

$$p = e^{-rT}XN(-d_2) - Se^{-\gamma T}N(-d_1)$$

Where,

$$d_1 = \frac{\ln(S/X) + (r - \gamma + \sigma^2/2)T}{\sigma\sqrt{T}}$$

d_2 can be expressed as $d_2 = d_1 - \sigma\sqrt{T}$

Value of a put option = $p + Se^{-\gamma T} = c + e^{-rT}X$

- $E(c_T) = Se^{(r-\gamma)T}N(d_1) - XN(d_2)$
- $E(p_T) = XN(-d_2) - Se^{(r-\gamma)T}N(-d_1)$
- The present value term is denoted as e^{-rT}.

The carry benefit adjusted BSM model can be described as having two components, a stock component and a bond component.

- For call options, the stock component is $Se^{-\gamma T}N(d_1)$ and the bond component is again $e^{-rT}XN(d_2)$.
- For put options, the stock component is $Se^{-\gamma T}N(-d_1)$ and the bond component is again $e^{-rT}XN(-d_2)$

Important to remember:

- If carry benefits increase, they lower the value of the call option and raise the value of the put option.
- The carry benefits tend to reduce d_1 and d_2, and consequently, the probability of being in the money with call options declines as the carry benefit rises.
- Dividends influence the dynamically managed portfolio by lowering the number of shares to buy for calls and lowering the number of shares to short sell for puts. Higher dividends will lower the value of d_1, thus lowering $N(d_1)$. In addition, higher dividends will lower the number of bonds to short sell for calls and lower the number of bonds to buy for puts.

BSM call model for a dividend-paying stock: The BSM call model for a dividend-paying stock can be expressed as follows:

$$Se^{-\delta T}N(d_1) - Xe^{-rT}N(d_2)$$

- The equivalent number of units of stock is $n_S = e^{-\delta T}N(d_1) > 0$ and the equivalent number of units of bonds remains $nB = -N(d_2) < 0$.

BSM put model for a dividend-paying stock: The BSM put model for a dividend-paying stock can be expressed as follows:

$$Xe^{-rT}N(-d_2) - Se^{-\delta T}N(-d_1)$$

The equivalent number of units of stock is $n_S = -e^{-\delta T}N(-d_1) < 0$ and the equivalent number of units of bonds again remains $n_B = N(-d_2) > 0$.

Practice: Example 11 & 12, Reading 40, Curriculum.

Foreign exchange options: For foreign exchange options, $\gamma = r^f$, which is the continuously compounded foreign risk-free interest rate.

Currency options: In currency options, the underlying instrument is the foreign exchange spot rate. Here, the carry benefit is the interest rate in the foreign country because the foreign currency could be invested in the foreign country's risk-free instrument. With currency options, the underlying and the exercise price must be quoted in the same currency unit. The volatility in the model is the volatility of the log return of the spot exchange rate.

BSM model applied to currencies: The BSM model applied to currencies can be described as having two components, a foreign exchange component and a bond component.

- For call options, the foreign exchange component is $Se^{-r^f T}N(d_1)$ and the bond component is $e^{-rT}XN(d_2)$, where r is the domestic risk-free rate.
  BSM call model applied to currencies = Foreign exchange component - Bond component
- For put options, the foreign exchange component is $Se^{-r^f T}N(-d_1)$ and the bond component is $e^{-rT}XN(-d_2)$.
  BSM put model applied to currencies = Bond component - Foreign exchange component

Practice: Example 13, Reading 40, Curriculum.

5. BLACK OPTION VALUATION MODEL

5.1 European Options on Futures

Model for European-style futures options is as below:

$$c = e^{-rT}[F_0(T)N(d_1) - XN(d_2)]$$

$$p = e^{-rT}[XN(-d_2) - F_0(T)N(-d_1)]$$

Where,

$$d_1 = \frac{\ln[F_0(T)/X] + (\sigma^2/2)T}{\sigma\sqrt{T}} \text{ and}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

- $F_0(T)$ denotes the futures price at Time 0 that expires at Time T, and σ denotes the volatility related to the futures price.

Futures option put–call parity can be expressed as

$$c = e^{-rT}[F_0(T) - X] + p$$

The Black model has two components, a futures component and a bond component.

- For call options, the futures component is $F_0(T)e^{-rT}N(d_1)$ and the bond component is again $e^{-rT}XN(d_2)$.
  Black call model = Futures component - Bond component
- For put options, the futures component is $F_0(T)e^{-rT}N(-d_1)$ and the bond component is again $e^{-rT}XN(-d_2)$.
  Black put model = Bond component - Futures component

Futures option valuation based on the Black model involves computing the present value of the ***difference between the futures price and the exercise price***.

- For call options, the futures price is adjusted by $N(d_1)$ and the exercise price is adjusted by $-N(d2)$.
- For put options, the futures price is adjusted by $-N(-d_1)$ and the exercise price is adjusted by $+N(-d_2)$.

Practice: Example 14, Reading 40, Curriculum.

5.2 Interest Rate Options

In interest rate options, the underlying instrument is a reference ***interest rate***, i.e. three-month Libor.

- An interest rate call option gains when the reference interest rate rises.
- An interest rate put option gains when the reference interest rate falls.

For an interest rate call option on three-month Libor with one year to expiration, the underlying interest rate is a forward rate agreement (FRA) rate that expires in one year. The underlying rate of the FRA is a 3-month Libor deposit that is investable in 12 months and matures in 15 months.

Interest rates are set in advance, but interest payments are made in arrears, which is referred to as *advanced set, settled in arrears*.

- The accrual period in FRAs is based on 30/360 whereas the accrual period based on the option is actual number of days in the contract divided by the actual number of days in the year (identified as ACT/ACT or ACT/365).

Example: In a bank deposit, the interest rate is usually set when the deposit is made, say t_{j-1}, but the interest payment is made when the deposit is withdrawn, say t_j. The deposit, therefore, has time until maturity = $t_m = t_j - t_{j-1}$.

Standard market model: In a standard market model, the prices of interest rate call and put options can be expressed as follows:

$$c = (AP)e^{-r(t_{j-1}+t_m)}\left[FRA(0,t_{j-1},t_m)N(d_1) - R_XN(d_2)\right]$$

And

$$p = (AP)e^{-r(t_{j-1}+t_m)}\left[R_XN(-d_2) - FRA(0,t_{j-1},t_m)N(-d_1)\right]$$

Where,

AP denotes the accrual period in years

$$d_1 = \frac{\ln\left[FRA(0,t_{j-1},t_m)/R_X\right] + (\sigma^2/2)t_{j-1}}{\sigma\sqrt{t_{j-1}}}$$

$$d_2 = d_1 - \sigma\sqrt{t_{j-1}}$$

- $FRA(0,t_{j-1},t_m)$ denote the fixed rate on a FRA at Time 0 that expires at Time t_{j-1}, where the underlying matures at Time t_j (= t_{j-1} + t_m), with all times expressed on an annual basis.
- RX denotes the exercise rate expressed on an annual basis.

- σ denotes the interest rate volatility. σ is the annualized standard deviation of the continuously compounded percentage change in the underlying FRA rate.
- Standard market model requires an adjustment when compared with the Black model for the accrual period, that is, $FRA(0,t_{j-1},t_m)$ or the strike rate, RX, are stated on an annual basis, as are interest rates in general.
- The actual option premium is adjusted for the accrual period.

Differences between Black Model and Standard Model:

1) The discount factor is applied to the maturity date of the FRA or t_j ($= t_{j-1} + t_m$), rather than to the option expiration, t_{j-1}.
2) The underlying is an interest rate, specifically a forward rate based on a forward rate agreement or $FRA(0,tj–1,tm)$. It is not a futures price.
3) The exercise price is a rate and reflects an interest rate, not a price.
4) The time to the option expiration, t_{j-1}, is used in the calculation of d_1 and d_2.
5) Both the forward rate and the exercise rate should be expressed in decimal form rather than as percent (for example, 0.01 and not 1.0).

Important to remember: In Black model, a forward or futures price is used as the underlying. In contrast, in BSM model, a spot price is used as the underlying.

Standard market model for calls:

$c = PV[E(c_{tj})]$

Standard market model for puts:

$p = PV[E(p_{tj})]$

Where,

- $E(c_{tj}) = (AP)[FRA(0,t_{j-1},t_m)N(d_1) – RXN(d_2)]$
- $E(p_{tj}) = (AP)\ [RXN(–d_2) – FRA(0,t_{j-1},t_m)N(–d_1)]$

Combinations created with interest rate options:

- If the exercise rate selected in interest rate option is equal to the current FRA rate, then long an interest rate call option and short an interest rate put option is equivalent to a receive-floating, pay-fixed FRA.
- If the exercise rate selected in interest rate option is equal to the current FRA rate, then long an interest rate put option and short an interest rate call option is equivalent to a receive-fixed, pay-floating FRA.
- An interest rate cap is a portfolio or strip of interest rate call options in which the expiration of the first underlying corresponds to the expiration of the second option and so forth. The underlying interest rate call options are called ***caplets***. Thus, a set of floating-rate loan payments can be hedged with a long position in an interest rate cap encompassing a series of interest rate call options.
- An interest rate floor is a portfolio or strip of interest rate put options in which the expiration of the first underlying corresponds with the expiration of the second option and so forth. The underlying interest rate put options are called ***floorlets***. Thus, a floating-rate bond investment or any other floating-rate lending situation can be hedged with an interest rate floor encompassing a series of interest rate put options.
- Long an interest rate cap and short an interest rate floor with the same exercise rate is equal to a receive-floating, pay-fixed interest rate swap. When the cap is in the money, the receive-floating counterparty will also receive an identical net payment. When the floor is in the money, the receive-floating counterparty will also pay an identical net payment.
- Long an interest rate floor and short an interest rate cap with the same exercise rate is equal to a receive-fixed, pay-floating interest rate swap. When the floor is in the money, the receive-fixed counterparty will also receive an identical net payment. When the cap is in the money, the receive-floating counterparty will also pay an identical net payment.
- If the exercise rate selected in interest rate option is set equal to the swap rate, then the value of the cap must be equal to the value of the floor. When an interest rate swap is initiated, its current value is zero and is known as an ***at-market swap***. When an exercise rate is selected such that the cap equals the floor, then the initial cost of being long a cap and short the floor is also zero.

Practice: Example 15, Reading 40, Curriculum.

5.3 Swaptions

A swap option or swaption is an option on a swap. It gives the holder the right, but not the obligation, to enter a swap at the ***pre-agreed*** swap rate (referred to as the exercise rate). Interest rate swaps can be either receive fixed, pay floating or receive floating, pay fixed.

Payer Swaption: A payer swaption is an option on a swap to pay fixed, receive floating.

Receiver Swaption: A receiver swaption is an option on a swap to receive fixed, pay floating.

Swap payments are ***advanced set, settled in arrears***.

Following equation represents the present value of an annuity matching the forward swap payment:

$$\text{PVA} = \sum_{j=1}^{n} PV_{0,t_j}(1)$$

Payer swaption valuation model is expressed as follows:

$$\text{PAY}_{\text{SWN}} = (\text{AP})\text{PVA}[R_{\text{FIX}}N(d_1) - R_X N(d_2)]$$

Receiver swaption valuation model is expressed as follows:

$$\text{REC}_{\text{SWN}} = (\text{AP})\text{PVA}[R_X N(-d_2) - R_{\text{FIX}}N(-d_1)]$$

where

$$d_1 = \frac{\ln(R_{FIX}/R_X)+(\sigma^2/2)T}{\sigma\sqrt{T}}, \text{ and as always,}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

The swaption model requires two adjustments, one for the accrual period and one for the present value of an annuity.

Differences between Swaption Model and Black Model:

i. The discount factor is absent in swaption model. The payoff is a series of payments. Thus, the present value of an annuity used here takes into account the option-related discount factor.
ii. The underlying is the fixed rate on a forward interest rate swap rather than a futures price,.
ii. The exercise price is expressed as an interest rate.
v. Both the forward swap rate and the exercise rate are expressed in decimal form and not as percent (for example, 0.02 and not 2.0).

Payer swaption model value is estimated as follows:

$$\text{PAY}_{\text{SWN}} = \text{PV}[\text{E}(\text{PAY}_{\text{SWN,T}})]$$

Receiver swaption model value is estimated as follows:

$$\text{REC}_{\text{SWN}} = \text{PV}[\text{E}(\text{REC}_{\text{SWN,T}})]$$

Where,

$\text{E}(\text{PAY}_{\text{SWN,T}}) = e^{rT}\text{PAY}_{\text{SWN}}$ and

$\text{E}(\text{REC}_{\text{SWN,T}}) = e^{rT}\text{REC}_{\text{SWN}}$.

The swaption model can also be described as having two components, a swap component and a bond component.

- For payer swaptions, the swap component is $(AP)PVA(R_{FIX})N(d_1)$ and the bond component is $(AP)PVA(R_X)N(d_2)$.
  Payer swaption model value = Swap component - Bond component
- For receiver swaptions, the swap component is (AP)PVA(RFIX)N(–d1) and the bond component is $(AP)PVA(R_X)N(-d_2)$.
  Receiver swaption model value = Bond component - Swap component

Combinations created with Swaptions:

- Long a receiver swaption and short a payer swaption with the same exercise rate is equivalent to entering a receive-fixed, pay-floating forward swap.
- Long a payer swaption and short a receiver swaption with the same exercise rate is equivalent to entering a receive-floating, pay-fixed forward swap.
- If the exercise rate is selected such that the receiver and payer swaptions have the same value, then the exercise rate is equal to the ***at-market forward swap rate***.
- A long position in a callable fixed-rate bond can be viewed as being long a straight fixed-rate bond and short a receiver swaption. The receiver swaption buyer will benefit when rates fall and the swaption is exercised. Thus, the embedded call feature is similar to a receiver swaption.

Practice: Example 16, Reading 40, Curriculum.

6. OPTION GREEKS AND IMPLIED VOLATILITY

6.1 Delta

Option delta is the change in an option value for a ***given small change*** in the value of the underlying stock, holding everything else constant. The option deltas for calls and puts are as follows, respectively

$$\text{Delta}_c = e^{-\delta T}N(d_1)$$

$$\text{Delta}_p = -e^{-\delta T}N(-d_1)$$

- The delta of long one share of stock is +1.0, and the delta of short one share of stock is –1.0.
- Delta is a static risk measure because it does not tell us how likely this particular change would be.
- The range of call delta is 0 and $e^{-\delta T}$ and the range of put delta is $-e^{-\delta T}$ and 0.
- As the stock price increases, the call option goes deeper in the money and the value of $N(d_1)$ moves toward 1.
- As the stock price decreases, the call option goes deeper out of the money and the value of $N(d_1)$ moves toward zero.
- When the option gets closer to maturity, the delta will drift either toward 0 if it is out of the money or drift toward 1 if it is in the money.
- As the stock price changes and as time to maturity changes, the deltas also changes.

Delta neutral portfolio: A delta neutral portfolio refers to setting the portfolio delta to zero. Theoretically, the value of delta neutral portfolio does not change for small changes in the stock instrument.

- Delta neutral implies that the portfolio delta plus $N_H\text{Delta}_H$ is equal to zero. The optimal number of hedging units, N_H, is

$$N_H = -\frac{\text{Portfolio delta}}{\text{Delta}_H}$$

Where,
N_H denote the number of units of the hedging instrument;
Delta_H denote the delta of the hedging instrument, which could be the underlying stock, call options, or put options.

- If N_H is negative, then we should short the hedging instrument.
- If N_H is positive, then we should go long the hedging instrument.

Example: Suppose a portfolio consists of 100,000 shares of stock at US$10 per share. In this case, the portfolio delta is 100,000. The delta of the hedging instrument, stock, is +1. Thus, the optimal number of hedging units, N_H, is –100,000 (= –100,000/1) or short 100,000 shares.

If the portfolio delta is 5,000 and a particular call option with delta of 0.5 is used as the hedging instrument, then to arrive at a delta neutral portfolio, we need to sell 10,000 call options (= –5,000/0.5). Alternatively, if a portfolio of options has a delta of –1,500, then we need to buy 1,500 shares of stock to be delta neutral [= –(–1,500)/1]. If the hedging instrument is stock, then the delta is +1 per share.

Practice: Example 17, Reading 40, Curriculum.

Delta approximation Equation:

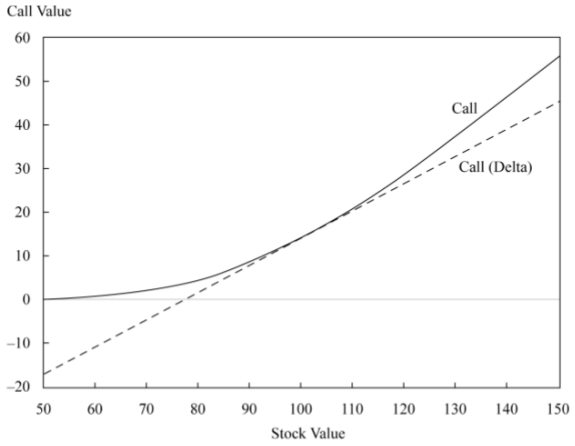
$$\hat{c} - c \cong \text{Delta}_c\left(\hat{S} - S\right) \text{ for calls}$$

or

$$\hat{c} = c + \text{Delta}_c\left(\hat{S} - S\right)$$

$$\hat{p} - p \cong \text{Delta}_p\left(\hat{S} - S\right) \text{ for puts}$$

The delta approximation is fairly accurate for ***very small*** changes in the stock. But as the change in the stock increases, the estimation error also increases. The delta approximation is biased low for both a down move and an up move.



The above chart shows that delta hedging is imperfect and gets worse as the underlying moves further away from its original value of 100.

Practice: Example 18, Reading 40, Curriculum.

6.2 Gamma

Option gamma refers to the change in a given option delta for a given small change in the stock's value, holding everything else constant. Option gamma is a ***measure of the curvature*** in the option price in

relationship to the stock price. Gamma approximates the estimation error in delta for options because the option price with respect to the stock is non-linear and delta is a linear approximation. This implies that gamma measures the ***non-linearity*** risk. A gamma neutral portfolio implies the gamma is zero.

- The gamma of a long or short position in one share of stock is 0 because the delta of a share of stock never changes. The delta of stock is always +1 and –1 for a short position in the stock.
- The gamma for a call and put option are the same and can be expressed as below:

$$\text{Gamma}_c = \text{Gamma}_p = \frac{e^{-\delta T}}{S\sigma\sqrt{T}} n(d_1)$$

Where, $n(d_1)$ is the standard normal probability density function.

- The gamma of a call equals the gamma of a similar put based on put–call parity or $c - p = S_0 - e^{-rTX}$. Note that neither S0 nor e–rTX is a direct function of delta. Hence, the right-hand side of put–call parity has a delta of 1.
- Gamma is always non-negative.
- Gamma is largest near at the money.
- Options deltas do not change substantially for small changes in the stock price if the option is either deep in or deep out of the money.
- As the stock price changes and as time to expiration changes, the gamma also changes.
- Buying options (calls or puts) will always increase net gamma.
- Gamma Risk: It is the risk associated with non-continuous and un smooth change in stock prices.

Important to remember: In delta neutral portfolio strategy, first we need to manage gamma to an acceptable level and then we neutralize the delta is neutralized. This hedging approach is more feasible because options, unlike stocks, have gamma. To alter the portfolio delta, we need to buy or sell stock. Because stock has a positive delta, but zero gamma, the portfolio delta can be brought to its desired level with no impact on the portfolio gamma.

Delta-plus-gamma approximation Equation:

$$\hat{c} - c \approx \text{Delta}_c\left(\hat{S} - S\right) + \frac{\text{Gamma}_c}{2}\left(\hat{S} - S\right)^2 \text{ for calls and}$$

$$\hat{p} - p \approx \text{Delta}_p\left(\hat{S} - S\right) + \frac{\text{Gamma}_p}{2}\left(\hat{S} - S\right)^2 \text{ for puts.}$$

The call value based on the delta approximation is estimated as below:

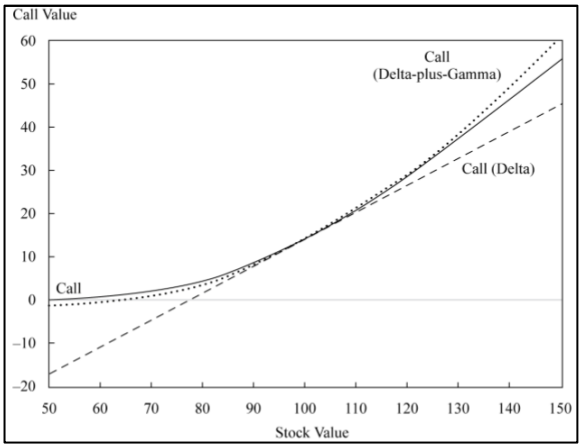
$$\hat{c} = c + \text{Delta}_c\left(\hat{S} - S\right)$$

Call value based on the delta-plus-gamma approximation is expressed as below:

$$\hat{c} = c + \text{Delta}_c\left(\hat{S} - S\right) + \frac{\text{Gamma}_c}{2}\left(\hat{S} - S\right)^2$$

- The delta approximation and the delta-plus-gamma approximations are fairly accurate for very small changes in the stock.

The chart below reflects that the call delta-plus-gamma estimated line is significantly closer to the BSM model call values. We can see that even for fairly large changes in the stock, the delta-plus-gamma approximation is accurate. As the change in the stock increases, the estimation error also increases. The chart also shows that the delta-plus-gamma approximation is biased low for a down move but biased high for an up move.



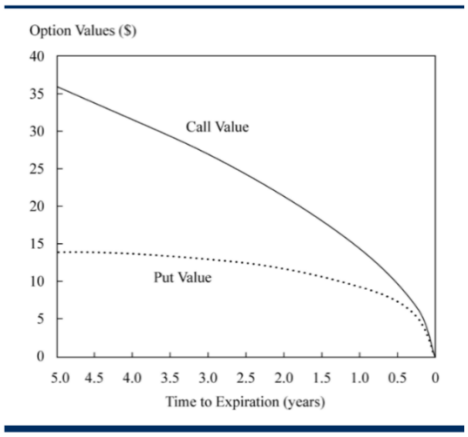
Practice: Example 19, Reading 40, Curriculum.

6.3 Theta

Option theta is the change in an option value for a given small change in ***calendar time***, holding everything else constant. In other words, Option theta is the rate at which the option time value declines as the option approaches expiration. Stock theta is zero because stocks do not have an expiration date. Like gamma, theta cannot be adjusted with stock trades. Typically, theta is negative for options. That is, as calendar time passes, expiration time declines and the option value also declines.

Time decay: It refers to the gain or loss of an option portfolio in response to the mere passage of calendar time. Particularly with long options positions, often the mere passage of time without any change in other variables, such as the stock, will result is significant losses in value.

Please refer to the chart below to assess how the speed of the option value decline increases as time to expiration decreases.



6.4 Vega

Vega is the change in a given portfolio for a given ***small change in volatility***, holding everything else constant. Thus, vega measures the sensitivity of a portfolio to changes in the volatility used in the option valuation model. The vega of an option is positive, i.e., an increase in volatility results in an increase in the option value for both calls and puts.

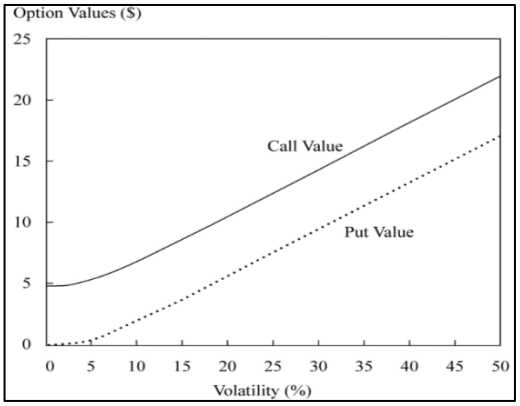
- Based on put–call parity, the vega of a call is equal to the vega of a similar put.
- Vega is high when options are at or near the money and are short dated.
- Volatility is usually only hedged with other options.
- Volatility is sometimes considered a separate asset class or a separate risk factor.

Unlike the delta, gamma, and theta, vega is based on an ***unobservable*** parameter, i.e. future volatility. Future volatility is a subjective measure similar to future value. Option's value is most sensitive to volatility changes.

When volatility is low, the option values tend toward their lower bounds.

- The lower bound of a European-style call option: Zero or the stock less the present value of the exercise price, whichever is greater.
- The lower bound of a European-style put option: Zero or the present value of the exercise price less the stock, whichever is greater.

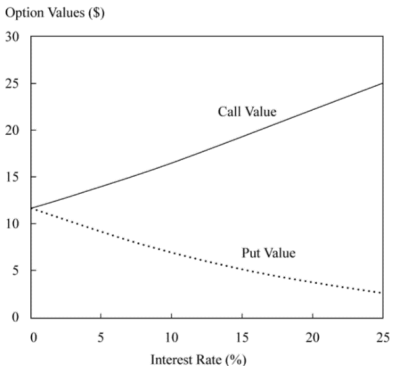
The chart given below shows that the call lower bound is 4.88 and the put lower bound is 0. The difference between the call and put can be explained by put–call parity.



6.5 Rho

Rho is the change in a given portfolio for a given small change in the ***risk-free interest rate***, holding everything else constant. Thus, rho measures the sensitivity of the portfolio to the risk-free interest rate.

- The rho of a call is positive because purchasing a call option allows an investor to earn interest on the money that otherwise would have gone to purchasing the stock. The higher the interest rate, the higher the call value.
- The rho of a put is negative because purchasing a put option rather than selling the stock deprives an investor of the potential interest that would have been earned from the proceeds of selling the stock. The higher the interest rate, the lower the put value.
- When interest rates are zero, the call and put option values are the same for at-the money options.
- As interest rates rise, the difference between call and put options increases.



The option prices not highly sensitive to changes in interest rates change when compared with changes in volatility and changes in the stock.

6.6 Implied Volatility

Implies volatility refers to the volatility estimated from option prices. Implied volatility is a measure of future volatility, whereas historical volatility is a measure of past volatility. The implied volatility can be estimated by using

BSM model. The that implied volatility provides us information regarding the perceived uncertainty going forward and thereby allows us to gauge collective opinions of investors on the volatility of the underlying and the demand for options.

- If the demand for options increases and the no-arbitrage approach is not perfectly reflected in market prices (e.g. due to transaction costs) then the option prices increase, and hence, the observed implied volatility also increases.
- If the implied volatility of a put increases, it indicates that it is more expensive to buy downside protection with a put. Hence, the market price of hedging rises.
- The original BSM model assumes constant volatility of underlying instrument. However, practically, the implied volatilities vary depending on exercise prices and observe different implied volatilities for calls and puts with the same terms. Implied volatility also varies across time to expiration as well as across exercise prices. Implied volatility is also not constant through calendar time.

There are two types of implied volatility:

1) **Term structure of volatility**: The implied volatility with respect to time to expiration is known as the term structure of volatility. The volatility surface is a ***three dimensional plot*** of the implied volatility with respect to both expiration time and exercise prices.
2) **Volatility smile**: The implied volatility with respect to the exercise price is known as the volatility smile or sometimes skew depending on the particular shape. The volatility smile is a ***two dimensional plot*** of the implied volatility with respect to the exercise price.

We can trade futures and options on various volatility indexes available in the market in order to manage our vega exposure in other options.

In the option markets, volatility can be used by investors as the medium in which to quote options. For example, rather than quote a particular call option as trading for €14.23, we may quote it as 30.00, where 30.00 denotes in percentage points the implied volatility based on a €14.23 option price. Quoting the option price in terms of implied volatility allows us to trade volatility.

Important to remember: Ignoring rounding errors, there is a one-to-one relationship between the implied volatility and the option price.

Uses of Implied Volatility:

- Implied volatility can be used to assess the relative value of different options, neutralizing the moneyness and time to expiration effects.
- Implied volatility can be used to revalue existing positions over time.
- Regulators, banks, compliance officers, and most option traders use implied volatilities to communicate information related to options portfolios because implied volatilities provide the "market consensus" valuation.

Example: The Chicago Board Options Exchange S&P 500 Volatility Index, known as the VIX, is a volatility index. The VIX is quoted as a percent and reflects the implied volatility of the S&P 500 over the next 30 days. VIX is often termed the ***fear index*** because it is viewed as a measure of market uncertainty. Thus, an increase in the VIX index is regarded as greater investor uncertainty.
Example: If a trader thinks that based on the current outlook, the implied volatility of S&P 500 (say 20%) should be 25%, it indicates that volatility is understated by the dealer. In this case, since the S&P 500 call is expected to increase in value. Hence, trader would buy the call.

Practice: Example 20 & 21, Reading 40, Curriculum.

1. INTRODUCTION

There are various types of derivative strategies; some of them are purely speculative which are designed to profit if a particular market change occurs, while other strategies are defensive, providing protection against an adverse event or removing the uncertainty around future events.

2. CHANGING RISK EXPOSURES WITH SWAPS, FUTURES, AND FORWARDS

Derivatives markets can be used to quickly and efficiently alter the underlying risk exposure of asset portfolios or forthcoming business transactions.

2.1 Interest Rate Swap/Futures Examples

Interest rate swaps and futures can be used to modify the risk and return of a fixed-income portfolio and can also be used in conjunction with an equity portfolio. Both interest rate swaps and futures are interest-sensitive instruments, so if they are added to a portfolio, they can increase or decrease the exposure of the portfolio to interest rates.

2.1.1.) Interest Rate Swap

It is an agreement to swap in which one party pays fixed interest rate payments (fixed rate is called ***swap rate***) and other party pays floating interest rate payments in exchange or when both parties pay floating-rate payments. When both parties pay floating rates then floating rates are different.

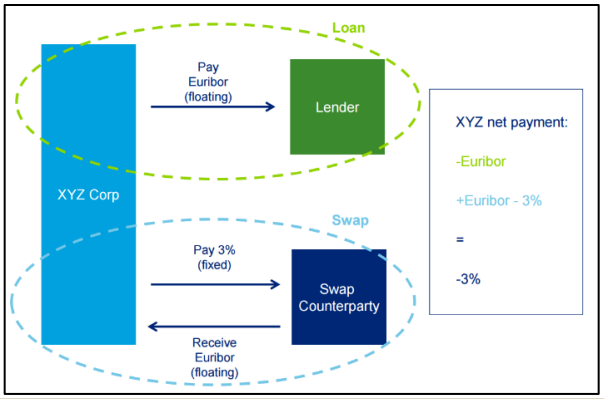
- Interest rate swaps have less credit risk relative to ordinary loans because interest payments are netted and there is no exchange of notional principal.
- However, it is important to note that netting reduces the credit risk but it does not prevent the LIBOR component of the net swap payment from offsetting the floating loan interest payment.

The period of time over which the payments are exchanged is called the ***swap tenor***. The swap expires at the end of this period.

Limitation: Swaps involve credit risk i.e. risk that counterparty may default on the exchange of the interest payments.

Example: XYZ Corp. has €100M of floating-rate debt at Euribor. XYZ would prefer to have fixed-rate debt. XYZ could enter a swap, in which they receive a floating rate and pay the fixed rate, which in the following example, is 3%.

- If a firm thought that rates would rise it would enter into a swap agreement to pay fixed and receive floating in order to protect it from rising debt-service payments.
- If a firm thought that rates would fall it would enter into a swap agreement to pay floating and receive fixed in order to take advantage of lower debt-service payments.
- The swap itself is not a source of capital but an alteration of the cash flows associated with payment.



Example: A portfolio manager has an investment portfolio containing $500 million of fixed-rate US Treasury bonds with an average duration of five years. He wants to reduce this duration to three over the next year but does not want to sell any of the securities.

- One way to do this would be with a pay-fixed interest rate swap in exchange for a floating-rate stream in order to lower the overall duration.
- Suppose the duration of the swap used by the manager is 1.5. This duration is less than the existing portfolio duration, so adding the swap to the portfolio will reduce the overall average duration.

2.1.2.) Interest Rate Futures

A forward contract is an agreement where one party promises to buy an asset from another party at a specified price at a specified time in the future. No money changes hands until the delivery date or maturity of the contract. The terms of the contract make it an

obligation to buy the asset at the delivery date. The asset could be a stock, a commodity or a currency.

A futures contract is very similar to a forward contract. Futures contracts are usually traded through an exchange, which standardizes the terms of the contracts. The profit or loss from the futures position is calculated every day and the change in this value is paid from one party to the other.
Forwards, like swaps, have counterparty risk and can be customized. Futures are standardized and come with greater regulatory oversight and with a clearinghouse that makes counterparty risk virtually zero. These contracts are also sometimes referred to as ***bond futures*** because the underlying asset is often a bond. Hence, the futures price fairly consistently and proportionately moves with the yield that drives the underlying bond.

- We can reduce duration of our portfolio by selling bond futures.

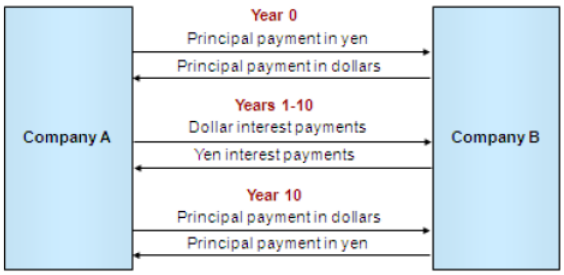
2.2 Currency Swap/Futures Examples

2.2.1.) Currency Swap

Currency swaps can be used by investors to manage exchange rate risk. In a currency swap, the interest rates are associated with different currencies and principal must be specified in each currency and the principal amounts are exchanged at the beginning and end of the life of the swap.

Currency Swaps can be used to transform a loan denominated in one currency into a loan denominated in another currency.

Example: Company B is a U.S. based firm and it borrows yen and engages in a swap with the company A that borrows dollars with parallel interest and principal repayment schedules:



Example:
A firm ABC needs £30 million expand into Europe. To implement this expansion plan, a firm needs to borrow Euros. Suppose current exchange rate is €1.62/£. Thus, a firm needs to borrow €48.60 million. Instead of directly borrowing Euros, a firm can use currency swap e.g. if a firm issues fixed rate pound denominated bond for 30 million pounds with interest rate of 5% (annual interest payments). A firm enters into a currency swap contract in which it will pay 30 million pounds to dealer and receives 48.60 Euros. The terms of a swap are i.e.

- Firm will pay 3.25% in Euros to a dealer.
- Firm will receive 4.50% in pounds from a dealer.

Exchange of principals at contract initiation:

- Firm ABC will receive €48.60 million from currency swap dealer.
- Currency swap dealer will receive £30 million from Firm ABC.

Cash flows at each settlement:

- Interest payments on pound-denominated bond = £30,000,000 × 0.05 = £1,500,000.
- Interest payments due to Firm ABC from swap dealer = £30,000,000 × 0.045 = £1,350,000.
- Interest payments that Firm ABC owes swap dealer = €48,600,000 × 0.0325 = €1,579,500.

At swap and bond maturity:

- Firm ABC will receive £30 million from currency swap dealer and uses that amount to discharge its liabilities.
- Firm ABC will pay €48.60 million to currency swap dealer.

Difference between currency swaps and interest rate swaps:

- Currency swaps involve the payment of notional principal. However, it is important to note that not all currency swaps involve the payment of notional principal.
- Unlike interest rate swaps, interest payments in currency swaps are not netted as they are in different currencies.

2.2.2.) Currency Futures

Foreign currency futures can also be used in managing risk. Following table summarizes the appropriate strategy to pursue in managing foreign currency risk.

Currency Exposure	Position in Foreign Currency	Action taken to hedge currency risk
Receiving Foreign Currency	Long	Sell Futures Contract
Paying Foreign Currency	Short	Buy Futures Contract

Example: A firm expects to receive a payment in British pounds worth £10 million. Payment will be received in 60 days. Current spot exchange rate = $1.45/ £ 60-days Futures exchange rate = $1.47/ £
A firm is long foreign currency because it expects to receive foreign currency. Therefore, a firm should take short position in a futures contract i.e. using futures contract a firm will receive (after 60 days): £10,000,000 × $1.47/£ = $14,700,000. This amount will be received by the firm irrespective of exchange rate at that time.

2.3 Equity Swap/Futures Examples

2.3.1.) Equity Swap

In an equity swap One party is obligated to make payments based on the total return of some equity index e.g. S&P 500 or an individual stock. The other party pays a fixed rate, a floating rate, or the return on another index.

- Equity swaps are created in the over-the-counter market, so they can be customized.

Strategies:

A. When investor has bearish outlook towards stock market and interest rates are falling → Swap equity return for fixed rate.

B. When investor has bearish outlook towards stock market and interest rates are increasing→ Swap equity return for floating rate.

Example: Consider the following table:

Pay the return on a $100 million equity portfolio

Receive six-month Libor, assumed to be 0.50%

Scenario 1: Equity portfolio rises 1%

Pay: $100 million × 1% =	$1,000,000
Receive: $100 million × 0.50% × 0.50 =	250,000
Net payment =	$750,000

Scenario 2: Equity portfolio declines 1%

Pay: $100 million × −1% =	($1,000,000)
Receive: $100 million × 0.50% × 0.50 =	250,000
Net receipt =	$1,250,000

- In the first scenario, the institutional investor would have an obligation to pay 1% × $100 million, or $1 million. On the Libor portion of the swap the investor would receive 0.50% × 0.50 × $100 million, or $250,000. The institutional investor would pay the netted amount of $750,000.
- In the second scenario, the return the institutional investor must pay is negative, which means it will receive money both from "paying" a negative return and from the Libor rate. It would receive $1 million from the "negative payment" and $250,000 from Libor, for a total of $1.25 million.

2.3.2.) Stock Index Futures

Stock index futures (unlike most other futures contracts) are cash settled at expiration. The market risk can be ***temporarily*** removed by selling stock index futures. One S&P 500 stock index futures contract is standardized as $250 times the index level.

Example: Assume that a one-month futures contract trades at 2,000 and that the portfolio carries average market risk, having a beta of 1.0. To fully hedge the $100,000,000 portfolio, the portfolio manager would want to sell $100,000 / ($250 × 2000) = 200 contracts.

- Suppose the S&P 500 stock index rises by 0.5% and thus, the index value is 2,012 at delivery time.

  Loss = −10 points per contract × $250 per point × 200 contracts = $500,000.
- If the stock index rises by 0.5%, the portfolio would also be expected to rise to = $100,000,000 × 0.5% = $500,000.

Practice: Example 1, Reading 41, Curriculum.

3. POSITION EQUIVALENCIES

3.1 Synthetic Long Asset

Synthetic long position = Buys a call + Writes a put = Long Call + Short Put

Where, both options have the same expiration date and the same exercise price.

- The long call creates the upside and the short put creates the downside of the underlying.
- The call exercises when the underlying is higher than the strike and turns into a synthetic position in the upside of the underlying.
- A short put obligates the writer to purchasing the stock at a higher price than its value from put buyer.

3.1 Synthetic Short Asset

Synthetic Short Position = Buy Put + Write Call = Long Put + Short Call

Where, both options have the same expiration date and the same exercise price.

3.3 Synthetic Assets with Futures/Forwards

Synthetic risk-free rate or Synthetic Cash = Long stock + Short futures

Or

Stock – Futures = Risk-free rate

Similarly, we can create a ***synthetic long position*** by investing in the risk-free asset and using the remaining funds to margin a long futures position, that is,

Stock = Risk-free rate + Futures

3.4 Synthetic Put

Synthetic Put = Short stock position + Long call

Important to Note: Any mispricing in a replicated put may make it cheaper or more expensive than a direct put.

3.5 Synthetic Call

Synthetic Call = Long stock position + Long put

- The long put eliminates the downside risk whereas the long stock leaves the profit potential unlimited.

3.6 Foreign Currency Options

Unlike forwards and future, options have asymmetrical payoffs. This implies that if someone wants to benefit from an appreciating currency "X" but do not want to lock in to a fixed rate, as with a futures or forward, he might buy a one-month call option on "X". Because the spot rate is quoted in "X", the strike will typically be quoted in "X". A foreign currency call option always has a put option that is an identical twin.

Practice: Example 2, Reading 41, Curriculum.

4. COVERED CALLS AND PROTECTIVE PUTS

Covered Call = Long stock position + Short call position

Covered Call[1] is appropriate to use when an investor:

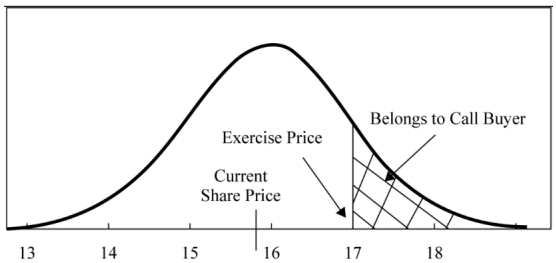
- Owns the stock and
- Expects that stock price will neither increase nor decrease in near future.

4.1 Investment Objectives of Covered Calls

Following are some of the investment objectives of Covered Call:

1) **Income Generation:** The most common motivation for writing covered calls is income generation as writing an option gives option writer option premium. There is a clear trade-off between the size of the option premium and the likelihood of option exercise. The option premium is higher for a longer-term option, but there is a greater chance that the option would move in the money, resulting in the option being exercised by the buyer.

Please refer to the return distribution below for a stock at 15.84, write 17-strike call[2]:



Note that if underlying goes up, the write of covered call bears opportunity loss.

2) **Improving on the Market:** If an investor has higher exposure in (say power sector) and wants to reduce it then he can write call option on those companies. By writing call option, he receives option premium. This income remains in his account regardless of what happens to the future stock price of those companies or whether or not the option is exercised by its holder. Hence, entering into covered call strategy provides her opportunity to reduce his exposure in power sector to desired level as well as generating additional income via option premium.

Option Premium:
The option premium is composed of two parts:

i. **Exercise value (also called intrinsic value):** The difference between the spot price of the underlying asset and the exercise price of the option is termed the intrinsic value of the option. E.g. the right to buy at 15 when the stock price is 15.50 is clearly worth 0.50. Thus, $0.50 is exercise value.

ii. **Time value:** The time value of an option is the difference between the premium of an option and its intrinsic value. E.g. say the option premium is $1.50, which is $1.0 more than the exercise value. This difference of $1.0 is called time value. Someone who writes covered calls to improve on the market is capturing the time value.

- When option is out of the money, the premium is entirely time value.

3) **Target Price Realization:** This strategy involves writing calls with an exercise price near the target price for the stock. Suppose a portfolio manager holds stock of company "X" in many of its accounts and that its research team believes the stock would be properly priced at 25/share, which is just slightly higher than its current price. So, if options trading is allowed, the portfolio manager may write ***near-term*** calls with an ***exercise price near the target price***, 25 in this case. Suppose an account holds 500 shares of "X". Writing 5 SEP 25 call contracts at 0.95 brings in 475 in cash. If the stock is above 25 in a month, the stock will be sold at its target price, with the option premium adding an additional 4% positive return to the account. If "X" fails to rise to 25, the manager might write a new OCT expiration call with the same objective in mind.

In short, covered calls can be used to *generate income*, to acquire shares at a lower-than market price, or to exit a position when the shares hit a target price.

Risks associated with this strategy: Although the covered call writing program potentially adds to the return, there is also the chance that the stock could fall substantially, resulting in an opportunity loss relative to the outright sale of the stock. The investor also would have an opportunity loss if the stock rises sharply above the exercise price and it was called away at a lower-than market price.

[1] If someone creates a call without owning the underlying asset, it is a **naked call**.
[2] 17-strike call" meaning a call option with an exercise price of 17.

4.1.4.) Profit and Loss at Expiration

Payoffs summary:

a) **Value at expiration** = Value of the underlying + Value of the short call = $V_T = S_T - \max(0, S_T - X)$

b) **Profit** = Profit from buying the underlying + Profit from selling the call = $V_T - S_0 + c_0$

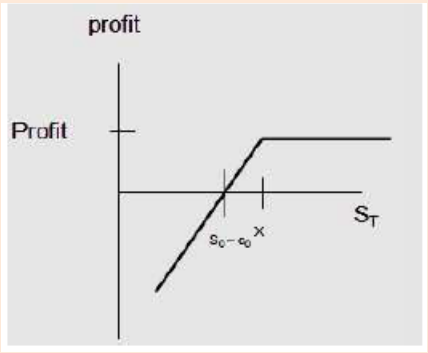
c) **Maximum Profit** = $X - S_0 + c_0$

d) Max loss would occur when $S_T = 0$. Thus, **Maximum Loss** = $S_0 - c_0$

- Even if the stock declines to nearly zero, the loss is less with the covered call because the option writer gets the option premium.

e) **Breakeven** = $S_T^* = S_0 - c_0$

Note that the breakeven price and the maximum loss are the same value.



The general shape of the profit and loss diagram for a covered call is the same as that of writing a put.

Practice: Example 3, Reading 41, Curriculum.

4.2 Investment Objective of Protective Puts

Protective Put = Long stock position + Long Put position

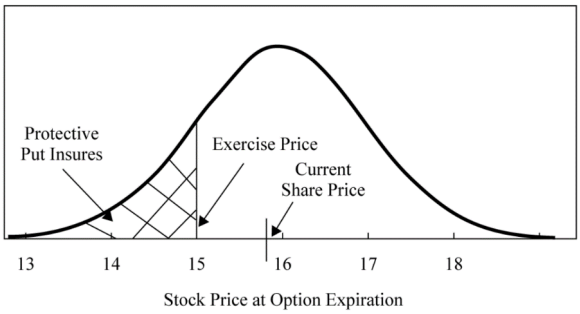
- This provides protection against a decline in value.
- It provides downside protection while retaining the upside potential.
- It requires the payment of cash up front in the form of option premium.
- The higher the exercise price of a put option, the more expensive the put will be and consequently the more expensive will be the downside protection.
- It is similar to "insurance" i.e. buying insurance in the form of the put, paying a premium to the seller of the insurance, the put writer.

Insurance Policy	Put Option
Premium	Time value
Value of asset	Price of stock
Face value	Exercise price
Term of policy	Time until option expiration
Likelihood of loss	Volatility of stock

- As with insurance policies, a put implies a deductible, which is the amount of the loss the insured is willing to bear. This implies that
  Deductible = Stock price - Exercise price
- The cost of insurance can be reduced by increasing the size of the deductible.
- Protective put strategy has a profit and loss diagram similar to that of a long call.

Protective put can be used when an investor expects a decline in the value of the stock in the near future but wants to preserve upside potential. The put value and its time until expiration does not have linear relationship. This implies that a two-month option does not sell for twice the price of a one-month option.

Please refer to the following diagram showing "**Protective Puts and the Return Distribution**"

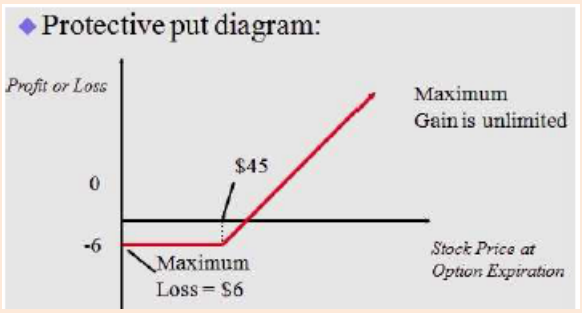


The above diagram shows that the put provides protection from the left tail of the return distribution. It is important to note that the continuous purchase of protective puts is expensive.

4.2.2.) Profit and Loss at Expiration

Payoffs summary:

a) Value at expiration: $V_T = S_T + \max(0, X - S_T)$
b) Profit = $V_T - S_0 - p_0$
c) Maximum Profit = ∞ or unlimited because the stock can rise to any level
d) The maximum loss would occur when underlying asset is sold at exercise price. Thus, Maximum Loss = $S_0 + p_0 - X$ ***or*** "deductible" + cost of the insurance.
e) In order to breakeven, the underlying must be at least as high as the amount paid up front to establish the position. Thus, Breakeven $=ST^* = S_0 + p_0$



Practice: Example 4, Reading 41, Curriculum.

4.3 Equivalence to Long Asset/Short Forward Position

Delta measures the change in option price due to the change in underlying asset price.

- A call option deltas range from 0 to 1 because call increases in value when value of underlying asset increases.
- A put option deltas range from 0 to -1 because put decreases in value when value of underlying asset increases.
- A long position in the underlying asset has a delta of 1.0, whereas a short position has a delta of –1.0.
- At-the-money option will have a delta that is ~0.5 (for a call) or ~–0.5 (for a put).
- Futures and forwards have delta of 1.0 for a long position and –1.0 for a short position.

4.4 Writing Cash-Secured Puts

Writing a cash secured put involves writing a put option and simultaneously depositing an amount of money equal to the exercise price into a designated account. This strategy is also called a fiduciary put. The escrow account provides assurance that the put writer will be able to purchase the stock if the option holder chooses to exercise. Cash in a cash-secured put is similar to the stock part of a covered call.

- This strategy is appropriate for someone who is bullish on a stock or who wants to buy shares at a particular price.
- When someone writes a put but does not escrow the exercise price, it is sometimes called a ***naked put***.

4.6 Collars

Collar refer to the strategy in which the cost of buying put option can be reduced by selling a call option. A collar is also called a ***fence or a hedge wrapper***. In a foreign exchange transaction, it might be called a risk reversal.

- When call option premium is equal to put option premium, no net premium is required up front. This strategy is known as a Zero-Cost Collar. For this reason, most collars are done in the over-the-counter market because the exercise price on the call must be a specific one.
- This strategy provides downside protection at the expense of giving up upside potential.

Typically,

- Put exercise price (e.g. X_1) < current value of the underlying.
- Call exercise price (e.g. X_2) must be > current value of the underlying.
- When price < X_1, put provides protection against loss.
- When price > X_2, short call reduces gains.
- When price lies between X_1 and X_2, both put and call are out-of-the-money.

Payoffs:

a) Initial value of the position = value of the underlying asset = $V_0 = S_0$
b) Value at expiration: V_T = Value of underlying S_T + Value of the put option + Value of the short call option = $S_T + \max(0, X_1 - S_T) - \max(0, S_T - X_2)$
c) Profit = $V_T - V_0 = V_T - S_0$
d) Maximum Profit = $X_2 - S_0$
e) Maximum Loss = $S_0 - X_1$
f) Breakeven $=S_T^* = S_0$

4.6.1.) Collars on an Existing Holding

A collar is typically established on an outstanding position. E.g. consider the risk–return trade-off for a shareholder who previously bought a stock at 12 and now buys the NOV 15 put for 1.46 and simultaneously writes the NOV 17 covered call for 1.44.

Stock price at expiration →	5	10	15	16	17	20
Profit/loss from long stock	–7.00	–2.00	3.00	4.00	5.00	8.00
Profit/loss from long 15 put	8.54	3.54	–1.46	–1.46	–1.46	–1.46
Profit/loss from short 17 call	1.44	1.44	1.44	1.44	1.44	–1.56
Total	2.98	2.98	2.98	3.98	4.98	4.98

- At or below the put exercise price of 15, the collar locks in a profit of 2.98.
- At or above the call exercise price of 17, the profit is constant at 4.98.

4.6.2.) Same-Strike Collar

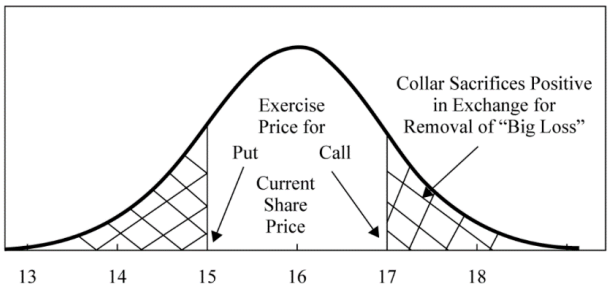
Long a put and short a call is a ***synthetic short position***. When a long position is combined with a synthetic short position, logically the risk is completely neutralized. Hence, if an investor combines a same-strike collar with a long position in the underlying asset, the value of combined position will be the option exercise price, regardless of the stock price at option expiration. *Please refer to the table below.*

Stock price at expiration →	0	20	40	50	60	80	100
Long 50-strike put payoff	50	30	10	0	0	0	0
Short 50-strike call payoff	0	0	0	0	–10	–30	–50
Long stock	0	20	40	50	60	80	100
Total payoff	50	50	50	50	50	50	50

4.6.3.) The Risk of a Collar

A collar forgoes the positive part of the return distribution in exchange for avoiding risk of adverse movement in stock price. See the diagram below (With stock at 15.84, write 17 call and buy 15 put):

- With the long put, the investor is protected against the left side of the distribution and the associated losses.
- With the short call option, the option writer sold the right side of the return distribution, which includes the most desirable outcomes.
- Hence, we can see that the collar tends to narrow the distribution of possible investment outcomes, which is risk reducing.



5. SPREADS AND COMBINATIONS

5.1 Bull Spreads and Bear Spreads

Spreads are classified in two ways, i) by market sentiment and ii) by the direction of the initial cash flows.

- **Bull spread:** A spread whose value increases when the price of the underlying asset rises is a bull spread.
- **Bear Spread:** A spread whose value increases when the price of the underlying asset declines.
- **Debit spread:** It is the spread which requires a cash payment. Debit spreads are effectively long because the long option value exceeds the short option value.
- **Credit spread:** If the spread initially results in a cash inflow, it is referred to as a credit spread. Credit spreads are effectively short because the short option value exceeds the long option value.

Any of these strategies can be created with puts or calls.

5.1.1.) Bull Spread

A spread strategy is appropriate to use with a volatile stock in a ***trending*** market.

Bull Call Spread: This strategy involves a combination of a long position in a call with a lower exercise price and a short position in a call with a higher exercise price i.e. Buy a call (X_1) with option cost c_1 and sell a call (X_2) with option cost c_2, where $X_1 < X_2$ and $c_1 > c_2$.

- Note that the lower the exercise price of a call option, the more expensive it is.

Rationale to use Bull Call Spread: Bull call spread is used when investor expects that the stock price or underlying asset price will increase in the near future.

- This strategy gains when stock price rises/ market goes up.
- Like covered call, it provides protection against downside risk but provides limited gain i.e. upside potential.
- It is similar to Covered call strategy i.e. in bull call spread, the short position in the call with a higher exercise price is covered by long position in the call with a lower exercise price.

Payoffs:

a) The initial value of the Bull call spread = $V_0 = c_1 - c_2$

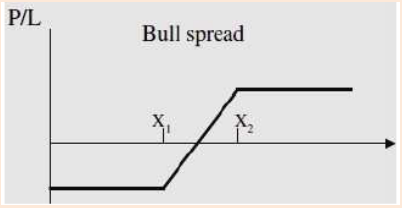
b) Value at expiration: VT = value of long call – Value of short call = max $(0, S_T - X_1)$ - max $(0, S_T - X_2)$

c) Profit = Profit from long call + profit from short call. Thus, Profit = $V_T - c_1 + c_2$

d) Maximum Profit = $X_2 - X_1 - c_1 + c_2$

e) Maximum Loss = $c_1 - c_2$

f) Breakeven $=S_T^* = X_1 + c_1 - c_2$



Bull Put Spread: In bull put spread, investor buys a put with a lower exercise price and sells an otherwise identical put with a higher strike price.

5.1.2). Bear Spread

Bear Put Spread: This strategy involves a combination of a long position in a put with a higher exercise price and a short position in a put with a lower exercise price i.e. Buy a put (X_2) with option cost p_2 and sell a put (X_1) with option cost p_1, where $X_1 < X_2$ and $p_1 < p_2$.

- Note that the higher the exercise price of a put option, the more expensive it is.

Rationale to use Bear Put Spread: Bear Put spread is used when investor expects that the stock price or underlying asset price will decrease in the future.

Payoffs:

a) The initial value of the bear put spread = $V_0 = p_2 - p_1$

b) Value at expiration: V_T = value of long put – Value of short put = max $(0, X_2 - S_T)$ - max $(0, X_1 - S_T)$

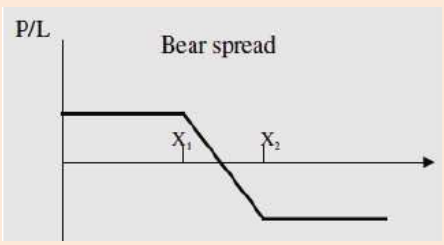
c) Profit = Profit from long put + profit from short put. Thus, Profit = $V_T - p_2 + p_1$

d) Maximum Profit occurs when both puts expire in-the-money i.e. when underlying price ≤ short put exercise price ($S_T \leq X_1$),

- Short put is exercised and investor will buy an asset at X_1 and This asset is sold at X_2 when long put is exercised. Thus, Maximum Profit = $X_2 - X_1 - p_2 + p_1$

e) Maximum Loss occurs when both puts expire out-of-the-money and investor loses net premium i.e. when $S_T > X_2$. Thus, Maximum Loss = $p_2 - p_1$

f) Breakeven $=S_T^* = X_2 - p_2 + p_1$



Bear Call Spread: In bear call spread, investor sells a call with a lower exercise price and buys an otherwise identical call with a higher strike price.

Important to remember:

- With either a bull spread or a bear spread, both the maximum gain and the maximum loss are known and limited.
- Bull spreads with American puts have an additional risk, because the short put gets exercised early, whereas the long put is not yet in the money. In contrast, if the bull spread uses American calls and the short call is exercised, the long call is deeper in the money, which offsets that risk. A similar point can be applied to bear spreads using calls. Thus, with American options, bull spreads with calls and bear spreads with puts are generally preferred (but not necessarily required).
- If puts and calls are bought with different exercise prices, the position is called a **strangle**.

5.1.3.) Refining Spreads

5.1.3.1.) Adding a Short Leg to a Long Position

Suppose, a speculator in September paid a premium of 1.50 for a NOV 40 call when the underlying stock was selling for 37. A month later, in October, the stock has risen to 48. He observes the following premiums for one-month call options.

Strike	Premium
40	8.30
45	4.42
50	1.91

- The call he bought is now worth 8.30. So, his profit at this point is 8.30 – 1.50 = 6.80.
- He thinks the stock is likely to stabilize around its new level; so, he writes another call option with an exercise price of either 45 or 50, thereby converting his long call position into a bull spread.
- At stock prices of 50 or higher, the exercise value of the spread is 10.00 because both options would be in the money, and a call with an exercise price of 40 would always be worth 10 more than a call with an exercise price of 50. The initial cost of the call with an exercise price of 40 was 1.50, and there was a 1.91 cash inflow after writing the call with an exercise price of 50. Thus, the profit is 10.00 – 1.50 + 1.91 = 10.41.
- At stock prices of 40 or lower, the exercise value of the spread is zero; both options would be out of the money. Thus, the profit is 0 – 1.50 + 1.91 = 0.41.
- Between the two striking prices (40 and 50), the exercise value of the spread rises steadily as the stock price increases. For every unit

increase up to the higher striking price, the exercise value of this spread increases by 1.0. For instance, if the stock price remains unchanged at 48, the exercise value of the spread is 8.00. Thus, the profit is 8.00 – 1.50 + 1.91 = 8.41.

The above example tells us that the Bull spread "locks in a profit," but it does not completely hedge against a decline in the value of his new strategy.

Practice: Example given in section "5.1.3.2. Multiple strikes"

Practice: Example 5, Reading 41, Curriculum.

5.1.4.) The Risk of Spreads

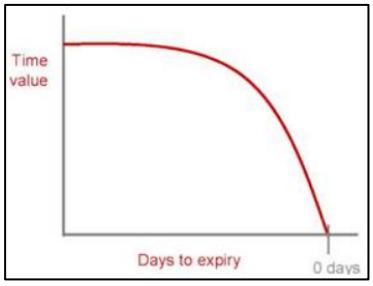
The shape of the profit and loss diagram for the bull spread is similar to that of the collar. Like collars, both the upside return potential and maximum loss is limited in bull spread.

5.2 Calendar Spread

Calendar spread involves ***selling (or writing) a near-dated*** call and ***buying a longer-dated*** call on the same underlying asset and with the same strike. Calendar spread can also be established using put options.

- When a more distant option is bought, it is a ***long calendar spread***.
- **Short calendar spread:** It involves buying a near-term option and selling a longer dated one.

Time value decays over time and approaches zero as the option expiration date approaches as reflected in chart below.



- Time decay is greater for a short-term option than that of a longer-term until expiration.
- A calendar spread trade seeks to exploit this characteristic by purchasing a longer-term option and writing a shorter-term option.

5.3 Straddle

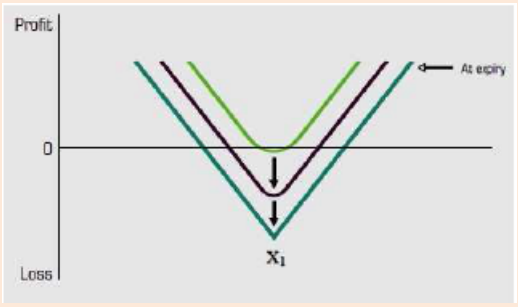
Long straddle: It involves buying a put and a call with same strike price on the same underlying with the same expiration; both options are at-the-money.

- Due to call option, the gain on upside is unlimited and due to put option, downside gain is quite large but limited.
- Straddle is a strategy that is based on the volatility of the underlying. It benefits from high volatility.
- A straddle is neither a bullish nor a bearish strategy; hence, the chosen options usually have an exercise price close to the current stock price.
- Straddle is a costly strategy because the straddle buyer pays the premium for two options. Hence, this implies that in order to make a profit, the underlying asset has to move either above or below the option exercise price by a significant amount (i.e. by the total amount spent on the straddle).
- In other words, in order to be profitable, the "true" underlying volatility of the underlying asset needs to be higher than the market consensus.

Rationale to use Straddle: Straddle is to be used only when the investor expects that volatility of the underlying will be relatively higher than what market expects but is not certain regarding the direction of the movement of the underlying price.

Payoffs:

a) Value at expiration: $V_T = \max(0, S_T - X) + \max(0, X - S_T)$
b) Profit = $V_T - p_0 - c_0$
c) Maximum Profit = ∞ or unlimited
d) Maximum Loss occurs when both call and put options expire at-the money and investor loses premiums on both options i.e. Maximum Loss = $p_0 + c_0$
e) Breakeven = $S_T^* = X \pm (p_0 + c_0)$



5.4 Consequences of Exercise

Options sellers (writers) have an obligation to perform if the option holder chooses to exercise the option. The option writer (seller) has no control over whether or not a contract is exercised, and he must recognize that exercise is possible at any time before expiration. The consequences of exercise can be significant. Hence, it is important to take into consideration those consequences before writing an option.

However, just as the buyer can sell an option back into the market rather than exercising it, a writer can purchase an offsetting contract to end their obligation to meet the terms of a contract provided they have not been assigned.

6. INVESTMENT OBJECTIVES AND STRATEGY SELECTION

6.1 The Necessity of Setting an Objective

For option holders, both the direction of the underlying and its volatility are important. A long call must have some upside volatility, and a long put must have some downside volatility.

Direction and Volatility with Options: The following table summarizes the direction and volatility of options.

		Direction		
		Bearish	**Neutral/No Bias**	**Bullish**
Volatility	High	Buy puts	Buy straddle	Buy calls
	Average	Write calls and buy puts	Spreads	Buy calls and write puts
	Low	Write calls	Write straddle	Write puts

6.2 Spectrum of Market Risk

Suppose a pension fund owns one million shares of ABC Holdings. The portfolio manager would like to temporarily reduce the position by 10%. There are a variety of ways to do this.

1) The manager can sell 100,000 shares, which is 10% of the holding. This strategy enables the manager to accomplish the goal of a 10% reduction. However, this action could create a tax problem for some investors or could result in inadvertently putting downward pressure on the stock price.
2) The manager can enter into a futures or forward contract to sell 100,000 shares. Forward contracts are simple and effective, but they involve counterparty risk and are difficult to cancel if later he needs to unwind this trade.
3) The portfolio manager can write call contracts sufficient to generate minus 100,000 delta points. By writing calls, he can earn cash premium, but this transaction exposes the manager to exercise risk.
4) The manager can buy put contracts sufficient to generate minus 100,000 delta points. Buying puts requires a cash outlay but protects the manager against exercise risk.
5) The manager can enter into a collar sufficient to generate minus 100,000 delta points.

6.3 Analytics of the Breakeven Price

The value of an option depends on variety of factors, including underlying market price, the exercise price of the option contract, the time left until option expiration, the current risk-free rate, and any dividends paid before expiration.

- The exercise price is known, and the underlying market price and risk-free rate are easily accessible.
- The time remaining until expiration is consistently changing but we know how much time is left until expiration.
- Dividends paid by companies are fairly stable.

However, the expected price movement or volatility of the underlying stock is not known with certainty. The higher the expected volatility, the higher the option premium would be.

Example: Suppose, a trader buying a ABC Mar 50 call and ABC Mar 50 put would be purchasing a ABC Mar 50 straddle for 1.95.

Buying a XYZ Mar 50 straddle would involve purchasing the XYZ Mar 50 call at 2.50 and XYZ Mar 50 put at 2.45 for a net cost of 4.95.

Breakeven at expiration for the ABC straddle occurs if the stock moves up or down 1.95 whereas breakeven for the XYZ straddle would require a move of 4.95.

In percentage terms, this means that breakeven for the ABC straddle is 3.90% (1.95/50.00) whereas breakeven for the XYZ straddle is 9.90% (4.95/50.00).

Suppose the underlying stock has an annual volatility of 30%. We know that in straddle, Breakeven occurs at $X \pm (p_0 + c_0)$. In order for the straddle to be profitable at expiration, the stock must move up or down by 2.29 (call option cost) plus 2.28 (put option cost) ➔ 4.57 units to allow the straddle buyer to breakeven (i.e. recover the cost of purchase of options) from the current price of 50, which is a 9.14% movement. With options, volatility is measured by the annual standard deviation[3]. Expiration is in 30 days, but this includes four weekends and possibly a holiday. Suppose there are only 21 trading days until expiration. We convert a 9.14% movement in 21 days to an annual volatility as follows:

[3] An annual variance (σ^2) can be converted into a daily variance by dividing by 252, and an annual standard deviation (σ) can be converted into a daily standard deviation by dividing by $\sqrt{252}$.

$$\sigma_{annual} = 0.0914x\sqrt{\frac{252}{21}} = 32.6\%$$

Practice: Example 6, Reading 41, Curriculum.

6.4 6.4. Applications (Read examples from Curriculum)

1. INTRODUCTION

Forms of Real Estate Investments:

a) **Private equity investment in real estate properties**. It refers to a *direct ownership* of real estate properties.

b) **Publicly traded debt investment**. It refers to an *indirect ownership* of real estate properties i.e. via investing in mortgage-backed securities; real estate investment trusts (REITs) etc.

Types of Lending in the Private real estate market:

a) **Direct lending:** It refers to mortgage-lending by banks or insurance companies. In a mortgage-lending, real estate properties serve as collateral for the loan.

b) **Indirect lending:** It refers to publicly traded debt investment i.e. mortgage-backed securities (MBSs).

2. REAL ESTATE INVESTMENT: BASIC FORMS

Forms of real estate investment market:

1) **Private market:** In a private market, an investor purchases a property (referred to as direct investment in an asset) or provides a mortgage loan to the buyer.

- Mortgage lending is regarded as less risky than equity investment or unsecured lending.
- Direct ownership in properties involves relatively larger investments because of the indivisibility of real estate property. However, it enables investors to have greater decision-making control.
- Private market investment is relatively more ***illiquid***.

Examples:

- Sole ownership
- Joint ventures
- A real estate limited partnerships
- A commingled real estate fund (CREF)
- Mortgages

2) **Public market:** In a public market, an investor indirectly invests in real estate e.g. through investment in:

- A real estate investment trust (REIT)
- A real estate operating company (REOC)
- A mortgage-backed security

- In a publicly traded real estate investment, the ownership or claim on the property can be easily divided. Thus, investors can easily diversify by having claims against more properties.
- In addition, it is more liquid compared to private real estate investment.
- In REOCs and REITs, real estate is professionally managed by managers.

Forms of real estate investment:

1) **Equity investment:** Equity investment refers to a direct ownership interest in a real estate or investment in securities of a company or a REIT that owns the real estate property.

Direct ownership: Direct investment in real estate is usually in large lots and is not easy to divide into smaller pieces; also, direct ownership is less liquid.

REITs: REITs represent shares of publicly-traded companies that buy and sell real estate. It is a form of pooled real estate investment.

- REITs are highly liquid;
- REITs involve professional management;
- REITs involve lower investment than direct ownership of real estate;
- REITs price reflect both the performance of the management of the company that owns the real estate and the value of the underlying properties.
- REITs can be used by investors with short investment horizons and higher liquidity needs;
- REITs have higher correlation with stocks and bonds than direct ownership of real estate; hence, it does not provide the same diversification benefits as that of private real estate.

When the returns to equity real estate investors is not perfectly positively correlated (i.e. $\rho < +1$) with the returns to stocks and/or bonds, then adding equity real estate investments to a traditional portfolio tends to provide diversification benefits.

Return of equity investors in real estate (in both private and public markets) = Cash flows from renting the property + Capital appreciation

Where, capital appreciation = Changes in the value of the underlying real estate.

2) **Debt investment:** Debt investment refers to lending funds to the buyer of real estate where the real estate property serves as collateral for a mortgage loan or investment in securities based on real estate lending e.g. mortgage-backed securities (MBSs).

- In a mortgage loan, the lender has a priority claim on the real estate. This implies that equity investors have higher risk and thereby require a higher rate of

return than lenders (debt investors).
- As the amount of debt on a property or financial leverage increases, then for both debt and equity:

- Risk increases (relatively more for equity investors)
- Investor's return expectations increase

Value of the equity investor's interest in the real estate = Value of the real estate – Amount owed to the mortgage lender

- Like other fixed-income investments (i.e. bonds), debt investors in real estate (in both private and public markets) earn *return in the form of promised CFs only*.
- Unlike other fixed-income investments, their income streams are secured on real estate assets and their default risks depend on the performance of the real assets and the ability of mortgages to pay interest.

Practice: Example 1, Volume 5, Reading 42.

3. REAL ESTATE: CHARACTERISTICS AND CLASSIFICATIONS

The performance of the real estate investment depends on the value of the underlying real estate property i.e.

- When value of a property increases → capital appreciates → return increases.
- When value of a property increases → probability to receive promised CFs increases → return increases.

3.1 Characteristics

A. Heterogeneity and fixed location: Unlike stocks and bonds, real estate properties are ***not homogeneous*** i.e. they differ in use, size, location, age, type of construction, quality, and tenant and leasing arrangements. In addition, they are ***immobile*** due to their fixed location.

B. High unit value: Due to large sizes and indivisibility, real estate investments have greater unit value compared to stocks and bonds; it implies a greater amount to invest and thereby discourages investment and limits the investors' ability to construct a diversified real estate portfolio.

C. Management intensive: Unlike stocks or bonds, a private real estate equity investment or direct ownership of real estate requires ***active management*** by investors or by hired property managers. As a result, investors need to have a property management expertise (e.g. maintaining the properties, negotiating leases, and collecting rents). It must be stressed that active management is both costly and time-consuming.

D. High transaction costs: Buying and selling real estate properties involve higher transactions costs and is more time-consuming.

E. Depreciation: Value of buildings decreases with use and the passage of time, changes in the desirability of its location, design etc.

F. Need for debt capital: As real estate investments require large amounts to invest, real estate values highly depend on the cost and availability of debt capital.

- When cost of debt capital (i.e. interest rates) is high (low), the value of real estate falls (increases).
- When debt capital is scarce (easily accessible), the value of real estate falls (increases).

G. Illiquidity: Real estate properties are relatively illiquid due to

- Large transaction sizes
- Lack of availability and timeliness of information which requires extensive valuation and due diligence.

H. Price determination: Heterogeneity of real estate properties, low volume of transactions and less informationally efficient markets relative to equity and bonds markets create difficulty in establishing valid benchmarks and performance appraisals. As a result, changes in real estate value or expected selling price over time are determined based on estimates of value or appraisals rather than transaction prices.

The aforementioned properties imply that private real estate investments are suitable for investors with long-term investment horizon and greater ability to tolerate relatively lower liquidity.

NOTE:

In a less efficient market, higher return can be earned through superior information and superior properties evaluation skill.

Practice: Example 2, Volume 5, Reading 42.

3.2 Classifications

A. Residential Properties: These properties provide housing for individuals or families. It is sub-divided as follows:

1) Single-family houses: These include owner-occupied or rental properties.

2) Multi-family properties: These include rental properties even if one of the units is owner-occupied i.e. apartments.

They are differentiated based on:

a) Location (urban or suburban)
b) Shape of structure (high-rise, low-rise or garden apartments)

The demand for multi-family space largely depends on population growth, population demographics (i.e. age segments for renters), ratio of home prices to rent, interest rates etc.

- As home prices increase (fall) → ratio of home prices to rent increases (fall) → demand for renting multi-family space (owning homes) increases.
- The higher (lower) the interest rates → the higher (lower) the cost of financing purchase of a home →the higher (lower) the opportunity cost of owning a home → the less (more) affordable homeownership and consequently, the lower (higher) the demand for homeownership relative to renting.

The length of multi-family properties lease ranges from 6 months to 2 years, (typically 1 year).

In multi-family properties leases, an owner is responsible for oversight, repair & maintenance and insurance of the common property; whereas, the tenants are responsible for cleaning that space rented and for insurance on personal property.

B. Non-residential Properties: These properties include

1) Commercial real estate properties: They are referred to as income-producing real estate properties. It represents the largest class of real estate for investment (by far).

Types of Commercial Real Estate Properties:

a) Multi-family properties: As discussed above.

b) Office properties: They include multi-tenant office buildings and single-tenant office buildings that serve the needs of a specific tenant e.g. medical office building.

- The demand for office properties heavily depends on employment growth of industries and strength of an economy i.e.
  - When economy is strong (weak), employment growth increases (decreases) and the amount of space used per employee tends to increase (decrease).
  - However, the average amount of space per employee tends to decrease with technological advancements as it reduces the need for permanent space.
- The average length of an office building lease varies globally based on the desirability of the property, the financial strength of the tenant, and other lease terms i.e. provisions for future rent changes, option to extend the lease etc.

(For details refer to section 4.3.1).

c) Industrial and warehouse properties: They refer to special purpose properties used for light or heavy manufacturing and the associated warehouse space. Generally, such properties are difficult to convert to another use.

- The demand for industrial and warehouse space heavily depends on the overall strength of the economy, economic growth and import/export activities in the economy.

d) Retail properties: Retail properties include small stores, large shopping centers with several stores, large department stores etc.

- Shopping centers require relatively high active management.
- The demand for retail space heavily depends on trends in consumer spending, which in turn depends on the overall strength of the economy, employment growth, population growth and savings rates.
- Retail lease terms (i.e. length of leases and rental rates) depend on the quality of the property, size and the importance of the tenant e.g.
  - For smaller tenants (e.g. in a shopping center), length of leases is 3-5 years.
  - For larger "***anchor***" tenants (e.g. department store), length of leases is longer than 5 years.

e) Hospitality properties: They include motels, smaller hotels, large hotels, destination resorts with different sizes and amenities available.

- Such properties require the ***most*** day-to-day management and are similar to operating a business.
- Hotels tend to have relatively higher risk because such properties have no leases and their performance may be highly sensitive to the business cycle.

f) Other types: They include parking facilities, restaurants, and recreational uses i.e. country clubs, marinas, sports complexities etc.

- Recreational facilities also require significant active management.

Important to Note: *Generally, the more active management is required, the greater the operational risks, the riskier the properties and as a result, the higher the rate of return required by investors.*

2) **Farmland:** They refer to properties that are used to produce crops or as pastureland for livestock. Crops can be produced annually.

3) **Timberland:** They refer to properties that are used to produce timber (wood) for industrial use purposes. Unlike crops production, timber (wood) production has relatively a longer growing cycle. The harvesting of timber depends on the favorable market conditions.

Return components on farmland and timberland:

i. Capital appreciation (i.e. sale of the commodities);
ii. Income streams from leasing the land to another entity;

- Revenue on farmland and timberland depends on harvest quantities, commodity prices, weather and demographics factors.

Mixed-use Development: A property that has more than one end users is referred to as a mixed-use development e.g. a building that contains both office and retail space.

Practice: Example 3, Volume 5, Reading 42.

4. PRIVATE MARKET REAL ESTATE EQUITY INVESTMENTS

Types of equity real estate investors:

- Endowments
- Pension funds
- Life insurance companies
- Sovereign wealth funds
- Publicly traded real estate companies

For simplicity in this reading, the term investor refers to an *equity investor* in real estate.

Benefits of equity real estate investment:

1. **Current income:** By investing in equity real estate, investors can earn return in the form of current income i.e. income streams (rents) resulting from letting, leasing, or renting the property. Current income return is affected by taxes and financing costs.

2. **Price appreciation (capital appreciation):** Besides current income, by investing in equity real estate investors can earn return in the form of capital (price) appreciation.

3. **Inflation hedge:** Unlike fixed-income securities, equity real estate investment may provide some inflation protection because:

- Both property rents and real estate prices tend to increase with inflation. Hence, the ***real rate of return*** in equity real estate investments tends to be less volatile.
- Construction costs for real estate tend to increase with inflation, leading to increase in real estate values.

4. **Diversification:** Typically, real estate performance is not highly correlated with the performance of other asset classes i.e. stocks, bonds, or money market funds; this implies that adding real estate to a traditional portfolio tends to provide diversification benefits (i.e. risk falls relative to the expected return).

5. **Tax benefits:** Mortgage interest, property taxes and other expenses are tax deductible which provide taxable benefits to owners of real estate e.g.

- For tax purposes, real estate can be depreciated over a shorter period than the actual deterioration period of the property.
- In some countries (e.g. in U.S.), publicly traded REITs are not subject to corporate income taxes.

Indices for various asset classes:

For private real estate equity investments ➔National Council of Real Estate Investment Fiduciaries **(NCREIF)** Property Index.

- NCREIF Index has *low correlation* with S&P 500 index and *negative correlation* with bonds. This indicates that adding private equity real estate investment to a stock and bond portfolio provides potential diversification benefits.
- NCREIF index has relatively a *higher correlation* with the CPI-U than the other alternatives (except T-bills). This indicates that private equity real estate investments may provide some inflation hedge.
- In NCREIF Index, property values are determined by property appraisals and conducted infrequently; as a result, this index lags changes in the transactions market.
- Due to use of appraised values:
  - Returns are smoothed i.e. volatility in underlying values is understated;
  - Correlations with other assets are understated;
  - Benefits of adding real estate to a traditional

portfolio is overstated;

For stocks ➔S&P 500 Index.

For bonds ➔Barclays Capital Government Bond.

For publicly traded real estate investments ➔The National Association of Real Estate Investment Trusts **(NAREIT)** Equity REIT index.

For All Urban Consumers (CPI-U) ➔90-day T-bills and all items U.S. Consumer Price Index.

Use of Real estate Indices: Real estate indices can be used to evaluate the performance (both risk and return) of properties relative to other asset classes and peers.

Practice: Example 4, Volume 5, Reading 42.

4.1 Risk Factors

Risk factors associated with investing in commercial real estate:

1) **Business conditions:** The demand for renting the property and thereby current income depends on numerous international, national, regional, and local economic factors i.e. GDP, employment, household income, interest rates, and inflation. Real estate values are also affected by changes in economic conditions.

2) **Long lead time for new development:** New development real estate projects are usually very time-consuming. Over the development time period, changes in market conditions may cause wide price swings for real estate e.g.

- During weak market conditions, demand can be lower than anticipated → rents reduce and vacancy rates increase; as a result, return decreases.
- During strong market conditions, demand can be greater than anticipated → rents increase to meet current demand and vacancy rates decrease; as a result, return increases.

3) **Cost and availability of capital:** As real estate investment requires substantial amount to invest, real estate investment depends on the availability of debt capital and the cost of that capital as well as the expected return on other investments (e.g. stocks and bonds).

- When cost of debt capital (i.e. interest rates) is high (low) → demand for real estate falls (increases) → the value of real estate decreases (increases).
- When debt capital is scarce (easily available), → demand for real estate falls (increases) → the value of real estate decreases (increases).

4) **Unexpected inflation:** Inflation hedge provided by real estate equity investments depends on responsiveness of net operating income (NOI) and values to ***unexpected*** changes in inflation. In addition, in a weak market with low demand, high vacancy rates and low rents, real estate values may not increase with inflation.

5) **Demographics:** The demand for real estate also depends on numerous demographic factors i.e.

- Size and age distribution of the population in the local market;
- Distribution of socio-economic groups;
- Rates of new household formation.

6) **Lack of liquidity:** Due to large transaction sizes and lack of availability and timeliness of information, real estate equity investment has low liquidity and thus, higher liquidity risk i.e. it is difficult to quickly sell a property at its current value without a significant price discount.

7) **Environmental:** Real estate values are affected by environmental conditions i.e. value reduces due to adverse environmental conditions, harm to property caused by prior owner or an adjacent property owner etc.

8) **Availability of information:** The lack of availability and timeliness of information increase the risk of the real estate equity investment.

9) **Management:** Real estate equity investments require considerable ***active management,*** which involves higher cost of monitoring investments. Investment management can be categorized into following two forms:

a) **Asset management:** It refers to monitoring the financial performance of an investment and making changes when required over time.
b) **Property management:** It refers to overall day-to-day operation, control, physical maintenance and oversight of real estate.

10) **Leverage:** Leverage refers to use of debt to finance some of the purchase price of an investment.

- Unlike other risk factors that affect both the ***value*** of the real estate property and the value of investment, leverage affects ***returns*** on investment in real estate only, not the value of the underlying real estate property).
- The higher the leverage, the greater the risk of investment i.e. a small changes in NOI can result in a relatively large changes in the amount of cash flow available to the equity investor after making the

mortgage payment.
- It must be stressed that lower leverage does not necessarily reduce the sensitivity of property to changes in inflation.

11) **Other risk factors:** Other risk factors include unobserved physical defects in the property, natural disasters (e.g. earthquakes and hurricanes), and acts of terrorism.

NOTE:

- Due to a number of risks associated with investing in commercial real estate, the expected return must be ≥ required return necessary to make the investments.
- It is important to note that *unidentified risks are more serious than identified risks* because identified risks can be managed to some extent e.g. through diversification or using derivative contracts to transfer risks to another party.

Practice: Example 5, Volume 5, Reading 3.

4.2 Real Estate Risk and Return Relative to Stocks and Bonds

The characteristics of real estate and the risk factors associated with investing in commercial real estate affect both the risk and return of equity real estate investments.

Real estate Lease: It is an agreement in which a tenant (like bond issuer/borrower) is legally obliged to make periodic payments to the owner of the property. However, unlike face value of a bond at maturity, the likelihood of lease renewal and the expected income associated with new rental rate leases renew can be quite uncertain and like stock prices depend on market conditions at the end of the lease term i.e.

- Demand/supply of real estate
- Factors that affect the profitability of the leasing companies
- Strength of the overall economy
  - Due to bond-like and stock-like characteristics, risk and return of a real estate ***portfolio*** tends to lie between the risk and return of stocks and bonds.
  - It must be stressed that risk of an ***individual*** real estate investment may be greater or less than that of an individual stock or bond.

4.3 Commercial Real Estate

The institutional investors' portfolio of real estate include the core property types i.e. office, industrial and warehouse, hospitality properties, retail and multi-family (apartments).

The portfolio risk is low when such properties:

- Are ideally located i.e.
  - Properties located in ***best*** locations with modern features, and functionality tends to have the ***highest value per unit of space***.
  - Properties located in ***adequate (not prime)*** locations with *slightly* outdated features tend to have ***moderate value per unit of space***.
  - Properties located in ***poor*** locations with outdated features tend to have the ***lowest*** value per unit of space.
- Have fiscally sound and responsible tenants
  - Typically, the greater (smaller) the number of tenants, the less (more) effect a single tenant can have on cash flows, and thus the lower (higher) the risk.
- Have low vacancies
- Have good rental terms

Lease Structures (Section 4.3.1)

The risk and return of equity real estate investments also depend on the structures of leases between the owner and tenants e.g. lease terms, lease renewal, rent reviews etc.

Forms of Lease:

1) **Net Lease:** In a net lease, a tenant is responsible for paying operating expenses.

- In a net lease, a detailed cost (expense) analysis is not required to estimate net operating income.
- Since the tenant (not owner) is subject to risk of increase in operating expense, *rent of net lease is lower than that of gross lease*.

Rent of Net Lease = Gross rent – Operating expenses

- Typically, *industrial leases are net leases*.

2) **Gross Lease:** In a gross lease, an owner is responsible for paying the operating expenses. In gross lease, a detailed cost (expense) analysis is highly important in estimating net operating income.

Example:

Gross rent = $25 per square foot and operating expenses
= $10 per square foot.
Rent of Net lease = $25 – $10
= $15 per square foot.

- In some leases, an owner is responsible for paying operating expenses in the 1st year of lease; after that, the owner pays ***fixed amount*** paid in the 1st year for every year of the lease and any increase in expenses is passed through to the tenant as an "***expense reimbursement***". In other words, a tenant

is subject to any increase in expenses.

Full repairing and insuring (FRI) basis Lease: In a FRI lease, a tenant is responsible for paying most of the expenses.

Upward-only rent reviews: When at rent review, rents are set at higher of the then market rent or contract rent, it is referred to as upward-only rent review. Rent reviews are upward only in the U.K.

Percentage Lease: In a percentage lease, a tenant must pay a "minimum rent" irrespective of the tenant's sales. However, once its sales reach a certain level (referred to as break-point), it is required to pay additional rent.

Lease Rent = Minimum rent + % of sales revenue above a certain level

When minimum rent calculated by multiplying specified % of sales by break-point is the same as minimum rent specified in a lease agreement, then the break-point is referred to as "***natural break-point***". In case of natural break-point, lease rent can be calculated as follows:

Lease Rent = % of sales revenue above a certain level × Tenant's sales

Mostly, *retail leases are percentage lease*.

Example:

- Minimum rent specified in a lease agreement = $30 per square foot.
- Break-point = $650 per square foot in sales.
- Specified % of sales above break point = 5%.
- Tenant's sales = $800 per square foot.
  Minimum rent = 5% × $650 = $32.50 ➔ Since it is ≠ $30 ➔ $650 is NOT the natural break-point.
  Rent per square foot = Minimum rent + % of sales revenue above a certain level = $30 + 5% ($800 - $650) = $37.50

Practice: Example 8 & 9, Volume 5, Reading 42.

5. OVERVIEW OF THE VALUATION OF COMMERCIAL REAL ESTATE

5.1 Appraisals

The value of the underlying real estate property affects the value of real estate investment. Since commercial real estate trade infrequently, the changes in value or expected selling price over time are determined using estimates of value or appraisals instead of transaction prices. Appraisals are performed either by hired independent appraisal experts or in-house appraisal experts.

Uses of Appraisals:

- The appraised value of underlying properties in REITs can be used to estimate the value of REITs.
- The appraised value of underlying property is used to estimate the value of the collateral for the mortgage loans.
- The appraised value of underlying property is used to estimate the reasonable offer (for buyers) and asking (for sellers) price for the property.
- The appraised value of underlying property can be used for measuring performance of real estate portfolio.

Limitations of Appraisal: Appraising value of properties is costly and time-consuming process.

5.1.1) Value

Value of property will depend on purpose for which it's being appraised.

A. Market value: The market value represents the ***objective*** value of a property to average buyer and sellers in a market. It can be considered as the most probable sale price.

- The objective of appraisal is to determine the market value of a property.
- It is also used to estimate the maximum amount received by a lender from sale of the property i.e.

Maximum amount received by a lender from selling the property = Market value - Transaction costs

B. Market price: It represents the price that is actually paid for a property by a potential buyer. The market price may differ from the market value i.e.

- Market price < market value when seller is under pressure to sell quickly.
- Market price > market value when buyer is under pressure to buy quickly.

C. Investment value: Investment value represents the value to a particular investor based on various factors i.e. particular investor's motivations, suitability to investor's portfolio, the investor's risk tolerance and the investor's tax circumstances etc. Investment value may differ from the market value.

D. Value in use: Value in use represents the ***subjective*** value of property to a particular owner/user e.g. the value of a manufacturing plant building for a particular company, assessed value of a property for

tax purposes etc. Value in use may differ from the market value.

E. Mortgage-lending value: It is the value of a property estimated based on prudent assessment of the future marketability of the property, long term sustainable aspects of the property, the normal and local market conditions, the current use and alternative appropriate uses. It represents a *conservative* value of a property for a lender in a mortgage loan. This value is different from the "mark-to-market" or "fair value".

Practice: Example 10, Volume 5, Reading 42.

5.2.1) Highest and Best Use

Value after construction: It refers to the selling price of a property once it is constructed and leased.

Value after construction = Cost to construct a building + Profit to the developer

Implied land value: It refers to the price of a land i.e.

Implied land value = Value after construction – Cost to construct a building

Highest and best use: The highest and best use of a vacant site refers to the use that would generate the highest value for the land.

- The property that has the *highest implied land value* (***not*** the highest total value) represents the highest and best use of the site.
- As long as the value under existing use > land value, the building should remain on the site.
- If the value under existing use < land value, any building on the site should be demolished* and the building that represents the highest and best use of the site should be constructed in its place.
- **NOTE: Only when cost of demolition is less than (Land value – value under existing use)*
- It must be stressed that when value under existing use < land value, then the value of the building, *not the land,* will be lower.

Example:

- Selling price of a site with an old warehouse = $2 million ➔It represents the Value of land + warehouse
- The value of a vacant land = $2.5 million
- Value of warehouse = -$500,000
- Since this value is negative, the building should be demolished, assuming demolition costs < 500,000

Practice: Example 11, Volume 5, Reading 42.

5.2 Introduction to Valuation Approaches

Three different approaches to estimate real estate value are as follows:

1) Income approach
2) Cost approach
3) Sales comparison approach

7. THE INCOME APPROACH TO VALUATION

Under the income approach, the value of a property depends on the expected rate of return required by investors. It is preferred to use for appraising commercial (income-producing) properties.

There are three types of Income Approach:

1) Direct capitalization method: It capitalizes the current NOI using a growth implicit capitalization rate.

$$\textbf{Appraised value of a property} = \frac{\text{NOI}}{\text{Capitalizationrate}}$$

where,

NOI = Net operating income for the subject property
Gross potential income = Rental income at full occupancy + Other income
Effective gross income = Gross potential income – Vacancy and collected loss

NOI = Effective gross income – Operating expenses
OR
NOI = Gross potential income – Estimated vacancy losses – Estimated collective losses – Insurance – Property Taxes – Utilities - Repairs and maintenance expenses

- Financing costs and federal income taxes are not subtracted to determine the NOI because the value of the property is estimated independently of its financing. It implies that NOI is a ***before-tax unleveraged*** measure of income.
- Depreciation is not subtracted to determine the NOI because it is assumed that repairs and maintenance facilitate the investor to keep the building in good condition forever.
- Estimates of NOI vary with lease terms e.g. in a ***net***

lease (discussed above), operating expenses are not deducted in calculating NOI. If such expenses are deducted, then the additional income associated with "expense reimbursement" is added in calculating the NOI.
- The direct cap method is not appropriate to use for multiple growth rates; rather, DCF or layer method should be used (discussed below).

Practice: Example 12, Volume 5, Reading 42.

Capitalization rate or Cap rate: It is the rate used to discount property's future cash flows (i.e. income, capital gains, tax benefits).

$$\textbf{Capitalization rate} = \frac{\textbf{First-year NOI}}{\textbf{Property value}}$$

- The Capitalization rate ***implicitly*** reflects the growth rate of future NOI, investors' expectations with regard to future changes and risk of income i.e.
  - The lower the expected future NOI (lower growth rate of NOI), the lower the value of a property and the higher the capitalization rate, reflecting less investor optimism.
- Since it is calculated using the current NOI, ***cap rate is < discount rate***. A cap rate can be viewed as a ***current yield*** for the property.

Discount rate: The discount rate is the return required by investors to invest in the property. It is not the same as cap rate.

Discount rate = Risk-free rate + Risk premium specific to the investment

- Discount rate is applied to current and future NOI.
- The growth rate is ***explicit*** in a DCF valuation.
- When rents (income) and value are expected to increase at a constant compound rate after every rent review,

Discount rate = Cap rate + Growth rate

where,

- Cap rate represents the return on 1st year income
- Growth rate represents the growth in income and value over time
  - It is important to note that income and value may not always change at the same compound rate each year.
  - When income and value are expected to remain unchanged, cap rate will the same as the discount rate.
  - If the growth rate is constant,

Value of property = NOI / (discount rate – Growth rate)

Important to Note:

- The cap rate is positively related to discount rate i.e. when discount rate is higher (lower), the cap rate will be higher (lower).
- The cap rate and terminal cap rate is negatively related to the growth rate i.e. when growth rate is higher (lower), the cap rate will be lower (higher).
- The discount rate is positively related to interest rates i.e. as interest rates increase, the discount rates increase.

Example:

NOI in the 1st year = $80,000
Growth rate in NOI = 3%
Growth rate of value = 2%
Discount rate = IRR = 10.5%

Value of the property today = $80,000 / (0.105 – 0.02)
= $941,176.47

When it is assumed that sale price for a comparable property represents a good proxy for the value of the subject property,

$$\textbf{Capitalization rate} = \frac{\textbf{NOI}}{\textbf{Sale price of comparable property}}$$

$$\textbf{Reciprocal of the cap rate} = \frac{\textbf{Price}}{\textbf{Current NOI}}$$

- Like stocks, the greater the income growth potential of a property, the higher the ratio of price to current NOI and the lower the cap rates.

All risks yield (ARY): For a property that is leased at market rent, a cap rate referred to as "all risks yield (ARY)" is used.

$$\boldsymbol{ARY} = \frac{\text{Rent}}{\text{Recent sales prices of comparables}}$$

And

$$\boldsymbol{Market\ value} = \frac{Rent}{ARY}$$

- ARY will differ from the total expected required rate of return (discount rate).
- ARY will be the same as total expected rate of return and will reflect an IRR or yield to maturity if rent is a fixed, level amount for a foreseeable future (like perpetuity).

Going-in cap rate: It is the rate based on the 1st year of ownership i.e. return that investor will accept today as a current yield of existing NOI. It is calculated as (discount rate – growth rate).

Terminal cap rate: It is the cap rate that is used to estimate the resale price or terminal value of a property. It is also called ***residual or resale cap rate***.

- The more uncertain the future growth in NOI, the higher the terminal cap rate.
- As interest rates increase, terminal cap rates may also increase.
- In general terminal cap rate ≥ going-in cap rate because at the time of sale, property will be older and thereby has less NOI growth.
- When terminal cap rate > going-in cap rate, it indicates that an investor is willing to pay higher price for current NOI as it is expected to increase.
- The terminal cap rate will be < going-in cap rate when:
  - Interest rates and thus discount rates are expected to fall in future at the time of sale;
  - NOI and property values are expected to increase at a higher growth rate compared to current market conditions.

Reversion value: The resale price of the property in the future is referred to as the reversion value.

Example:

NOI for the next 3 years = $100,000 per year
NOI in year 4 = $115,000
Growth rate of NOI after Year 4 = 2%
Growth rate of property value after Year 3 = 2%
Discount rate = 10.5%
Holding period = 3 years

Terminal cap rate = 10.5% – 2% = 8.5%
Resale price after 3 years = NOI in Year 4 / Terminal cap rate = $115,000 / 0.085 = $1,352,941

- *This value is also referred to as **reversion value**.*

Using financial calculator, **Current value of the property** is estimated as follows:

PMT = NOI for the next 3 years = $100,000
FV = Resale price = $1,352,941
n = 3 years
i = 10.5%

Solve for PV ➔ $2,860,618 ➔**Current value of property**

Implied Going-in Cap rate = NOI for the next 3 years / Current value of the property = $100,000/$2,860,618 = 3.50%

Practice: Example 16 & 17, Volume 5, Reading 42.

6.2.3) Stabilized NOI

A stabilized NOI refers to a ***level (stable)*** pattern of income i.e. NOI of a non-renovated property or post-renovated NOI of a property.

- During renovation time period, NOI will be lower, resulting in lower or understated value of a property.
- Once the renovation is complete, NOI will be higher, resulting in higher value of a property.

Hence, it implies that stabilized NOI should be used to estimate the value of a property.

Post-renovation Value = Post-renovation NOI of a property / (Discount rate – growth rate)

Loss in income due to renovation = Post-renovation NOI of a property - NOI of a property during renovation time-period

PV of the lost income ➔ Loss in value = Loss in income due to renovation / (1 + discount rate)

Value of a property = Post-renovation Value - Loss in value

OR

Value of a property = {NOI of a property during renovation time-period + [Post-renovation NOI of a property (1 + growth rate)] / (Discount rate – growth rate)]} / (1 + discount rate)

Example:

- Post-renovation NOI of a property = $10 million
- NOI of a property during renovation time-period = $4.5 million
- Post-renovation Growth rate of income = 3.5%
- IRR (discount rate) = 10%

Post-renovation Value = $10 million / (0.10 – 0.035)
= $153.85 million

Loss in income due to renovation = $10 million - $4.5 million = $5.5 million

PV of the lost income = Loss in value
= $5.5 million / 1.10 = $5 million

Value of a property = $153.85 million – $5 million
= $148.85 million

OR

Value of a property = {$4.5 million + [$10 million (1.035)] / (0.10 – 0.035)]} / (1.10)
= $148.85 million

Practice: Example 14, Volume 5, Reading 42.

2) **Discounted cash flow method (DCF):** In this approach, after-tax cash flows from operations and after-tax proceeds from future property disposition (i.e. resale price) are forecasted into the future. These cash flows are then discounted at the investor's required rate of return (i.e. discount rate) on equity to estimate the Present Value of the property.

- The DCF approach is preferred to use for real estate income-producing properties, particularly when number of tenants is large and leases are complex.

Steps to a DCF analysis (Section 6.4):

1) **Project income from existing leases:** The projected income from existing leases would include income from expense reimbursement on leases.
2) **Make lease renewals assumptions:** Market leasing assumptions include:

a) ***Lease renewal probabilities***: They are estimated based on historical data, economic conditions and subjective judgment e.g. 60% probability that the lease will be renewed and a 40% probability that it will not be renewed. The lease renewal probabilities affect property cash flows i.e. rent received from an existing tenant that renews a lease may be lower than that of a new tenant to:

- Avoid risk of vacancy loss until a new tenant is found;
- Avoid tenant improvements expenses for new tenant;
- Avoid lease commissions to a broker for finding new tenants.

b) ***Number of months vacant if a lease is not renewed, and the length for a new lease***. These assumptions can be used to estimate a vacancy rate for the property.

Average number of months vacant until the lease is renewed = Lease Non-renewal probability × Number of months vacant if not renewed

Vacancy rate = Average number of months vacant until the lease is renewed / (Lease term + Average number of months vacant until the lease is renewed)

Example:

- Lease Renewal probability = 60%
- Lease Non-renewal probability = 40%
- Number of months vacant if not renewed = 8
- The lease term = 2 years or 24 months

Average number of months vacant until the lease is renewed = 40% × 8 months = 3.2 months.

Vacancy rate = 3.2 / (24 + 3.2) = 11.76% = 12%

3) **Make assumptions about operating expenses:** Operating expenses include property taxes, insurance, maintenance, management, marketing, and utilities. Even in a net lease, such expenses are paid by the owner and then the owner is reimbursed by the tenant.

Operating expenses are categorized as follows:

a) ***Fixed expenses:*** These expenses are fixed and do not depend on the level of occupancy e.g. insurance and property taxes. Fixed expenses can change (increase) over time with inflation.
b) ***Variable expenses:*** Variable expenses depend on the level of occupancy e.g. management fee.
c) ***Hybrid expenses:*** Hybrid expenses are those which are partially fixed and partially variable e.g. utilities i.e. utility expenses increase with increase in tenants but there is some fixed amount of utility expense even a property is vacant.

Example:

- % of fixed utilities expenses = 30%
- % of variable utilities expenses = 70%
- Utility expense if 100,000 square foot building is fully occupied = $5 per square foot

Utility expense if the building is 75% occupied is estimated as follows:

Fixed utility expense = ($5 × 0.30 × 100,000) / (0.75 × 100,000) = $2.00 per occupied square foot

Variable utility expense = $5 × 0.70 = $3.50 per occupied square foot

Total utility expense = $3.50 + $2.00
= $5.50 per occupied square foot

4) **Make assumptions about capital expenditures:** Capital expenditures (or capex) are non-ordinary expenses e.g. new heating and air conditioning system, replacement cost of a roof, tenant improvements expenses, renovation expenses etc. The capex are deducted from NOI to calculate cash flows used in a DCF analysis.

- Analysts can use average amount of capex instead of separately projecting capex each year.
- When capex is included as part of operating expenses, it is named as "***replacement reserve***".
- ***Important to Note:*** Since future income pattern and effect of growth is explicit in a DCF, it explicitly takes into account capex, which is not reflected in NOI.

5) **Make assumptions regarding absorption of any vacant space:** Vacancy rate is estimated for a property until it is leased or its lease is renewed.

6) **Estimate resale value (reversion):** It is relatively easy to estimate resale value using holding period longer than expiration period of existing leases so that all leases will be at market rents and have normal rent growth after that.

- If the holding period is N years, then the expected NOI in year N+1 will be used to estimate resale value because that NOI is the 1st year NOI for the next buyer.
- When a contract rent of a lease < market rent, its income would be expected to increase when the lease ends. In such case, using before the lease expiration income to estimate resale price gives an underestimated resale price because it will ignore the growth in income and value associated with lease renewal.

7) Select discount rate to find PV of cash flows: Investing in the property is usually regarded as riskier than making a loan on the property as lender has a higher claim on property's cash flows. Hence, the discount rate should be > mortgage loan rate.

- Sometimes, lower discount rate is used for less risky cash flows i.e. rent income from existing leases; higher discount rate is used for more risky CFs i.e. income from lease renewals and resale.

Practice: Question 22, Volume 5, Reading 42.

6.5 Advantages and Disadvantages of the Income Approach

Advantages:

- DCF approach takes into account all important cash flows.
- DCF approach is not based on current transaction from comparable sales.

Disadvantages:

- DCF approach requires detailed information.
- DCF approach requires making future projections about growth rate etc.
- Simple DCF approach lacks a detailed lease-by-lease analysis.
- DCF approach critically depends on the assumptions, appropriate discount rate and cap rate.

6.6 Common Errors associated with assumptions in DCF Approach

1) Using an inappropriate discount rate that does not reflect the risk.
2) Using growth rate of income > expense growth rate.
3) Using the terminal cap rate is not commensurate with the implied going-in cap rate.
4) Applying the terminal cap rate to abnormal or non-typical income.
5) Ignoring the affects of business cycles on real estate markets.

Practice: Example 22, Volume 5, Reading 42.

3) Gross income multiplier (GIM):

$$GIM = \frac{\text{Sales Price}}{\text{Expected Gross Income of a proeprty in the 1st year after sale}}$$

Steps:

i. Identify recent sales of similar properties.
ii. Determine gross income of these properties.
iii. Calculate GIM for each comparable property.
iv. Calculate market GIM by calculating average of GIM of all comparable properties.
v. Estimate value of a subject property as follows:

Value of a subject property = Gross income × GIM (from step 3)

Limitations of GIM approach: This method assumes similar ratio of vacancy and expenses to gross income for the comparable and subject properties. Hence, it is considered as less reliable method than a direct capitalization method.

6.3.3) Adapting to Different Lease Structures

Lease structures vary across countries.

Using ARY (all risks yield) to value a "Fully let" property: A ***fully let property*** is a property that is fully leased at current market rents. In a fully let property, the tenants (s) are responsible for paying all operating expenses.

If the appraisal date lies between the initial letting (or the last rent review) and the next rent review, the contract rent (referred to as passing rent) ≠ current market rent (referred to as the open market rent) e.g.

- If the current market rent > contract rent, the rent is adjusted ***upward*** at the time of rent review.
- In this case, the property is said to have "reversionary potential" and the expected increase in rent must be included in the appraisal.

Term and reversion approach: In this approach, income is divided into two components. The values of these two components are appraised individually using different cap rates. The two components include:

1) Term rent: It is the fixed passing rent (current contract rent) from the date of appraisal to the next rent review.

- The discount rate used for term rent is lower than that of reversion because term rent is considered as less risky as it is protected by existing leases.
- It is important to note that when contract rent < (>) market rent, tenants are less (more) likely to default.

2) Reversion: It is the estimated rental value (ERV) of a property.

- The cap rate used for the reversion is derived from sales of comparable fully let properties.
- By convention, the rate used to ***discount*** the future reversionary value is the same as the cap rate used to ***calculate*** the reversionary value; however, they are not necessarily same.

Total value of a property = PV of the income until the rent review + PV of what the property could be sold for at rent review

Example:

- The term rent of a property for two years = $350,000 per year
- Estimated rental value (ERV) = market rent = $400,000
- ARY on comparable fully let properties = 4%
- Since the property is less risky than market rent (ERV), discount rate = 3%

Using financial calculator:

a) Calculating PV of the term rent:

N = 2, PMT = 350,000, I/Y = 3%, FV = 0, CPT → PV = $669,714.3934 ➔ PV of the term rent for two years.

PV of ERV *at the time of the rent review* at 4% ARY = $400,000/0.04 = $10,000,000

b) Calculating PV ERV at present:

N = 2, PMT = 0, I/Y = 4%, FV = 10,000,000, CPT → PV = $9,245,562.130 ➔ PV of ERV at present (i.e. 2 years back).

Total value = PV of the term rent + PV of ERV at present = $9,245,562.130 +$669,714.3934 = $9,915,276.523

- The property could be sold at the time of rent review for $10 million.

Layer Method: This method is used in U.K. Under this method,

Value of a property = PV of current contract rent in perpetuity + PV of expected incremental rent after the rent review

- Since contract rent is viewed as secure income and is less risky, it is discounted at cap rate close to or equal to ARY.
- The expected incremental rent after the rent review is more risky than contract rent, therefore, it is discounted at a cap rate > ARY.

Example:

Discount rate for current contract term rent = 4%
Discount rate for incremental rent = 5%

The term rent of a property for two years = $300,000
Reversion to ERV = $350,000

PV of current contract term rent *in perpetuity*
= $300,000/0.04 = $7,500,000

PV *in perpetuity* at 5% = (Reversion to ERV – Term rent) / 0.05 = ($350,000 - $300,000) / 0.05 = $1,000,000

PV for 2 years back (at present) at 5% = PV in perpetuity at 5% × [1 / $(1.05)^2$] = $907,029.4785

Total Capital Value = $7,500,000 + $907,029.4785
= $8,407,029.478

Practice: Example19, Volume 5, Reading 42.

6.3.4) The Equivalent Yield

Equivalent yield is the single discount rate that can be applied to both current contract rent and market rent to be received at rent review and generate the same value.

- It must be stressed that an equivalent yield will represent an IRR only if it is assumed that rent will not increase after 1st rent review; otherwise, it simply represents an average (although not a simple average) of the two separate cap rates.

7.1 The Cost Approach

In the cost approach, value of the property (i.e. building) is estimated based on adjusted ***replacement cost.*** The basic idea behind cost approach is that value of a property should not be > cost of buying vacant land and developing a comparable property.

- The cost approach is preferred to use for unusual properties or special-purpose properties for which market comparables are difficult to obtain.
- ***Replacement cost*** refers to the cost to construct the building with same utilities but using current construction costs and standards.
- ***Reproduction cost*** refers to the cost to construct the exact replica of the building using original building materials. It is higher than the replacement cost because it is not economical to construct the building using the original materials.
- **Depreciated replacement cost:** It is the replacement cost that is adjusted for different types of depreciation (loss in value).

Types of Depreciations:

1) **Physical deterioration:** It refers to physical wear and tear of components of the property over time as the property gets older. There are two types of physical deterioration:

a) Curable depreciation: It refers to the physical deterioration that when cured increases value of a property by an amount ≥ the cost of the cure e.g. replacing a roof.

- Cost to cure the curable depreciation is deducted from the replacement cost of the property.

b) Incurable depreciation: It refers to the physical deterioration that when cured does not increase value of the property as much as the cost of cure e.g. a structural problem with the foundation of the building.

- Cost to cure the incurable depreciation is NOT deducted from the replacement cost of the property; rather, it is estimated to evaluate its effect on property's value.
- When the effective age of the property < its economic life, then
  Physical deterioration = % worn out
  = Effective age / Economic life
- To avoid double counting, incurable depreciation is deducted as follows:

Incurable depreciation deduction = (Replacement cost + Developer's profit – curable depreciation costs) × Physical deterioration

2) **Functional obsolescence:** It refers to loss in property value due to features, design, and other elements of the building that are not up to modern standards. As a result, NOI and rent are relatively lower and operating expenses are relatively higher.

Amount of functional obsolescence = Income loss due to the functional obsolescence / cap rate

- Amount of functional obsolescence is deducted from the replacement cost.

Example:

- Loss in NOI due to *poor elevator design* = $30,000 per year.
- Cap rate = 8.5%

Amount of functional obsolescence = $30,000 / 0.085 = $352,941.18

3) **External obsolescence:** It refers to loss of value due to influences from outside factors that are external to the property e.g. factors associated with location, economic conditions etc.

a) Locational obsolescence: It is the loss in value due to non-desirable, non-optimal location of the property which makes it no longer the highest and best use of the site e.g. when a factory is constructed near a luxury apartment, the location becomes less desirable for a luxury apartment building.

- Total loss in the value of the building will include the loss in land value i.e. land value after factory was built will be < land value before construction of the factory.

Amount of locational obsolescence associated with building only = Total Loss in the value - Loss in land value

- The loss is land value is not deducted from the replacement cost of the building.
- Only the amount of locational obsolescence associated with building is deducted from the replacement cost of the building.

Example:

- Total Loss in the value of the luxury apartment building (land + building) due to the factory = $150,000.
- Land value before construction of the factory = $100,000
- Land value after factory was built = $70,000
  Loss in land value = $100,000 - $70,000 = $30,000
  Amount of locational obsolescence associated with building only = $150,000 - $30,000 = $120,000

b) Economic obsolescence: It is the loss in value associated with constructing ***new*** buildings due to non-desirable, infeasible economic conditions (e.g. low or insufficient current market rent levels), resulting in higher replacement costs.

Total depreciation = Curable depreciation + functional obsolescence + Locational obsolescence + Economic obsolescence

Depreciated building value = (Replacement cost + Developer's profit) – Total depreciation – incurable depreciation

Final Appraisal value ➔Estimated value of the property = Depreciated building value + Land value

Advantages:

- The cost approach is relatively easy to use because it is based on current construction costs.
- It is preferred method for unique properties or when there are limited sales of comparables.
- The cost approach is appropriate for newer properties.

Disadvantages:

- The cost approach requires estimating the value of land, which is difficult to estimate.
- Under cost approach, value of a property should not be > cost to buy land and build a comparable building. Hence, under the cost approach, value of the property is underestimated.
- The market value of an existing property may depend on other factors (e.g. prestigious and stable tenants) besides construction costs.
- The cost approach is not appropriate for older properties with volatile markets because it is difficult to estimate the depreciation for a property that is older and/or has much obsolescence.
- It is not appropriate to use when new construction is not feasible and depreciation is difficult to estimate.

Cost approach example:

- The replacement cost of a building = $15 million
- Developer's profit = $700,000
- Land value = $5 million based on comparable sales of other parcels of land.
- Curable maintenance expense = $1 million
- Effective age = 15 yrs
- Economic life = 50 yrs
- The functional obsolescence of the property = $1.50 million
- Locational obsolescence = $1 million
- Economic obsolescence loss in value = $1 million
- Physical deterioration = % worn out = 15/50 = 30%

Incurable depreciation deduction = ([$15,000,000 + $700,000] – $1,000,000) (0.30) = $3,990,000

Total depreciation = $1 million + $1.50 million + $1 million + $1 million = $4.50 million

Depreciated building value = ($15,000,000 + $700,000) – $4,500,000 – $3,990,000 = $7,210,000

Estimated value of the property = $7,210,000 + $5,000,000
= $12,210,000

Practice: Example 23, Volume 5, Reading 42.

7.2 The Sales Comparison Approach

In sales comparison approach, sales (transaction) prices of similar (comparable) properties are compared to a subject property. The basic idea in sales comparison approach is that the value of the property should not be > value of similar properties.

- The sales comparison approach is preferred to use for estimating value of single-family homes, which are non-income producing and when sales data for comparable properties is easily available.
- Sales prices are adjusted for each of the comparables to account for the differences between the comparables and the subject property with respect to size, age, location, quality of construction, amenities, view, condition of the property, market conditions at the times of sale and the difference between the date of sale of comparables and the date of the appraised value.
  - Sales price of the comparable property is adjusted UP when the subject is SUPERIOR to the comparable property.
  - Sales price of the comparable property is adjusted DOWN when the subject is INFERIOR to the comparable property.

Adjustments:

Depreciation adjustment: Subject property is older than the comparable → price per square foot of the comparable is adjusted *downward*.

E.g. depreciation rate is 2% and subject property is 5 years older.

Depreciation adjustment = 5 × 2% = - 10% → price of comparable is reduced by 10%.

Condition adjustment: Subject property is in "average" condition but comparable property is in "good" condition → price per square foot of the comparable is adjusted *downward*.

E.g. suppose condition adjustment for average is 8%→ price of comparable is reduced by 8%.

Location adjustment: Subject property is in "prime" location but comparable property is in "secondary" location → price per square foot of the comparable is adjusted *upward*.

E.g. suppose location adjustment for secondary is 15% → price of comparable is increased by 15%.

Market conditions adjustment: Market has been *rising* (falling)→ price per square foot of the comparable is adjusted *upward* (downward).

E.g. suppose market has been rising by 0.7% per month → price of comparable is increased by 0.7%.

After making all the adjustments, weighted average of price per square foot for each of the comparables is estimated, by giving more weight to more similar comparables.

Estimated value of the property = Weighted average of price per square foot for each of the comparables × Square feet of the subject property

Example:

	Subject property	Comparable property
Size (square feet)	15,000	20,000
Age (years)	9	7
Condition	Average	Good
Location	Secondary	Secondary
Date of sale (months ago)	-	1 month ago
Sale price	-	$3 million

Additional data:

Depreciation rate = 2%
Condition adjustment for good: none; average: –10%
Location adjustment for prime: none; secondary: 15%
Market is increasing by 0.65% per month.

Sale price per square foot of a comparable property = *$3,000,000 / 20,000 = $150*

- Since subject property is 2 years older, Depreciation adjustment = $150 × (2×2%) = -$6.
- Since subject property is in average condition, condition adjustment = $150 × 10% = -$15.
- Since both subject and comparable properties are in secondary location, no adjustment is needed.
- Since market is increasing by 0.65% per month, price is adjusted as = $150 × 0.65% = $0.975

Adjusted price per square foot = $150 - $6 - $15 + $0.975 **=** $129.975

For details, Volume 5, Reading 42, Exhibit 10.

Practice: Example 24, Volume 5, Reading 42.

Advantage of sales comparison approach: It is appropriate to use when the market is strong and active and there are many comparable properties transactions e.g. apartment buildings market.

Limitations of sales comparison approach:

- It is inappropriate to use when the market is weak and when there are fewer comparable properties transactions. In such case, the most reliable approach is income approach.
- It is not appropriate to use for special purpose properties.
- This approach wrongly assumes that investors are rational and prices paid by them represent the current market prices/values.

8. RECONCILIATION

Three valuation approaches use different assumptions and data to estimate value, and therefore, they do not necessarily provide the same value. Hence, it is recommended that final estimate of value for the subject property should be determined after reconciliation of the differences in the estimates of value from each approach.

9. DUE DILIGENCE

Due diligence is a process that involves verifying and analyzing legal, environmental, physical and other unanticipated facts and conditions that may negatively affect the value of the property but either have been ignored by appraiser and/or have not been disclosed by the seller. It helps to reduce the risk of acquiring a property or lending funds in a mortgage loan.

Due diligence process includes:

- Reviewing the leases for the major tenants, history of rental payments and any defaults or late payments.
- Collecting copies of bills and analyzing cash flow statements of the previous owner to verify operating expenses.
- Performing an environmental and physical/engineering inspection to verify compliance with regulations.
- Reviewing the ownership history to identify any issues related to the seller's ability to transfer free and clear title.
- Reviewing service and maintaining agreements.
- Verifying payments of property taxes, insurance, special assessments etc.

Limitations: Adequate due diligence is both costly and time-consuming.

Letter of intent: It is a legal document that describes the investor's intent to acquire the property at a specified price *but after performing due diligence*. In other words, when a contract has a "***conditional clause***" or "***letter of intent***", then the contract is not a binding contract i.e. if any issues are identified during due diligence process,

then an investor can either try to renegotiate the price or cancel the deal.

10. VALUATION IN AN INTERNATIONAL CONTEXT

Different international markets operate in different regulatory environments, have different training, use different definitions of the key concepts, apply different interpretations of common concepts, use different valuation methods, and have different availability of key data.

- These differences affect appraised values in different countries.
- In some countries (e.g. in Germany and U.S.), land and building are valued separately even in the income approach i.e. land is assumed to be leased and
  Income to the building = NOI – assumed land lease payment
  PV of building = Income to the building / Cap rate or discount rate
  Total value = PV of Building + Value of the land (from sales comparison approach)

11. INDICES (Section 11.1 – 11.2)

There are two major types of Real estate Indices.

1) **Appraisal-Based Indices:** These indices use ***appraised value***s of individual real estate properties rather than their transaction prices due to lack of actual comparable property transactions or higher costs associated with obtaining information on actual transaction prices.

- They provide information about market movements.
- These indices are constructed as an average of the current appraised values of the properties for each time period when the index is reported.

The NCREIF Property Index (NPI): NPI is a type of *appraisal-based index*. NPI measures the performance of real estate properties on quarterly basis. It is constructed using information on the appraised value, NOI, capex, occupancy rates etc. Such information is provided by members of NPI (mostly investment managers and pension fund plan sponsors) every quarter.

The return for all the properties is calculated as follows:

$$\textit{Total Return of individual Property} = \frac{\text{NOI} - \text{Capex} + (\text{Ending market value} - \text{Beginning market value})}{\text{Beginning market value}}$$

Income return = Cap rate = NOI / beginning value
Amount of cash flows available each quarter = NOI – Capex

$$\textit{Capital return} = \frac{(\text{Ending market value} - \text{Beginning market value})}{\text{Beginning market value}}$$

Total Index Return = Value-Weighted average return for individual properties

- Unlike stocks and bonds returns, which are based on actual transaction prices, the beginning and ending market values in properties return represent ***appraised values*** of the properties.
- Return is also known as holding period return and is equivalent to a single-period IRR.
- Capital return reflects the change in value ***net of*** capex.

Uses of NPI:

- NPI facilitates investors to compare the performance of real estate with other asset classes i.e. stocks and bonds.
- By providing information on quarterly returns, NPI facilitates investors to measure risk i.e. by estimating volatility of S.D. of quarterly returns.
- NPI can be used as a benchmark against which the returns for individual funds can be compared.

Disadvantages of Appraisal-based indices:

- They suffer from appraisal lag due to use of appraised values because
  - They take into account any increase or decrease in price after such changes are already reflected in transaction.
  - They appraise properties infrequently i.e. not necessarily every quarter.
- They are not appropriate to use for comparison with other publicly traded asset classes i.e. stocks and bonds.
- They reflect a "smoothed" index; as a result, they underestimate the variability of returns in the property market.
- Due to appraised values, they tend to exhibit lower correlation with other asset classes; as a result, the allocation to real estate tends to be overestimated.

There are two general ways to adjust the appraisal lag:

1) "***Un-smoothing" the appraisal-based index.*** An unsmoothed index will have more volatility and higher correlation with other asset classes.

2) ***Using a transaction-based index***.

Practice: Example 27, Volume 5, Reading 42.

2) **Transaction-Based Indices:** Transaction-based indices are based on actual transactions rather than appraised values. Transaction-based indices are constructed when more information on property transaction prices is available.

- These indices are mostly used for *residential properties*.
- They are constructed as an average of the transaction prices of the properties.

There are two approaches to construct transaction-based indices:

a) **Repeat sales index:** This index is based on ***repeat sales*** (i.e. more than once) of the ***same*** property e.g. Moody's REAL index.

- For example, if the same property sold twice, then the difference in value between the two sales dates indicates changes in market conditions over time.
- The more repeat sales, the more reliable the index.

b) **Hedonic index:** This index is based on only ***one sale***. To capture the affect of differences in the characteristics of the property (i.e. size, age, quality of construction, and location etc.) on value, the hedonic index use various independent variables in the regression model.

Advantage of Transaction-based indices: Transaction-based indices are based on actual transaction prices and thus are free from appraisal lags and smoothing effect.

Disadvantage of Transaction-based indices:

- Transaction-based indices are very data intensive and thus are most reliable to use at the national level for major types of properties.
- Transaction-based indices are reliable only when sufficient transactions are available.
- Since transaction-based indices are constructed using statistical techniques, they may suffer from noise i.e. include random elements in the observations.

12. PRIVATE MARKET REAL ESTATE DEBT

Use of debt to finance real estate investment magnifies both risk and return for an investor.

- The risk increases because investor has secondary claim relative to the lender on cash flows of a property.
- The greater the debt, the higher the risk.

When debt financing is not explicitly considered in property valuation:

Discount rate = Weighted average of the required rate of return of an equity investor and debt interest rate

- **Loan-to-value (LTV):** It the ratio of borrowed funds to total purchase price.
  LTV = Loan / value of the property
- The higher the ratio, the greater the leverage and thus, the greater the risk.
  Maximum loan amount based on LTV ratio = LTV ratio (in %) × Appraisal value of property (in $)
- **Debt serve coverage ratio (DSCR):** It is the ratio of the 1st year NOI to the loan payment (referred to as debt service for commercial real estate). It reflects the amount of NOI from the property that is available to meet debt service payments.
- The higher the ratio, the lower the leverage and thus, the lower the risk.
- Typically, lenders require DSCR of ≥ 1.2.

DSCR= NOI/Debt service

where,

Debt service = Interest + Principal payments on the mortgage.

Principal payments = Part of the loan payment that amortizes the loan over the loan term.

Maximum *debt service* based on DSCR = NOI/DSCR

When the loan is interest-only (discussed below):

Maximum *loan amount* based on DSCR= Maximum debt service based on DSCR / Debt interest rate

Important to Note: *The maximum amount of debt that an investor can obtain will be the lower of the loan amount based on LTV and DSCR.*

Interest only loan: The loan in which borrower makes no principal payments i.e. only interest payments are made and entire loan balance is repaid at loan maturity is called "interest-only loan".

- In an interest-only loan, the loan balance remains

constant over time.
- The entire loan balance payment is referred to as "***balloon payment***".
- Such loans may revert to amortizing loans at some future point in time or may have pre-specified maturity date.

Participation Loans: In "***Participation loans***", lenders receive some of the price appreciation but at the cost of ***lower*** interest rate.

Equity dividend rate: It reflects the amount of cash flows available to an investor as a % of its equity investment. It is also called the ***"cash-on-cash" return***.

Equity dividend rate or Equity yield rate = Cash flow / Equity

where,

Cash flow = NOI – Debt Service
Equity = Price – Mortgage

Calculating *Leveraged* IRR:

Cash flows received by the equity investor from the sale = Sale price – Mortgage balance

PV = – Initial investment
PMT = Cash flow
n = Holding period
FV = Cash flow received from sale
CPT→ I/Y ➔ Leveraged IRR.

Calculating *Unleveraged* IRR:

Cash flow received by the equity investor from the sale = Sale price + NOI in the 1st year

PV = – Initial investment
PMT = NOI in the 1st year
n = Holding period
FV = Sale price
CPT → I/Y ➔Unleveraged IRR.

Practice: Example 30, 31, 32 &33, Volume 5, Reading 42.

Practice: End of Chapter Practice Problems for Reading 42.

2. TYPES OF PUBLICLY TRADED REAL ESTATE SECURITIES

Major types of publicly traded real estate securities are as follows:

1) Real estate investment trusts (REITs): REITs are classified into three types:

1. ***Equity REITs:*** Equity REITs are tax-advantaged entities (companies or trusts) that generally hold, own, operate, manage and develop *income-producing* real estate property. They represent the predominant form of REITs.

- *Primary source of revenue:* Rent income from properties.
- They are actively managed i.e. they seek to expand ownership by investing in additional properties and tend to selectively develop, improve, or redevelop properties.
- Examples: Shopping mall, apartment complex, commercial office, hotel and other similar REITs.

2. ***Mortgage REITs:*** Mortgage REITs finance real estate investments by making mortgage loans to real estate owners or invest (typically 75% or more of their assets) in existing mortgages or mortgage backed securities.

- *Primary source of revenue:* Interest on mortgages.
- The total market value of mortgage REITs is relatively less than that of equity REITs.

3. ***Hybrid REITs:*** Hybrid REITs own & operate income-producing real estate properties and make loans as well. It involves investment strategies of both equity and mortgage REITs.

2) Real estate operating companies (REOCs): Like REITs, REOCs are real estate ownership companies. Unlike REITs, REOCs are ordinary ***taxable*** entities.

- *Primary source of revenue:* Cash inflows from sales of developed or improved properties.

3) Mortgage-backed securities (MBS): MBS are asset-backed securitized debt obligations backed by portfolios of mortgage loans e.g. on commercial properties (i.e. CMBS) or residential properties (i.e. RMBS).

- *Primary source of income:* Cash flows from portfolios of mortgage loans.
- The market capitalization of MBS is substantially less than that of publicly traded real estate ***equity*** securities.

Privately held real estate securities:

1) Private REITs: They are similar to equity REITs with regard to business model but unlike equity REITs, they do not trade on active exchanges. As a result,

- They are less liquid.
- They are mostly purchased by institutional investors.

2) Private REOCs
3) Privately held mortgages
4) Private debt issues
5) Bank debt

Advantages of Income-producing Real Estate:

- It provides stable, predictable, recurring, contractual rental income. As a result, investors can use above-average financial leverage (i.e. mortgage debt) and can generate attractive returns when rate of return on assets exceeds the interest rate on the mortgage. However, use of substantial financial leverage also increases risk.
- It tends to generate capital appreciation over the long-term and provides protection against inflation i.e. value increases with
  - Increase in replacement cost of buildings and prices for land.
  - General price inflation.
  - Decrease in supply of well-located property.
  - Increase in local population.
  - Increase in growth in economic activity.

Value of income-producing real estate is also affected by changes in discount rates.

3. PUBLICLY TRADED EQUITY REITS

3.1 Market Background

Investors of REITs:

- Institutional investors (i.e. stock funds, income funds, hedge funds) → represent 30% ownership
- Index funds →represent 25% ownership
- REIT sector dedicated funds → represent 15% ownership
- Individual investors →represent 15% ownership
- Pension funds → represent 10% ownership
- Insiders → represent 5% ownership

3.2 REIT Structure

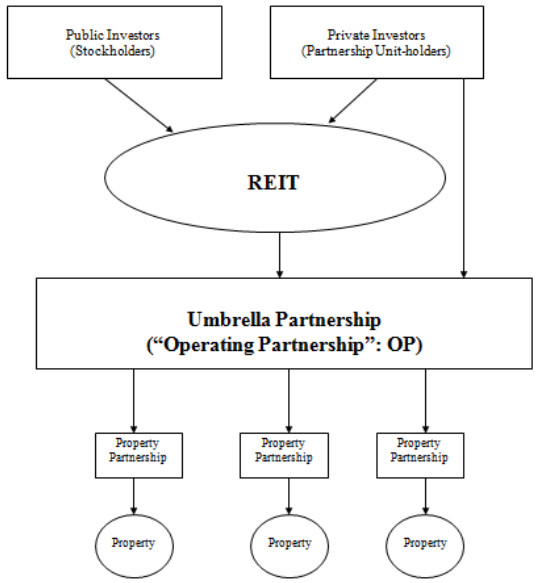
REITs can either acquire & hold properties directly or can create two types of subsidiary acquisition entity structure:

1. **Umbrella Partnership REITs (UPREITs):** An UPREITs is a partnership that owns and operates all or most of the properties and offer shares (referred to as "***operating partnership units***") to property sellers that can be convertible into REIT shares. In UPREITs,

- REIT has a controlling interest in a limited partnership that owns the real estate, as opposed to a traditional REIT structure in which the REIT owns the real estate.
- REIT serves as the general partner (i.e. responsible for managing operations).
- Property sellers have to pay capital gain taxes only when their limited-partnership units are converted into REIT shares.

Advantages of UPREITs:

1) They provide tax deferral advantage;
2) They provide diversification benefits;
3) They are highly liquid;
4) They are professionally managed;



2. **DOWNREIT:** In a DOWNREIT, the REIT owns more than one partnership and may own properties at both the REIT level and the partnership level. In addition, the REIT can create partnerships for each property it acquires.

3.3 Investment Characteristics

Investment characteristics for both public and private REITs: The following characteristics make REITs stable savings, retirement, and income-producing investments.

Exemption from income taxes at the corporate/trust level: Typically, REITs are exempt from income taxation at the corporate (or trust) level i.e. in U.S.,

- Distributions to shareholders are classified as ordinary income for tax purposes.
- Return of capital (portion of distributions > REIT's earnings) is treated as return of capital; for tax purposes, it is deducted from the investor's share cost basis.
- Sales proceeds in excess of the cost basis of the shares are treated as capital gains. Capital gains are subject to lower capital gains tax rates.

However, to qualify for income tax exemption, REITs must

a) Distribute at least 90% of their taxable income to shareholders in the form of dividends.
b) Have ≥ 100 shareholders
c) Not have ≥ 50% of shares held by five or fewer shareholders.
d) Invest at least 75% of total assets in real estate investments, mortgage loans or shares in other REITs.
e) Have limited non-rental property assets.
f) Earn 75% of income from real estate investments i.e. from rent, mortgage interest or gains from sale of property.
g) Must hire independent real estate professionals to execute certain management activities.

High income distributions: As a result of the distribution requirement, REITs typically offer higher dividend yields than other publicly traded equities with similar risk profiles.

Relatively low volatility of reported income: REITs are required to generate at least 75% of income from real estate investments (i.e. contractual rents). As a result, they tend to have relatively stable revenue streams, except hotel REITs.

More frequent secondary equity offering compared with industrial companies: Due to dividends distribution requirement and consequently limited retained earnings, REITs need to issue equity to finance property acquisition.

NOTE:

REITs are subject to the same regulatory, financial reporting, disclosure, and governance requirements as that of other public companies.

3.3.1)Advantages of Publicly Traded Equity Real Estate Securities

Advantages of publicly traded equity real estate securities (both REITs and REOCs) *relative to private real estate investments*:

1) **Greater liquidity:** Relative to private real estate investment, publicly traded equity real estate securities are more readily accessible, easier to buy and sell, involve less transaction costs and are less time consuming. As a result, they are highly liquid.

2) **Lower investment requirements:** Investment in publicly traded equity real estate securities can be done with a much lower investment amount than needed to acquire a single commercial property.

3) **Limited liability:** Like in other public companies, investors of REIT and limited partnership interest real estate investments have limited liability. However, in general partnership interests, investors are exposed to potential liability beyond their original capital investment.

4) **Access to superior quality and range of properties:** By purchasing shares of REITs, individual investors can gain access to institutional quality properties (e.g. super-regional shopping malls).

5) **Active professional management:** Unlike direct real estate ownership, publicly traded real estate equity securities are professionally managed and thus do not require investors to have real estate expertise or asset/property management skills.

6) **Diversification:** REITs facilitate investors to achieve greater diversification (by property, geography and property types) with only modest investment amounts.

Advantages of REITs relative to REOCs:

1) **Taxation:** Typically, REITs are exempt from the double taxation of income (i.e. once at corporate level and then at shareholders' level).

2) **Earnings predictability:** As a result of requirement of majority of income comprised of rent payments, REITs tend to have relatively stable and predictable earnings.

3) **High income payout ratios and yields:** REITs typically offer higher dividend yields than other publicly traded equities due to dividend distribution requirement.

Practice: Example 1, Volume 5, Reading 43.

3.3.2) Disadvantages of Publicly Traded Equity Real Estate Securities

1) **Taxation:** Unlike direct property ownership and partnership investments in some countries, REITs and REOCs cannot treat tax losses as deductions from their taxable income. In addition, REIT shares investors cannot defer tax when a property investment is exchanged or sold and replaced by similar property within a short period of time.

2) **Control:** Minority shareholders in a publicly traded REIT tend to have less control than direct property owners with regard to property-level investment decisions.

3) **Costs:** The structure of publicly traded REIT is costly to maintain.

4) **Stock market determined pricing and returns:** Since REITs are publicly traded, the returns and the stock market value of a REIT are determined by the market and as a result are more volatile than the appraised net asset value of a REIT. In addition, REITs have higher correlation with stocks than real estate prices. However, it must be stressed that appraised net asset values tend to exhibit underestimated volatility.

5) **Structural conflicts and related costs:** The UPREIT and DOWNREIT structures are exposed to conflicts of interest between the partnership and REIT shareholders with regard to property disposition and company debt levels decisions. In addition, such structures involve greater administrative costs.

6) **Relatively moderate income growth potential:** Due to low rate of income retention, REITs tend to have low rate of reinvestment and thus low income growth potential. As a result, during periods of high demand for fast-growing companies, REITs shares tend to underperform.

7) **Potential for forced equity issuance at disadvantageous prices:** Due to use of higher financial leverage and low rate of income retention, REITs may face pressures to issue equity at dilutive prices, particularly during periods of weak credit availability.

NOTE:

REIT shareholders benefit from:

- Timely debt and equity financing;
- Share repurchases during periods when market price < intrinsic values;
- Use of retained portion of operating cash flows or the proceeds of debt issuance or property sales;

Practice: Example 2 & 3, Volume 5, Reading 43.

3.4 Consideration in Analysis and Due diligence

Remaining lease terms: Hotels and multi-family residential properties tend to have the *shortest* lease terms whereas shopping centers, offices, and industrial buildings tend to have the *longest* lease terms. Generally, *properties with shorter leases are more sensitive to economic cycles.*

- During expansionary economy and/or rising rental rate environment, the shorter the remaining lease terms, the better it is.
- During declining economy and/or falling rental rate environment, the shorter the remaining lease terms, the worse it is.

Inflation protection: Leases that have pre-set periodic increases in rent or have minimum or base rents linked to the local inflation rate tend to provide greater inflation hedge.

Market rent analysis: Leases with low rents compared to current market rents are more attractive as they tend to provide upside potential to rental income upon lease re-negotiation. In contrast, leases with high rents compared to current market rents are regarded as risky.

Costs of re-leasing space: Costs to re-lease space typically include brokerage commissions, allowances for tenants' improvements to their space, free rent, and downtime between leases. Such costs can be substantial for landlords.

Tenant concentration: Properties with large and significant tenants with respect to space and the percentage of rents paid are regarded as less risky.

Availability of new competitive supply: Properties with less potential for new competitive supply and with lengthy construction lead times are regarded as less risky.

Balance sheet/leverage analysis: The REIT's balance sheet i.e. its leverage levels, cost of debt, and debt maturity profile should be thoroughly analyzed. In general, the higher the financial leverage, the higher the risk.

Management: Investors should also analyze the REIT's senior management's background, skill sets, track records, years of experience and length of time with the REIT.

3.5 Equity REITs: Property Subtypes (Section 3.5.1-3.5.8)

1) **Shopping Center/Retail REITs:** Shopping center or retail REITs invest in and manage retail properties i.e. regional shopping malls, community/neighborhood shopping centers (i.e. food and groceries centers) etc.

Features of Regional shopping malls:

- They represent large spaces.
- Lease terms range from 3-10 years.
- Tenants (except largest "anchor" tenants) are required to pay the greater of a fixed-minimum rental rate and a % of their sales.

  Rent paid by Tenants = Net rent + Proportionate share of the common area costs of the mall (based on space leased)
- Such REITs have relatively stable revenue streams as more than 90% of revenue is comprised of high levels of minimum rent.
- Anchor tenants tend to have very long-term, fixed rent leases or own their premises.

Features of Community shopping centers:

- Lease terms range from 3-10 years.
- Non- participatory rents are subject to periodic rent increases.

Factors need to consider for shopping center REITs:
Rental rates and sales per square foot/meter for the rental property portfolio.

2) **Office REITs:** Office REITs invest in and manage multi-tenanted office properties.

- Lease terms are long i.e. range from 5-25 years.
- Tenants pay fixed contractual base rents that are subject to upward revision (typically every 5-10 years) + proportionate share of operating expenses, common area costs, and property taxes.
- On year-to-year basis, rental income tends to be relatively stable. However, over long-term, it is sensitive to changes in office market vacancy and rental rates.

Factors need to consider for office REITs:

- New space under construction in a REIT's local market
- Site locations
- Access to public transportation and high-ways
- Business conditions for a REIT's principal tenants
- Quality of a REIT's office space i.e. location, convenience, utilitarian and architectural appeal, and the age and durability of the building.

3) **Industrial REITs:** Industrial REITs invest in and manage portfolios of single-tenant or multi-tenant industrial properties i.e. warehouses, distribution centers, light manufacturing facilities, and small office etc.

Features of Industrial property:

- Industrial property and industrial REITs are less sensitive to economic cycles than other property/REIT types i.e. hotel, health care, and storage.
- Industrial properties tend to have long-term *net leases* (i.e. 5-25 years), short construction periods (i.e. <1 year), are often pre-leased and build to meet particular tenants demands. As a result, they are less exposed to any rapid change of rental income and values.

Factors need to consider for Industrial REITs:

- Trends in tenants' requirements and their potential impact on the obsolescence of existing space;
- Trends in new supply and demand in the local market;
- Strategic property locations i.e. near airport, highway etc.;
- Changes in the composition of national and local

industrial bases;
- Changes in the composition of trade;

4) Multi-family/Residential REITs: Multi-family/residential REITs invest in and manage rental apartments.

Features of Rental apartments:

- Leased to individual tenants, typically for one-year.
- Demand tends to be relatively stable.

Factors need to consider for Multi-family/residential REITs: Rental income are exposed to fluctuations resulting from:

- Local demographics
- Trends in income, age, competitive appeal, cost and availability of homeownership in local markets
- Competition from condominium construction
- Tenant (move-in) inducements
- Regional economic strengths and weaknesses
- Effects of inflation on operating costs i.e. fuel & energy, utility, taxes and maintenance costs.
- Degree of government control

5) Storage REITs: Storage REITs invest in and manage self-storage properties or mini-warehouse facilities.

Features of Storage REITs:

- They are leased to individuals and small businesses under *gross lease* terms, on a monthly basis.
- They have shorter construction periods, which implies ease of entry and may result in over supply.

Factors need to consider for storage REITs:

- Rate of construction of new competitive facilities
- Trends in housing sales activity and their affect on the demand for temporary storage
- Trends in local demographics, new business start-up activity
- Seasonal trends in demand for storage facilities

6) Health Care REITs: Health care REITs invest in and own skilled nursing facilities, assisted living and independent residential facilities for retired persons, hospitals, medical office buildings, and rehabilitation centers.

- To maintain their tax-advantaged REIT status, REITs are not allowed to operate such properties themselves.
- They usually represent *net leases*.
- They are less sensitive to economic recessions.

Factors need to consider for health care REITs:

- Trends in population demographics
- Rate of construction of new competitive facilities
- Trends in government funding programs for health care
- Trends in construction cycles
- Financial condition of health care facilities operators/lessees
- Litigation settlements and insurance costs
- Prospects for acquisitions

7) Hotel REITs: Hotel REITs invest in and own hotel properties. Like health care REITs, they are prohibited from operating their properties themselves to maintain their tax-advantaged REIT status. In hotel REITs,

- Properties are typically leased to taxable REIT subsidiaries (or to third-party lessees) and are managed by hotel management companies.
- The hotel REIT parent receives passive rental income that represents the major portion of a hotel's net operating cash flow.
- A small % of net operating cash flow from hotel properties may be subject to income taxation.

Features of hotel REITs:

- Due to short lease terms, the hotel properties are highly sensitive to business-cycle driven short-term changes with respect to regional, national, and international business and leisure travel. As a result, they tend to have volatile revenue streams.
- Hotel properties have long construction periods (typically 1.5 to 3 years).

Factors need to consider for hotel REITs:

- Trends in occupancies
- Trends in average room rates, and operating profit margins
- Trends in ***revenue per available room*** i.e. (Average room rate × Average occupancy)
- Trends in hotel room forward bookings by category (i.e. individual, corporate, group, and convention)
- Trends in food and beverage and banqueting sales
- Maintenance and improvement expenditures
- Rates of new room construction and completion in local markets
- Level of financial leverage: Due to volatile income of Hotel REITs, use of greater financial leverage is highly scrutinized.

8) Diversified REITs: Diversified REITs own and operate in more than one type of property.

Factors need to consider for diversified REITs:

- Management's experience with regard to each property type
- Degree of local market presence

3.6 Economic Drivers

A. Economic growth or national GDP: Economic growth affects all types of property and REITs i.e. the higher the national GDP, the greater the demand for all types of property.

B. Retail sales growth rate: The higher the retail sales growth rate → the higher the sales growth of shopping center, retail and industrial REITs → the higher the rental rates (i.e. through rental rates based on a % of sales and increase in tenant's ability to pay higher rents) and the higher the occupancies in shopping centers.

C. Employment growth rate and job creation: The higher the employment growth rate→ the greater the demand for office space, multi-family accommodation, hotel room (due to increase in leisure and business travel) and storage properties (due to increase in personal and small business demand for space).

D. New space Construction time period: The longer the construction period, the more sensitive the property to changes in demand and supply e.g. office, hotel and health care properties and the REITs that invest in such properties.

E. Population growth: The higher the population growth rate → the higher the demand for multi-family accommodations, storage and health care facilities.

Other factors include:

A. Tenant/occupant Demand for space: Properties with fluctuating demand for space in the short-term are regarded as risky e.g. hotel properties.

B. Dislocations between supply and demand: Properties which are more subject to dislocations between supply and demand tend to have higher risks e.g. office, hotel, and health care.

C. Occupancy rate: It is an important factor for shopping centers, offices and hotel REITs.

D. Quality and locations of properties held by a REIT

E. Leasing terms and financing status

Read Exhibit 6, Curriculum Volume 5, Reading 43.

Practice: Example 4, Volume 5, Reading 43.

4. REAL ESTATE OPERATING COMPANIES

Like REITs, real estate operating companies (REOCs) operate in real estate industry; however, unlike REITs, REOCs are ***taxed like ordinary corporations*** either as a result of operating in countries with non tax-advantaged REIT regime or engaging in ineligible activities with regard to REIT framework.

- Like private real estate investments, both REOCs and REITs are exposed to operating and financial risks i.e. leasing, operating, financing, market risks and economic risks.
- REOCs can choose to covert to REIT status but only after they meet the general requirements of REITs.

Advantages of REOCs relative to REITs: REOCs have more operating flexibility compared to REITs i.e.

1) REOCs can invest in any kind of real estate or related activity without losing their tax status.
2) REOCs are not required to distribute 90% of their income. So, they can reinvest their retained earnings back into new development activities to maximize their returns.
3) REOCs are not subject to distribution requirement; as a result, they can retain more of their income to finance growth.
4) REOCs are allowed to use a wider range of capital structures and degrees of financial leverage in their activities.
5) Due to high leverage and use of accelerated depreciation, REOCs frequently have relatively low cash tax liabilities.

Disadvantages of REOCs relative to REITs:

1) Investors tend to prefer REITs (due to their tax-advantage, high income distributions and precisely defined operating and financial mandates) to REOCs.

2) Due to investors' low preference, REOCs tend to have less access to equity capital and higher cost of equity (i.e. lower market value) compared to REITs.

Practice: Example 5, Volume 5, Reading 43.

5. VALUATION: NET ASSET VALUE APPROACH

There are two approaches to value shares of REITs and REOCs:

1) **Book value per share (BVPS):** BVPS represents the reported accounting value for assets.
2) **Net asset value per share (NAVPS):** NAVPS represents the market values for assets.

- NAVPS is regarded as the most relevant and superior measure to estimate values of REITs and REOCs.
- NAVPS is often used as a fundamental benchmark for the value of a REIT or REOC.

Components of the intrinsic value of a REIT or REOC:

i. Net asset value→ it represents the largest % of the intrinsic value of a REIT or REOC.
ii. Assessed value of any non-asset based income streams (e.g. fee or management income)
iii. Value added by management of the REIT or REOC
iv. Value of any contingent liabilities

When REIT share price < REIT's NAVPS ➔ it indicates that REIT is undervalued.

When REIT share price > REIT's NAVPS ➔ it indicates that REIT is overvalued.

5.1 Accounting for Investment Properties

When accounting values are based on fair values rather than historical cost values, then accounting values can be used for asset-based valuation.

Under IFRS: **"Investment properties"** refer to the building owned by a company and leased to tenants. They do not include long-lived tangible assets held for sale for ordinary business purposes e.g. houses and property owned by a housing construction company, owner-occupied property, plant and equipment used for company's goods and services etc.

- To value investment properties, companies are permitted to use either a ***cost model*** or a ***fair value model***.
- However, a company is required to use its chosen model to all of its investment property and must continue to use that chosen model until it disposes of the property or changes its use.
- The investment property is recorded as separate line item on the balance sheet.
- In addition, companies are required to disclose the model used, method used to estimate fair value, reconciliation between the beginning and ending carrying amounts of investment property, depreciation method and useful lives.

Cost model: It is similar to the cost model used for property, plant and equipment.

Fair value model: A company can use fair value model only if it is able to reliably determine the property's fair value on a continuing basis. In a fair value model, net income is affected by any changes in the fair value of the asset.

Under U.S. GAAP: Unlike IFRS, the investment property has no specific definition. Under U.S. GAAP, companies are allowed to use ***historical cost accounting model*** to value investment properties.

Historical cost accounting model: In this method, assets (including buildings) are recorded at depreciated historical cost.

- Values of assets are written done when their economic values are permanently impaired.
- Values of assets can be written up only under exceptional circumstances i.e. mergers, acquisitions or reorganizations.

Limitation: Historical cost model does not accurately represent the economic values of assets and liabilities and current economic return during periods of fluctuating prices and costs i.e. carrying values on appreciating long-held property assets are understated while depreciation on such assets are overstated, resulting in inaccurate economic income and asset values.

5.2 Net Asset Value per Share: Calculation

Net asset value per share:

NAVPS = (Market value of Real estate company's assets – Market value of Real estate company's liabilities) / Number of shares outstanding

- For valuing REITs and REOCs, analysts can use existing appraisals (if available).
- When such appraisals are not available or analysts disagree with the assumptions or methodology of the appraisals, value can be estimated as follows:

Appraised value = Net operating income (NOI) / Cap rate

where,

- Net operating income (NOI) represents the rental income streams of REITs or REOCs.
- Cap rate represents the rate used in the market in recent similar transactions.

Cap rate = NOI of a comparable property or portfolio of comparable properties / Total value of the comparables

- The properties are recorded at their estimated asset values instead of book values on the balance sheet and necessary adjustments are made e.g.
  - To determine the "***hard economic value of total assets***", the value of goodwill, deferred financing expenses, and deferred tax assets are removed from value of assets.
  - If face value of debt is significantly different from its market value, then debt is recorded at its market value instead of face value.
  - To determine "***hard economic value of total liabilities",*** any "soft liabilities" (e.g. deferred tax liabilities) are removed from value of total liabilities.

  NAV = (Hard economic value of total assets – Hard economic value of total liabilities) / Number of shares outstanding

  OR

  NAV = Revised net worth of the company / Number of shares outstanding
- Although it represents the ***pre-tax value***, it is used as a primary net worth benchmark due to lack of information about the tax circumstances and strategies of a would-be acquirer.

Estimating NAVPS:

Pro forma cash NOI = NOI – Non cash rents* + Adjustments for full impact of acquisitions

*Non-cash rent = Average contractual rent over the leases' terms – Cash rent actually paid

Estimated future expected cash NOI = Pro forma cash NOI + Expected growth in NOI

Estimated value of operating real estate = Estimated future expected cash NOI / Cap rate

Estimated gross asset value = Estimated value of operating real estate + Book value of Cash & equivalents + Book value of Land held for future development + Book value of Account receivables + Book value of Prepaid/other assets

Net asset value = Estimated gross asset value – Total debt – Other liabilities (but not deferred taxes)

NAVPS = Net asset value / Number of shares outstanding

5.3 Net Asset Value per Share: Application

- An active private investment market exists for REITs and REOCs, which facilitates estimation of net asset value of REITs or REOCs.
- NAV valuation approach for REITs is similar to the sum-of-the parts valuation approach for a company with multiple business lines.

Uses of NAV valuation approach:

- The NAV approach to valuation is preferred by sector-focused real estate investors as REITs and REOCs represent liquid forms of commercial real estate ownership.
- The NAV approach to valuation is preferred by value-oriented investors during periods when value of stocks is lower than the underlying value of the assets.
- The NAV approach to valuation is also useful during periods of greater leveraged buyout (LBO) activity in the broader market i.e. during such times,
  - When REITs are trading at large discounts to NAV → LBO sponsors buy REITs to realize their underlying real estate value.
  - When REITs are trading at large premiums to NAV → IPO activity and stock issuance activity increases to benefit from higher value of real estate in public markets compared to private markets.

5.3.1) Important Considerations in a NAV-Based Approach to Valuing REITs

Discount or cap rate: The discount rate used by a private owner/operator of commercial real estate may be different from the discount rate used by investors purchasing shares of REITs

NAV: The value of a REIT's assets represented by NAV may be different from the value estimated by public equity investors.

Long-term v/s short-term focus: Unlike stock markets, property market tends to focus on long term changes in income and value of assets.

NAV valuation approach ignores the going concern assumption: Under the NAV approach, a company is treated as either an individual asset or static pool of assets. Hence, it is recommended that an analyst must incorporate the value added (reduced) by a management team to current NAV estimates.

Substantial subjective judgments are required to estimate NAV when:

- Property markets are illiquid;
- There are very limited transactions;
- A large number of properties are owned by REITs and REOCs.

These subjective judgments make estimates of NAV less useful and reliable.

It is difficult to estimate an accurate and reliable measure of NAV for:

- Undeveloped land;
- Significantly large properties with few comparable assets;
- Special-purpose properties;
- Service businesses;
- Joint ventures;

5.3.2) Further Observations on NAV

- REITs tend to trade at premiums to their underlying NAVPS when a company has good quality management team, which adds value to the company.
- Theoretically, NAV is an *absolute* valuation metric.

Nevertheless, in practice, it can be used as a relative valuation tool.

- When ***all REITs*** are trading at ***premiums*** to NAV → it suggests that the ***REIT stock,*** which is trading at the *smallest discount* to NAV should be *purchased*.
- When ***all REITs*** are trading at ***discounts*** to NAV → it suggests that the ***REIT stock,*** which is trading at the *smallest discount* to NAV should be *sold*.

NOTE:

We can calculate the ***implied cap rate*** using NAV i.e. by using current price and solving for cap rate by backward calculation.

6. VALUATION: RELATIVE VALUE (PRICE MULTIPLE) APPROACH (Section 6.1 -6.4)

Value of shares of REITs and REOCs can also be estimated using ***Market-based or relative value*** approaches.

Common Relative value measures used in valuing REIT and REOC shares:

1) **Price-to-funds from operations (P/FFO):** P/FFO of a REIT is compared with average P/FFO of comparable REITs. It is the most commonly used measure.

P/FFO = Current stock prices / Year-ahead estimated FFO

where,

Funds from operations (FFO): It represents the operating income of a REIT or REOC.

FFO = Net earnings + Depreciation expense on real estate + Deferred tax charges – Gains/losses from sales of property and debt restructuring + Losses on sales of property and debt restructuring

OR

FFO = EBITDA – Interest Expense

NOTE:

- Depreciation is excluded from net earnings because it is believed that real estate tends to maintain its value to a greater extent than other business assets and often its value appreciates over long-term.
- Deferred tax liability and the related periodic deferred tax charges are excluded from net earnings because they do not have reliable economic value.
- Gains and losses from sales of property and debt restructuring are excluded from net earnings because they do not represent sustainable, normal income.

2) **Price-to-adjusted funds from operations (P/AFFO**): P/AFFO of a REIT is compared with average P/AFFO of comparable REITs.

P/AFFO = Current stock prices / Year-ahead estimated AFFO

where,

Adjusted funds from operations (AFFO): AFFO is also referred to as funds available for distribution (FAD) or cash available for distribution (CAD).

AFFO = FFO – Non cash rent* – Recurring Maintenance type Capital expenditures – Leasing costs (i.e. leasing agent's commissions – Tenants' improvement allowances)

Straight-line rent: It refers to the average contractual rent over a lease term. It is recognized as revenue under IFRS and U.S. GAAP.

**Non-cash rent = Straight-line rent - Cash rent paid during the period*

Advantage of AFFO: It is considered as a more accurate and superior measure of current economic income because it considers the capital expenditures necessary to maintain the economic income of a property portfolio.

Limitations of AFFO:

- AFFO is a more volatile measure of current economic income because it is subject to more fluctuations and errors in estimation than FFO.
- Corporations do not tend to report actual AFFO measures on a consistent basis.

The higher the cap rate → the lower the estimated value of operating real estate and thus the lower the NAV

The lower the cap rate →the lower the NOI → the lower the FFO and AFFO →the higher the P/FFO and P/AFFO ratios.

3) Enterprise value-to-earnings before interest, taxes, depreciation, and amortization (EV/EBITDA):

EBITDA = NOI - General and administrative expenses

Major Factors that affect (P/FFO), (P/AFFO) and (EV/EBITDA):

a) Expectations for growth in FFO/AFFO: The higher the expected growth, the higher the multiple or relative valuation. Growth depends on following factors:

i. ***Business model:*** The more successful business models of REITs and REOCs, the higher the FFO/AFFO growth over time.
ii. ***Geography:*** The greater the concentration of properties in *primary, supply-constrained markets* (i.e. London), the more pricing power of landlords and the higher the cash flow growth rate.
iii. ***Other factors*** i.e. management skill or lease structure.

b) Risk associated with the underlying real estate: Apartments properties are regarded as less risky due to their more stable income streams relative to hotels properties. Thus, apartment REITs tend to trade at relatively high multiples than hotel REITs.

c) Risk associated with company's capital structures and access to capital: The greater the financial leverage → the higher the risk → the higher the required rate of return ➔ the lower the equities' FFO and AFFO multiples.

6.3 Advantages of P/FFO and P/AFFO multiples

1. P/FFO and P/AFFO are widely accepted for valuing shares across global stock markets and industries.
2. Due to wide acceptance of such multiples, these multiples can be used to compare REITs and REOCs with other assets.
3. P/FFO is easy to estimate because estimates of FFO are readily available in the market i.e. such estimates are provided by Bloomberg and Thomson Reuters.
4. P/FFO and P/AFFO multiples can be used in conjunction with other measures i.e. expected growth and leverage.
5. P/FFO and P/AFFO multiples facilitates investors to compare value of REIT's shares with other REIT shares or to compare the current valuation level of a REIT's shares with historical levels.

6.3 Disadvantages of P/FFO and P/AFFO multiples

1. P/FFO and P/AFFO multiples do not consider the intrinsic value of all real estate assets held by the REIT or REOC e.g. properties (like land and empty buildings) that do not currently generate any income may have value but are not included in FFO.
2. P/FFO ignores the capital expenditures necessary to maintain the economic income of a property portfolio.
3. There is no universally accepted methodology for computing AFFO.
4. Changes in accounting rules make it difficult to compute P/FFO and P/AFFO and to compare them across different companies.
5. FFO and AFFO do not consider differences in leverage.

Practice: Examples 6 & 7, Volume 5, Reading 43.

7. VALUATION: DISCOUNTED CASH FLOW APPROACH (Section 7.1-7.2)

Generally, REITs and REOCs shares have high dividend yields. As a result, dividend discount models can be used to value them. Typically, two or three step dividend discount models with near-term, intermediate-term and /or long-term growth assumptions are used.

Factors that affect longer-term growth rates:

1) Internal growth potential that stem from rent increases over time: The more supply-constrained (saturated) markets and higher (lower) demand for real estate → the more (less) pricing power of landlords → the greater (lower) the cash flow growth rate.

2) Investment activities: Investment activities include acquisitions, new developments, re-development, or dispositions of assets.

- The more successful new development activities → the higher the returns on invested capital → the higher the long-term growth rate.
- Weaker assets with below-average growth rates are generally disposed off (sold) at cap rates > re-investment rate, resulting in reduced earnings and poor cash flow growth in the near term.

3) **Capital structure:** In general, the higher the leverage, the higher the risk. However, when REITs and REOCs have positive leverage spread i.e. going-in cap rates on property investments > cost of debt, leverage may positively affect growth rate.

4) **Retention and reinvestment of free cash flow:** REITs in many countries are permitted to use high rates of depreciation for tax purposes, which facilitate them to retain sufficient cash flow that in turn positively affects growth rate.

It is important to note that growth is negatively affected by adverse changes in business cycle, operational and investment missteps and new equity issuance.

Required returns in dividend discount models: Due to their relatively low risk (as reflected by contractual revenue streams), the risk premium and thus the discount rate associated with REITs and REOCs is lower than that of average stock in the broader market. In addition, betas of REIT shares have tended to be less than 1.0.

Discount rate or Cost of equity of capital
= Risk-free rate + Company's historical beta to the broader equity market beta (equity risk premium)

Limitation of Dividend discount model (DDM) for valuing REITs and REOCs: DDM is highly sensitive to changes in assumptions and inputs of growth and discount rates.

Practice: Example 8, Q4 & Q12, Volume 5, Reading 43.

Important to Note:

- When a REIT has ***higher*** P/FFO and P/AFFO relative to peers or all other REITs but ***lower*** growth rate relative to peers or all other REITs ➔ it implies that the REIT is overvalued compared to its peers and/or all other REITs.
- When a REIT has ***higher*** P/FFO and P/AFFO relative to its historical values, ➔ it implies that the REIT is overvalued/expensive based on historical multiples.
- When a REIT has ***higher*** P/FFO and P/AFFO relative to peers or all other REITs but ***lower*** financial risks (.e. lower leverage, attractive geographic exposure, positive growth trend during weak markets, higher dividends, higher interest coverage ratios etc.) relative to peers or all other REITs ➔ it suggests that REIT may trade at some premium.

Estimated Value of a REIT company in year N = (P/FFO of overall REIT group for year N) × REIT company's expected FFO in year N

Or

Estimated Value of a REIT company in year N = (P/AFFO of overall REIT group for year N) × REIT company's expected AFFO in year N

- When estimated value of a REIT company < current market value ➔ it implies that the REIT is trading at premium to NAV and indicates that the REIT's shares are overvalued.
- When estimated value of a REIT company > current market value ➔ it implies that the REIT is trading at discount to NAV and indicates that the REIT's shares are undervalued.
- The lower the % of amount retained by a REIT with a given growth rate, the cheaper or more attractively priced the REIT is.

For details, refer to Curriculum Volume 5, Reading 43, Section 8 and Exhibit 16 and 17.

NOTE:

When a property is *not fully leased*, net operating income can be increased if REIT can attract additional tenants.

8.4 Selection of Valuation Methods

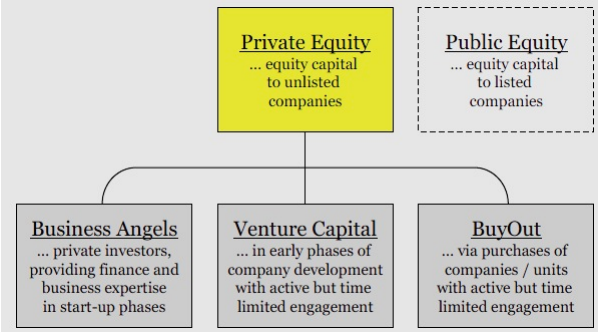
Different valuation methods generate different values. Analysts tend to select methods based on assumptions used and analyst's own investment philosophy or view of value.

Practice: Example 9 & 10, Volume 5, Reading 43.

Practice: End of Chapter Practice Problems for Reading 43.

1. INTRODUCTION

Private equity is defined as any type of equity investment in an asset in which the equity is not quoted on stock markets or freely tradable on a public market.



PRIVATE EQUITY V/S PUBLIC EQUITY

Private Equity	Public Equity
• Illiquid investment	• Liquid investment
• Ownership is concentrated	• Ownership is dispersed
• Valuation is difficult • The price paid for a private equity stake is determined through a negotiation process between two or more parties. • Private equity valuation is time bound and depends on the respective motives and interests of buyers and sellers.	• Valuation is relatively easy because shares of public companies are traded regularly on regulated market.
• Finance is accompanied by control and mentoring.	• Finance is often separated from control and monitoring
• Leverage is typically measured as a multiple of EBITDA instead of equity. • Relative to comparable publicly quoted companies, there is a much greater use of debt in a typical buyout transaction. • Unlike public companies, private equity firms have better control over management; therefore, they may have a better ability to raise higher levels of debt.	

1. Venture Capital (VC):

- VCs are early stage companies with no revenues
- Less mature companies
- Potentially good ideas or technology

Characteristics of VC:

- Long time horizon
- Lack of liquidity
- High risk
- Equity participation
- Participation in management

2. Buyouts: It is a form of private equity transaction in which a controlling stake in the equity of a target company is acquired by a buyer from a seller. It involves following techniques:

a) Management buyouts (MBOs): In this technique, a company is put in the hands of the current management with the use of leverage.
b) Leveraged buyouts (LBOs): It involves acquisition of a company using leverage.
- It involves a negotiation between the providers of equity capital, senior debt, high yield bonds and mezzanine finance (a hybrid form of financing that can be considered as a bridge between equity and debt).

c) Takeovers

Buy-outs are associated with later-stage financing. These transactions involve:

- Privately owned companies
- A particular division of an existing company
- Companies that are mature and generate operating cash flows.

After the buyout, control of the company is concentrated in the hands of the LBO firm and management, and there is no public stock outstanding.

Investors of Private Equity include:

- Institutional investors i.e. pension funds, endowments, and insurance companies and
- High-net-worth individuals (who generally invest in fund of funds intermediaries or directly).

Reason to invest in PE

- Diversification benefits due to low correlation with traditional assets.
- Relatively good returns of PE over the last years.

NOTE:

One unique characteristic of private equity investment is a buy-to-sell orientation.

Private equity funds can also be classified as follows:

a) Geographical focus (national, regional, or global) and/or
b) Sector focus (diversified industries, telecommunications, biotechnologies, health care, industries etc.).

See: Exhibit 1, Volume 5, Reading 44.

2. Introduction to Valuation Techniques in Private Equity Transactions

1. **Income approach e.g. discounted cash flows (DCF):** Value of a company is estimated by discounting expected future cash flows at an appropriate cost of capital.

- It can generally be applied across the broad spectrum of company stages.
- It is the most preferable valuation approach for companies with sufficient operating history i.e. companies operating from the expansion up to the maturity phase.
- This method typically derives the highest values.
- It is the most assumption-laden method, all based on the future.

2. **Relative value e.g. Earnings multiples:** It involves application of an earnings multiple to the earnings of a portfolio company.

- The earnings multiple is frequently obtained from a set of Peers/Comparables.
- It is preferred to be used for companies with a *significant operating history* and predictable stream of cash flows.
- It can be used with caution to companies operating at the expansion stage.
- It is not preferred to be used for early stage or start-up companies.
- Commonly used multiples include P/E, EV/EBITDA, EV/Sales.

3. **Real option:** Real options represent a right to undertake a business decision (call or put option).

- It is based on judgmental assumptions regarding key option parameters.
- This approach is generally applied when the management or shareholders have significant flexibility in making different strategic decisions (i.e. option to undertake or abandon a high risk, high return project).
- It is preferred to be used for seed or start-up phase companies (immature firms).

4. **Replacement cost:** Replacement cost refers to an estimated cost that is incurred to recreate the business as it stands as of the valuation date.

- It is generally applied to:
  - Early (seed and start-up) stage companies (immature firms).
  - Companies operating at the development stage
  - Companies generating negative cash flows
- It rarely applies to mature companies because it is difficult to estimate the cost to recreate a company with a long operating history.

NOTE:

Due to the lack of liquidity of private equity investments, it is recommended to use all valuation techniques simultaneously.

Key considerations in evaluating a private equity transaction include the following:

- Value of control
- Impact of illiquidity
- Extent of any country risk (e.g. investing in emerging markets)

Control Premium: The control premium is an incremental value associated with a large bloc of shares that can be helpful in gaining control of a company.

- In most buyouts, the entire equity capital is acquired by the private equity purchasers.
- In venture capital, minority positions are acquired by the private equity purchasers.

In the case of venture capital deals, control premium significantly depends on the:

- Relative strength i.e. control premium is large when only a limited number of investors are able to acquire control relative to when there is a dominant controlling shareholder along with a large number of much smaller shareholders.
- Alignment of interest of shareholders willing to gain control.

Cost of illiquidity: The cost of illiquidity refers to the cost associated with finding prospective buyers and the speed of converting assets to cash. The cost of illiquidity is closely related to control premium because illiquidity

increases with the increase in willingness to acquire control.

Size of the illiquidity discount depends on following factors:

- Shareholding structure,
- Level of profitability and its expected sustainability,
- Possibility of IPO in the near future, and
- Size of the private company

Because of the difficulty of determining the importance of each factor, estimation of illiquidity discount is based on judgment.

Cost of marketability: It is the cost associated with the right to sell the assets.

NOTE:

Practically, the discount for illiquidity and premium for control are incorporated through adjustments to the preliminary value estimate instead of the cost of capital.

1) If valuing a private business for sale (in whole or part) to another individual (to remain private), it is necessary to estimate:

- An illiquidity discount because private businesses cannot be easily bought and sold
- A control premium (if more than 50% of the business is being sold)

2) If valuing a business which is taken public, it is necessary to estimate the effects of:

- Creating different classes of shares in the IPO.
- Options or warrants on the issuance price per share.

3) If valuing a business for sale (in whole or part) to a *publicly traded firm*:

- There should be no illiquidity discount
- There can be a premium associated with the publicly traded firm being able to take better advantage of the private firm's strengths.

Use of Private equity valuation:

- Helps to determine transaction price.
- Serves as a monitoring tool to identify new opportunities or protect from losses.
- Serves as a performance reporting tool to investors while the company remains in the fund portfolio.

2.1 How Is Value Created in Private Equity?

Value can be created by improving the business's financing, operations, management and marketing.

Economic advantages of private equity over public equity governance:

1) Ability to re-engineer the private firm to generate superior returns.
2) Ability to access credit markets on favorable terms.
3) Better alignment of interests between private equity firm owners and managers of the firms they control.

Value in LBOs is created through restructuring i.e.

- Financial restructuring of balance sheet i.e. improved combination of debt and equity.
- Operational restructuring i.e. improving operations to increase cash flows.
- Lower agency costs of free cash flows i.e. debt from LBO commits cash flows to debt

Term Sheet: Term sheet represents investments terms and agreements structured to create a balance of rights and obligations between the private equity firm and the management team. It consists of the following contractual clauses to better align the interests of management and firm owners.

1) Tag-along and drag-along rights: A tag along provision creates an obligation to ensure that the potential future acquirer of the company may not acquire control without extending an acquisition offer to all shareholders including the management of the company. This right may have the effect of making the shares more difficult to sell. Tag-along rights protect the interest of minority shareholders.

2) Corporate board seats: This provision is used to ensure private equity control in case of major corporate events i.e. company sale, takeover, restructuring, IPO, bankruptcy or liquidation.

3) Noncompete clause: This clause is used to impose an obligation on founders and prevents them from restarting the same business during a predefined period of time.

4) Preferred dividends and liquidation preference: Distributions are first made to private equity firms and they are generally offered a guaranteed minimum multiple of their original investment before return is received by other shareholders.

5) Reserved matters: It refers to a provision that requires approval or veto by the private equity firm in case of important strategic decisions i.e. changes in the business plan, acquisitions, or divestitures etc.

6) Earn-outs (mostly in VC): It refers to a clause in which part of the price of a transaction is conditional on the future financial performance of the company following the deal (over a predetermined time horizon i.e. not > 2 to 3 years).

Advantages of Effective contractual structuring of the investment terms:

Effective Contractual Structuring:

- Allows venture capital firms to significantly increase their level of control over time.
- Allows VC firms to seize control when a company fails to achieve the agreed goals.

2.2 Using Market Data in Valuation

There are two important ways in which the market data can be used to estimate the value of the entity being acquired i.e.

1) **Comparable Public Companies:** Value can be determined by analyzing publicly-traded companies in the same industry with similar operating and financial statistics.

- It involves use of valuation/trading multiples i.e. enterprise value to EBITDA of comparable public companies which can be applied to private company earnings.
- Since stock prices represent minority positions, control premiums are incorporated.
- This method is appropriate to use when similar/comparable companies are easily available.

2) **Comparable Transactions: In this method,** valuations are implied by recent transactions involving similar entities i.e. if there are recent M&A transactions in the related sector then the transactions multiples paid could be used to estimate value of target firm.

Issues with using market data in estimating cost of equity: Due to lack of historical public data on share prices and returns, the cost of capital for private companies cannot be directly estimated using the WACC formula for public companies. Beta must be estimated by means of a proxy, by estimating the beta for comaparable companies and adjusting for financial and operating leverage.

Ways to estimate Terminal Value:

In DCF valuation approach, future financial performance forecasts are generally only available for a few years ahead. Therefore, it is necessary to estimate the terminal value of the company beyond this forecasting horizon. There are two ways to use estimate terminal value:

i. Using a perpetual growth rate assumption.
ii. Using trading multiple that exists in public markets and applying this multiple to the last years' forecast values.

2.3 Contrasting Valuation in Venture Capital and Buyout Settings

VC	Buyout (LBO)
• Generally invest in specialized industry	• Generally invest in a portfolio of firms with more predictable cash flow patterns
• Seek revenue growth	• Focus more on EBIT or EBITDA growth

IMPORTANT:

See: Exhibit 3, Volume 5, Reading 40.

2.4 Valuation Issues in Buyout Transactions

Valuation Issues	VC	Buyout
1. DCF valuation technique	Less commonly used due to uncertain cash flows	Commonly used
2. Relative value valuation technique	Difficult to apply due to difficulty in finding true comparable firms.	Generally apply to analyze reliability of value estimated from DCF approach.
3. Use of leverage	Low use of leverage as equity represents dominant form of financing.	High use of leverage.
4. Key Value drivers	• Pre-money valuation • Investment • Subsequent dilution	• Earnings growth • Multiples expansion upon exit • Debt reduction

NOTE:

For buyouts, the level and pattern of leverage over the investment period should be taken into account in valuation techniques.

2.4.1) The LBO Model

The LBO model is a method of determining the maximum price (or range of acceptable prices to conclude the transaction) that can be paid to the seller and which is also appropriate to meet target returns for the providers of financing. Thus, LBO model is not a separate valuation technique. The LBO model has three main input parameters:

1. Cash flow forecast of the target company
2. Expected return from the providers of financing
3. Amount of financing available for the transaction.

Exit Value:

The exit value is determined by using expected range of exit multiples that are estimated from a peer group of comparable companies (i.e. Enterprise value-to-EBITDA).

Value creation depends on the following factors:

- Operational improvements that lead to earnings growth.
- Improved corporate governance system.
- Multiple expansion i.e. Difference between entry and exit multiples;
- Debt reduction = Entry net debt – exit net debt

Example:

- Investment in a private equity transaction = $5,000 million
- Financed with 50% debt and 50% equity.
- The $2500 equity investment is further divided into:
  - $2400 of preference shares owned by the private equity fund
  - $95 of equity owned by the private equity fund
  - $5 of management equity.
- 12% annual return (paid at exit) is promised to preference shares.
- The private equity firm equity is promised 95% of the residual value of the firm after creditors and preference shares are paid.
- The remaining 5% is promised to management equity holders.
- Assume that the exit value is 1.6 *times the original costs* (5 years after investment).

Thus, the initial investment of $5000 has an exit value = 1.6 × $5000 = $8000.

The specific payoffs for the four claimants are as follows:

1) Senior debt has been partially retired using operational cash flows i.e. debt is reduced from $2500 to $1600. So debt-holders get $1600.

2) Preference shares receive = $2400 $(1.12)^5$ = $4230.

- Value of Preference shares increases over time due to capitalization of their preferred dividend.

3) PE Fund equity receives = 0.95 [8000 – (4230 + 1600)] = $2061

4) Management equity receives = 0.05[8000 – (4230 + 1600)] = $109.

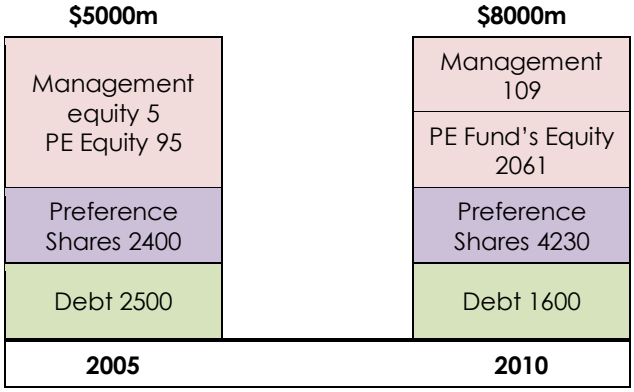
- The increase in equity held by the PE Fund and by the management ***significantly*** depends on the ***total enterprise value upon exit*** i.e. the larger the exit multiple, the larger the upside potential for both the MEP and the equity held by the private equity firm.

IRR realized by the management on its investment = 85% per annum (i.e. n = 5, PV = -5, FV = 109, PMT= 0, compute I = 85%).

IRR realized by the private equity fund holders = 20% per annum (i.e. n = 5, PV = -(2400+95) = -2495, FV = (4230+2061) = 6291, PMT= 0, compute I = 20%). Note that the private equity firm also earns 12% per annum on its preference shares.

	Invested	Proceeds	Multiple	IRR
Management	$5m	$109m	21.8x	85%
PE fund	$2495m	$6291m	2.5x	20%

$5000m enterprise value
12% rolled up dividend to preference shares
Management contribute $5m of equity
PE funds $95m equity and all the preference shares
Lenders fund the debt



2.5 Valuation Issues in Venture Capital Transactions

Dilution of ownership: Ownership is diluted by additional financing rounds and the issuance of stock options to the management of a company.

Limitations of DCF method: DCF valuation method is difficult to use in VC transactions because of significant uncertainty in projected future cash flows.

NOTE:

In buyouts due to the significant predictability of cash flows, the income based approach (i.e. DCF, adjusted PV, LBO model, target IRR) is commonly used.

Limitations of comparable companies approach: Comparable company approach involves difficulty due to unique features of start-up companies and difficulty in finding comparable quoted company in the same sector/industry.

Alternative valuation methods: Due to the limitations of other valuation techniques, VC approach or the real option method are recommended to use to determine value in private equity.

2.6 Exit Routes: Returning Cash to Investors

The most critical mechanism to unlock value in private equity is the "exit". There are four exit routes/options available to private equity investors for their investments:

1. IPOs:

Advantages: It results in higher valuation multiples due to:

- Enhanced liquidity
- Access to larger amounts of capital
- Possibility to attract higher caliber managers

Limitations:

- IPO involves a cumbersome process
- Less flexibility
- Significant costs

Appropriate to use:

IPO is an appropriate exit route for private companies with:

- An established operating history
- Excellent growth prospects and a sufficient size

Note that **timing** of the IPO is considered an important factor.

2. Secondary Market: It refers to sale of equity stake that is held by a financial investor to other financial investors or to strategic investors.

These secondary market transactions are very common and represent a significant proportion of exits (particularly in the buyouts).

Advantages:

- In the absence of an IPO, it results in the highest valuation multiples.

3. Management Buyout (MBO): MBOs refer to takeover by the management of the company with the use of significant amount of leverage to finance the acquisition of the company.

Advantage: It facilitates alignment of interest between management and shareholders.

Limitation: It involves excessive use of leverage, which can result in reduction in company's flexibility.

4. Liquidation: It refers to liquidation of the company by the controlling shareholders when the company is no longer viable.

Advantage: This mechanism generally creates a floor value for the portfolio company.

Limitation:

- It results in a low value.
- It may result in negative publicity for the private equity.

Two important investment management decisions include:

1) Timing the exit.
2) Determining the optimal exit route i.e.
   - When the exit is expected in the near future (1-2 years), current valuation multiples inferred from comparable quoted firms will provide good guidance.
   - When the exit is expected in a longer time horizon, the current valuation multiples become less relevant; thus, stress tests should be conducted on a wider range of values to determine the expected exit multiple.

3. PRIVATE EQUITY FUND STRUCTURES AND VALUATION

The Fund: Funds structured as limited partnerships formed for the specific purpose of investing money. A Private Equity Fund is comprised of two parties:

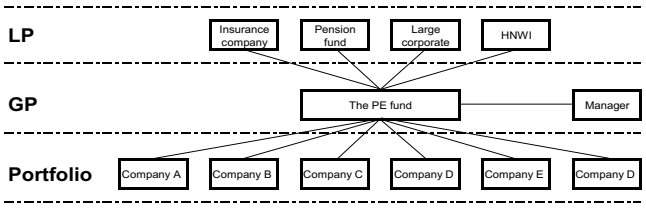
1) General Partner: GP is fund manager.

- The general partner in a limited partnership is responsible for all management decisions of the partnership.
- The GP has a fiduciary responsibility to act in the best interest of the limited partners (LPs) and is fully liable for its actions.

- The GP is paid primarily through two types of fees i.e. a Management Fee and a Carried Interest Fee.

2) Limited Partner (LP): LP is an investor in a limited partnership.

- LP's have limited liability and usually have priority over GP's in case of liquidation of the partnership.
- LPs have no control over the daily management of the Fund.



NOTE:

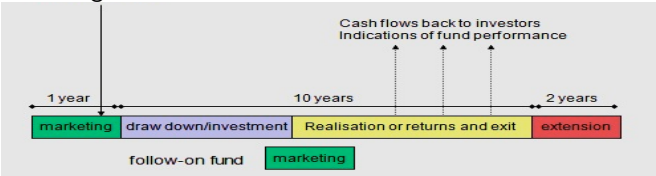
Other corporate structures include a company limited by shares, which is similar to the limited partnership in functioning but it offers relatively a better legal protection to the GP and to some extent the LPs.

- Most of the private equity fund structures are "closed end" in which existing investors are restricted from redeeming their shares over the lifetime of the fund and new investors are allowed to enter the fund only at predefined time periods at the discretion of the GP.
- Term of a Private Equity Fund: Typically 10 years, with 2-3 year extensions with the approval of the Limited Partners.
- Investment Period: Typically 5-6 years.
- Funds are available to qualified prospective investors according to some wealth criteria i.e. exceeding US$ 1 million and/or minimum subscription threshold (minimum 125,000) Euros.
- Private equity firms perform two functions i.e. operate the business of managing private equity investments and raise funds.

How are private equity funds structured?

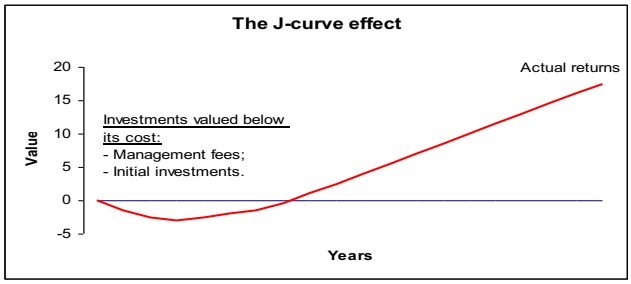
Commitments by investors multiple "closings"

Most private equity is invested via partnerships of a limited duration



Private equity v/s Public Equity:

- In private equity investing, investors initially commit only a certain amount to the private equity fund that is subsequently drawn by the fund as the fund's capital is used in making investments in target portfolio companies.
- In public market investing, investors fully invest total amount of capital committed at the time the orders are settled on the markets.
- Private equity investments follow a "j-curve" pattern.
- **The "j-curve" effect:** It refers to a curve realized by plotting the returns generated by a private equity fund against time (from inception to termination). It shows that a private equity fund will initially show a negative return (due to payment of management fee and start-up costs) followed by positive (increased) returns thereafter.



Following are the terms that are used to ensure alignment of interests between the GP and LP and defining the GP's incentives (transaction fees, profit shares etc.).

Economic Terms:

1. **Management fees:** The Management Fee is paid annually to the GP and is calculated as a percent of total Committed Capital.

- The most common Management Fee ranges from 1.5% to 2.5% of Committed Capital, paid annually.
- Management fees can also be calculated on the basis of the net asset value or on invested capital.
- Management Fees are typically used to pay GP salaries and overhead costs.

2. **Transaction fees:** These fees are paid to GPs for providing investment banking services for a transaction (mergers and acquisitions, IPOs) that benefits the fund.

- When there is fee-sharing agreement (i.e. 50/50% split between the GP and LPs) then these fees are treated as a deduction to the management fees.

3. **Carried interest**: It is the GP's share in the profits generated by a private equity fund.

- 20% is the most common; however it can vary across funds.
- The GP is not required to commit capital to earn its Carried Interest.
- Typically, the GP is allowed to share in the profits of fund only when a fund returns LPs the capital provided by them plus any preferred return.

4. **Ratchet**: It is a mechanism that is used to determine the allocation of equity between shareholders and the management team of the portfolio company. A ratchet effectively increases the percentage of equity allocated to management team depending on the future performance of the company or the rate of return achieved by the private equity firm.

5. **Hurdle rate**: It is the internal rate of return (IRR) that a private equity fund must achieve before its GP receives any carried interest. The hurdle rate is typically ranges from 7% to 10%. The objective of hurdle rate is to align the interests of the GP and LPs.

6. **Target fund size:** It refers to an absolute amount in the fund prospectus or information memorandum. Target fund size indicates GP's capacity to manage a portfolio of a predefined size and ability to raise funds. A fund that closed with a significantly lower size relative to the target size indicates negative signal about GP's ability to raise funds.

7. **Vintage year:** It is the year in which the venture firm (private equity firm) began making investments. Vintage year helps investors in comparing performance of funds of the same stage and industry focus.

8. **Term of the fund:** Term of equity firm is typically 10 years, with 2-3 year extensions with the approval of the Limited Partners. When funds have unlimited duration, they are often quoted on stock markets i.e. *investment trusts*.

Corporate Governance Terms:

1. **Key man clause:** When a certain specified number of key named executives expected to play an active role leave the fund or cease to devote sufficient amount of time to the management of fund then under key man clause, the manager of the fund is temporarily prohibited from making any further ***new*** investments until a new key executive is appointed.

2. **Disclosure and confidentiality:** Private equity firms are not required to disclose their financial performance publicly. In addition, disclosable information is only limited to financial performance of the underlying funds and the information on the companies in which the funds invest is not required to be disclosed.

3. **Clawback provision**: A clawback provision represents the GPs obligation to return LPs any amounts received in excess of the contractually agreed profit share. GPs are generally required to return any *over-distributed* amount (excess distribution) at *liquidation* of the fund; but GPs may be subject to annual reconciliation (or true-up).

4. **Distribution waterfall:** A distribution waterfall is a mechanism providing an order of distributions and sets a priority how these distributions are paid i.e. what amount must be distributed to LPs before any carried interest is paid to GPs.
   i. Deal by Deal (allows for earliest distribution of carried interest) after each individual deal (mostly used in U.S.).
   ii. Total Return (results in earliest distributions to LPs). In this method, carried interest is calculated on the profits of the entire portfolio (mostly used in Europe and FOFs). It involves two ways to determine carried interest:
      a) ***Using committed capital:*** Carried interest is paid to GP only after the fund has returned the entire ***committed*** capital to LPs.
      b) ***Using invested capital:*** Carried interest is paid to GP as long as the value of the investment portfolio exceeds a certain threshold (i.e. 20%) above the ***invested*** capital.

5. **Tag-along, drag-along rights:** In case of Tag-along, drag-along provisions, any potential future acquirer of the company is not allowed to acquire control without extending an acquisition offer to all shareholders including the management of the company.

6. **No-fault divorce:** According to no-fault divorce, a GP can be removed without cause if the removal is approved by a super majority of LPs (i.e. greater than 75%).

7. **Removal for "cause":** This clause allows either a removal of the GP or an earlier liquidation of the fund for a "cause" i.e. gross negligence of the GP, a "key person" event, a felony conviction of a key management person, bankruptcy of the GP or a material breach of the fund prospectus.

8. **Investment restrictions:** These investment restrictions refer to a minimum level of diversification of the fund's investments, a geographic and/or sector focus or limits on borrowing.

9. **Co-investment:** LPs generally enjoy the first priority co-investing along with the GP.

Advantages:

- The first right to co-investing benefits LPs due to lower fees (or zero) and profit share on co-invested capital.
- It prevents "cherry picking" by GP and its affiliates i.e. GP has annual Fixed Percentage of co-investment in all deals.
- It prevents conflicts of interest by prohibiting GP using capital from different funds to invest in the same portfolio company.

Practice: Example 2 & 3, Volume 5, Reading 40.

3.2 What Are the Risks and Costs of Investing in Private Equity?

Private equity investing is typically restricted to "qualified investors" i.e. institutions and high-net-worth individuals who satisfy certain wealth criteria.

Risks of investing in Private Equity include:

1. General private equity risk factors
2. Investment strategy specific risk factors (buyout, VC, mezzanine)
3. Industry specific risk factors
4. Risk factors specific to the investment vehicle
5. Regional or emerging market risks

Types of General Private Equity Risk Factors:

1. **Illiquidity of investments:** Private equity investments are generally not traded on any securities market and therefore they do not have any market to value them. This results in:

- Risk of inconsistency i.e. quarterly marked-to-market valuation, significant degree of subjectivity.
- Risk of stale valuation i.e. infrequent valuation results in understated S.D. and correlation to other asset classes.

2. **Unquoted investments:** Since private equity investments are generally not traded on any securities market, they are risky relative to investing in securities quoted on a regulated securities exchange.

3. **Competition for attractive investment opportunities:** High competition exists for finding attractive investment opportunities.

4. **Reliance on the management of investee (agency risk):** Agency risk refers to uncertainty that the management of the investee companies will run the company in the best interests of the private equity firm. This risk is particularly higher in earlier stage deals because of controlling stake enjoyed by management.

5. **Loss of capital:** Private equity investing involves high business and financial risks, which can result in substantial capital loss.

6. **Government regulations:** Changes in government regulations may adversely affect business model of the investee companies.

7. **Taxation risk:** Taxation risk refers to changes in tax treatment of capital gains, dividends or limited partnerships over time.

8. **Valuation of investments:** Valuation of private equity investments is significantly based on subjective judgment and can be biased if it is not conducted by an independent party.

9. **Lack of investment capital:** Investee companies may face lack of investment capital when they require additional future financing.

10. **Lack of diversification:** Private equity investment portfolios may be highly concentrated and may result in significant losses due to lack of diversification.

11. **Market risk:** Private equity investments are adversely affected by changes in general market conditions (i.e. interest rates, currency exchange rates etc.). Due to long-term horizon of private equity firms, market risk has long term impact. Hence, temporary short-term market fluctuations are generally irrelevant.

Costs of investing in Private Equity: Private equity investing involves higher costs relative to public market investing. These costs may be classified as follows:

1. **Transaction fees:** Transaction fees are associated with due diligence, bank financing costs, legal fees etc.

2. **Investment vehicle fund set up costs:** Setup of the investment vehicle incurs legal costs. These costs are typically amortized over the life of the investment vehicle.

3. **Administrative costs:** These costs include custodian fee, transfer agent fee, and accounting costs generally charged annually as a % of the investment vehicle's NAV (net asset value).

4. **Audit costs:** Audit costs generally represent a fixed annual fee.

5. **Management and performance fees:** Typically, 2% management fee and a 20% performance fee are common in the private equity industry.

6. **Dilution:** It is a cost associated with the dilution of ownership as a result of stock option plans granted to the management and to the private equity firm and from additional rounds of financing.

7. **Placement fees:** These fees refer to fundraising fees that may be charged up-front (e.g. 2%) or trailer fee that is charged by a fund raiser (i.e. charged annually as a % of amount invested by limited partners as long as these amounts remain invested in the investment vehicle).

3.3 Due Diligence Investigations by Potential investors

A thorough due diligence should be conducted by investors before investing in private equity fund. Following are some fundamental characteristics of private equity funds that indicate importance of the due diligence process.

- Private equity funds tend to exhibit a strong

persistence of returns over time i.e. top performing funds tend to continue to outperform and poor performing funds tend to continue to underperform or disappear.
- The range of performance between funds is significantly large e.g. the difference between top quartile and third quartile fund IRRs can be around 20% points.
- Private equity has limited liquidity and investors are locked for the long-term.
- Private equity investments have shorter duration (i.e. less than maximum life of the fund) as cash is immediately returned to investors when private equity fund exit an investment.

3.4 Private Equity Fund Valuation

It is difficult to value investments in private equity fund as there is no market for securities issued by private equity companies.

Private equity valuation is generally associated with estimating NAV (i.e. NAV = fund asset – fund liabilities). These private equity valuations are mostly done by GPs according to their valuation policies*.

*NAV can be valued in six different ways:

- At cost with adjustments for subsequent financing or deterioration events.
- At lower of cost or market value.
- By revaluation in case of new financing.
- At cost without any interim adjustments.
- Using a discount for restricted securities.
- Marking to market by reference to a peer group of public comparables and applying liquidity discounts.

Issues in estimating NAV:

- NAV provides stale valuations when it is adjusted infrequently i.e. adjusted only for subsequent financing rounds.
- NAV valuations are based on subjective judgments because of uncertainty of market value of portfolio companies.
- Undrawn commitments are not part of the NAV; however, they should be treated as unfunded liabilities. The value of these commitments is low when GP face difficulty in raising funds.
- Different valuation techniques are used for different strategies and maturities.
- These private equity valuations are mostly done by GPs and thus may be subject to bias.

3.5 Evaluating Fund Performance

Each private equity has unique characteristics. Therefore, following are some of the typical factors that should be considered in evaluating private equity fund's financial performance.

3.5.1) Analysis of IRR and Multiples Gross and Net of Fees since Inception

Benchmarks are used to measure and monitor the performance of private equity transactions. There are two benchmarks used in private equity fund's financial performance evaluation:

1. ***Internal Rate of Return (IRR):*** It represents the average annual return generated by an investment. It provides investors an idea of the results of the fund over time. IRR is greatly affected by time i.e. an investment that has doubled in value over 3 years will have a much higher IRR relative to the one that has doubled in value over 8 years. IRR (a cash flow weighted rate of return) is generally considered the most appropriate measure of private equity performance by the GIPS, VC and Private equity valuation principles etc.

Assumptions: IRR assumes that the private equity fund is liquid but in reality a significant portion is not.

Types of IRR:

a) Gross IRR: A gross IRR or gross value multiple relates to the performance of a private equity fund before fees are paid to GP. It is a measure of cash flows between the portfolio companies and the fund.

b) Net IRR: The net IRR or value multiple relates to the performance of a private equity fund that is enjoyed by LPs (investors) after all the fees have been paid. It is a measure of cash flows between the fund and LPs.

2. ***Multiples:*** They measure the total return to investors relative to the total sum invested.

Advantages:

- Easy to calculate.
- Have ability to differentiate between "realized" actual proceeds from divestments and "unrealized" portfolio subject to GP valuation.

Limitation: They ignore time value of money.

Definitions of terms:

Net of fees: Net of fees refer to net of management fees, carried interest, or of any other financial arrangements that accrue to the GP.

Total current invested capital: The total current invested capital is the amount of the paid-in capital that is actually invested in private equity assets.

Distributions: The total distribution is the total amount of capital/income that has been returned to investors. This measure facilitates prospective investors to compare an amount of initial invested capital returned relative to other composites with similar vintage years and strategies.

Committed Capital: Committed Capital is the total dollar amount that an individual LP agrees to commit to a fund over the life of the fund.

- It is a legally binding commitment that is made during the fundraising process.
- The committed capital is not drawn at once.
- Committed Capital is not paid by the LP's when the fund is created; it is drawn dawn over time as the fund makes investments.

Operating results: It corresponds to the sum of realized returns upon exit of portfolio companies and unrealized returns from the revaluation of investments held in portfolio companies.

Performance Measures: Performance measures include:

1. **PIC (paid in capital):** The paid-in capital to date is the amount of the total committed capital that the firm has drawn down (called) from investors.

PIC = Cumulative capital called down
PIC Multiple = PIC / Committed capital

- PIC multiple indicates the proportion of capital called by a GP.

2. **DPI (distributed to paid in):** DPI is the ratio of cumulative distribution paid out to LPs as a proportion of the cumulative invested capital (or capital called down). This ratio is also known as "cash-on-cash return" or realization multiple.

DPI = Sum of distributions / cumulative capital called down (or PIC)

- It indicates ***realized*** return on investment of the private equity fund.
- It indicates how much of the return has actually been realized and returned to investors.
- DPI will be zero in the early life of an independent fixed-life fund until distributions are made.
- DPI will increase as the fund matures.
- When the DPI = 1.00, it represents break-even point of the fund. A DPI of greater than one indicates that the fund has generated capital gains.
- DPI is reported net of management fees and carried interest.

3. **RVPI (residual value to paid in):**

RVPI = NAV after distributions / cumulative capital called down (or PIC)

- RVPI indicates how much of the return is unrealized.
- RVPI will increase to a maximum point as the fund matures and then decreases to a residual market value of zero when the fund liquidates.
- A residual market value of zero indicates that the entire return of the fund has been distributed to investors.
- RVPI is reported net of management fees and carried interest.

4. **TVPI (total value to paid in):** It is also known as investment multiple.

TVPI = DPI + RVPI

- It provides prospective clients information regarding the value of the composite relative to its cost basis.
- TVPI is reported net of management fees and carried interest.

Note:

Paid in capital and distributions are cash flow items while residual value is non-cash flow item.

Calculations:

Management fees = % fee × PIC

Carried interest*= % × (NAV before distributions – committed capital)

*Carried interest is first paid in the year NAV before distributions > Committed capital

- Thereafter, (provided that NAV before distribution > committed capital):
- Carried interest = [20% × (increase in NAV before distributions)]

NAV before distributions$_t$ = NAV after distributions$_{t-1}$ + called down capital$_t$ – management fees$_t$ + operating results$_t$

NAV after distributions$_t$ = NAV before distributions$_t$ – carried interest$_t$ – distributions$_t$

Cash flows for Gross IRR$_t$= – Capital called down at the beg of period$_{t+1}$ + Operating result$_t$

- For example Cash flows for Gross IRR (for year 2003) = – Capital called down in 2004 (beg) + Operating result (2003)

Cash flow for Net IRR$_t$ = – Capital called down at the beg of period$_{t+1}$ + Operating result$_t$ – management fees$_t$ – carried interest$_t$

- For example Cash flows for Net IRR (for year 2003) = – Capital called down in 2004 (beg) + Operating result (2003) – management fees (2003) – carried interest (2003)

NOTE:

Beginning of period t+1 =End of period t

NOTE:

In addition to these quantitative performance measures, some qualitative measures are also used e.g. analysis of realized investments, unrealized investments, cash flow forecast at both company and portfolio level, portfolio valuation, audited financial statements and NAV.

Benchmarks:

IRR measure is necessary for private equity assets because cash in-flows and out-flows are controlled by the firm. Therefore, time-weighted rate of return (TWRR) is not an appropriate measure to use in private equity funds as it does not incorporate the effects of cash flow management within the control of the private equity manager.

However, IRR returns exhibit definitive trends; therefore, Net IRR should be compared relative to comparable private equity funds i.e. funds with same vintage and strategy.

IRR is cash-flow-weighted measure whereas performance of most other asset classes is measured in terms of time-weighted rate of return.

Solution: Publicly traded equity benchmark returns can be converted to cash weighted returns using the same cash flow pattern as a private equity fund e.g. Public Market Equivalent (PME) is the cash flow weighted rate of return of an index (S&P 500 or any other index). It is regarded as an index return measure.

Venture Capital Method:

Let,

V = terminal value (at time of exit)
t = time to exit event (When VC gets money back)
I = amount of investment or Amount being raised from VC
r = discount return used by investors (VC's required return (IRR))
x = number of existing shares (currently owned by the entrepreneurs) = 1 million

Numbers we need to calculate:

- F = Fraction of company VC would need to own to achieve their required return on investment.
- Post-Money Valuation = Value of company ***after*** funding is received
- Pre-money Valuation = Value of company ***before*** funding is received
- P = Price per share.
- y = Number of shares required by the VC to obtain F

The General Case: NPV Method

Step 1: Post money valuation = POST = $V / (1 + r)^t$

Step 2: Pre money valuation = PRE = POST - I

Step 3: Required ownership fraction for the investor = F = I / POST

Step 4: Number of shares the investors require to achieve their ownership fraction = $y = x [F / (1 – F)]$

Step 5: Price per share = $p_1 = I / y$

Example:

ABC.com is asking for $5 million investment, Projected income in year 5 is $4 million and expected exit multiple is 25x. Company currently has 1 million shares all owned by the entrepreneur. What share of company would a VC require today if VC's required return is 50%?

- Exit Value = $4 x 25 = $100 million
- Post money valuation = $100/(1+50\%)^5 = 13.169$ million
- Pre money valuation = 13.169 – 5 = $8.169 million

Since 1 million shares outstanding Price per share = $8.17

- Alternatively VC must get = 5/13.17 = 37.97% of the company
- $y/(1{,}000{,}000+y) = F = 37.97\%$; algebraic manipulation yields $y = 612{,}123$ shares.
- Price per share = 5,000,000/612,123 = $8.17

Alternative Method using IRR:

Step 1: Amount of wealth investors expect to accumulate = $W = I (1 + r)^t$
Step 2: Fraction of share ownership required by investors = F = W / V

where,

V is the projected terminal value of the company at exit (given the projected earnings at exit and an appropriate Price Earnings ratio (PER) for the company):

Step 3: Number of shares the investors require to achieve their ownership fraction = $y = x [F / (1 – F)]$

Step 4: Price per share = $p_1 = I / y$

Step 5: Post-money valuation = POST = I / F or $p_1 \times (x + y)$

Step 6: Pre-money valuation = PRE = POST – I or PRE = $p_1 \times x$

Example:

ABC.com is asking for $5 million investment, Projected income in year 5 is $4 million and expected exit multiple is 25x. Company currently has 1 million shares all owned by the entrepreneur. What share of company would a VC require today if VC's required return is 50%?

- Exit Value of ABC.com = $4 x 25 = $100 million

- Exit Value of VC has to be =$5(1+50%)^5 = 37.97 million
- Fraction of Company Needed = 37.97/100=37.97%
- Implied POST MONEY Valuation=5/0.3797=13.17 million
- Implied PRE MONEY Valuation= 13.17-5=8.17 million
- $y/(1{,}000{,}000+y)$ = F =37.97%; algebraic manipulation yields y= 612,123 shares.
- Price per share = 5,000,000/612,123 = $8.17

When there are two rounds of financing:

Step 1: Define appropriate compound interest rates e.g. for time between dates T_1 and T_2, compound interest = $(1 + R_1)$. For time between dates T_2 and T_3, compound interest = $(1 + R_2)$.

Step 2: Post money valuation = $POST_2 = V / (1 + R_2)$

where,

$POST_2$ = *post-money valuation at the time of the second round*
V = *terminal value*
R_2 = *compound discount rate between the time of the second round and the time of exit.*

Step 3: Pre money valuation = $PRE_2 = POST_2 – I_2$

where,

PRE_2 = *pre-money valuation at the time of the second round of financing*
I_2 = *amount raised in the second round.*

Step 4: Post money valuation = $POST_1 = PRE_2 / (1 + R_1)$

where,

$POST_1$ = *post-money valuation at the time of the first round*
R_1 = *compound discount rate between the time of the first round and the second round.*

Step 5: Pre money valuation = $PRE_1 = POST_1 – I_1$

where,

PRE_1 = *pre-money valuation at the time of the first round of financing*
I_1 = *amount raised in the first round.*

Step 6: Required ownership fraction for the investor in the second round= $F_2 = I_2 / POST_2$

Step 7: Required ownership fraction for the investor in the first round= $F_1 = I_1 / POST_1$

Step 8: Number of new shares the investors in the 1st round require to achieve their ownership fraction = $y_1 = x_1[F_1/ (1 – F_1)]$

where,

x_1 *is the number of existing shares*

Step 9: Price per share in the 1st round = $p_1 = I_1/ y_1$

Step 10: Number of existing shares at the time of the second round = $x_2 = x_1 + y_1$

Step 11: Number of new shares the investors in the 2nd round require to achieve their ownership fraction = $y_2 = x_2[F_2/ (1 – F_2)]$

Step 12: Price per share in the 2nd round = $p_2 = I_2/ y_2$

Accounting for Risk in Venture Capital: The current valuation is strongly affected by discount rate and estimate of terminal value. There are two ways to account for risk in VCs.

1) **By adjusting Discount Rate** i.e. a high discount rate reflects both the probability of failure and lack of diversification.

$$\mathbf{r^*} = \left(\frac{1+r}{1-q}\right) - 1$$

where,

r * = adjusted discount rate
r = discount rate unadjusted for probability of failure
q = probability of failure

2) **By adjusting terminal value using Scenario analysis** i.e. TV is adjusted for the probability of failure or poor results.

Adjusted TV = (% probability for scenario 1 × expected earnings × expected price-earnings multiple) + (% probability for scenario 2 × expected earnings × expected price-earnings multiple) + … + (% probability for scenario n × expected earnings × expected price-earnings multiple)

Limitation: It is difficult to apply due to difficulty of finding true comparables.

VC valuations significantly depend on assumptions and treatment of risk. In addition, the final transaction price in VC investments greatly depends on the negotiating power of the buyer and seller.

Practice: End of Chapter Practice Problems for Reading 44 & FinQuiz Item-set ID# 11382.

1. INTRODUCTION

Commodities trade in physical (spot) markets and in futures and forward markets.

- **Spot Market**: In spot markets, goods are physically transferred between buyers and sellers. The prices in spot markets reflect current (or very near term) supply and demand conditions.
- **Futures Market**: Futures markets involve trading of standardized futures contracts in financial exchanges. In these markets, a price is established in the market today for the sale of some defined quantity and quality of a commodity at a future date of delivery. The futures contract can be based on cash settlement or physical delivery.

Commodity futures exchanges facilitate transfer of risk, price discovery, and liquidity in the market. Exposure to commodities is also traded in forward markets and in swap markets. Forward markets are suitable for investors that require customization in contract terms.

Commodities have historically low average return correlation with stocks and bonds. Therefore, in a multi-asset class portfolio, commodities offer the potential for diversification benefits.

2. COMMODITIES OVERVIEW

Commodities are valued differently than traditional financial assets (i.e. equities and bonds). Unlike traditional financial assets, whose intrinsic value is calculated as the present discounted value of their expected future cash flows, the value of commodities is derived from either their use as consumables or as inputs to the production of goods and services.

2.1 Commodity Sectors

Commodities can be segmented by sector, i.e. energy, grains, industrial (base) metals, livestock, precious metals, and softs (cash crops). The supply and demand of each commodity is determined based on different factors, including weather and geo-political and geo-economic events, frequency/timing of consumption, spoilage, insurance, ease and cost-effectiveness of storage, and ease of transportation to consumers.

2.1.1.) Energy

The energy sector is the most economically valuable of all the commodities. Typically, there are three distinct products and value chains in the energy sector:

i. **Crude oil**: Unlike other commodities, crude oil does not require special storage facilities. Since it has limited use by itself, it must be extracted, transported, and refined into useful products. Liquid crude oil is comparatively easily transported via ship.
   - Crude oil that has relatively low density and flows freely at room temperature is called light. Light oil is easier to process, hence, it is traded at a substantial premium to heavier oils. Oil prices vary due to differences in the quality of crude oil. Crude oil from the North Sea (represented by Brent Crude), Middle East (typically represented by a price benchmark, such as "Abu Dhabi Light"), Nigeria (Bonny Light), or the southern United States (represented by the West Texas Intermediate or Louisiana Light price benchmarks) can all sell at a substantial premium to heavier oils, such as benchmark Mayan Crude (from Mexico).
   - Sweet oil is oil with low sulfur content. High sulfur content oil is cheaper because it destroys metal piping.

Drivers of global oil supply and demand:

- **Technology**: Technology affects oil usage in three forms: the level of technology for extraction, the level of technology and efficiency in which oil is transformed into useful products, and the efficiency with which these products are used by the engines that burn them. Technological revolution has increased the supply of oil by minimizing cost of drilling, pressurization, and extraction (e.g. shale oil). The demand for oil tends to decrease as the cost of substitutes becomes more affordable with improvement in technology.
- **Efficiency by the end users**: Efficiency by the end users of refined products also affects the supply and demand balance for oil. E.g. improvement in mileage efficiency in automobile engines tend to reduce the demand of oil.
- **Politics:** The supply of oil is impacted by geopolitical conflicts. Environmental limits on the extraction and/or transportation of oil also impacted the supply of crude oil.

- **Business cycle:** In recession, demand for oil tends to decrease due to the destruction of credit and diminished business confidence.
- **Weather:** Demand for oil tends to rise during summer (cooling requirements) and winter (heating needs).

ii. **Natural gas:** Natural gas is typically categorized by its source as either "associated gas" (coming up from an oil well) or "unassociated gas" (coming from a gas field or shale rock where oil is not present). Because associated gas is a co-product of oil production, its supply is driven by the demand for oil. Sometimes, associated gas is collected and re-injected into the oil field to maintain pressure and keep oil production costs low. The global drivers of natural gas supply and demand are similar to crude oil.
   - Unlike crude oil, natural gas can be used cost-effectively and directly for transportation and electrical generation after cleaning out impurities (carbon dioxide).
   - Heavier compounds (called natural gas liquids or NGLs) are also extracted and are highly useful in chemical production.
   - Due to the need to keep the gas under pressure, storage and transportation costs are relatively high for natural gas.

iii. **Refined products**: Refined products are end-use fuels, such as heating oil, gas oil, jet fuel, propane, gasoline, and bunker fuel. The refined product is used in cars, trucks, planes, and ships, used as raw materials for chemical production, used in roads construction in the form of asphalt. Typically refined products have a short shelf life, with availability measured in days. This imply that in order to ensure adequate supplies, refineries must run continuously and coordinate scheduled maintenance. Due to tightening of environmental mandates pertaining to pollution standards, processing costs for refined products have increased while demand has decreased.

2.1.2.) Grains

Generally, grains have a long enough storage period as they can be stored for multiple seasons. Some crops can be grown multiple times per year. The supply of grains is affected by weather as levels of heat and precipitation determining yields and acreage. Disease and pests also impact the supply.

2.1.3.) Industrial (Base) Metals

Industrial metals include copper, aluminum (or aluminium), nickel, zinc, lead, tin, and iron. These commodities are used in industrial production, including construction, infrastructure development, and durable goods (cars, planes, ships, household goods, and military products) manufacturing.

- Demand for these metals is directly linked with GDP growth.
- Most industrial metals have longer storage life, hence, their supply is not affected by weather conditions. However, demand may be affected by weather and seasonal factors because building construction requires suitable conditions.
- The supply of industrial metals is impacted by politics; e.g., labor strikes limit supply which lead to increase in prices.

2.1.4.) Livestock

The supply and price of livestock sector depend on conditions in grain markets, GDP per capita, and weather conditions.

- If grain prices increase significantly, animals are slaughtered more quickly to avoid the higher cost of maintaining (i.e., feeding) them. Resultantly, supply increases in the near term, temporarily lowering prices. Opposite occurs when grain prices fall.
- The supply of livestock is also impacted by weather. In winter, cattle tend to suffer more than hogs and chickens because of their height. In summer, heat and humidity limit the weight gain in hogs.
- The supply of livestock is affected by disease as well. Diseases which can be transmitted to humans tend to drive the price of livestock down. Diseases which may create shortage of animals tend to increase the prices.
- Supply of livestock is influenced by government-permitted use of drugs and growth hormones.
- The demand for livestock depends on global (notably emerging market) wealth owing to increasing import/trade activity and cross-border transactions (i.e. mergers and acquisitions).

2.1.5.) Precious Metals

Gold, silver, and platinum can serve as stores of value (like currencies) and can be used as inputs in electronics, auto parts, and jewelry. Typically, gold acts as a hedge to the value of paper currencies. The precious metals have longer storage period (i.e. decades, centuries, and even millennia).

- The supply and demand of precious metals is not affected by weather.
- Factors that affect supply and demand of previous metals include inflation expectations, fund flows, and industrial production.

2.1.6. Softs (Cash Crops)

Soft commodities include cotton, coffee, sugar, and cocoa. Currently, softs are generally cash crops (i.e., sold for income). Unlike hard commodities which are mined, soft commodities are grown.

- Proper storage is very essential for soft commodities because quality and weight of the commodity are determined by freshness of the commodity.
- Supply of soft commodities is impacted by weather conditions because these are dependent on high levels of properly timed rainfall. Cold weather and diseases can adversely impact growth of soft commodities.
- Like livestock, demand for softs depends on global (notably emerging market) wealth.

Practice: Example 1 & 2, Reading 45, Curriculum.

2.2 Life Cycle of Commodities

The market demand and supply can be easily adjusted for commodities having shorter life cycles. Whereas, it is hard to react to changes if commodities have long life cycle. The shifts in demand and supply of commodities result in changes in valuation and shape of the commodity supply and demand curves and their respective price elasticities of demand and supply.

- Among the food commodities, agriculture and livestock have well-defined seasons and growth cycles that are specific to geographic regions.
- Energy and metals sectors are extracted all year around; hence, their life cycles are not specific to discrete time or season.
- Crude oil and metal ore have seasonal demands depending on weather (e.g., gasoline demand in the summer and heating oil demand in the winter) that affect the life cycle and usage of the underlying commodity.

The life cycles of several key commodity sectors are as follows.

Energy:

- *Extraction of oil or gas takes around 50-100 days to begin*. This process involves selection of drilling location, digging of well, siphoning of crude oil or natural gas, and fracturing process or fracking (for shale oil).
- Transportation (i.e. shipping via pipe, ship, train or truck to a storage facility, refinery, or consumer) takes around 1 – 10 days.
- Crude oil is stored for a few months on average.
- At the stage of trading, natural gas can be consumed but crude oil requires further processing (referred to as refining).
- During refining stage, crude oil is distilled into its component parts via a process generally called cracking. This process takes 3-5 days to complete.
- After being refined, crude oil is transported to various locations for consumption. Transportation takes 5 – 20 days. Commodity futures trading also occurs at this stage.

Note: US-based crude oil (West Texas Intermediate, or WTI, crude oil) and the UK-located Brent crude oil from the North Sea contracts represent a high-quality refinery input that can be used as a hedging device by exploration and production companies.

Practice: Example 3, Reading 45, Curriculum.

Industrial/Precious Metals: The life cycle of both precious and industrial metals is highly flexible because if stored properly, these metals can be stored for months (if not years). The purification process of copper is discussed below:

- Copper (ore) is extracted via mine or open pit and then grounded into powder so that it can be purified and unwanted material can be removed. Afterwards, ore is gone through roasting (heated to 500 – 700 degrees Celsius), resulting in a solid called calcine. The solid calcine is heated to 1200 degrees Celsius and melts (smelting process) to remove further impurities. Then, the calcine is converted into liquid forming blister copper by blowing air. The blister is reshaped and purified through electrolysis (electro-refining process). This purified metal is stored in bonded warehouse until it is transported to end users.

Large scale of facility and higher capacity utilization bring economies of scale in smelting and roasting processes by declining the marginal costs (i.e., the cost to convert the last pound or kilogram of processed ore into a useful metal). As a result, when supply is greater than demand, it is difficult to cut back production of industrial metals – leading to overproduction in the market. When utilization (and profit) is high, metal producers have an incentive to invest in new capacity.

Practice: Example 4, Reading 45, Curriculum.

Livestock: The timing to maturity typically increases with size. Poultry matures in few weeks, hogs in months, and cattle in a few years. Ranchers and slaughterhouses trade hog and cattle futures to hedge against their

processed meat commitments. Cattle can be of two types, i) feeder cattle that are young cattle on pasture, and ii) live cattle that are fattened for butchering.

Practice: Example 5, Reading 45, Curriculum.

Grains: Grains in the northern hemisphere follow a similar growth cycle but opposite growth cycle in the southern hemisphere.

	Corn	Soybeans	Wheat*
Planting	April–May	May–June	Sept–Oct
Growth	June–Aug	July–Aug	Nov–Mar
Pod/Ear/Head Formation	Aug–Sept	Sept	April–May
Harvest	Sept–Nov	Sept–Oct	June–July

Farmers and consumers can trade futures to hedge their exposure to the crop. In such contracts, the contract delivery months reflect the different times of the growing cycle. Grain futures can also be traded by ranchers to hedge against the cost of feeding an animal.

Softs: E.g. Coffee is harvested somewhere all year around, but the best quality coffees are from high-altitude plantations and are picked in the middle of the harvest periods.

Timetable for Coffee Harvesting

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Brazil							Harvest	Harvest	Harvest	Harvest	Harvest	
Central America	Harvest	Harvest	Harvest							Harvest	Harvest	Harvest
Vietnam	Harvest	Harvest	Harvest								Harvest	Harvest
Java						Harvest	Harvest	Harvest	Harvest	Harvest		
Kenya	Harvest	Harvest	Harvest			Harvest	Harvest			Harvest	Harvest	Harvest
Jamaica	Harvest	Harvest	Harvest									Harvest

There are two main varieties of Coffee:

1) *Robusta:* Robusta beans are lower quality with less flavor. Robusta variety futures contract is traded in London.
2) *Arabica:* These are high quality coffee beans. Arabica futures contract is traded in New York.

Farmers and distributors can sell futures contracts to hedge the sales price of production, and coffee roasters can buy futures contracts to hedge coffee bean purchase costs. Brazil is the world's largest coffee producer and produces a mix of both beans, whereas Vietnam, the second largest producer, predominantly produces robusta.

2.3 Valuation of Commodities

Commodities are tangible items with an intrinsic (but variable) economic value. But, unlike financial assets, valuation of commodities is not based on the estimation of future profitability and cash flows but a discounted forecast of future possible prices based on supply and demand of the physical item or the expected volatility of future prices. The value of commodity derivative contracts depends on the price of the underlying commodity. The shape of a forward price curve of the commodity derivative contracts is affected by the transportation and storage costs associated with owning a commodity. If there are large storage and transportation costs, the prices for a longer term commodity futures contract will likely be higher.

Practice: Example 6 & 7, Reading 45, Curriculum.

3. COMMODITY FUTURES MARKETS

- Public commodity markets are structured as futures markets – centralized exchange where participants trade standardized contracts to make and take delivery at a specified place at a specified future timeframe. Futures contracts are standardized agreements traded on public exchanges, such as the Chicago Mercantile Exchange (CME), Intercontinental Exchange (ICE), and Shanghai Commodities Exchange. The exchange provides regulatory oversight by overseeing trading and margin requirements. Future contracts are marked to market, that is, daily cash movements occur in the futures price. Future contracts are less risky because the exchange acts as guarantor of the payments. Futures contracts have maturities extending up to about a year (e.g., livestock) to several years (e.g., crude oil).
- Forward contracts are bilateral, OTC (or over the counter), agreements between known parties. These contracts can be customized according to the specific needs of the hedging (or speculating) party and involve less regulatory oversight. Forward contracts are usually only settled upon expiration.

Commodity markets versus Markets for stocks and bonds: The commodity markets are net zero in terms of aggregate futures positions (futures contract longs equals futures contract shorts). In contrast, stocks and

bonds market may have net long position. Shorting commodity futures is much simpler as compared to shorting an equity because short investors in commodities only need to post the same margin that long investors post.

3.1 Futures Market Participants

There are a number of participants in commodity futures markets as discussed below.

1) **Commodity Hedgers**: Hedgers trade in the markets to hedge their exposures related to the commodity. The primary motivation behind using futures is to mitigate the risk of cash flow. Hedging is not the same as speculating.

	Long Position	Short Position
Hedging	Snack chips manufacturer can hedge the price of corn needed for snack chips by taking long position in futures contract.	Gold mining company can short future contract by to hedge against likely fall in gold prices in future.
Speculating	Integrated oil company can make bets on future price of oil using its knowledge of physical oil markets.	Commodity trading adviser (CTA) can go short a commodity future contract to earn a profit for clients via a macro commodity investment fund

2) **Commodity Traders and Investors**: Commodity traders and investors include three primary types:
   i. Informed investors: These include hedgers and speculators, including index and institutional investors.
   ii. Liquidity providers: These include speculators willing to provide liquidity to generate some profit, i.e., buying when the producer wants to sell and selling when the consumer is willing to buy.
   iii. Arbitrageurs: These traders generate profit by capitalizing on mispricing between the commodity and the futures price. They can keep physical commodities inventory.
3) **Exchanges (or clearing houses):** Exchanges set trading rules and provide the infrastructure of transmitting prices and payments. These exchanges also provide centrally established, publicly available pricing.
4) **Commodity Market Analysts**: Analysts use the exchange information for non-trading purposes, i.e., making research reports, evaluating commodity businesses, creating products based on commodity futures (e.g., exchange-traded funds, swaps, and notes), and making public policy decisions. Analysts also include brokers and other financial intermediaries who participate in the markets for non-trading purposes.
5) **Regulators**: Regulators of both the exchange and traders monitor the markets for any market abuse. In U.S., commodity and futures markets are regulated by Commodity Futures Trading Commission (CFTC). The CFTC delegates much of the direct monitoring to the National Futures Association (NFA) – a self-regulatory body whose members are the authorized direct participants in the markets with customer responsibilities (e.g., clearing firms, brokers, advisers). Other countries usually have a unified regulatory structure, i.e. one commission regulates both futures and securities. E.g. China Securities Regulatory Commission.

Uses of Commodity prices:

- Commodity prices are used in making projections about inflation;
- Commodity prices are used in other indexes that indicate quality of life for consumers and households.
- Commodity prices are used by governments for controlling natural resource extraction.

Practice: Example 8, Reading 45, Curriculum.

3.2 Spot and Futures Pricing

Commodity prices are typically represented by spot prices in the physical markets and futures prices for later delivery. Spot prices are highly localized and associated with physical delivery. In contrast, futures prices can be global and can be used for hedging and speculating motives.

Benefits of Futures Prices:

- Futures prices promote liquidity;
- Futures prices can act as a reference price point for customized (i.e., forward) contracts;
- Futures prices can be used by market participants and governments to judge supply and demand and to make planning decisions.

Basis: It is the difference between spot and futures prices.

- **Backwardation**: When the spot price > futures price, the situation is called backwardation. Similarly, when the near-term (i.e., closer to expiration) futures contract price is higher than the longer-term futures contract price, the futures

market is in backwardation. As a long owner of a futures contract in backwardation, value increases over time as the contract pricing moves closer to the spot price.

- **Contango**: When the spot price < futures price, the situation is called contango. When the near-term futures contract price is lower than the longer-term futures contract price, the futures market is in contango. As a long owner of a futures contract in contango, value decreases over time as the contract pricing moves closer to the spot price.

Calendar Spread: The price difference (whether in backwardation or contango) between spot price and futures price is called the calendar spread. Like single-month (nearest to expiration) futures contract, the calendar spreads are traded with their own bid–ask prices, trading range, and order book.

- When futures market is in backwardation, calendar spread is positive.
- When futures market is in contango, calendar spread is negative.

Settlement of Commodity Futures: Commodity futures are settled by either cash or physical delivery.

- Cash-settled contracts do not have any value after the maturity date. Cash settled contracts are popular among speculators and arbitrageurs.
- Physical-settled commodity futures contracts require transfer of the title of the actual commodity by the seller of the futures contract to the buyer at a particular place, on or by a particular date, and of a particular quality specification. Unlike cash-settled contracts, physical delivery contracts ensures a convergence of the futures and spot market. Complete convergence is limited due to trading costs and other factors. Due to complexity associated with quality or variety differences in the commodity, physical delivery contracts require a premium or discount associated with specifications.

Practice: Example 9,10,11 & 12 Reading 45, Curriculum.

3.3 Futures Returns

3.3.1.) Theories of Futures Returns

1) **Insurance Theory**: This theory says that commodity producers who are long the physical good can use commodity futures markets (taking short position) for insurance by locking in prices to hedge their production price risk exposure. This theory is based on concept of normal backwardation, that is, futures price must be than the current spot price as a form of profit to the speculator who assumes price risk and provides price insurance to the commodity seller. However, the presence of backwardation does not guarantee positive returns.
2) **Hedging Pressure Hypothesis**: This theory states that producers along with consumers seek to protect themselves from commodity market price volatility by entering into price hedges to stabilize their projected profits and cash flow. Producers of commodities tend to sell commodities forward and thus sell commodity futures. Whereas, consumers of commodities tend to buy commodity futures to lock in price of the commodity.

- When number of producers seeking price protection is greater than consumers, the shape and structure of the futures price curve will reflect backwardation.
- When number of consumers seeking price protection is greater than producers, the shape and structure of the futures price curve will reflect contango.
- When number of consumers seeking price protection is equal to number of producers, future commodity curve will be flat in shape.

Generally, producers have greater exposure to commodity price risk than consumers. It is usually difficult to measure the asymmetry in hedging pressure between buyers and sellers of a commodity.

3) **Theory of Storage:** This theory focuses on supply and demand dynamics of commodity inventories, including the concept of "convenience yield." Under this theory, shape commodity futures price curves is determined by the level of commodity inventories. Storing physical assets involve costs (e.g. rent, insurance, inspections, spoilage, etc.). Therefore, to account for these storage costs, a commodity that is regularly stored should have a higher price in the future (contango). In contrast, when commodities require minimal storage (low level of inventories), demand dominates supply and current prices are higher than futures prices (i.e., backwardation). Convenience yield refers to a benefit of having inventory of commodity available which acts as a buffer to a potential supply disruption. The convenience yield is low when commodity has abundant supply. Opposite is true when inventories are low. Under Theory of Storage, futures prices can be calculated as follows:

Futures price = Spot price of the physical commodity + Direct storage costs (such as rent and insurance) – Convenience yield

Practice: Example 13,14,15 & 16 Reading 45, Curriculum.

3.3.2.) Components of Futures Returns

The total return on commodity futures can be divided into three components:

1) **Price return (or spot yield):**

Price return = (Current price – Previous price)/Previous price

As investors roll over the futures contract by selling the current contract as it approaches expiration and buying the next contract (assuming a long position), there is likely a difference between two prices of future contracts depending on the shape of the futures curve.

- In case of backwardation, investor needs to buy more far contracts than the near contracts being sold.
- In case of contango, investor needs to buy fewer far contracts than the near contracts.

Example: Assume an investor has $100 of exposure in wheat futures and the near contract is worth $10 of exposure (so, the investor has $100 exposure divided by $10 per contract, or 10 contracts), but the far (i.e., longer expiration date) contract is worth only $8 of exposure. To roll forward this contracts and maintain a constant level of exposure, the investor needs to roll the 10 contracts forward and also buy an additional 3 contracts to keep the post-roll exposure close to the pre-roll exposure ($100 exposure divided by $8 per contract = 12.5 , or 13 contracts rounded).

Example: Assume an investor has $100 of exposure in gasoline futures and the near contract is worth $ 10 of exposure (so, the investor has $100 exposure divided by $10 per contract, or 10 contracts), but the far contract is worth $11 of exposure. Thus, in order to roll forward contracts and maintain a constant level of exposure, investor needs to roll only 9 contracts and sell the extra 1 near contract to keep the post-roll exposure close to the pre-roll exposure ($100 exposure divided by $11 per contract equals 9.09, or 9 contracts rounded).

2) **Roll return (or roll yield):** Roll return is effectively the accounting difference (in percentage terms) between the near-term commodity futures contract price and the farther-term commodity futures contract price. It is calculated as follows:

Roll return = [(Near-term futures contract closing price – Farther-term futures contract closing price)/Near-term futures contract closing price] × Percentage of the position in the futures contract being rolled

Example: Consider the following contract.

- March contract closing price: $99.88/barrel
- April contract closing price: $99.35/barrel
- The contract rolls its positions over a 5-day period, i.e. 1/5 = 20% per day.

Roll return = ($99.88 – $99.35)/$99.88 = 0.53% gross roll return × 20% rollover portion = 0.11% net roll return

It is argued that roll return is approximately equal to a risk premium. This means that positive long-run average returns are associated with positive roll return (i.e., in backwardation) and negative long-run average returns are associated with negative roll return. However, because 40% of the commodities examined had negative roll returns but positive total returns, it cannot be concluded that backwardation earns a positive total return.

Roll return is very sector dependent; thus, an investor can improve its overall roll return by investing in a diversified portfolio of commodity futures.

3) **Collateral return (or collateral yield):** Long derivative positions require margin, which acts as collateral. Collateral yield is the return earned bonds or cash used as margin. It's generally equivalent to T-bill returns.

Of the three sources of excess return to a commodity futures investor, price return and the roll yield are the most important.

Total Return on a fully collateralized commodity futures contract = Spot price return + Roll return + Collateral return (risk free rate return)*

**With an index, a return from rebalancing the index's component weights (called a rebalance return) would also be added.*

Practice: Example 17,18 & 19 Reading 45, Curriculum.

3.3.3). Contango, Backwardation, and the Roll Return

- Industrial metals, agriculture, livestock, precious metals, and softs tend to have statistically strong negative mean roll returns. This is because these commodities have longer storage life.
- Energy has a reasonable statistically strong positive mean roll return and a lower or negative convenience yield because energy is consumed on a real-time basis, with minimal storage buffer.

Practice: Example 20, Reading 45, Curriculum.

4. COMMODITY SWAPS

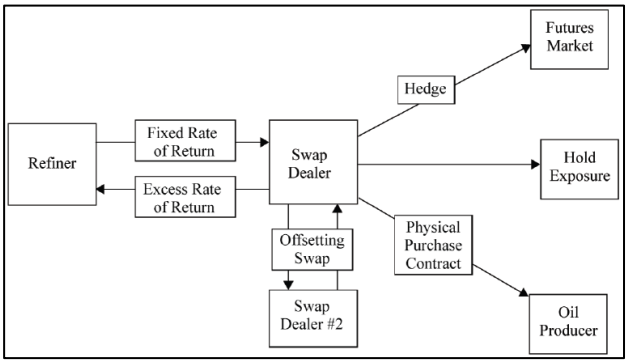
A Commodity Swap is an agreement involving the exchange of a series of commodity price payments (fixed amount) against variable commodity price payments (market price) resulting exclusively in a cash settlement (settlement amount).

- Both streams of payment (fixed/variable) are in the same currency and based on the same nominal amount. While the fixed side of the swap is constant, the variable side is related to the trading price of the relevant commodities quoted on a stock exchange or otherwise published on the commodities futures market on the relevant fixing date or to a commodity price index.
- Swaps are arguably the most popular hedging instrument used by oil and gas producers to hedge their exposure to volatile oil and gas prices as hedging with swaps allows them to lock in or fix the price they receive for their oil and gas production. In addition, swaps are also utilized by companies seeking to hedge their exposure to agriculture commodities, metals, foreign exchange rates and interest rates, among others.

Benefits of Swaps:

- Swaps facilitate risk management and risk transfer;
- Swaps provide customization because they are not standardized, exchange-traded contracts like futures.

Swap Market Participant Structure



Types of Swaps:

1) **Excess return Swap:** In excess return swap, payments are made or received by either party based on a return calculated by changes in the level of the index relative to a benchmark or fixed level. The commodity index is typically an index comprised of futures contracts on various commodities. Indices can be broad-based (i.e., comprised of futures contracts on a wide range of commodities) or based upon one or more particular sectors (e.g., energy) or based on a particular commodity (e.g., crude oil). One party will receive a payment based upon the change in the level of the index between two valuation dates (multiplied by the notional amount of the swap). If the level of the index increases, the buyer of the swap will be entitled to a payment based on this performance (net of fee paid to the seller). If the level of the index decreases, the seller of the swap will be entitled to a payment based on this performance (plus the fee charged to the buyer). In an excess return swap, the change in the level of the index will be equal to the returns generated primarily by the changes in price of each of the futures contracts that comprise the index.
2) **Total return Swap:** In a total return swap, the change in the level of the index will be equal to the returns generated by the change in price of each of the futures contracts that comprise the index plus a return based upon interest earned on any cash collateral posted upon the purchase of the futures contracts comprising the index. This type of swap is generally used by large institutional investors (e.g., pension plans) as opposed to commodity producers or buyers.
3) **Basis Swap:** In a commodity basis swap, periodic payments are exchanged based on two floating commodity reference prices. The two floating commodity reference prices are often related to each other, but are not perfectly correlated. E.g., one of the floating commodity reference prices may be for a liquid, highly-traded commodity or contract (i.e. Brent Oil) while the other is for a similar but less frequently traded reference commodity or contract (i.e. Gulf of Mexico Oil). Alternatively, one of the floating prices may be a spot price for a commodity while the other is a futures price (or forward price) for the same commodity. Another example would be one of the floating prices may be for a futures contract for delivery of a commodity to a particular location and the other is a futures contract for delivery to a different location.

Important to Note: The meaning of basis depends on the commodity in question. E.g., in grains, the basis may refer to the difference between the soybean contract and physical soybeans available for delivery at the Mississippi River.

3) **Variance swaps**: Variance swap is a forward contract that allows investors to trade future realized variance[1] (floating leg) against predefined implied variance (fixed leg). If this difference (floating versus fixed) is positive, the variance swap buyer receives a payment; if it is negative, the variance swap seller receives payment. Often the variance differences are capped to limit upside and losses. In the absence of a cap, the variance seller's potential loss under a variance swap is not quantifiable and is potentially unlimited.
4) **Volatility commodity swaps**: A volatility swap is a forward contract on future realized price volatility rather than the direction of prices. If volatility is higher (lower) than expectations, volatility buyer (seller) receives a payment.

Practice: Example 21 & 22, Reading 45, Curriculum.

[1] Variance being the square of volatility.

5. COMMODITY INDEXES

Commodity indexes reflect the aggregate movement of commodity prices, investment vehicles, and investing approaches. These indexes can be used as a reference pricing for financial contracts. The commodity indexes usually have high (> 70%) correlation with each other and low (roughly 0%) correlations with traditional asset classes (e.g., US large-cap stocks, US bonds, international stocks). Like equity indexes, there are various commodity indexes providers.

Role of Commodity indexes: Commodity indexes play three primary roles in commodity sector investments.

a) Commodity index can be used as a ***benchmark*** for commodity pricing.
b) Commodity index can be used for ***macroeconomic or forecasting purposes*** as it helps examining relationships between movements in commodity index and other macroeconomic variables.
c) Commodity index can act as the ***basis for an investment vehicle*** – helping in evaluating price changes that affect contract value.

Key characteristics of Indexes:

A. Breadth of coverage: The breadth of coverage refers to number of commodities and sectors included in each index.

B. Criteria for Selection of components: Some indexes focus on liquidity measures (minimum level of trading volume and available historical pricing) while others select components based on significance in worldwide consumption or value of global production.

C. Relative weighting: The relative weightings refers to weights assigned to each component/commodity, and the related methodology of determining these weights. E.g. equal weighting, weights based on average dollar value of their production, or weights determined by index committee. The weighting method determines the diversification provided by the index. E.g. a production value-weighted index will be more concentrated in the energy sector because of its higher value.

D. Rolling methodology: The rolling methodology refers to method used to roll over the contracts that are about to expire. This decision has a direct impact on the roll return (or yield) of the overall commodity. Indexes that contain contracts trading in backwardation generate positive roll returns, whereas those that contain contracts trading in contango generate negative roll returns.

E. Methodology and frequency for rebalancing: The methodology and frequency for rebalancing the weights of the individual commodities, sectors, and contracts in the index to maintain the relative weightings assigned to each investment. E.g. some indexes are rebalanced annually on a price-percentage basis, some are rebalanced annually by committee, while some are rebalanced monthly. The rebalancing frequency tends to influence the opportunity to earn positive rebalance returns.

F. Governance of indexes: Some indexes are rules-based, whereby a quantitative methodology is used to pick the commodities; some indexes are selection-based, whereby an index committee picks the commodities. The governance also oversees the independence of index providers to ensure that asset price remain independent from the index provider, which, in turn, should be independent from the product provider (e.g., the exchange-traded fund or swap provider).

Important to Note: A viable and useful index is the one that is investable, i.e., easily replicated by investors. E.g. an index that covers investments in exchanges all over the world is more difficult and expensive to replicate. This implies that index providers and investors must consider the venues (physical or electronic) for trading each commodity index, the liquidity and turnover of contracts based on each commodity index (illiquid contracts lead to higher transaction costs), and the term structure of each index (i.e., how far into the future the index extends and which months it covers).

Commonly used Indexes in the market: Currently, following are the most frequently used commodity indexes:

1) S&P GSCI: It is the oldest index. It is based on 24 commodities and select components based on liquidity measures. The weights are assigned based on production value weighting scheme (assigning the largest weight to the most valuable commodity on the basis of physical trade value). As a result, like market-capitalization indexes, this index can be highly concentrated in few commodities (e.g. energy). The rolling methodology focuses on owning the front (i.e., near term) contracts (having highest liquidity). This index tends to outperform during rising energy prices and when markets are in backwardation. The index is rebalanced on annual basis. In this index, individual investor funds are available.

2) Bloomberg Commodity Index (BCOM), formerly known as the Dow Jones–UBS Commodity Index (DJ

UBS): This index is based on 22 commodities. Although the index is selection-based, the weights are assigned based on liquidity measures as well as based on production value. The index places caps on the size of the sectors (33% maximum) and floors on individual commodities (2% minimum). Resultantly, the index composition and weights may vary. The index is heavily concentrated in natural gas, which has highest roll cost (negative roll return) due to inventory storage costs. Hence, this index requires to find other sources of return (e.g., price return and rebalance return) to overcome this loss. The rolling methodology focuses on owning the front (i.e., near-term – front month and second month) contracts. The index is rebalanced on annual basis. In this index, individual investor funds are available. DJ UBS tends to outperform when price of agriculture and metals rise more than that of energy and when agriculture and metals are more in backwardation than energy.

3) **Deutsche Bank Liquid Commodity Index (DBLCI):** The DBLCI uses a fixed-weighting scheme to allocate weights. It is based on 14 commodities. The rolling methodology focuses on optimization based on the time value of maximized backwardation/minimized contango for the contracts that fall within the next 12 calendar months. This implies that unlike other indexes, DBLCI takes an active decision with regards to roll return positioning*. The index is rebalanced on annual basis. In this index, individual investor funds are available. Since DBLCI has low diversification, it tends to outperform when it subset of commodities perform well. It can benefit when markets are in contango as it trades a few days ahead of DJUBS.

   *E.g. Suppose, a June 2015 ore futures contract is at 1% backwardation versus May 2015 copper contract. But if the July 2015 ore contract is at a 3% backwardation (1.5% per month, or 3% divided by 2 months) versus the 1% backwardation per month on the June 2015 contract, then the DBLCI will roll to the July 2015 contract.

4) **Thomson Reuters/Core Commodity CRB Index (TR/CC CRB):** The TR/CC CRB is based on 19 commodities. The weights are assigned by index management committee using a fixed-weighting scheme based on various factors, including diversification, sector representation, liquidity, and economic importance. The fixed weights are further clustered into a number of tiers and constituents thus move from tier to tier as they gain or lose relative importance as seen by the committee. The rolling methodology focuses on owning the front (i.e., near-term, typically front month or second month) contracts. The index is rebalanced on monthly basis. The individual investor funds are available in this index; however, there is an exchange-traded fund on a related index. CRB tends to outperform when agriculture and metals prices are rising. It can benefit when markets are in contango as it trades a few days ahead of DJUBS.

5) **Rogers International Commodities Index (RICI):** The RICI is based on 37 different commodities and uses a fixed-weighting scheme to allocate exposure. The weights are assigned by the index management committee based on a number of factors, including diversification, sector representation, liquidity, and economic importance. The rolling methodology focuses on owning the front (i.e., near-term, typically front month or second month) contracts. The index is rebalanced on monthly basis. The individual investor funds are available in this index. RICI tends to outperform when the agriculture sector performs better than energy and metals and when natural gas declines.

All five above discussed indexes have broad sector coverage (including energy, grains, livestock, precious metals, industrial metals, and softs), except for DBLCI, which does not have any softs or livestock exposure. Similarly, RICI has exposure in exotic (and thus illiquid) commodities, such as lumber, oats, and rubber.

Important to Note: Some energy constituents are denominated in non-US dollar terms; this potentially creates a foreign exchange exposure element in the index returns.

5.6 Rebalancing Frequency

Index returns are impacted by rebalancing frequency.

- Theoretically, rebalancing is more important if a market is frequently mean reverting because there are more peaks to sell and valleys to buy.
- In a trending market, frequent rebalancing can lead to underperformance. This is because the outperforming assets are sold (which continue to rise), whereas the underperforming assets are purchased (which continue to fall).
- The relative performance of the monthly rebalanced indexes (i.e. TR/CC CRB and RICI) versus the annual rebalanced indexes depend on the length of time of price trends. This means that if a market is frequently mean reverting, monthly rebalanced indexes tend to outperform. In contrast, if a market is trending, the annually rebalancing indexes will outperform.
- If an index uses a floating weighting scheme (i.e. production value), the higher (lower) futures prices usually coincide with higher (lower) physical prices. In this case, the magnitude of rebalancing weights

are generally lower than a fixed-weight scheme because the post-rebalance weights will generally drift in line with the current portfolio weights. This implies that indexes using floating weighting scheme tend to incur lower rebalancing costs and tend to outperform fixed weight index

Practice: Example 23 & 24, Reading 45, Curriculum.

1. INTRODUCTIONS

The portfolio management process is "an integrated set of steps undertaken in a consistent manner to create and maintain an appropriate portfolio (combination of assets) to meet client's stated goals".

Portfolio management is an ongoing process in which:

- Investor and market characteristics are evaluated:
  i. The first step is to determine and specify objectives and constraints of the investor.
  ii. The second step is to evaluate the economic environment i.e. macro issues which include growth prospects, inflation, unemployment etc.
- IPS is developed.
- Strategic asset allocation is determined.
- Portfolio performance is measured and evaluated.
- Dynamic investor objectives and capital market conditions are monitored.
- Portfolio decisions are initiated by portfolio managers and implemented by traders.
- Any necessary rebalancing is implemented.

The Portfolio Management Process is based on three steps:

1. **Planning**

2. **Execution:**

i. ***Portfolio selection or composition decision:*** The manager combines investment strategies with capital market expectations to select specific assets for the portfolio. In making the portfolio selection/composition decision, portfolio managers often use the techniques of portfolio optimization.

ii. ***Portfolio implementation decision****:* The portfolio implementation decision is as important as the portfolio selection/composition decision because poorly managed portfolio executions result in transaction costs that reduce performance.

Types of transaction costs:

- Explicit Transaction Costs i.e. commissions, fees and taxes.
- Implicit Transaction Costs i.e. bid-ask spread, market price impacts, and opportunity costs.

3. **Feedback:**

Any changes suggested by the feedback are examined carefully to ensure that they represent long-run considerations. The feedback step has two components:

i. Monitoring and Rebalancing.
ii. Performance Evaluation.

5.3.1) Monitoring and Rebalancing

Portfolio manager monitors and evaluates risk exposures to investment opportunities and compares it with strategic asset allocation. The objective is to ensure that client's current objectives and constraints are continuously met. Two types of factors are monitored:

1) ***Investor related factors*** i.e. changes in investor's circumstances. For example termination of a pension plan, death of a spouse.

2) ***Economic and market input factors.*** When making rebalancing decisions, portfolio manager must take into consideration factors that affect investors' returns. It is important that rebalancing accounts for transaction costs and taxes.

5.3.2) Performance Evaluation

Investment performance must be regularly evaluated by the investor to assess progress toward the achievement of investment objectives as well as to assess portfolio management skill. The assessment of portfolio management skill has three components.

1) Performance measurement:

It involves the calculation of the portfolio's rate of return.

2) Performance attribution:

It examines why the portfolio performed as it did and determines the sources of a portfolio's performance.

3) Performance appraisal:

It assesses how well the portfolio manager performed on a risk-adjusted basis, relative to a benchmark.

Portfolio performance can be examined in two ways:

i. Absolute returns.
ii. Relative returns i.e. with reference to a benchmark.

NOTE:

- The focus of this reading is the "Planning" step only.
- Broadly speaking, investors can be described as institutional or individual. Institutional investors include pension funds, foundations, endowments, insurance companies and banks etc.

3. THE PORTFOLIO PERSPECTIVE

According to the portfolio perspective, individual investments should be evaluated on the basis of how much risk they add to a portfolio rather than on how risky they are on a stand-alone basis. In other words, the risk-return trade-off of the portfolio must be analyzed as a whole not on individual basis.

4. PORTFOLIO MANAGEMENT AS A PROCESS

Portfolio management is a continuous (on-going) and systematic process to create and maintain appropriate combinations of investment assets.

5. THE PORTFOLIO MANAGEMENT PROCESS LOGIC

5.1 The Planning Step

It consists of the following tasks:

1) Analyzing objectives and constraints.
2) Developing an IPS.
3) Forming capital market expectations.
4) Selecting an appropriate asset allocation.

5.1.1) Identifying and Specifying the Investor's Objectives and Constraints

The first task in investment planning is to identify and specify the investor's objectives and constraints.

- Investment objectives are desired investment outcomes. They are related to return and risk.
- Constraints are limitations on the investor's investment decisions or investment choices.

5.1.2) Creating the Investment Policy Statement (IPS)

After the objectives and constraints have been specified, the next task is to formulate the investment policy statement.

Investment Policy Statement (IPS): An IPS is a written document that clearly states client's return objectives and risk tolerance, client's relevant time horizon, and applicable constraints. When IPS is combined with capital market expectations, it sets out a guideline for a strategic asset allocation and investment decisions (e.g. whether the portfolio will be actively managed or passively managed).

Role of an IPS is to:

- Be readily and easily implemented by current or future investment advisers.
- Promote long-term discipline for portfolio decisions.
- Protect investors against short-term shifts in investment strategy when either market conditions or portfolio performance cause panic or overconfidence.

Elements of an IPS include:

- A client description.
- Investment policies, objectives, goals, restrictions and portfolio limitations.
- Identification of duties and responsibilities of parties involved.
- The formal statement of objectives and constraints.
- Asset allocation ranges, investment strategy and the desired investment style or styles of investment managers.
- Reporting requirements.
- Portfolio performance and IPS review pre-specified schedule.
- Guidelines for portfolio adjustments and rebalancing.

NOTE:

When investor circumstances or capital market expectations change, the portfolio needs to be revised. If changed circumstances become permanent, the investor's IPS must be updated and the temporary asset allocation plan effectively becomes the new strategic asset allocation.

Tactical Asset Allocation: Tactical Asset Allocation responds to changes in short-term capital market expectations rather than to investor circumstances.

The planning process also involves the details of investment strategy.

Types of investment strategy:

1. Passive Investment Approach:

In a passive investment approach, portfolio composition does not react to changes in capital market expectations. For example, Indexing and a strict buy-and-hold strategy i.e. fixed portfolio of bonds to be held to maturity.

2. Active Investment Approach:

In an active investment approach, a portfolio manager responds to changes in capital market expectations. In Active portfolio management, active portfolio's holding differs from the portfolio's benchmark in order to earn positive excess risk-adjusted returns i.e. positive alpha.

3. Semi-active, risk-controlled active or enhanced index approach:

In this approach, portfolio managers attempt to match risk characteristics of a benchmark portfolio, but also deviate from the exact benchmark portfolio weights in order to earn higher returns.

5.1.3) Forming Capital Market Expectations

The third task in the planning process is to form capital market expectations. In this task, long-run forecasts of risk and return characteristics for various assets are made to choose portfolios that maximize expected return for given level of risks or minimize risk for given level of expected returns.

5.1.4) Creating the Strategic Asset Allocation

The fourth and final task in the planning process is to determine the strategic asset allocation. In this step, the IPS is combined with capital market expectations to determine long-term target asset class weights to be included in the portfolio. When portfolio risk-return profile significantly deviates from the investor's stated objectives, the strategic asset allocation must be reviewed.

6. INVESTMENT OBJECTIVES AND CONSTRAINTS

6.1 Objectives

The two objectives i.e. risk and return are interdependent because the risk objective limits how high the investor can set the return objective.

6.1.1) Risk Objective

Expected asset risk is generally positively correlated with expected asset return. To determine a risk objective, there are several steps:

1) ***Specify risk measure:*** Risk measurement is a key issue in investments. Risk can be measured in absolute terms or in relative terms.

- Absolute risk objectives include specified level of S.D or variance of total return.
- Relative risk objectives include a specified level of tracking risk.
- Relative risk measures are easier to quantify from an individual investor's perspective whereas, absolute risk measures are often used in qualitative forms.
- Downside risk concepts i.e. value at risk (VAR), as well as other risk exposures such as: Exposures to specific economic sectors are also an important risk measure.

2) ***Investor's willingness to take risk:*** Institutional investors and individual investors have different willingness to take risk. For an individual, willingness to take risk is determined by behavioral or psychological factors, whereas ability is generally determined primarily by financial or practical limitations. For example:

- **Spending Needs:** Investors with high levels of wealth relative to expected spending needs can take more risk.
- **Long-term wealth targets or obligations:** Investors with high levels of wealth relative to long-term wealth targets or obligations can take more risk.
- **Liabilities/Pseudo liabilities:** The higher the liabilities (i.e. future payments to beneficiaries) or pseudo liabilities (i.e. future retirement spending needs) faced by an investor, the less risk can be taken.
- **Financial Strength:** More financial strength means more risk can be taken.

3) ***Investor's ability to take risk:*** Even if an investor is willing to take risk, practical or financial limitations limit the amount of risk that investor can take. This concept is known as ***ability*** to take risk. For example,

- A short time horizon negatively affects the investor's ability to take risk.
- If the investor's portfolio is greater than spending and obligation, he/she has greater ability to take risk.

Determine investor's risk tolerance: Risk tolerance or the capacity to accept risk is a function of both an investor's willingness and ability to take risk. Risk tolerance can also be defined in terms of risk aversion i.e. the degree of an investor's inability and unwillingness to take risk.

	Ability to Take Risk	
Willingness to Take Risk	**Below Average**	**Above Average**
Below Average	Below-average risk tolerance	Resolution needed**

Above Average*	Resolution needed	Above-average risk tolerance

*When an investor's willingness to take risk exceeds the ability to do so, ability prudently places a limit on the amount of risk the investor's should take.
**When ability to take risk exceeds willingness, willingness is preferred; and it is documented under the heading of "unique circumstances".

6.1.2) Return Objective

To determine a return objective, there are several steps:

1) ***Specify a return measure*** e.g. total return which is the sum of the return from price appreciation and the return from investment income.

- Return can be stated in ***absolute term*** i.e. 15% a year or ***relative term*** i.e. benchmark return plus 3% a year.
- Return can be stated in ***nominal*** or ***real*** terms. Real returns are adjusted for inflation while nominal returns are not.
- Pre-tax returns must also be distinguished from after-tax returns.

NOTE:

Even if a substantial income component is required from the portfolio, it is recommended that the return objective should be evaluated by the total return perspective.

2) ***Determine the investor's stated return desire:*** Desired return states how much return the investor wants to earn e.g.an investor may want to have a return higher than average.

3) ***Determine the investor's required rate of return:*** Required return states how much return the investor needs to achieve (on average) e.g. a retired investor must have to earn a return on his/her investment portfolio that will cover his/her living expenses. *Note that requirements are more rigid than desires.*

Example:

Suppose that a retiree must achieve a 5% after-tax return on his current investment to meet his current annual living expenses. Thus, his return requirement on a real, after-tax basis is 5% a year.

If he expects inflation to be 1.5% per year and 35% tax rate applies to investment returns, his **Pre-tax Nominal Return Requirement** is estimated as follows:

= (After-tax real return requirement + Expected Inflation Rate) / (1 – Tax rate) = (5% + 1.5%) / (1 – 0.35) = 10%

Thus, Pre-tax Nominal Return Requirement = 10%

4) ***Determine the investor's specific return objectives:*** An investor's return objective should be consistent with that investor's risk objective i.e. high return objective requires an asset allocation with an expected high level of risk. Return objectives can be stated in two ways:

- Absolute return objective i.e. 5%.
- Relative return objective i.e. 1.5% higher than the benchmark.

NOTE:

Benchmark's total return is an effective return objective for the investment manager.

See Table 2 "Return Requirements and Risk Tolerance of Various Investors", Reading 46, Curriculum, Volume 6.

6.2 Constraints

Constraints are those factors that limit or restrict certain decisions or investment choices. Constraints are either ***internal*** i.e. client's specific liquidity needs, time horizon, and unique circumstances or ***external*** i.e. tax issues and legal and regulatory requirements.

6.2.1) Liquidity

A liquidity requirement is a need for cash in excess of new contributions (e.g. for pension plans and endowments) or savings (e.g. for individuals) at a specified point in time. Such needs can be anticipated or unanticipated. The liquidity requirement can be met by holding cash or cash equivalents in the portfolio or by selling assets and converting them into cash.

- Sensitivity to liquidity negatively affects the willingness to take risk. Thus, higher liquidity requirements indicate a lower tolerance to take risk.

Liquidity risk is the risk of bearing economic loss when a relatively less liquid asset is sold to meet liquidity requirements. Liquidity risk arises for two reasons:

i. An asset-side reason (asset liquidity).
ii. A liability-side reason (liquidity requirements).

Portfolio managers control asset selection but not liquidity requirements. Hence, managers use asset selection to manage liquidity risk.

- If portfolio's asset and income base > potential liquidity requirements, relatively less liquid assets can be held.

6.2.2) Time Horizon

Investment objectives and associated time horizons can be short term (< 10 years), long term (≥ 10 years) or multi-stage. A multi-stage horizon is a combination of shorter term and longer term horizons e.g. funding children's education in 5 years is a shorter term and the investor's retirement in 30 years is a longer term. Other constraints i.e. unique circumstance or a specific liquidity requirement can also affect an investor's time horizon.

The length of time horizon influences the investor's asset allocation and investor's ability to take risk.

- The longer the time horizon, the greater is the investor's ability to take risk (but not necessarily the willingness to take more risk).
- Investors with long-term time horizons often prefer to allocate a greater proportion of funds to risky assets i.e. equities. While the investors with a shorter investment time horizon prefer to choose less risky strategic asset allocation.

Important:

- Large asset base, long time horizon, plenty of income to cover expenses, and lack of need for liquidity or cash flow indicate an ***above-average ability*** to take risk.
- And desire for high returns indicates substantial ***willingness*** to take risk.

6.2.3) Tax Concerns

Tax concerns should be considered by investors and portfolio managers because (for taxable investors) tax payments reduce the amount of the total return earned by investors. Tax policy changes regarding security prices, affect both taxable and tax-exempt investors. When different tax rates are applied to investment income and capital gains, tax considerations will influence the *choice* of investment as well.

6.2.4) Legal and Regulatory Factors

Legal and regulatory factors are external factors imposed by government and/or regulatory authorities that affect investment decisions e.g. government agency may limit the use of certain asset classes in retirement portfolios.

6.2.5) Unique Circumstances

Unique circumstances are internal factors that may constrain portfolio choices e.g. an investor may avoid investment in tobacco companies.

9. THE ETHICAL RESPONSIBILITIES OF PORTFOLIO MANAGERS

Ethical conduct is the foundation requirement for managing investment portfolios. The portfolio managers must have ethical conduct towards the public, clients, prospects, employers, employees and fellow workers. CFA institute members are *required* to follow Code of Ethics and Standards of Professional Conduct Statement.

Practice: End Of Chapter Practice Problems For Reading 46 & FinQuiz Item-set ID# 11277.

1. INTRODUCTION

Multifactor models have more explanatory power and flexibility compared to single-factor models (typically based on a market risk factor). Multifactor models allow investors to

- build portfolios that replicate or modify in accordance with a particular index;
- establish desired exposures to one or more risk factors in portfolios;
- do detailed risk and return attribution analysis of actively managed portfolios;
- do comparative risk analysis of exposures of equity, fixed-income, and other asset class returns;
- identify active decisions relative to a benchmark and measure the quantum of those decisions; and
- set the active risk and return objectives of aggregate portfolio in accordance with active fees.

2. MULTIFACTOR MODELS AND MODERN PORTFOLIO THEORY

Under **Modern Portfolio Theory (MPT),** portfolios of securities are constructed by quantitatively considering each investment in terms of its impact on a portfolio, rather than in isolation.

- Under modern portfolio theory, asset returns can be modelled using a multivariate normal distribution. In a multivariate normal distribution, distribution of returns are defined in terms of mean returns, return variances, and return correlations.
- According to MPT, if correlation among asset returns is less than 1, then risk can be reduced by means of diversification.

Capital asset pricing model (CAPM): The capital asset pricing model is a model for predicting returns of risky assets assuming that in equilibrium all investors hold a portfolio of risky assets that has the same weights as the market portfolio. According to CAPM,

- The primary determinant of expected return for a security is its beta or systematic risk (non-diversifiable risk). In other words, investors should not be compensated for assuming diversifiable risk (i.e. risk that can be mitigated by holding an asset in a portfolio).
- An asset's systematic risk is a positive function of its beta and any difference in mean return are explained by a single factor, the market portfolio return.
- Beta of an asset measures the sensitivity of an asset's return to the market's return. Higher beta indicates greater risk with respect to market factor and resultantly, higher return.

Assumptions of CAPM:

1) Investors are risk-averse, utility-maximizing, rational individuals.
2) Markets are frictionless, including no transaction costs and no taxes.
3) Investors plan for the same single holding period.
4) Investors have homogeneous expectations or beliefs.
5) All investments are infinitely divisible.
6) Investors are price takers.

Limitation of CAPM: CAPM provides an incomplete description of risk and return because it uses just a single factor (market portfolio return) as a source of systematic risk

3. ARBITRAGE PRICING THEORY

The arbitrage pricing theory (APT) describes the expected return on an asset (or portfolio) as a linear function of the risk of the asset with respect to a set of factors as a source of systematic risk. Unlike the CAPM, the APT does not indicate the identity or even the number of risk factors. According to APT, the asset's expected return can be expressed as:

$$R_i = a_i + b_{i1}I_1 + b_{i2}I_2 + \ldots + b_{iK}I_K + \varepsilon_i,$$

Where
R_i = the return to asset i
a_i = an intercept term. It is the expected return of asset i given that all the factors take on a value of zero.
I_k = the return to factor k, k = 1, 2, ..., K
b_{ik} = the sensitivity of the return on asset i to the return to factor k, k = 1, 2, ..., K

ε_i = an error term with a zero mean that represents the portion of the return to asset i not explained by the factor model

Arbitrage is a risk-free profits made by investors by exploiting security mispricing, without risk and without a net investment.

When there are many stocks in a portfolio, then the asset-specific or non-systematic risk of individual stocks makes almost no contribution to the variance of portfolio returns.

Like the CAPM, the APT assumes that financial markets are in equilibrium, but the APT makes less strong assumptions. The three assumptions of APT are as follows:

1) A factor model describes asset returns.
   - Implication: It is important to note that in APT, the number of factors is not specified.
2) There are many assets, so investors can form well-diversified portfolios that eliminate asset specific risk.
   - This assumption allows investors to form portfolios with factor risk but without asset-specific risk.
3) No arbitrage opportunities exist among well-diversified portfolios.
   - This assumption implies that financial market is in equilibrium.

If the above three assumptions hold, then the expected return to Portfolio can be estimated as follows:

$$E(R_p) = RF + \lambda_1\beta_{p,1} + \ldots + \lambda_K\beta_{p,K}$$

Where,
E(Rp) = the expected return to portfolio p
RF = the risk-free rate
λj = the expected reward for bearing the risk of a portfolio with a sensitivity of 1 to factor j and a sensitivity of 0 to all other factors
βp,j = the sensitivity of the portfolio to factor j
K = the number of factors

- The above equation shows that expected return on any well-diversified portfolio is linearly related to the factor sensitivities of that portfolio.
- The **risk premium for the market factor** is the expected return of the market in excess of the risk-free rate.
- The factor risk premiums for the other factors are the mean returns of the specific portfolios held long (e.g., the portfolio of small-cap stocks for the "small minus big" factor) minus the mean return for a related but opposite portfolio (e.g., a portfolio of large-cap stocks).

Pure Factor Portfolio: A portfolio with a sensitivity of 1 to factor j and a sensitivity of 0 to all other factors is called a pure factor portfolio for factor j (or simply the factor portfolio for factor j).

Practice: Example 1, Curriculum, Reading 47.

Example: Suppose there are three portfolios i.e. A, B, and C (shown in the table below). Portfolio D is a new portfolio.

Sample Portfolios for a One-Factor Model

Portfolio	Expected Return	Factor Sensitivity
A	0.0750	0.50
B	0.1500	2.00
C	0.0700	0.40
D	0.0800	0.45
0.5A + 0.5C	0.0725	0.45

- The factor sensitivities given relate to the one-factor APT model E(Rp) = 0.05 + 0.05βp,1
- An arbitrage opportunity will exist if a portfolio can be formed from Portfolios A, B, and C that has the same factor sensitivity as Portfolio D but a different expected return.
- If return of Portfolio D would be relatively higher (lower), it would be undervalued (overvalued).

The expected return and factor sensitivity of Portfolio D are calculated as weighted averages of the expected returns and factor sensitivities of A and C as follows:

Expected return of Portfolio D = (0.50)(0.0750) + (0.50)(0.07) = 0.0725, or 7.25%

Factor sensitivity = (0.50)(0.50) + (0.50)(0.40) = 0.45

- The factor sensitivity of 0.45 is same as the factor sensitivity of Portfolio D.

According to the assumed APT model, the expected return on Portfolio D should be as follows:

$$E(R_D) = 0.05 + 0.05\beta_{D,1} = 0.05 + (0.05 \times 0.45) = 0.0725, \text{ or } 7.25\%$$

Suppose, portfolio D offers 8% expected return; this return is higher given its factor sensitivity. Since, the return of Portfolio D is higher, it is undervalued relative to its factor risk.

- Thus, investor will buy D (hold it long) in the portfolio using the proceeds from selling short an equally weighted portfolio of A and C with exactly the same 0.45 factor sensitivity as D.
- E.g. an investor buy $10,000 of Portfolio D and sell $10,000 of an equally weighted portfolio of

Portfolios A and C.
Initial net cash flow = $0

Expected value of investment in Portfolio D at the end of one year = $10,000(1 + 0.08) = $10,800

Expected value of short position in Portfolios A and C at the end of one year = −$10,000(1.0725) = −$10,725

Combined expected cash flow from our investment position in one year = $10,800 −$10,725 = $75

- Purchasing D and selling short an equally weighted portfolio of A and C creates a portfolio with a factor sensitivity of 0.45 – 0.45 = 0.
- According to APT, if Portfolio D actually had an expected return of 8%, its price will increase until the expected return fell and the arbitrage opportunity no more exists.

Carhart four-factor model: The Carhart four-factor model, also known as the four-factor model or simply the Carhart model, assumes following four factors as a source of systematic risk:

1) Small minus big (SMB) i.e. small-capitalization stocks
2) High minus low (HML) i.e. low price-to-book-ratio stocks, commonly referred to as "value" stocks.
3) Winners minus losers (WML) i.e. stocks with rising prices (called momentum stocks).
4) Market risk factor

In a Carhart model, the excess return on the portfolio is explained as a function of the portfolio's sensitivity to a market index (RMRF), a market capitalization factor (SMB), a book-value-to price factor (HML), and a momentum factor (WML).

$$R_p - R_F = a_p + b_{p1}\text{RMRF} + b_{p2}\text{SMB} + b_{p3}\text{HML} + b_{p4}\text{WML} + \varepsilon_p$$

Where,

- Rp = Return on the portfolio
- RF = Risk-free rate of return
- a_p = "alpha" or return in excess of that expected given the portfolio's level of systematic risk
- b_p = the sensitivity of the portfolio to the given factor
- RMRF = the return on a value-weighted equity index in excess of the one-month T-bill rate
- SMB = small minus big, a size (market capitalization) factor;

SMB = Average return on three small-cap portfolios - Average return on three large-cap portfolios

- HML = high minus low

HML = Average return on two high book-to-market portfolios - Average return on two low book-to-market portfolios

- WML = winners minus losers, a momentum factor

WML = Return on a portfolio of the past year's winners - Return on a portfolio of the past year's losers

- εp = an error term that represents the portion of the return to the portfolio, p, not explained by the model

Since the expected value of alpha is zero, the Carhart model can also be stated as follows:

$$E(R_p) = R_F + \beta_{p,1}\text{RMRF} + \beta_{p,2}\text{SMB} + \beta_{p,3}\text{HML} + \beta_{p,4}\text{WML}$$

The Carhart model is a multifactor extension of the CAPM that explicitly incorporates size, value, and momentum systematic risk factors as drivers of differences in expected returns among assets. This implies that investors are compensated for assuming exposure to these risk factors. These factors are viewed as anomalies from a pure CAPM perspective.

4. MULTIFACTOR MODELS: TYPES

4.1 Factors and Types of Multifactor Models

Macroeconomic factor models assume that returns are explained by surprises in macroeconomic risk factor. Macroeconomic risk factors include interest rates, inflation risk, business cycle risk, and credit spreads etc.

Fundamental Factor Models assume that asset returns are explained by the returns from multiple firm-specific factors. Fundamental factors include book-value-to-price ratio, market capitalization, the price-to- earnings ratio, and financial leverage.

Statistical Factor Models use statistical methods to identify multiple statistical factors that explain the covariation among asset returns. In statistical factor models, the factors are portfolios of the securities in the group under study and are therefore defined by

portfolio weights. E.g. a portfolio with weights that are similar to market index weights will be "the market factor".

- The major advantage of Statistical models is that they make minimal assumptions.
- The ***limitations of statistical factor model*** include following:
  i. Statistical factors are difficult to interpret.
  ii. They lack economic interpretation.
  iii. Use of Statistical factor models requires understanding of quantitative methods.

Types of Statistical Factor Models: Two major types of statistical factor models are

1) ***Factor analysis models:*** In factor analysis model, the factors are the portfolios of securities that best explain (reproduce) historical return ***covariances***.
2) ***Principal components models:*** In principal components model, the factors are portfolios of securities that best explain (reproduce) the historical return ***variances***.

Advantages of fundamental and macroeconomic models:

i. Fundamental and macroeconomic models are easier to interpret;
ii. Fundamental and macroeconomic models rely less on data-mining approaches;

4.2 The Structure of Macroeconomic Factor Models

In macroeconomic factor models, the returns to each asset are correlated with only the surprises in some factors related to the aggregate economy, i.e. inflation or real output.

- Surprise is defined as the actual value minus predicted (or expected) value.
- The component of the factor's return that is unexpected is referred to as a factor's surprise.

In Macroeconomic Factor Model, the return of asset i is expressed as:

$$R_i = a_i + b_{i1}F_1 + b_{i2}F_2 + \ldots + b_{iK}F_K + \varepsilon_i$$

Where,
R_i = the return to asset i
a_i = the expected return to asset i
b_{ik} = the sensitivity of the return on asset i to a surprise in factor k, k = 1, 2, ..., K
F_k = the surprise in the factor k, k = 1, 2, ...,K
ε_i = an error term with a zero mean that represents the portion of the return to asset i not explained by the factor model.

Example of analyzing monthly returns for stocks:
Suppose, at the beginning of month, inflation is forecasted to be 0.4%; but at the end of month, inflation was 0.5%.

Actual inflation = Predicted inflation + Surprise inflation
0.5% = 0.4% + Surprise inflation
Surprise inflation = 0.5% - 0.4% = 0.1%

It is important to note that in forecasting stock returns using macroeconomic factor model, predicted values of macroeconomic factors (e.g. inflation, GDP) are not used for analysis because the predicted values should already be reflected in stock prices and thus in their expected returns. The new information about the variable is reflected in surprise in the macroeconomic variables during the month.

Macroeconomic Factor Model analyses the return to an asset in three components:

i. Asset's expected return
ii. Asset's unexpected return resulting from new information about the factors, and
iii. Error term

E.g. assume that the returns for a stock i are correlated with surprises in inflation rates and surprises in GDP growth. The return to this stock can be modelled as follows:

$$R_i = a_i + b_{i1}F_{INFL} + b_{i2}F_{GDP} + \varepsilon i$$

Where,
R_i = the return to stock i
a_i = the expected return to stock i
b_{i1} = the sensitivity of the return to stock i to inflation rate surprises
F_{INFL} = the surprise in inflation rates
b_{i2} = the sensitivity of the return to stock i to GDP growth surprises
F_{GDP} = the surprise in GDP growth (assumed to be uncorrelated with FINT)
ε_i = an error term with a zero mean that represents the portion of the return to asset i not explained by the factor model; it reflects a part of return that is unexplained by expected return or the factor surprises

- The factor model shows that a 1 percentage point surprise in inflation rates (GDP growth) will contribute b_{i1} (b_{i2}) percentage points to the return to stock i.
- Factor sensitivities are also known as factor betas or factor loadings.
- If the surprises in both inflation rates and GDP growth are zero, then the return to asset i will be equal to a_i.

- If all sources of systematic risks have accurately considered into the model, then εi must represent an asset-specific risk or return an unanticipated company-specific event.

GDP Growth and Inflation Factors:

- The risk premium for the GDP growth factor is typically positive.
- In contrast, the risk premium for the inflation factor is typically negative. This implies that an asset whose returns tend to increase with unexpectedly high inflation (i.e. positive sensitivity to the inflation factor) would have a lower required return because of that asset's inflation-hedging ability.

Important to Note: In macroeconomic factor models, the time series of factor surprises are constructed first and regression analysis is used afterwards to estimate assets' sensitivities to the factors.

Investment Guidelines for Inflation and GDP Growth Factors:

Low Inflation / Low Growth:
i. Cash
ii. Government bonds

Low Inflation / High Growth:
iii. Equity
iv. Corporate debt

High Inflation / Low Growth:
v. Inflation-linked bonds
vi. Commodities
vii. Infrastructure

High Inflation / High Growth:
viii. Real assets (real estate, timberland, farmland, energy)

Practice: Example 3, Curriculum, Reading 47.

4.3 The Structure of Fundamental Factor Models

Fundamental Factor Model is stated as follows:

$$R_i = a_i + b_{i1}F_1 + b_{i2}F_2 + \ldots + b_{iK}F_K + \varepsilon_i$$

Fundamental factors used in fundamental factor models describe either attributes of the securities themselves or attributes of the securities' issuers. Fundamental factors include industry membership, price-to-earnings ratio, book-value-to-price ratio, size, and financial leverage.

In fundamental factor models,

- The factors are stated as returns rather than return surprises; therefore, the expected value is not zero in fundamental factor model.
- The factor sensitivities are attributes of the security. An asset's sensitivity to a factor is expressed using a ***standardized beta,*** calculated as below.

$$b_{ik} = \frac{\text{Value of attribute } k \text{ for asset } i - \text{Average value of attribute } k}{\sigma(\text{Values of attribute } k)}$$

Difference between Macroeconomic multifactor models and fundamental factor models:

- In macroeconomic multifactor models, the factor (surprise) are estimated first and then factor sensitivities are estimated through regressions.
- In fundamental factor model, the factor sensitivities (attributes) are estimated first and then factor returns are estimated through regressions.
- Unlike fundamental factor models, the factors in macroeconomic factor models are individually backed by statistical evidence that they represent systematic risk.
- Fundamental factor model involves more factors compared to macroeconomic factor models. Therefore, fundamental factor model provides more detailed picture of risk and return of a portfolio manager.

Use of Fundamental Factor Model:

i. Fundamental factor models can be used for portfolio performance attribution and risk analysis. The sources of portfolio performance can be easily described using fundamental factor models compared to statistical factor models.
   - ***Return attribution*** involves identifying the sources of the excess return of a portfolio against its benchmark.
   - ***Risk attribution*** involves identifying the sources of risk (e.g. tracking error), identifying the sources of portfolio volatility for absolute mandates and the sources of tracking risk for relative mandates.

ii. Fundamental factors are relatively easy to understand and interpret.

iii. Investment style choices and security characteristics can be expressed more directly and often in greater detail compared to macroeconomic factor models.

Practice: Example 4, Curriculum, Reading 47.

Three broad group of Factors of Fundamental factor models for equities:

1) ***Company fundamental factors*** include factors related to the company's internal performance e.g. earnings growth, earnings variability, earnings momentum, and financial leverage.
2) ***Company share-related factors*** include valuation measures and other factors related to share price or the trading characteristics of the shares (i.e. share price momentum, share price volatility, and trading activity). Unlike company fundamental factors, company share-related factors directly take into account investors' expectations concerning the company. E.g. price multiples such as earnings yield, dividend yield, and book to market, and market capitalization.
3) ***Macroeconomic factors*** include sector or industry membership i.e. CAPM beta, other similar measures of systematic risk, and yield curve level sensitivity.

Global factor models use classification of country, industry, and style as factors. Style factors include those related to earnings, risk, and valuation that define types of securities depending on its styles of investing.

5. MULTIFACTOR MODELS: SELECTED APPLICATIONS

Applications of multifactor models in investment:

a) Return attribution
b) Risk attribution
c) Portfolio construction
d) Strategic portfolio decisions
e) Asset allocation purposes

5.1 Factor Models in Return Attribution

Multifactor models can be used to identify the sources of a manager's returns relative to a benchmark.

Active return = Portfolio return - Benchmark return = $R_P - R_B$

Using factor model, a portfolio manager's active return can be decomposed as the sum of following two components.

1. **Return from Factor Tilts:** Portfolio manager's factor tilts (over- or underweights relative to the benchmark factor sensitivities) × factor returns;
2. **Security Selection:** Manager's skill in individual asset selection (ability to overweight securities that outperform the benchmark or underweight securities that underperform the benchmark);

$$\text{Active return} = \sum_{k=1}^{\cdots} \left[(\text{Portfolio sensitivity})_k - (\text{Benchmark sensitivity})_k\right] \times (\text{Factor return})_k + \text{Security selection}$$

Where, k represents the factor or factors represented in the benchmark portfolio:

Important to Note:

- If the sensitivity to factor RMRF (return on a value-weighted equity index in excess of the one-month T-bill rate) is 1, it indicates the assigned benchmark has average market risk, reflecting broad-based index.
- If there is a negative sensitivity to factor SMB (small minus big, a size (market capitalization) factor), it indicates that a portfolio manager has large-cap orientation.
- If there is a positive sensitivity to HML factor, it indicates that a portfolio manager has a value orientation.

Practice: Example 5, Curriculum, Reading 47.

5.2 Factor Models in Risk Attribution

Active risk can be represented by the standard deviation of active returns (also known as tracking error (TE) or Tracking risk).

$$TE = s(R_P - R_B)$$

Where, s(Rp – RB) indicates sample standard deviation (indicated by s) of the time series of differences between the portfolio return, Rp, and the benchmark return, R_B.

Note: Active return and tracking error should be stated on the same time basis.

Tracking error and investment strategy:

- A passive investment strategy tends to have a tracking error of 0.10% or less per annum.
- A low-risk active or enhanced index investment strategy, which involves tightly controlled use of managers' expectations, tends to have a tracking error of 2% per annum.
- A diversified active large-cap equity strategy that might be benchmarked to the S&P 500 Index tends to have a tracking error in the range of 2%–6% per annum.

- An aggressive active equity manager tends to have a tracking error in the range of 6%–10% or more.

The information ratio (IR) reflects mean active returns generated by a portfolio manager against per unit of active risk. The historical or ex post IR is expressed as follows:

$$\text{IR} = \frac{\overline{R}_p - \overline{R}_B}{s(R_p - R_B)}$$

Where, R bar p and R bar B stand for the sample mean return on the portfolio and the sample mean return on the benchmark, respectively.

Practice: Example 6, Curriculum, Reading 47.

For the purpose of risk attribution, it is more convenient to use variances rather than standard deviations because the variances of uncorrelated variables are additive.

Variance of Active Risk = Active risk squared = s^2(Rp – RB)

A portfolio's active risk squared can be decomposed into two components as follows:

Active risk squared = Active factor risk + Active specific risk

- Contribution to active risk squared that results from the portfolio's different-from-benchmark exposures relative to factors specified in the risk model is called ***Active factor risk***. In other words, active factor risk represents contribution to active risk squared resulting from portfolio's active factor exposures.
- The active non-factor or residual risk assumed by the portfolio manager is known as ***Active specific risk or security selection risk***.

Active specific risk can be expressed as

$$\text{Active specific risk} = \sum_{i=1}^{n} \left(w_i^a\right)^2 \sigma_{\varepsilon_i}^2$$

Where,
W^a_i = i^{th} asset's active weight in the portfolio (asset's weight in the portfolio - its weight in the benchmark)
$\sigma^2\varepsilon i$ = Residual risk of the i^{th} asset (variance of the i^{th} asset's returns left unexplained by the factors).

Practice: Example 7, Curriculum, Reading 47.

5.3 Factor Models in Portfolio Construction

Multifactor models can be used in portfolio construction as they allow portfolio manager to make focused bets, to control portfolio risk relative to the benchmark's risk, or to establish desired risk profiles.

For example, if a portfolio manager wants hedge any risk or speculate on it, he/she can construct a pure factor portfolio, having sensitivity of 1 for that factor and a sensitivity (or weight) of 0 for all other factors.

- In passive management, multifactor models can be used to replicate an index fund's factor exposures.
- In active management, multifactor models can be used to predict alpha (excess risk-adjusted returns) or relative return (the return on one asset or asset class relative to that of another).

Rules-based active management (alternative indexes). Alternative Index strategies seek to take exposure against factors (i.e. size, value, quality, or momentum) while taking some exposure against systematic risk as well.

Practice: Example 8, Curriculum, Reading 47.

5.4 How Factor Considerations Can Be Useful in Strategic

Portfolio Decisions

Multifactor models facilitate investors in making various strategic decisions. For example, unlike an investor with independent wealth, a salaried individual is sensitive to business cycle risk. Therefore, he would demand higher risk premium for investing in pro-cyclical assets.

End of Reading Practice Problems:

1. INTRODUCTION

Market Risk: Market risk is the risk associated with unfavorable changes in stock prices, interest rates, exchange rates, and commodity prices.

Managing Market Risk: Risk management is the process of identifying and measuring risk and ensuring that the risks being taken are consistent with the desired risks. The process of managing market risk relies heavily on the use of models.

2. UNDERSTANDING VALUE AT RISK

2.1 Value at Risk: Formal Definition

VaR is a measure of market risk. It is the minimum loss which can occur with X% of the time over a holding period of n days. For example if a daily VaR is stated as $100,000 to a 95% level of confidence, this means that during the day there is only 5% chance that the loss the next day will be greater than $100,000.

How to interpret VaR: A 5% VaR of a portfolio of $3 million over a one-day period is interpreted as follows:

- $3 million is the minimum loss that we would expect 5% of the time; or
- 5% of the time, the losses would be at least $3 million; or
- We would expect a loss of no more than $3 million 95% of the time.

Three important points to consider in VaR:

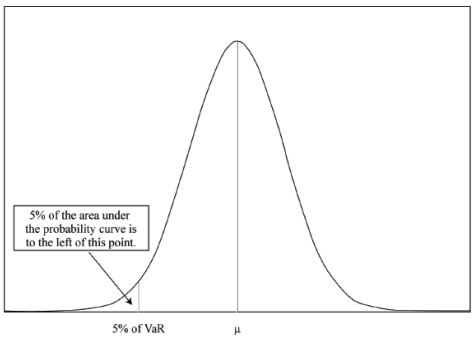
i. ***VaR can be measured in either currency units or in percentage terms***. E.g. if portfolio value is $300 million, the VaR expressed in percentage terms would be $3 million/$300 million = 1% or 0.01.

ii. ***VaR is a minimum not maximum loss.*** In a $300 million portfolio, with zero leverage, the maximum loss one can incur is $300 million.

iii. ***VaR statement always refers to a certain time horizon:*** In the above example, that period of time is one day. VaR can be measured on a weekly, bi-weekly, monthly, quarterly, semiannually, or annual basis. If VaR is measured on a daily basis, and a typical month has 20–22 business days, then 5% of the days means ➔ 5% X 20 days = one day per month.

This implies that for one month, we would say that a loss of at least $3 million would be expected to occur once every month.

Illustration of 5% VaR in the Context of a Probability



Considering a normal distribution, a 5% VaR is equivalent to the point on the distribution that is 1.65 standard deviations below the expected value. Similarly, a 1% VaR is equivalent to the point on the distribution that is 2.33 standard deviations from the expected value.

Practice: Example 1, Reading 48, Curriculum.

2.2 Estimating VaR

Steps of calculating VaR:

1) Convert the set of holdings in the portfolio into a set of exposures to risk factors. This process is referred to as risk decomposition.
2) Collect a data history for each of the risk factors in the VaR model.
3) Using the data collected to make an estimate of the VaR.

	Daily		Annualized	
	Average Return	Standard Deviation	Average Return	Standard Deviation
SPY	0.08%	0.70%	20.72%	11.14%
LWC	0.02%	0.55%	4.21%	8.68%

Correlation of LWC and SPY = –0.1902

Methods to estimate VaR: There are typically three methods to estimate VaR (discussed below in detail):

1) Parametric (variance– covariance) method:
2) Historical simulation method:
3) Monte Carlo simulation method:

Each method provides an estimate of VaR and is highly sensitive to the input parameters.

Example: Suppose, there is a portfolio with a market value of $150 million, consisting of two ETFs SPDR S&P 500 ETF (SPLY), representing the US equity exposure, and SPDR Barclay's Long-Term Corporate Bond ETF (LWC), representing a corporate bond exposure. Assume that we allocate 80% of the portfolio to SPY and 20% of the portfolio to LWC. Suppose that we have collected a set of two years of daily total return data, reflecting both capital appreciation and dividends on each security. The period used for this historical data set is called the **lookback period**.

The table shows that SPY produced an annualized average return of 21% with a standard deviation of about 11%. Suppose, the long-term historical performance of the S&P 500 Index is ~0.5% average return and 20% standard deviation. LWC produced an annualized average return of 4.2% with a standard deviation of about 8.7%. Suppose an average annual return for long-term corporate bonds is ~6% and a standard deviation is 8.5%. If we assume that future performance will be the same as historical one, we can use returns of 10.5% for SPY and 6% for LWC, with standard deviations of 20% for SPY and 8.5% for LWC.

2.2.1.) The Parametric Method of VaR Estimation

The parametric method of estimating VaR is also known as analytical method or the variance–covariance method. In parametric method, we assume that the return distributions for the risk factors in the portfolio are normal. We then uses the expected return and standard deviation of return for each risk factor to estimate the VaR.

A normal distribution with expected value μ and standard deviation σ can be converted to a standard normal distribution (using the formula below), in which the expected value is zero and the standard deviation is one. A standard normal distribution is also known as a ***z-distribution***.

$$z = \frac{R - \mu}{\sigma}$$

In a standard normal (z) distribution, a 5% VaR is 1.65 standard deviations to the left of the expected return. To estimate this VaR, we need the expected return and volatility of the portfolio. The **expected return** is estimated from the following equation:

$$E(R_p) = w_{SPY}E(R_{SPY}) + w_{LWC}E(R_{LWC})$$

Where,
E(Rp) = expected return of the portfolio
wSPY = portfolio weights of SPY
wLWC = portfolio weights of LWC
E(RSPY) = expected return of SPY
E(RLWC) = expected return of LWC

The volatility of the portfolio, σp, is estimated from the following equation:

$$\sigma_p = \sqrt{w_{SPY}^2\sigma_{SPY}^2 + w_{LWC}^2\sigma_{LWC}^2 + 2w_{SPY}w_{LWC}\rho_{SPY,LWC}\sigma_{SPY}\sigma_{LWC}}$$

Where,
σSPY and σLWC = standard deviations (volatilities) of SPY and LWC, respectively
ρSPY,LWC = correlation between the returns on SPY and LWC, respectively; and
ρSPY,LWCσSPYσLWC = covariance between SPY and LWC.

The expected return and standard deviation of our portfolio, consisting of an 80% position in SPY and a 20% position in LWC, is calculated as follows:

$$E(Rp) = 0.8(0.105) + 0.2(0.06) = 0.096000 = 9.6\%$$

$$\sigma_p = \sqrt{(0.8)^2(0.2)^2 + (0.2)^2(0.085)^2 + 2(0.8)(0.2)(-0.20)(0.2)(0.085)} = 0.157483$$

$$= 15.75\%$$

Important to Note: The above inputs are based on annual returns. To estimate an annual VaR, annual data must be used. But in order to have sufficient data points, a longer lookback period is needed.

For a one-day VaR, the expected returns and volatilities should be adjusted to their daily counterparts. Assuming 250 trading days in a year, the expected return is adjusted by dividing by 250 and the standard deviation is adjusted by dividing by the square root of 250. Thus, the daily expected return and volatility are

$$E(Rp) = 0.096 / 250 = 0.000384$$

$$\sigma p = 0.157483 / \sqrt{250} = 0.009960$$

With the distribution having an expected return of 0.0384%, and a one standard deviation move equal to 0.996%, a 5% VaR is calculated by identifying the point on the distribution that lies 1.65 standard deviations to the left of the mean.

Step-by-step procedure to derive VaR: The following step-by-step procedure shows how the VaR is derived:

$$[(E(Rp) - 1.65\sigma p)(-1)]x(\text{Portfolio Market Value in Currency})$$

$$[(E(Rp) - 1.65\sigma p)(-1)]X(\$150{,}000{,}000)$$

Step 1: Multiply the portfolio standard deviation by 1.65
That is, 0.009960 × 1.65 = 0.016434

Step 2: Subtract the answer obtained in Step 1 from the expected return. That is, 0.000384 – 0.016434 = –

0.016050

Step 3: Because VaR is expressed as an absolute number, change the sign of the value obtained in Step 2. That is, change –0.016050 to 0.016050

Step 4: Multiply the result in Step 3 by the value of the portfolio. That is $150,000,000 × 0.016050 = $2,407,500

VaR estimate is $2,407,500, which implies that on 5% of trading days the portfolio would be expected to incur a loss of at least $2,407,500.

Practice: Example 2, Reading 48, Curriculum.

Advantages of Parametric Method:

i. Parametric method is simple and straightforward.
ii. Due to an assumption of the normal distribution, the parameters can be easily estimated using historical data.
iii. The parametric method is most preferred method when returns have normal distribution.

Disadvantages of Parametric Method:

i. Under the parametric method, VaR is very sensitive to the parameter estimates, especially the covariance.
ii. The parametric method is difficult to use when the investment portfolio contains securities with non-linear payoffs (when the distribution is non-normal), e.g. options.
iii. All observations are weighted equally in parametric method.

2.2.2.) The Historical Simulation Method of VaR Estimation

Historical simulation implicitly assumes that the distribution of past returns is a good and complete representation of expected future returns. In this approach, the VaR for a portfolio is estimated by creating a hypothetical time series of returns on that portfolio. In this method, the current portfolio is repriced given the returns that occurred on each day of the historical lookback period and then the results are sorted from largest loss to greatest gain. To estimate a one-day VaR at a 5% confidence interval, we choose the point on the resulting distribution beyond which 5% of the outcomes result in larger losses.

Assume 100 days of past data and arrange portfolio outcomes from largest loss to largest profit. The VaR at 95% will be 5% * 100 = 25th observation from the last. A VaR with 99% of confidence level, can also be calculated using the following formula:

=Percentile (Returns Range, 1%)

- Note that 1% level means 10th observation from the last.

Under the historical method, daily return can be converted to an annual return to estimate the annual VaR. Typically, an annual VaR is preferred. In order to calculate annual VaR, we need a much longer lookback period.

Advantages of Historical Simulation Method:

- Historical simulation method estimates VaR based on what actually happened.
- Historical simulation method can be used for non-normal distribution (e.g. options).

Disadvantages of Historical Simulation Method:

- All observations are weighted equally. However, this problem can be avoided by assigning more weight to more recent observations and less weight to more distant observations.
- Since historical method is based on historical data, there is uncertainty that a historical event will re-occur, or that it would occur in the same manner or with the same likelihood as represented by the historical data.

Practice: Example 3, Reading 48, Curriculum.

2.2.3.) The Monte Carlo Simulation Method of VaR Estimation

In Monte Carlo simulation method, VaR is estimated by simulating random scenarios and revaluing instruments in the portfolio. Unlike historical simulation method, Monte Carlo method does not rely on past data in forecasting future data.

E.g. generate 10,000 outcomes, sort them from worst to best, and select the outcome at the 5th percentile for a 5% VaR, or the outcome at the 1st percentile for a 1% VaR, or the outcome at the 16th percentile if we want to evaluate the impact of one standard deviation move.

Important to Note: The VaR estimated using historical simulation method is typically smaller than that of parametric method. Whereas, VaR calculated using Monte Carlo simulation method is fairly close to VaR under the parametric VaR method.

The Monte Carlo simulation only samples from a population with certain parameters, and the parametric method assumes those parameters. The results obtained from Monte Carlo simulation will be close to the results obtained from parametric method when sample size is large, i.e. much larger than the 10,000.

Advantages of Monte Carlo simulation Method:

- Monte Carlo simulation do not require estimation of a large number of parameters.
- Monte Carlo simulation is not constrained by the

assumption of normal distributions and thus it can be used for any distribution.
- Monte Carlo method can handle more complex distributions.
- Unlike parametric method, Monte Carlo and historical simulation methods are more appropriate to use for option positions or bond positions with embedded options.

Practice: Example 4, Reading 48, Curriculum.

2.3 Advantages and Limitations of VaR

2.3.1.) Advantages of VaR

- **Simple concept.** VaR is relatively easy to understand.
- **Easily communicated concept**. VaR provides very significant and practical piece of information to risk managers.
- **Provides a basis for risk comparison.** VaR can be used to identify high risk asset classes in the portfolio and across trading units.
- **Facilitates capital allocation decisions.** VaR can be used as a benchmark for making capital allocation decision across trading units or portfolio positions. *Example:* A proprietary trading firm has VaR of $20mln in equity trading and its VaR in fixed-income trading is $10mln. If its equity trading portfolio is not expected to take more risk than its fixed-income trading portfolio, then a higher VaR of the equity trading activities indicate that equity trading activities are either taking too much risk or there is too much capital allocated to equity trading. This implies that the firm should either make adjustments to realign its VaR or allocate capital in proportion to the relative risks.
- **Helpful in performance evaluation.** VaR can be used to evaluate risk-adjusted performance of portfolio.
- **Reliability can be verified.** VaR can be easily verified through back-testing. For example, if the daily VaR is $2 million at 1%, we can determine over a historical period of time whether losses of at least $2 million were incurred on 1% of trading days.
- **Widely accepted by regulators.** Regulators typically require that the risk of derivatives positions be disclosed either by sensitivity analysis or by VaR.

2.3.2.) Limitations of VaR

- **Subjectivity.** VaR is a subjective method and requires judgments to be made regarding inputs, source of data, etc.
- **Underestimating the frequency of extreme events.** Use of the normal distribution in the parametric method and sometimes in the Monte Carlo method tend to underestimate the likelihood of extreme events (ignore events in the left tail of the distribution).
- **Ignore liquidity.** VaR is understated if some assets in a portfolio are relatively illiquid, even under normal market conditions. In non-normal market, VaR tends to underestimate the magnitude of potential losses.
- **Sensitivity to correlation risk.** VaR does not take into account correlation risk (risk of rising correlations among all assets in the portfolio during stressed market conditions).
- **Vulnerability to trending or volatility regimes.** VaR ignores the risk of accumulation of substantial losses over time without technically breaching the VaR limit on daily basis.
- **Underestimating the losses during periods of low volatility:** VaR tends to underestimate the losses during periods of low volatility.
- **VaR does not reflect a worst-case scenario.** VaR does not reflect a worst-case scenario as it tells us the minimum loss that one can expect at a certain confidence.
- **Oversimplification.** VaR as a single number provides significant information but this number is often not interpreted properly.
- **Disregard of right-tail events and ignores overall risk–reward trade-off.** VaR only focuses on the left tail (the losses) while the right tail (potential gains) are ignored.

Practice: Example 5, Reading 48, Curriculum.

2.4 Extensions of VaR

Conditional VaR (CVaR): It is the average loss that would be incurred if the VaR cutoff is exceeded. CVaR tells us total amount of loss that one can expect to bear if VaR is exceeded. CVaR is also referred to as the expected tail loss or expected shortfall. CVaR can be estimated using the historical simulation or Monte Carlo methods.

Example: We have a sample of 500 historical returns. We will sort them from lowest to highest. Suppose, the 5% VaR comes out to be $1,300,000. In a sample of 500 returns, there are 25 observations (5% of 500) that lie below the VaR estimate. Suppose, the average of these losses is $1,950,000. Thus, when the VaR is exceeded, we would expect an average loss of about $1.950million.

For the Monte Carlo method, suppose we generated 10,000 random values and obtained a 5% VaR of $2mln. Given 10,000 random values, 5% VaR means there are 500 observations in the lowest 5% of the VaR distribution (10,000*0.05). The CVaR using the Monte Carlo method would be the average of the 500 lowest values.

Important to remember: Due to lower volatility in the historical data series, CVaR derived using the historical simulation method would be lower than the CVaR derived using the Monte Carlo method.

Incremental VaR (IVaR): IVaR tells us change in portfolio VaR if a position size is changed relative to the remaining positions. It is calculated as the difference between the VaR calculated under proposed allocation and VaR calculated under increased allocation to risky component (i.e. equities). E.g. a portfolio manager increases the portfolio risk by increasing the investment in equity index to 90% of the portfolio. VaR under proposed allocation is $2,505,000 and VaR under new allocation is $2,760,000.

IVaR = $2,760,000 - $2,505,000 = $255,000

Marginal VaR (MVaR): MVaR reflects the effect of an anticipated a very small change in the position in the portfolio. In a diversified portfolio, marginal VaR can be used to determine the contribution of each asset to the overall VaR. A total VaR can be estimated by summing the weighted average marginal VaRs for all positions.

Uses of IVaR and MVaR: Both IVaR and MVaR can be used to determine the anticipated change on the total VaR due to change in the portfolio holdings. In addition, both measures can be useful to evaluate the potential effect of a trade before the trade is done.

Relative VaR: Relative VaR (also known as ex ante tracking error) is a measure used to evaluate the degree to which the performance of a portfolio differs from it benchmark. If performance of portfolio is the same as that of its benchmark, it reflects zero ex ante tracking error. The greater the difference between portfolio and its benchmark, the larger the ex-ante tracking error will be.

Practice: Example 6, Reading 48, Curriculum.

3. OTHER KEY RISK MEASURES— SENSITIVITY AND SCENARIO MEASURES

3.1 Sensitivity Risk Measures

Sensitivity measures are used to determine the impact of a single change in an underlying risk factor on the portfolio performance. There are various risk factors, i.e. equity exposure measures and fixed income exposure measures.

3.1.1.) Equity Exposure Measures

Beta: It is the primary equity exposure measure. It measures the sensitivity of the security's expected return to the equity risk premium. In a single risk factor CAPM model, the expected return on a stock is given by following equation.

$$E(Ri) = RF + \beta i[E(RM) - RF]$$

Where,
E(Ri): Expected return on the asset or portfolio i
RF: Risk-free rate
E(Rm): Expected return on the market portfolio, and
E(Rm) – RF: Equity risk premium. Equity Risk Premium is the return investors demand for investing in equities rather than risk-free instruments.
βi: Beta, which is the risk measure. It is the covariance of the asset return with the market return divided by the variance of the market return.

- The broad market beta (an average of all individual betas) is 1.0.
- Assets with betas > (<) 1 are considered more (less) volatile than the market.

3.1.2.) Fixed-Income Exposure Measures

The primary sensitivity exposure measures for fixed-income investments are duration and convexity.
Duration: Duration is the weighted-average time to maturity of a bond. The duration is a measure of the sensitivity of the bond price to the change in interest rate.

Rate of return or percentage price change for the bond = $\Delta B / B \approx -D(\Delta y / 1 + y)$

Where,
B: Price of Bond
Δy: Change in yield (small changes, in particular)
D: Duration
The ≈ sign reflects "approximately equal"

Limitations of Duration Measure: Duration cannot capture change in bond price against large changes in yield and for longer periods.
Convexity: Convexity describes the sensitivity of a bond's duration to large changes in interest rates. It is estimated using the following equation.

$$\frac{\Delta B}{B} \approx -D\frac{\Delta y}{1+y} + \frac{1}{2}C\frac{\Delta y^2}{(1+y)^2}$$

3.1.3.) Options Risk Measures

Option risk measures include the following:

i. **Delta:** Delta describes sensitivity of the option

price to the change in price of the underlying. It is calculated using the following equation.

$$\Delta \quad (\text{delta}) \approx \frac{\text{Change in value of option}}{\text{Change in value of underlying}}$$

- Call option deltas range from a value of 0 to 1. A call delta of 1 means that the price of the call option changes in unison with the underlying. This condition is approximately true for very deep in-the-money calls.
- Put option deltas range from a value of 0 to –1. A put delta of –1 means that the price of the put option changes in unison with the underlying but in the opposite direction. This condition is approximately true for very deep in-the-money puts.
- A value of 0 means that the option value does not change when the value of underlying changes. This condition is true for a very *deep out-of-the-money* option.
- As expiration approaches, an in-the-money call (put) delta approaches 1 (–1) and an out-of-the-money call (put) delta approaches 0.

Delta can be used to determine an approximate new price of an option as the underlying changes. For a call option, we can use the following formula:

$c + \Delta c \approx c + \Delta c \Delta S$

Where,
c: Original price of the option
Δc: Change in the price
ΔS: Change in the value of the underlying.
Note: The same relationship would hold for puts.

Limitation of Delta: Delta captures the effect of *only small changes* in the value of the underlying security over a short period of time.

ii. **Gamma:** Gamma measures the sensitivity of an option's delta to a change in the underlying. Like convexity measure, it is a second-order effect.

$$\Gamma \quad (\text{gamma}) \approx \frac{\text{Change in delta}}{\text{Change in value of underlying}}$$

Example: Suppose, a value of the underlying increases by 0.15 and the gamma is 0.04, then the delta would increase by 0.006 (0.15 × 0.04).

- As the option approaches maturity, value of gammas increases.
- Gammas are large when options approach expiration, unless the option is deeply in or out of the money. This is because when an option is close to expiration and roughly at the money, a small change in the price of the underlying will determine whether the option expires worthless or in the money.

The estimated change in the call price is determined as follows:

$$c + \Delta c \approx c + \Delta_c \Delta S + \frac{1}{2}\Gamma_c (\Delta S)^2$$

Where Γc is the gamma of the call option.

iii. **Vega:** Vega is a first-order effect reflecting the relationship between the option price and the volatility of the underlying. Vega is expressed by the following relationship:

$$\text{Vega} \approx \frac{\text{Change in value of option}}{\text{Change in volatility of underlying}}$$

Using delta, gamma, and Vega, the new value of an option given an old value, a change in the value of the underlying, and a change in the volatility can be estimated as follows:

$$c + \Delta c \approx c + \Delta_c \Delta S + \frac{1}{2}\Gamma_c (\Delta S)^2 + \text{vega}(\Delta\sigma)$$

Where Δσ is the change in volatility.

Practice: Example 7, Reading 48, Curriculum.

3.2 Scenario Risk Measures

Unlike sensitivity risk measures, scenario tests measure the cumulative effect of adverse movement in a number of risk factors, e.g. equity prices, foreign exchange rates and interest rates on a bank's financial position. They typically consider larger movements in risk factors. Like VaR measure, scenario risk measures focus on extreme outcomes; but unlike VaR, they do not have any constraints related to recent historical events or assumptions about parameters or probability distributions.
The two types of scenario risk measures – historical scenarios and hypothetical scenarios:

3.2.1.) Historical Scenarios

Historical scenarios employ shocks that occurred in the past.

Historical scenario measure for Bonds: Using historical prices of a bond to determine risk of current portfolio position is not appropriate because the volatility of the bond decreases as it approaches its maturity. Hence, it is preferred to use the historical yields, spreads, implied volatilities, prices of the underlying assets in derivative contracts, and the other input parameters that drive the pricing of these instruments.

Method of doing Historical scenario analysis: In the case of standard bonds, the terms and conditions of these instruments (e.g., coupon, call features, put features, any amortization or sinking fund features, and maturity) are entered into fixed-income pricing models. For a convertible bond, the bond's terms and conditions (e.g., coupon, conversion ratio, and maturity) are entered into a convertible bond pricing model. These modeled fixed-income or derivative holdings, together with the equity holdings, are then repriced under the historical conditions. The value of each position is recorded before and after these changes in order to arrive at the gain or loss that would occur under the chosen scenario. The output of the scenario can include

- the total return of the portfolio;
- for long-only asset managers, the total return of the portfolio relative to its benchmark;
- for pensions, insurers, and others whose liabilities are not already incorporated into the portfolio, the total return of the portfolio relative to the change in liabilities under the scenario; and
- any collateral requirements and other cash needs that will be driven by the changes specified in the scenario.

Important to remember:

- Gamma and convexity are inadequate to use for scenario analysis.
- It is also important to determine how to treat securities or markets that did not yet exist at the time of the scenario are modeled. Such securities and market need to be mapped to a relevant index or to a similar company, or decomposed into the relevant statistical factors (such as growth, value, volatility, or momentum) by using a factor model before beginning the analysis.
- Stress tests and scenarios analyses are used to understand a portfolio's risk exposures. These measures cannot to eliminate such exposures.
- A portfolio that has no sensitivity to any stress event would earn only the risk-free rate, or in the case of long-only asset managers, could not outperform the benchmark index.
- Parties that use leverage, such as banks and hedge funds, are more likely to use single-factor stress tests rather than multifactor scenario analyses. It is because for banks if the capital falls below an acceptable level, it could set off a chain reaction of margin calls, withdrawal of financing, and other actions that threaten the viability of the business.

3.2.2.) Hypothetical Scenarios

Hypothetical scenarios account for plausible changes in circumstances that have no historical precedent.

Reverse stress testing: It is a process whereby we first identify the portfolio's most significant exposures and then target these material exposures to assess their behavior in various environments. Reverse stress testing is particularly helpful in estimating potential losses if more than one important exposure is affected in a market crisis.

Practice: Example 8, Reading 48, Curriculum.

3.2 Sensitivity and Scenario Risk Measures and VaR

Comparison:

- VaR is a measure of losses and the probability of large losses.
- Sensitivity risk measures determine changes in the value of an asset against change in other factors, i.e. a market index, an interest rate, or an exchange rate.
- Sensitivity risk measures do not provide us the probability of occurrence of a given change in value.
- Betas and deltas do not capture an anticipated change in the underlying risk factors, but given a change, they tell us how change in value of an asset or derivative against that change.

Note: Rather than only focusing on methods to modify risk measures, it is equally important to determine the factors (e.g. high beta stocks, high duration bonds, high delta options etc) that are leading to large values of risk measure (e.g. VaR).

3.3.1.) Advantages and Limitations of Sensitivity Risk Measures and Scenario Risk Measures

Advantages of Scenario measures:

- Such measures can reflect the impact of extreme hypothetical market movements as well as historical market movements.
- Such measures are not constraint by assumptions regarding normality or correlation.
- Scenario analysis can incorporate the impact of worst case situations when the correlation amongst assets temporarily increases substantially.

Limitation of Scenario measures:

- In scenario analysis, it is difficult to determine the probability of a given scenario.
- Relying on historical scenarios do not necessarily provides accurate result.
- Hypothetical scenarios may incorrectly specify co movement in assets, the magnitude of those movements, and the effects of liquidity and concentration.
- Hypothetical scenarios can be very difficult to create and maintain.
- The more extreme the scenario, and the farther from historical experience, the less reliable the

scenario measures are.

Advantages of sensitivity analysis:

- Sensitivity measures, e.g. duration measures the level of interest rate risk. Option delta and duration (for fixed income) helps to assess risk in a portfolio that has hedging or short positions with optionality or interest rate risk.

Limitation of sensitivity analysis:

- Sensitivity measures do not distinguish assets by standard deviation/volatility or other higher confidence loss measures. E.g. a high yield bond portfolio might have the same sensitivity to a 0.02% credit spread movement as an investment-grade portfolio, but they do not have the same risk because the credit spreads of the high-yield portfolio are more likely to move 0.02%, or more, than are the credit spreads of the investment-grade bonds. However, this short coming can be overcome by measuring sensitivity to a one standard deviation movement in an asset's price or yield.

Granularity: Granularity reflects the degree to which individual OR exposures are modeled. In other words, it is the width or the narrowness of the risk-factor buckets used to portray sensitivity. A **bucket** is a risk factor description, e.g. sensitivity of portfolio to world equity markets (e.g. using MSCI world index) is a broad bucket. A narrower bucket would be sensitivity of a portfolio to Developed or Emerging equity markets (e.g. using MSCI developed and MSCI emerging market indices). An even narrower bucket can be used by measuring sensitivity against equity markets of individual counties.

- ➢ The correlation risk can be avoided by using a broader.

Practice: Example 9, Reading 48, Curriculum.

4. APPLICATIONS OF RISK MEASURES

4.1 Market Participants and the Different Risk Measures They Use

The type of risk measures used by market participants depend on following three factors:

i. The ***degree of leverage*** and the resulting need to assess minimum capitalization/maximum leverage ratios. Market participants who use a high degree of leverage focus on potential loss measures with a high confidence interval or to focus on rare events that might occur in a short period of time. Market participants who use a minimal (or no) leverage, such as long-only asset managers are more interested in avoiding underperformance,

ii. The ***mix of risk factors*** to which the business is exposed (e.g., the degree of equity or fixed-income concentration in their portfolios). For portfolios dominated by fixed-income investments, risk managers focus on sensitivity of portfolios to instantaneous changes in price and yield and thus use duration, credit spread duration, and key rate duration measures. When portfolios are dominated by equities, risk managers typically categorize the equities by broad country markets, industries, and market capitalization levels, and then the returns of their portfolios are regressed against fundamental factor histories (i.e. growth, value, momentum, and capitalization size) in order to understand their exposure to such factors.

iii. The ***accounting or regulatory requirements*** that govern their reporting. Portfolios with full fair value accounting (e.g. mutual funds, the held-for-sale portfolios of banks) prefer to use risk measures as VaR, economic capital (the amount of capital a firm needs to hold if it is to survive severe losses from the risks in its businesses), duration, and beta etc. Portfolios with book value accounting typically uses asset/liability gap models.

4.1.1.) Banks

Banks focus on following risk factors:

a) **Liquidity gap:** It refers to the extent of mismatch between asset and liability of the bank.

b) **VaR:** The value at risk for the held-for-sale or trading (fair value) portion of the balance sheet.

c) **Leverage:** A leverage ratio weight risk assets using a variety of methods and rules and divide this weighted asset figure by equity. The riskier assets are assigned a greater weighting, and less risky assets a lower weighting. This implies that more equity is required to support riskier assets.

d) **Sensitivities:** For the held-for-sale portion of banks' balance sheet, banks measure duration, key rate duration or partial duration, and credit spread duration for interest rate risk positions. Banks also measure foreign exchange exposure and any equity or commodity exposures. All these exposure

measures include delta, gamma and Vega exposures of options.

e) **Economic capital:** This is measured by combining the company's market, credit, and operational risk measures to estimate the total loss the company could suffer at a very high level of confidence (e.g., 99% to 99.99%), usually in one year's time. Economic capital measures are applied to the full balance sheet, including both the held-for-sale and held-for-investment portfolios.

f) **Scenario analysis:** Stress tests are applied to the full balance sheet. They are used to identify whether capital is sufficient for targeted, strong negative shocks. Scenario analysis is used to examine how the full balance sheet might be affected by different interest rate, inflation, and credit environments.

4.1.2.) Asset Managers

In asset management portfolios, risk management efforts are focused primarily on volatility, probability of loss, or probability of underperforming a benchmark rather than insolvency.

- In a diversified, unleveraged, long-only fund, the asset values are unlikely to decline below zero in the absence of a wholesale withdrawal of assets by the firm's clients. Although use of derivatives create leverage in a portfolio, these positions are typically balanced by amount of cash present in the portfolio which is equal to notional exposure created by the derivative.
- For leveraged portfolios, insolvency is an important threat.

How different Funds manage risk exposure:

- Long-only asset managers: Long-only asset managers need to aggregate the firm investments to assess its risk exposures across portfolios.
- Hedge funds: A hedge fund manager needs to aggregate the adviser's side-by side investment in the various funds it advises.
- Funds of funds: Manager of funds of funds need to aggregate the risks of the underlying hedge funds to the master fund level.

Backward-looking returns-based measures: These include standard deviation, ex post tracking error, Sharpe ratio, information ratio, and historical beta.

- Current risk exposures can be assessed only analyzing the current portfolio holdings. Measures that use current holdings typically include VaR, ex ante tracking error, duration and forward-looking beta, stress tests, and scenario analyses.

4.1.2.1.) Traditional Asset Managers

- For absolute return strategies, the benchmark is typically cash or a cash-like hurdle rate. When cash is the benchmark, VaR and ex ante tracking error, if measured using the same holding period and confidence interval, will be effectively the same.
- Although banks, insurers, and other market participants favor measuring VaR in currency terms relevant for the institution (e.g., dollars for a US-based insurer, yen for a Japanese bank) and measure duration and similar statistics as the value change for a 1 bps interest rate change, long-only asset managers generally prefer to express VaR in percentage terms and will divide VaR and duration by the net assets of the portfolio being analyzed.

A typical sample of risk measures used by asset managers includes the following:

a) **Position limits:** Position size limit is appropriate to use for homogeneous, long-only portfolios. However, position size measure is less useful for assessing interest rate risk, risk of a multi-asset class portfolio, and assessing net risk in a portfolio that uses hedging instruments, short positions, and liabilities. Position limits include restrictions on country, currency, sector, and asset class; they may measure them in absolute terms or relative to a benchmark, and they are almost always expressed as a percentage of the portfolio's value.

b) **Sensitivities:** Asset managers use the full range of sensitivity measures, including option adjusted duration, key rate duration, credit spread duration, delta. Measures can be expressed in absolute terms as well as relative to a benchmark.

c) **Beta sensitivity:** Beta is frequently used for equity-only accounts.

d) **Liquidity:** Asset managers often look at the liquidity characteristics of the assets in their portfolios. It is commonly used for equity portfolios.

e) **Scenario analysis:** Long-only asset managers use stress tests or scenario analyses to verify that the risks in the portfolio are as they have been disclosed to investors and to identify any unusual behavior that could arise in stressed markets.

f) **Active share:** Active share is the measure of that percentage of the portfolio that differs from the benchmark index.

g) **Redemption risk:** Open-end fund managers often assess what percentage of the portfolio could be redeemed at peak times and track this behavior across the funds and asset classes they manage.

h) **Ex post versus ex ante tracking error:** Traditional asset managers typically use limits on ex ante tracking error. It provides an estimate of the degree to which the current portfolio could underperform its benchmark.

- Ex post tracking error measures the historical

deviation between portfolio returns and benchmark returns. Ex post tracking error is a useful tool for assessing sources of performance, manager skill and behavior.
- Ex ante tracking error is used to identify whether today's positions could give rise to unexpected potential performance. Ex ante tracking error takes today's benchmark-relative position.

i) VaR: VaR measure is typically used for portfolios that are characterized as "absolute return" strategies, for which a given market benchmark may not serve as the portfolio objective. VaR is less commonly used as a risk measure than ex ante tracking error by traditional asset managers.

4.1.2.2.) Hedge Funds

Similar to banks, hedge funds that use leverage need to focus on sources and uses of cash through time, including timing of withdrawal of credit lines, interplay between market movements, margin calls, and the redemption rights of investors in order to understand worst-case needs for cash.

Hedge fund market risk measures includes the following:

a) Sensitivities: All types of sensitivity measures are useful for hedge fund risk management.
b) Gross exposure: Long–short, market neutral, and arbitrage strategies will typically measure long exposure, short exposure, and gross exposure (the sum of the absolute value of long plus short positions) separately.
c) Leverage: Hedge funds commonly use leverage measures.
d) VaR: Hedge funds that use VaR measures tend to focus on high confidence intervals (more than 90%) and short holding periods. Hedge funds do not use a benchmark-relative measure.
e) Scenarios: Hedge funds commonly use scenario/stress tests that are well tuned to the specific risks of their strategy, such as for merger arbitrage strategies, they focus on likelihood of occurrence of a merger.

Drawdown: Drawdown is the worst-returning month or quarter for the portfolio or the worst peak-to-trough decline in a portfolio's returns. This measure is preferred to use for the following strategies:

- Strategies that focus on credit risk taking, such as long–short credit, credit arbitrage, or bankruptcy investing;
- Strategies that focus on events, such as merger arbitrage;
- Strategies that invest in non-publicly issued assets or other assets that do not reliably have a daily, independent fair value determination;
- Strategies that invest in illiquid asset classes or take large positions relative to market size in any asset class;
- Strategies that sell options or purchase bonds with embedded options;
- Strategies that heavily rely on correlation relationships, such as equity market neutral;

4.1.3.) Pension Funds

The risk management goal for pension funds is to remain sufficiently funded to make future payments to pensioners. The pension fund should neither be over-funded and under-funded.

- Overfunding occurs when the funding ratio (the assets divided by the present value of the liabilities) is greater than 100%. Overfunding can be mitigated over time by stopping of regular contributions by the plan sponsor while the number of employees and their salary levels may be growing.
- Underfunding occurs when the funding ratio is under 100%. Underfunding can be mitigated by growth in the assets in the fund over a suitable time horizon or by additional contribution by the plan sponsor.

Important market risk measures or methods for pension funds often include the following:

a) Interest rate and curve risk: The expected future cash flows to the fund are grouped by maturity and, in the case of an international pension fund that must make future payouts in multiple currencies, they may also be grouped by currency. When a particular fixed-income instrument or curve is used to provide the discount rate for arriving at the present value of the pension liability, the liability cash flows will be expressed as a short position at the relevant points on the curve.
b) Surplus at risk: This measure is an application of VaR. It is computed by entering the assets in the portfolio into a VaR model as long positions and the pension liabilities as short fixed-income positions. It estimates how much the assets might underperform the liabilities, usually over one year. If the value of assets in the portfolio is equal to the liabilities, such pension fund would have zero surplus at risk.

- The more volatile the investments in the pension fund, and the lower the correlation between these assets and liabilities, the higher the surplus at risk will be.
- When the pension fund's surplus at risk exceeds a certain limit, pension staff will change the fund's asset allocation to make the assets in the fund matching the liabilities. This liability-focused form of pension investing is commonly referred to as "***liability driven investing***."

c) Glide path: A glide path is a tool for managing surplus at risk. It charts multi-year stages to change the portfolio from its current state to its target state.

d) **Liability hedging exposures versus return generating exposures**: Matching liabilities is not the only goal of pension funds. Pension fund managers may separate their investment portfolio into investments designed to match the pension liability and the investments that generate excess returns. The return-generating portion of the portfolio can help to hedge the underfunding risk.

4.1.4.) Insurers

Property and casualty insurance, including home, auto, corporate liability insurance, and health insurance, are typically not highly correlated with financial asset markets. Insurers do not focus on matching assets with liabilities in their property and casualty lines of business. Premium income is used for meeting insurance claims while investment portfolios of insurers are used to achieve a good absolute return within the constraints imposed under regulatory reserve requirements. The risk metrics of property and casualty insurance differ significantly from those used for life insurance and annuity products.

Property and casualty insurance companies: The market risk management measures in the property and casualty lines of business include the following:

a) **Sensitivities and exposures:** Insurers set a target asset allocation for the investment portfolios and then these current exposures continuously monitored in order to keep the target ranges set forth in the target asset allocation within range.
b) **Economic capital and VaR:** Insurers focus on capital at risk and VaR measures. The premiums earned by insurance companies are used for the expected payouts, so capital is tapped only in cases of greater-than-expected payouts. Assessment of the risk to economic capital will include the market risks in the portfolio, as well as characteristics of the insurance exposures and reinsurance coverage.
c) **Scenario analysis:** For the property and casualty lines, these scenarios may stress the market risks and the insurance risks in the same scenario.

Life Insurance Companies: Life liabilities are very long and the reserves that insurers are required to maintain by insurance regulators are highly dependent on discount rate assumptions. For life portfolios, market risk measures include the following:

a) **Sensitivities:** Insurers set a target exposures of the investment portfolio and the annuity liability and these exposures are measured and monitored continuously.
b) **Asset and liability matching:** The investment portfolio is not designed to be a perfect match to the liabilities; but unlike property and casualty insurance companies, it is more closely matched to liabilities.
c) **Scenario analysis:** The main focus of risk measurement for the life lines of insurance are measures of potential stress losses based on the differences between the assets in which the insurance company has invested and the liabilities driven by the insurance contracts it has written to its customers. Scenario analyses need to stress both market and non-market sources of cash flow change (in which non-market changes can include changes in longevity).

Practice: Example 10, Reading 48, Curriculum.

5. USING CONSTRAINTS IN MARKET RISK MANAGEMENT

Effective market risk management requires use of suitable constraints. If constraints are too tight, they may result in lower returns or profitability to a sub-optimal level. If constraints are too loose, they may result in substantial losses. There are two types of limits, restrictive and non-restrictive. Unrestrictive limits are typically set far from current risk levels and permit larger losses than restrictive limits.

Example: Assume a leveraged portfolio in which insolvency could occur if cumulative daily losses exceed $15 million, and the portfolio's current two week, 1% VaR measure is $2 million, then an unrestrictive limit for this portfolio can be set at $15 million. If the limit is set below $15 mln, e.g. $6 million, the portfolio might under-allocate the capital it has to invest and fail to make a high enough return on equity.

- The risk exposures of a firm should be analyzed at aggregate level rather than exclusively. E.g. in a bank with four trading desks, assume that an overall VaR tolerance is $8 million. This implies that each trading desk might get a limit of $2mln. But, if these desks have lower than perfect correlation, that is, if one desk has a short position that to some degree serves as an offset to another desk's long position, the firm will never be able to use its $8 million risk appropriately. This problem can be resolved by over allocation to a particular desk.

- Some firms also use MVaR for each trading desk, allocating each desk a VaR budget such that the total VaR is equal to the sum of each individual desk's MVaR. This practice helps each trading desk to "reinvest" the diversification benefits obtained at the aggregate level.

Types of Constraints (section 5.1 – 5.5):

1) **Risk Budgeting:** In risk budgeting, the total risk appetite of the firm or portfolio is set at the highest level of the entity and then allocated to sub-activities. Risk budgeting typically uses VaR or ex ante tracking error. E.g.

- A bank set a limit on total economic capital or VaR, and then allocate this risk appetite among the basic risk types (market, credit, and operational) and different business units, geographies, and activities.
- Similarly, a pension fund sponsor may establish a broad asset allocation and then further establish its tolerance for underperformance in a given asset class and allocate that tolerance to the asset managers selected to manage the assets by assigning each an ex ante tracking error budget.
- A portfolio manager might have an ex ante tracking error budget explicitly provided by the client, or it might herself set a tracking error budget based on her investment philosophy and market practice. Given this budget, the portfolio manager seeks to optimize the portfolio's exposures relative to the benchmark to ensure that the strategies generating most tracking error for the portfolio are also generating highest reward.

2) **Position Limits:** Position limits are limits set on the market value of any given investment, or the notional principal amount for a derivative contract. They can be expressed in currency units or as a percentage of some other value (i.e. net assets). Position limits do not take into account duration, volatility, and correlation, as VaR does, but they are useful for controlling overconcentration. Position limits should not be overly prescriptive but should address the event risk and single name risk (that VaR handles does not handle properly) as follows:

- limits per issuer;
- limits per currency or country;
- limits on a given strategy, such as high yield credit or emerging market equities;
- limits on gross size of long–short positions or derivative activity; and
- limits on asset ownership that correspond to market liquidity measures, such as daily average trading volume.

3) **Scenario Limits:** A scenario limit is a limit on the estimated loss for a given scenario. A tolerance level for each scenario is developed to determine whether results are within the established risk tolerance. Afterwards, the risk manager observes portfolio's sensitivity to the scenarios periodically. It is not appropriate to set same limits to all scenarios. This implies that for potential loss under the most extreme scenarios, a higher risk tolerance should be established.

4) **Stop-Loss Limits:** A stop-loss limit describes a reduction in the size of a portfolio, or its complete liquidation, when a loss of a particular size occurs in a specified period. This measure can be used to avoid problem of "trending" associated with VaR, whereby a portfolio remains under its VaR limit each day but cumulatively loses more than expected. Rather than using stop-loss limits, a firm can use drawdown control or portfolio approach. Under this approach, a firm is required to undertake a hedging activity (i.e. purchase of protective options) after losses of a given magnitude, with the magnitude of the hedge increasing as losses increase. Drawdown control is more sophisticated than the simpler stop-loss limit.

5) **Risk Measures and Capital Allocation:** Capital allocation involves placing limits on each of a company's activities in order to ensure that the areas in which it expects the greatest reward and has the greatest expertise are given the highest resources. The company's actual, physical on-balance-sheet capital must always be greater than value of its economic capital, and a minimum level of economic capital must be established to ensure that the company does not take on a risk of loss that will exceed its available capital. Economic capital is the amount of capital needed by a firm to survive severe losses associated with its businesses.

- Under this approach, the company first establishes its overall risk appetite in economic capital terms, and then subdivides this appetite among its units. This exercise is similar to risk budgeting, but in the case of corporations, banks, insurers, or hedge funds, it is more likely to be called "capital allocation."
- Risk budgeting more commonly focuses on losses at the one standard deviation level whereas, capital allocation focuses on losses at a very high confidence level.
- Capital allocation is preferred to use when leverage is used by the portfolio or in which the strategy has greater-than-expected tail risk (e.g. sell options, sell insurance, take substantial credit risk, or have unique liquidity or exposure concentration risks).

To optimize the use of capital, the "owner" of the capital will typically establish a hurdle rate over a given time horizon; this is often expressed as the expected rate of return per unit of capital allocated.

Example: Suppose there are two portfolios, A and B.

- Portfolio A requires €325,000, and its expected return is €50,000 per year (i.e. 50,000 / 325,000 = 15.4%).
- Portfolio B require €1,000,000 in capital and is expected to return €100,000 per year (i.e. 100,000 / 1,000,000 = 10%.

If the investor has an annualized hurdle rate of 15%, Portfolio A will exceed the hurdle rate and is considered to be better than Portfolio B, even though the absolute income for Portfolio B is higher.

Binding and Non-Binding Constraint:

Binding constraint is a constraint when the current measure of economic capital is a greater number than the portfolio's cash or regulatory capital needs, otherwise it is not a binding constraint.

Practice: Example 11, Reading 48, Curriculum.

Practice: End of Chapter Practice Problems for Reading 48

1. INTRODUCTION

The state of the economy and financial market activity are interrelated. Moreover, all financial instruments represent claims on an underlying economy. Savers, by deferring consumption into future, helps generating future economic growth and employment by facilitating governments to raise the capital needed for creation of a secure society and allows corporations to raise capital to invest in profitable investment opportunities.

2. FRAMEWORK FOR THE ECONOMIC ANALYSIS OF FINANCIAL MARKETS

2.1 The Present Value Model

The fundamental present value formula for the value at time t of any financial asset i, V^i_t, which is assumed to be equal to its current market price, P^i_t is estimated using following formula.

$$P_t^i = \sum_{s=1}^{N} \frac{E_t\left[\widetilde{CF}^i_{t+s}\right]}{\left(1 + l_{t,s} + \theta_{t,s} + \rho^i_{t,s}\right)^s}$$

where:

P_t^i = the value of the asset i at time t (today)

N = number of cash flows in the life of the asset

$\widetilde{CF}^i_{t+s}$ = the uncertain, nominal cash flow paid s periods in the future

$E_t\left[\widetilde{CF}\right]$ = the expectation of the random variable $\widetilde{CF}$ conditional on the information available to investors today (t)

$l_{t,s}$ = yield to maturity on a real default-free investment today (t), which pays one unit of currency s periods in the future

$\theta_{t,s}$ = expected inflation rate between t and $t + s$

$\rho^i_{t,s}$ = the risk premium required today (t) to pay the investor for taking on risk in the cash flow of asset i, s periods in the future

The state of the economy affect asset prices by affecting the asset's expected cash flows and the discount rate(s) applied to the asset's expected cash flows. It is basically the degree of certainty that investors have about future cash flows that distinguishes one financial asset class from another.
The uncertainty about future cash flows of an asset is reflected in the discount rate. The discount rate has three different components:

1) $l_{t,s}$ represents return required by an investor on a real default-free fixed income security (e.g. inflation-linked bond issued the government of a developed economy) at time 0 (today) against a cash flow to be paid 's' periods in the future.
2) $\theta_{t,s}$ represents additional return required by an investor, above $l_{t,s}$, for investing in a nominal default-free fixed income security. In other words, it represents compensation for the inflation which affects real purchasing power of investments in the future.
3) $\rho^i_{t,s}$ represents the additional return required by an investor for investing in financial assets with uncertain future cash flows.

Discount rate, for particular assets i.e. commercial real estate and high-yield corporate bonds/loans, also represent liquidity risk. Liquidity risk is the risk that a

financial asset cannot be converted quickly into cash at close to its fair value.
The discount rates applied to the cash flows of financial assets tends to vary over time with changes in perceptions of expected economic growth, inflation, and cash flow risk. For example, during recessions, investors demand higher risk premium for investing in non default-free (risky) assets.

2.2 Expectations and Asset Values

Asset values depend both on past cash flows and on the expectation of future cash flows which are based on current information. Information that has been anticipated is already reflected in asset prices, but information that is different from expectations constitutes real news that need to be adjusted in the asset prices. Due to this adjustment, the holding-period return of an asset is different from its expected return. For example, if the expectation was for better (worse) news but real news are good (bad), prices may fall (rise).
Unlike economic factors that affect asset values through direct effects on cash flows and/or discount rates, investor sentiment affects assets values through ***direct*** effects on discount rates via higher or lower risk premiums and possibly ***indirect*** effects on future cash flows.

3. THE DISCOUNT RATE ON REAL DEFAULT-FREE BONDS

3.1 Real Default-Free Interest Rates

Investing in a bond that is both default-free and unaffected by future inflation has no risk of losing money over the investment period in either nominal or real terms but it involves opportunity cost of forgoing consumption today. The price and return of such an asset is determined by the aggregated opportunity cost of all investors.

- The ***marginal utility of consumption*** is the additional satisfaction or utility that a consumer derives from one additional unit of consumption. The marginal utility of consumption of investors diminishes with increase in their wealth because their fundamental needs have been satisfied.

Inter-temporal rate of substitution = $\widetilde{m}_{t,s}$ =

$$\frac{\text{Marginal utility of consumption 's' periods in the future}}{\text{Marginal utility of consumption today}}$$

- Inter-temporal rate of substitution represents investor's marginal willingness to trade consumption t time t for (real) wealth at time t + s.
- During "good" economic times, levels of current income are high and so is the current consumption. Consequently, the utility derived from an additional unit of consumption today is relatively low.
- Conversely, during "bad" economic times, levels of current income are low and so is the current consumption. Consequently, the utility derived from an additional unit of consumption today is relatively high.
- When the price of a bond is less (greater) than investor's expectation of the inter-temporal rate of substitution, investor would prefer to buy more (less) of the bond today. As a result, today's consumption falls (rises) and marginal utility of consumption today rises (falls) while expectations conditional on current information of the inter-temporal rate of substitution falls (rises). Consequently, the inter-temporal rate of substitution would fall (rise) and demand and price of the bond would increase (fall). Hence, this process continues until the rate of substitution is equal to the bond price.

Practice: Example 1 and 2, Reading 49, P.10 and 12.

Relationship between one-period real risk-free rate and inter-temporal rate of substitution: The one-period real risk-free rate is inversely related to the inter-temporal rate of substitution. That is, the higher the one-period real risk-free rate, higher will be the return that can be earned by an investor, and consequently, the lower will be the inter-temporal rate of substitution. This implies that current consumption becomes more important relative to future consumption.

Yield to maturity on a real default-free investment today which pays one unit of currency s periods in the future

$$(I_{t,1}) = \frac{1}{\text{Inter-temporal rate of substitution}} - 1$$

Practice: Example 3, Reading 49, P.13.

3.1.1.) Uncertainty and Risk Premiums

- An investor's expected marginal utility associated with a given expected payoff is inversely related to the uncertainty of the pay-off. That is, as the uncertainty of the pay-off increases, investor's expected marginal utility associated with a given

expected payoff decreases.

- Similarly, investor's marginal utility falls as his/her wealth or income increases. This implies that as investor's wealth or income increases, his/her absolute risk aversion[1] decreases and thus, investment in risky assets increases. In other words, when wealth or income is high (low), the risk premium for a given risk is low (high). As investment in risky assets increases, the expected marginal utility for wealthier investors decreases. Hence, when market is in equilibrium, the willingness to invest in risky assets is same for both the wealthier and poorer investors.
- As investor's wealth or income increases, his/her absolute risk aversion decreases, leading to higher demand for risky assets and lower demand for riskless assets. As a result, price of riskless assets decreases and the equilibrium return on the riskless asset increases.
- When uncertainty in future income increases → savings increase → future resources increase → future marginal utility declines and investors demand higher expected return.

Practice: Example 4, Reading 49, P.14.

3.1.2.) Risk Premiums on Risky Assets

Assume that the investor is only holding the security for one period. Its current price is Pt,s. In this case, the bond has value $\widetilde{P}_{t+1,s-1}$ in one period because the term to maturity of the bond has been reduced by one period relative to its original maturity date.

$$P_{t,s} = E_t\left[\widetilde{P}_{t+1,s-1}\,\widetilde{m}_{t,1}\right]$$

Practice: Example 5, Reading 49, P.15.

Equation 6 expresses the value of a risky asset as the sum of two terms. The first term is the asset's expected future price discounted at the risk-free rate. It may be called the risk neutral present value because it represents a risky asset's value if investors did not require compensation for bearing risk.

Value of Risky Asset = Asset's expected future price discounted at the risk-free rate + Covariance term (i.e. discount for risk)

Or

$$P_{t,s} = \frac{E_t\left[\widetilde{P}_{t+1,s-1}\right]}{1+l_{t,1}} + \operatorname{cov}_t\left[\widetilde{P}_{t+1,s-1}, \widetilde{m}_{t,1}\right]$$

Note: Asset's expected future price discounted at the risk-free rate is also known as risk neutral present value because it represents a risky asset's value if no compensation is demanded by investors for bearing risk.

Covariance term represents the covariance between an investor's inter-temporal rate of substitution and the random future price of the investment at t+ 1, based on the information available to investors today (t).

- For one-period default-free bond, the covariance term is zero because the future price is a known constant ($1) and the covariance of a random quantity with a constant is zero.
- For two-period default-free bond, the covariance term is not zero because the future price of $1 two periods in the future is known with certainty, but the price one period in the future is not.
- When the covariance is positive — that is, payoffs are high and expected utility from consumption is low, the risk premium will be low.
- Generally, for risk-averse investors, most risky assets tend to have negative covariance term and so the price of asset is lower because when the expected future price of the investment is low or during bad economic times, the marginal utility of future consumption relative to that of current consumption and hence the inter-temporal rate of substitution is high. Moreover, during bad economic times, pay-outs (earnings, dividends, for shares and defaults for bonds) of risky assets decline leading to declining asset prices. This implies that negative covariance term results in a positive risk premium.

Important to note: All else equal, the more negative covariance term is, the higher the risk premium term and the required return for an asset and the lower its current market price is.

Practice: Example 6, Reading 49, P.18.

The risk premium can be computed as follows:

$$= -\left(1+l_{t,s}\right)\operatorname{cov}_t\left[\widetilde{m}_{t,1}, \frac{\widetilde{P}_{t+1,s-1}}{P_{t,s}}\right]$$

- Even a default-free bond has risk premium because of the uncertainty of its price before maturity.
- An asset that generates high returns during bad times, when the marginal value of consumption is high, provide a type of hedge against bad times and bear a ***negative*** risk premium. Negative risk

[1] Absolute risk aversion refers to the amount held in risky assets at different levels of wealth.

premium implies relatively high price and low required rate of return.

Summary:
- When the covariance term is zero (there is no relationship), the asset is risk-free.
- When the covariance term is positive, the asset is a hedge and will have a rate of return less than the risk-free rate and the asset's risk premium will be negative.

3.2. Default-Free Interest Rates and Economic Growth

When independent change in the real GDP growth is known or can be anticipated perfectly, any increase in real GDP growth rate leads to decrease in saving and increase in borrowing as more goods and services will be available in the future. Due to increase in borrowing, real default-free interest rate increases.

- This implies that ***real interest rates are positively related to the expected growth rate of GDP.*** That is, an economy with higher trend real economic growth, other things being equal, tends to have higher real default-free interest rates because during growth periods, the marginal product of capital (the additional output resulting from the addition of one unit of capital, holding all else constant) would be expected to be higher, so the real default-free interest rate should also be expected to be higher.
- All else equal, real interest rates are positively related to the volatility in GDP growth. That is, the more volatile GDP growth is, greater the risk that the income available for consumption will be lower than expected, the higher real rate of return will be required by investors in compensation, and hence the higher the real interest rates.

Practice: Example 7,8,9, & 10, Reading 49, P.21,23,29,32 respectively.

3.3.2.) Real Default-Free Interest Rate Summary

$$P_t^i = \sum_{s=1}^{N} \frac{CF_{t+s}^i}{(1 + l_{t,s})^s}$$

The above equation shows that it is only changes in $l_{t,s}$ that will affect the price of such a bond. In turn, $l_{t,s}$ is determined by real economic growth and the volatility in economic growth over time as a result of the aggregation of the consumption and saving decisions of individual investors.

4. THE YIELD CURVE AND THE BUSINESS CYCLE

Since investors are generally assumed to be risk-averse, they demand compensation for assuming risk, for expected inflation (denoted as θt,s), as well as for uncertainty related to future inflation.
The risk premium associated with uncertainty related to future inflation is denoted by $\pi_{t,s}$, which is distinct from the risk premium ($\rho^i_{t,s}$) as shown in the equation below.

$$P_t^i = \sum_{s=1}^{N} \frac{CF_{t+s}^i}{(1 + l_{t,s} + \theta_{t,s} + \pi_{t,s})^s}$$

Practice: Example 11, Reading 49, P.36.

4.1 Short-Term Nominal Interest Rates and the Business Cycle

Treasury bills (T-bills) are very short-dated ***nominal*** zero-coupon government bonds. As T-bills are short-dated and used to implement monetary policy, their yields are usually very closely related to the central bank's policy rate. Price of T-bill is estimated as below.

$$P_t^i = \frac{CF_{t+s}^i}{(1 + l_{t,s} + \theta_{t,s})^s}$$

- It is important to note that because T-bill has only one payment, summation term is not needed in the equation above.

4.2 Treasury Bill Rates and the Business Cycle

- Short-term nominal interest rates are positively related to short-term real interest rates and to short-term inflation expectations.
- All else equal, short-term nominal and real interest rates are positively related to GDP growth rate , volatility in GDP growth, and average levels of inflation over time.

Taylor Rule: It is a rule that helps monetary authority to gauge whether their policy rate is at an "appropriate" level.

$$pr_t = l_t + i_t + 0.5\,(i_t - i^*_t) + 0.5\,(Y_t - Y^*_t)$$
$$= l_t + 1.5i_t - 0.5i^*_t + 0.5\,(Y_t - Y^*_t)$$

Where,

- pr_t is the policy rate at time t
- l_t is the level of real short-term interest rates that balance long-term savings and borrowing in the economy
- i_t is the rate of inflation
- i^*_t is the target rate of inflation
- Y_t is logarithmic levels of actual GDP
- Y^*_t is logarithmic levels of potential real GDP.

The policy rate is often referred to as the ***neutral policy rate***.
The policy rule has a larger weight on inflation (1.5) relative to the weight on output (0.5) in order to stabilize inflation over the longer term near the targeted inflation rate.
All else equal,

- When inflation is close to the targeted rate, and when the output gap is zero, the appropriate policy rate will be equal to the level of the short-term real interest rate 'lt', that balances long-term savings and borrowing in the economy, plus the targeted/preferred rate of inflation.
- When inflation is above (below) the targeted level, the policy rate should be above (below) the neutral rate; and when the output gap is positive (negative), the policy rate should also be above (below) the neutral rate.

Output Gap = $Y_t - Y^*_t$

- When the output gap is positive, it implies that the economy is producing beyond its sustainable capacity. Positive output gap usually occurs during high and/or rising inflation.
- When the output gap is negative, it implies that the economy is producing below its sustainable capacity. Negative output gap usually occurs during high levels of unemployment.

Impact of policy errors on business cycle:

- If a central bank sets interest rates too low for too long, it may create a credit bubble in an economy.
- If a central bank sets interest rates too high for too long, it may lead to recessionary or even depression-like economic conditions.

4.3 Short-Term Interest Rate Summary

- Short-term default-free interest rates tend to be very heavily influenced by the inflation environment, inflation expectations over time, and growth in real GDP.
- Growth in real GDP is affected by saving and investment decisions of households.
- Short-term default-free interest rates are also affected by central bank's policy rate, which also vary with the level of real economic growth, with the expected volatility of that growth, and with the changes in the level of targeted inflation.

4.4 Conventional Government Bonds

4.4.1.) Break-even Inflation Rates

- The greater the uncertainty about the real value of the bond's payoff, the higher the risk premium will be demanded by investors. Therefore, investors demand compensation for bearing risk of loss in value of their investment due to inflation.

Break-even Inflation (BEI) Rate = Yield on zero-coupon default-free nominal bond – yield on zero-coupon default-free real bond

- Break-even inflation rates provide an independent view about future inflation that can be compared with the judgment of the central bank.
- Break-even inflation rates also include a risk premium to compensate investors for their uncertainty largely about future inflation.

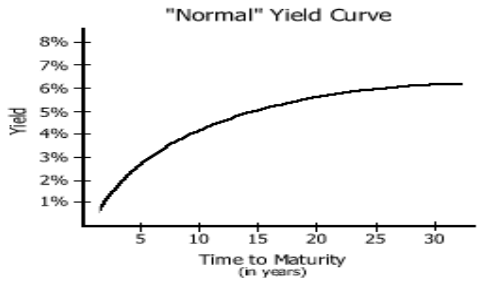
4.5 The Default-Free Yield Curve and the Business Cycle

The yield curves have following three distinct characteristics.

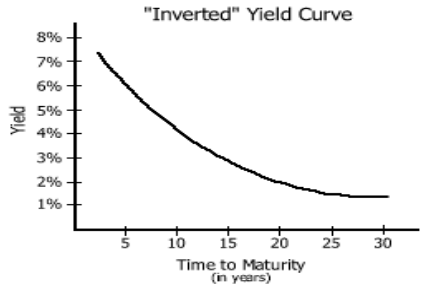
1) **Level of interest rates:** Level of interest rates indicates upward or downward shift in the yield curve over time. The majority of the movement in the yield curve is in the level.
2) **Slope:** Slope indicates steepness of the curve, i.e. how quickly or slowly rates change with maturity. Changes in slope contribute a smaller proportion of the change in the yield curve over time. The slope of the yield curve is influenced by the magnitude of the risk premium between the price of the bond and the inter-temporal rate of substitution over the investor's time horizon. This implies that during good (bad) economic times, the slope of the yield curve tends to decrease (increase).
3) **Curvature:** Curvature indicates how much the curve is different from a straight line. Changes in curvature contribute the least towards changes in the yield curve over time.

The shape of the yield curve and its three factors can provide valuable information for both central banks and investors. E.g. if policy rates and short-term risk free rates are expected to revert back to normal as the recession ends, then the yield curve will become steeper for the short-term maturities but flatter for the long-term maturities, leading to increase in the curvature as well.
Three primary shapes of a yield curve:

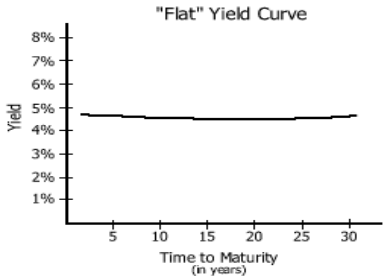
1) **Upward sloping yield curve:** If short-term yields are lower than long-term yields, then the curve is referred to a positive (or "normal") yield curve. The upward sloping yield curve represents that short-dated bonds are less positively (or more negatively) correlated with bad economic times than are long-dated bonds.



2) **Inverted Yield curve:** If short-term yields are higher than long-term, then the curve is referred to as an inverted (or "negative") yield curve.



3) **Flat yield curve:** When there is little or no difference between short- and long-term yields, then the curve is referred to as a flat yield curve.



Practice: Example 12, Reading 49, P.56.

4.5.1.) The Slope of the Yield Curve and Investor Expectations

The shape of the yield curve and the relative performance of bonds with different maturities over the business cycle depends on a complex mixture of interest rate expectations and risk premiums considerations. For example,

- If investors expect future interest rates to decline, then the yield curve will be downward sloping or inverted. If interest rates decline, then reinvestment of the principal amounts of maturing short-term bonds at declining interest rates would offset the initial yield advantage of the shorter-dated bonds.
- An upward sloping yield curve may result due to bond risk premiums that are positively related to maturity (also known as term premium), expectations of future increase in interest rates, or due to a combination of expected rate increases in future interest rate and positive term premiums. This implies that if positive risk premiums are large enough to offset expectation of declining interest rates, then the yield curve will be upward sloping.

4.5.2.) The Term Spread and the Business Cycle

Generally, during the late staged of business cycles, inflation and short-term interest rates are relatively high. Subsequently, over time, as demand for credit declines and inflation falls, the yield curve becomes flatten or invert, reflecting decline in longer-term yield.

Practice: Example 13, Reading 49, P.61.

4.5.3.) Evidence on Risk Premiums for Default-Free Bonds

- Typically, the average yield difference (yield on longer-term bonds – shorter-term bonds) is positive. This indicates that bond risk premium (also referred to as term premium) increases with term to maturity.

Bond Risk Premium = Yield on a conventional government bond - Yield on an index-linked government bond with a similar maturity – survey-based measure of inflation expectations

- The correlation between total return on government bonds and economic growth is negative, i.e. during bad economic times, investors demand low return from government bonds and pay higher price for them because they act as partial hedge against "bad" consumption outcomes.

Note: When consumption-hedging properties of government bonds are given less (more) value, bond risk premiums tend to rise (fall).

- The correlation between short-dated government bond returns and economic growth in is relatively more negative than the equivalent correlations for long-dated government bonds. This implies that, bond risk premium for short-dated bonds is lower than for long-dated bonds because short-dated bonds act as reliable hedge against bad economic times than long-dated bonds. Resultantly, the yield curve tends to be upward sloping.

Summary: Government bond risk premiums
- are positive;
- probably are influenced by consumption hedging benefits of government bonds i.e.

during periods of economic uncertainty, price of government bonds increases and risk premium falls; and
- are positively related to bond maturity, implying that "normal" shape for the yield curve is upward sloping.
- are not constant over time, like other risk premiums.

5. CREDIT PREMIUMS AND THE BUSINESS CYCLE

Credit premium is the risk premium demanded by investors for investing in any corporate or government bond for assuming the risk of default of the issuer on its obligations. In the following equation, it is represented by $\gamma^{i}_{t,s}$.

$$P_t^i = \sum_{s=1}^{N} \frac{E_t\left[\widetilde{CF}^{\,i}_{t+s}\right]}{\left(1 + l_{t,s} + \theta_{t,s} + \pi_{t,s} + \gamma^i_{t,s}\right)^{s}}$$

Practice: Example 15, Reading 49, P.72.

5.1 Credit Spreads and the Credit Risk Premium

Credit Spread = Yield on a corporate bond – Yield on a government bond with the same currency denomination and maturity

- Both corporate bonds and government bonds are subject to interest rate risk (change in the price of a bond due to parallel shift up in the yield curve). This implies that is the credit risk component of a corporate bond that differentiates it from government bonds.
- During bad economic times, when the probability of a corporate default and bankruptcy is highest, credit risk premium tends to increase.

Expected loss = Probability of default × (1 – Recovery rate)

- In general, secured debt holders have higher recovery rates compared with unsecured debt holders because secured debt is secured by a lien or other claim against some or all of the company's assets, whereas unsecured debt has no explicit claim to the company's assets in the event of bankruptcy.
- Similarly, senior debt holders have higher recovery rates compared subordinated debt holders because subordinated debt holders have an inferior claim on the company's assets compared with senior debt holders.
- Loss rates are negatively related to economic activity, i.e. as economic activity increases (decreases), loss rates decline (increase).

5.2 Industrial Sectors and Credit Quality

- The impact of the business cycle or economic environment on spreads depends on issuers' industrial sector and rating. When spreads widen, the spreads on lower-rated corporate bonds and/or bonds issued by corporations that belong to cyclical sector tend to widen the most.
- When credit spreads between corporate and government bonds widen (narrow), the spreads between higher-rated and lower-rated bond also widen (narrow). During these periods, corporate bonds tend to underperform government bonds and lower-rated corporate bonds tend to underperform higher rated bonds.
- Corporate bond spreads vary depending on the business cycle. E.g. during recession, the spread on the consumer cyclical sector tend to widen more as opposed to corporate bonds in the consumer non-cyclical sector.

5.3 Company-Specific Factors

Credit spreads are also influenced by company-specific factors. E.g. issuers that are profitable, have low debt interest payments, and that do not heavily rely on debt financing will tend to have a high credit rating because of their high ability to meet debt obligations. Hence, due to high credit rating, bonds issued by such issuers have lower credit spreads. If this ability declines relative to other issuers in their sector, then the spread demanded on their debt will rise, relative to the sector average, and their rating may be lowered by the rating agency.

5.4 Sovereign Credit Risk

Sovereign credit risk refers to risk of default by the governments on their debt obligations. Such defaults are mostly country specific in character, but are also

triggered by the global economic environment, oil prices, and the evolution of global trade.

Sovereign credit risk spread for bonds issued by emerging markets governments = Yield on bonds issued by emerging markets governments - Yield on bonds with comparable maturity issued by the US Treasury

Practice: Example 16, Reading 49, P.86.

6. EQUITIES AND THE EQUITY RISK PREMIUM

In equities, both the size and timing of the cash flows are uncertain; moreover, there is a risk that cash flows may not materialize at all. This is because in equity investment, dividend payment is not promised, may increase or decrease over time, and if the issuing corporation becomes bankrupt, investors may not receive any dividend payment.
Following is the equation to calculate price of equity security.

$$P_t^i = \sum_{s=1}^{\infty} \frac{E_t\left[\widetilde{CF}^i_{t+s}\right]}{\left(1 + l_{t,s} + \theta_{t,s} + \pi_{t,s} + \gamma^i_{t,s} + \kappa^i_{t,s}\right)^s}$$

- It is important to note that there is no maturity to the cash flows, so investors are essentially buying cash flows (dividends) into perpetuity (∞).
- **$K^i_{t,s}$** is the equity premium relative to credit risky bonds.

Equity risk premium is demanded by investors because of the following reasons:

- If a company's financial condition deteriorates, equity holders will receive the residue (which could be zero), once company's debt-holders have been paid, because the company's debt holders have the senior claim on the company's cash flow.
- If a company's becomes bankrupt, both debt and equity investors are exposed to risk, but the potential loss is greater for the equity investor.

Equity Risk Premium: Equity risk premium is the addition to return required by investors over and above the compensation for risk that they require for holding a default-free government bond of the same currency.
In the following equation, it is denominated as $\lambda^i_{t,s}$, which is equal to $\gamma^{i}_{t,s} + k^i_{t,s}$.

$$P_t^i = \sum_{s=1}^{\infty} \frac{E_t\left[\widetilde{CF}^i_{t+s}\right]}{\left(1 + l_{t,s} + \theta_{t,s} + \pi_{t,s} + \lambda^i_{t,s}\right)^s}$$

Ex-post risk premium = Annualized return generated by the stock – Nominal Risk-free rate

6.1 Equities and Bad Consumption Outcomes

The magnitude of equity risk premium depends on consumption hedging properties of equities. That is, if consumption hedging properties of equities are good, investors will demand low risk premium.

6.2 Earnings Growth and the Economic Cycle

Corporate profitability is considered as an important leading indicator of the business cycle and as it provides useful information about future growth.

- When demand declines, corporate profits shrinks, resulting in lay off of workers by companies, reducing their cost base, and thereby adding to the recessionary backdrop.
- When demand rises, demand growth on a lower cost base can lead to a sharp increase in corporate profits; as a result, companies increase their and hire more staff, and so on.

The company's earnings and its equity performance over the business cycle depend on the type of product sold or service provided by the company.

- **Non-cyclical defensive Companies:** Companies that make products or provide services that are relatively insensitive to general economic conditions are referred to as non-cycle companies. E.g. toothpaste.
- **Cyclical companies:** Companies that make products or provide services that are extremely sensitive to general economic conditions are referred to as cycle companies. E.g. airlines. Increase in the earnings of cyclical companies after a period of decline indicates likely improvement in wider economic growth in the future.

Other factors that determine the earnings growth of an equity or equity sector include the financial structure of the company, the quality and experience of its management, and the ease with which new entrants can establish themselves to compete away any abnormal profits.

6.3 How Big is the Equity Risk Premium?

The growth in real earnings of companies and growth in real GDP are positively correlated; i.e. as economy grows, companies' profits also grow. Hence, equities are not considered a good hedge against bad consumption outcomes; resultantly, investors demand a risk premium.

6.4 Valuation Multiples

Various valuation multiples are used to compare equities within and among sectors, i.e. the price-to-earnings ratio (P/E) or the price-to-book ratio (P/B).

$$\text{P/E} = \frac{\text{Ratio of Current Share Price}}{\text{Earnings per Share}}$$

- If a stock is trading with a high (low) P/E relative to the rest of the market, it implies that investors are willing (not willing) to pay a high price for a dollar's worth of the company's earnings, may be because the prospect of strong earnings growth in the future is high (low).

$$\text{Historical or Trailing P/E} = \frac{\text{Ratio of Current Share Price}}{\text{Last year's Earnings per Share}}$$

$$\text{Leading or Forward P/E} = \frac{\text{Ratio of Current Share Price}}{\text{Estimated Future Earnings per Share}}$$

- If a company's EPS is expected to grow, then its forward P/E will be less than its historic P/E.

The high P/E may occur due to a number of factors, including

a) an increase in expectation of future real earnings growth.
b) falling real interest rates, likely due to decline in volatility in real GDP growth;
c) a fall in inflation expectations ($\theta_{t,s}$);
d) a decline in uncertainty about future inflation ($\pi_{t,s}$); or
e) a fall in the equity risk premium ($\lambda^{i}_{t,s}$).

Real cyclically adjusted P/E (CAPE): In CAPE, the 'P' represents the real (or inflation-adjusted) price of the equity market and the 'E' is a 10-year moving average of the market's real (or inflation-adjusted) earnings. Using a moving average of real earnings removes the short-term volatility in this indicator over time.

$$\text{Price to book ratio (P/B)} = \frac{\text{Ratio of Current Share Price}}{\text{Net Assets or Assets minus Liabilities}}$$

- P/B ratio indicates the value of investor's shares that is "covered" by the company's net assets.
- P/B ratio also indicates the strength of investors' expectations about the company's ability to generate a high return, or not, on its net assets, adjusted for risk. The higher the ratio, the greater the growth expectations but the lower the safety margin if things occur against expectations.

6.5 Investment Strategy

Growth stocks: In growth stocks, the prospects for strong earnings growth are good. Growth stocks tend to have a high P/E and a very low dividend yield; in fact, they often have very low (or no) earnings.

- Growth companies operate in immature markets where the prospects for higher sales in the future are relatively high.
- Growth stocks tend to outperform value stocks during times of expansion.

Value Stocks: In value stocks, the prospects for strong earnings growth are not good. Value stocks tend to have a low P/E and a high dividend yield.

- Value stocks operate in more mature markets where the prospects for higher sales in the future are relatively low.
- Value stocks tend to outperform growth stocks in the aftermath of a recession.

Equity investors also focus on company size while making investment decisions. Typically, small stocks tend to underperform large stocks in recession. Therefore, investors may demand a higher equity premium on small, relative to large, stocks.

Similarly, for small stocks, the average expected utility of the investor is lower, so that smaller stocks require a bigger risk premium because small-company stock would lead to bigger increases and decreases in wealth relative to the investor's initial wealth.

Practice: Example 17 & 18 Reading 49, P.109 and 110.

7. COMMERCIAL REAL ESTATE

Commercial real estate investments have both equity and bond like elements.

- Like bond holders, investors in commercial real estate receive regular cash flows (like coupon income) derived from the rents paid by tenants.

However, in commercial real estate when the lease on a property expires, the investors (acting as landlords) will take back possession of the property and will have to decide whether to re-rent it to another tenant, to sell it to another investor, or to

redevelop it for a future sale; whereas, in bonds where the investor generally receives the face value of the bond along with the final coupon upon maturity. The decision to re-rent, sell, or to redevelop depends on the value of the property at that time. The value of the property depends on two key factors: the property's location and the state of the underlying economy.

- If, during the time of the lease, the area in which the property is situated has become more popular, then it may be profitable to sell the property, or to redevelop it.
- If, during the time of the lease, general economic activity is high, and thus there is strong demand for property, then it may be profitable to sell the property, or to redevelop it. Opposite is true when the location is deemed to be less desirable or the economy is weak.

- Like bonds, wherein the credit quality is determined by the credit ratings of the bond issuers of the constituent bonds, the credit quality of a commercial property portfolio is determined by the credit quality of the underlying tenants.
- Like equity securities, investment in commercial real estate has potential for profit or loss, and the uncertainty related to this profit from redevelopment.

7.1.2.) Illiquidity and Investment in Commercial Real Estate

Unlike developed-economy government bonds, investment-grade corporate debt, or publicly traded equities, investment in commercial real estate suffers from illiquidity and high transaction costs.

7.2 The Pricing Formula for Commercial Real Estate

The pricing formula for commercial real estate is shown below.

$$P_t^i = \sum_{s=1}^{N} \frac{E_t\left[\widetilde{CF}^{\,i}_{t+s}\right]}{\left(1 + l_{t,s} + \theta_{t,s} + \pi_{t,s} + \gamma^i_{t,s} + \kappa^i_{t,s} + \phi^i_{t,s}\right)^s}$$

The discount rate in this formula takes into account following factors:

1. A developed economy government tenant that agrees to pay rental income that is indexed to inflation, $(1 + l_{t,s})$. It is similar to the purchase of a real default-free government bond.
2. A developed economy government tenant that agrees to pay fixed nominal rental income, $(1+ l_{t,s} + \theta_{t,s} + \pi_{t,s})$. It is similar to the purchase of a nominal default-free government bond.
3. A corporate tenant that agrees to pay a fixed nominal rental income, $(1+ l_{t,s} + \theta_{t,s} + \pi_{t,s} + \gamma^i{}_{t,s})$. It is similar to the purchase of a credit risky nominal bond.
4. Since a real estate does not trade in public markets, investors will demand a risk premium, expressed as $\phi^i{}_{t,s}$, for lack of liquidity.

The relative sizes of the components listed above will vary depending on the length of the lease, the quality of the tenant, and the location of the property.

Practice: Example 19 Reading 49, P.115.

7.3 Commercial Real Estate and the Business Cycle

Although the nominal rental income is relatively stable, the capital values of commercial property are highly sensitive to the economic environment, implying that the commercial properties are pro-cycle in nature.

- During recessions (expansion), capital values of commercial property tend to fall (rise). This implies that commercial property does not act as very good hedge against bad economic outcomes. Hence, investors demand a high risk premium for investing in commercial properties.
- The value for the property risk premium varies over time with economic conditions and is relatively highly and positively correlated with the risk premiums on corporate bonds and equities.

Practice: Example 20 Reading 49.

End of Reading Practice Problems:

1. INTRODUCTION

Active management theory involves construction of a portfolio assuming competitive advantage over other because of skill in predicting returns. Active management theory is based on the assumption that financial markets are not perfectly efficient. In active management, risk and return are compared against a benchmark portfolio.

2. ACTIVE MANAGEMENT AND VALUE ADDED

The objective of active management is to outperform the benchmark portfolio or passively managed portfolio.

- If the investor outperforms the benchmark portfolio, value added is positive.
- If the investor underperforms the benchmark portfolio, value added is negative.

2.1 Choice of Benchmark

Benchmark or passive portfolio should have following qualities to serve as a relevant comparison for active management:

- The benchmark should be representative of the assets from which the investor will select.
- Positions in the benchmark portfolio can actually be replicated at low cost.
- Benchmark weights are verifiable ex ante, and return data are timely ex post.

Examples of indices that are used as benchmark portfolios include the MSCI All Country World Index for global equities and the Barclays Global Aggregate Bond Index for global bonds.

Market capitalization-weighted indices: Commonly, the market indices weight the individual assets by their market capitalization. Capitalization weighted indices are generally self-rebalancing and can be simultaneously held by many investors.

Float-adjusted market capitalization-weighted indices: In Float-adjusted market capitalization-weighted indices, the individual assets are weighted by a percentage that is not privately held and thus available to the general investing public. A market portfolio in a float-adjusted market capitalization weighted index represents the average performance across all investors that own securities before costs; hence, active investors as a group cannot outperform the market (i.e., active management is a zero-sum game).
In contrast, an active management would not generally be a zero-sum game with respect to the narrower benchmark because investors might invest in assets outside the benchmark.

The return on the benchmark portfolio, R_B is estimated using the following formula:

$$R_B = \sum_{i=1}^{N} w_{B,i} R_i$$

Where,
Ri = Return on security i,
$w_{B,i}$ = Benchmark weight of security i
N = Number of securities.

The return on an actively managed portfolio, R_P is estimated as follows:

$$R_P = \sum_{i=1}^{N} w_{P,i} R_i$$

Where,
Ri = Return on security i,
$W_{P,i}$ = Portfolio weight of security i
N = Number of securities.

Value added or Active Return (R_A) = Return on Actively Managed Portfolio – Return on Benchmark Portfolio = $R_P - R_B$

A risk-adjusted calculation of value added, which we will refer to as the managed portfolio's alpha, incorporates some estimate of the managed portfolio's risk relative to the benchmark, often captured by the Managed Portfolio's Alpha or Active Return, estimated as follows:

Managed Portfolio's Alpha or Active Return = $\alpha P = R_P - \beta_P R_B$

Where, β_P = Portfolio's beta

Portfolio alpha or active return implicitly assumes that the beta of the managed portfolio relative to the benchmark is 1.

The important principle that value added is ultimately driven by the differences in managed portfolio weights and benchmark weights:

Difference from Benchmark weights = $\Delta w_i = wP,_i - wB,_i$

Value added is the sum product of the active weights and asset returns as shown in the following equation:

$$R_A = \sum_{i=1}^{N} \Delta w_i R_i$$

Where

$R_{Ai} = R_i - R_B$.

- The above equation indicates that positive value added is generated when securities that have returns greater than the benchmark are over-weighted and securities that have returns less than the benchmark are underweighted.
- Value added is positive if and only if end-of-period realized active asset returns are positively correlated with the active asset weights established at the beginning of the period.

Example:
Suppose that in a benchmark portfolio, weight of stocks is 60% and weight of bonds is 40%. The investor believes that over the next year stocks will outperform bonds, so the investor holds a portfolio that is 10% over weighted in stocks and 10% underweighted in bonds.
Suppose, ex-post return on the stock market is 14.0% and the return on the bond market is 2.0%.

Return on the managed portfolio = 0.70(14.0) + 0.30(2.0) = 10.4%
Return on the benchmark = 0.60(14.0) + 0.40(2.0) = 9.2%
Value added as 10.4 – 9.2 = 1.2%

Value added showing the contributions from each segment = 0.10(14.0 – 9.2) – 0.10(2.0 – 9.2) = 0.5 + 0.7 = 1.2%

The above calculation indicates that a 0.5% return relative to the benchmark was generated by being overweight stocks and a 0.7% return was generated by being underweight bonds, for a total of 1.2%.

Practice: Example 1, Curriculum, Reading 50.

2.3 Decomposition of Value Added

Value added can be decomposed into two factors i.e. value added due to asset allocation and value added due to security selection.

In a Composite portfolio of stocks and bonds where the asset allocation weights differ from a composite benchmark and each asset class is actively managed by selecting individual securities, the total value added is calculated as the difference between the actual portfolio return and the benchmark return as follows:

$$R_A = \sum_{j=1}^{M} w_{P,j} R_{P,j} - \sum_{j=1}^{M} w_{B,j} R_{B,j}$$

- The first summation has both portfolio weights and the returns on actively managed portfolios, designated by the "P" subscript.
- The second summation has both benchmark weights and benchmark returns, designated by the "B" subscript.
- The subscript j = 1 to M counts the number of asset classes
- The subscript i = 1 to N counts the securities within each asset class.

Value added can be estimated as follows:

Value added = Active asset allocation decisions + Weighted sum of the value added from security selection

$$R_A = \sum_{j=1}^{M} \Delta w_j R_{B,j} + \sum_{j=1}^{M} w_{P,j} R_{A,j}$$

Where,
Value added from security selection = $R_{A,j} = R_{P,j} - R_{B,j}$

Using stocks and bonds as the subscripts, the above equation can be re-written as:

$$R_A = \left(\Delta w_{stocks} R_{B,stocks} + \Delta w_{bonds} R_{B,bonds}\right) + \left(w_{P,stocks} R_{A,stocks} + w_{P,bonds} R_{A,bonds}\right)$$

For a given asset class, the performance attribution system might also include value added from the selection of industries or sectors relative to the benchmark.

- For an equity portfolio, value added can be measured from over- and underweighting different industry sectors, as well as individual stock selection within those sectors.
- For a fixed-income portfolio, value added can be decomposed into mix of sovereign government bonds versus corporate bonds, as well as individual bond selection.

Sources of Value-added: Value added can be generated from a variety of active portfolio management decisions, i.e.
a) Security selection
b) Asset class allocation
c) Decompositions of portfolio into economic sector weightings and geographic or country weights.

3. COMPARING RISK AND RETURN

3.1 The Sharpe Ratio

Sharpe Ratio = (Portfolio Return – Risk Free Rate) / Standard Deviation

$$SR_P = \frac{R_P - R_F}{STD(R_P)}$$

- The Sharpe ratio is used to compare the portfolio return in excess of a riskless rate with the volatility of the portfolio return.
- The ratio indicates the return an investor is receiving in excess of a riskless rate for assuming the risk of the portfolio.

Ex-ante Sharpe ratio = (Expected Portfolio Return – Risk free Rate) / Forecasted Volatility or Standard Deviation

Sharpe Ratio for multiple time periods = (**Average Realized Portfolio Return – Average Risk free Rate) / Sample Standard Deviation**

Important to Note:
- In Sharpe ratios both the portfolio average return and the portfolio risk are annualized. For example, if the past return data are measured monthly, then

Annualized Monthly Return = Average monthly return × 12
Annualized Monthly return Volatility = Monthly return volatility × Square root of 12

- The Sharpe ratio of one fund over one five-year period should be compared with that of another fund over same five-year period.
- Sharpe ratio is unaffected by the addition of cash or leverage in a portfolio.

Suppose, a combined portfolio has a weight of w_P on the actively managed portfolio and a weight of $(1 – w_P)$ on risk-free cash.

Return on the combined portfolio is $R_C = w_P R_P + (1 – w_P)R_F$

Volatility of the combined portfolio = STD $(R_C) = w_P STD(R_P)$ because the $(1 – wP)RF$ portion is risk free

Thus, Sharpe ratio for the combined portfolio is equal to the Sharpe ratio of the actively managed portfolio

$$SR_C = \frac{R_C - R_F}{STD(R_C)} = \frac{w_P(R_P - R_F)}{w_P STD(R_P)} = SR_P$$

Two-Fund Separation: According to two-fund separation proposition, independent of preferences, investors should form portfolios using two funds, i.e. risk-free asset and the risky asset portfolio with the highest Sharpe ratio.

Volatility of the risky asset portfolio can be reduced (increased) by holding more cash (leverage) and less of risky portfolio (cash).

Suppose, volatility of risky asset portfolio is 20% and we want to reduce it to 10%. In order to reduce it, the weight of risky stocks in the combined portfolio must be 10/20 = 50%, leaving a 50% weight in risk-free cash. With that amount of cash, the volatility of the combined portfolio will be 0.50(20%) = 10.0%, the same as desired.

Note: The Sharpe ratio of a fund which is being actively managed but is similar to an index fund is closer to the benchmark because the excess return and volatility will be similar to the benchmark.

Practice: Example 2, Curriculum, Reading 50.

3.2 The Information Ratio

The information ratio tells an investor how much excess return (or active return) is generated from the amount of excess risk (referred to as active risk or benchmark tracking risk) taken relative to the benchmark.

Information ratio of an actively managed portfolio (IR) = Active Return / Active Risk = ($R_P - R_B$) / STD ($R_P - R_B$) = R_A / STD (R_A)

- Like Sharpe ratio, both the active return and the active risk are annualized in information ratio.

Ex-ante Information Ratio = Expected Active Return / Expected Active Risk

- If active return is negative, ex-post information ratios will be negative.
- The information ratio of a closet index fund will likely be close to zero or slightly negative if value generated by it is less than the management fees. Due to the zero-sum property of active management, the average realized information ratio across investment funds with the same benchmark should be near to zero.
- For a market-neutral long-short fund (a fund with offsetting long and short positions that has a beta of zero with respect to the market) and if the benchmark is risk-free rate, the Sharpe ratio and the information ratio will be same because the excess return will be same as active return and total risk will be same as active risk.
- Unlike the Sharpe ratio, the information ratio is affected by the addition of cash or the use of leverage. Generally, if cash is added to a portfolio of risky assets, the information ratio for the combined portfolio will decrease.
- In unconstrained portfolio, the information ratio does not change by the increase in active weights because if the active weights are doubled, the expected active return (or realized average active return) would be doubled, along with the expected or realized active risk. Hence, the information ratio will remain unchanged.

Adjusting the active risk of a fund: Active risk of a fund can be adjusted by taking positions in the benchmark portfolio. E.g. suppose active risk of a fund is 7%, combining that fund in a 70/30 mix with the benchmark portfolio will result in an active risk of the combined portfolio of 0.70(7.0%) = 4.9%.

Similarly, if an investor wants to increase the active risk and return, he can short sell the benchmark portfolio and use the proceeds to invest in the actively managed fund.

3.3 Constructing Optimal Portfolios

For an investor, the optimal portfolio is the one with the maximum Sharpe ratio, whatever the investor's risk aversion.

Property in active management theory: Given the opportunity to adjust active risk and return by investing in both the actively managed and benchmark portfolios:

Squared Sharpe ratio of an actively managed portfolio = Squared Sharpe ratio of the benchmark + Information ratio squared

- For any given asset class, an investor should choose the manger with the highest information ratio, because the highest information-ratio produces the highest Sharpe ratio for the investor's portfolio.

For unconstrained portfolios, the level of active risk that leads to the optimal result in standard deviation of the benchmark as follows:

$$\text{STD}(R_A) = \frac{\text{IR}}{\text{SR}_B}\text{STD}(R_B)$$

The optimal amount of active management that maximizes a portfolio's Sharpe ratio is positively related to the assumed forecasting accuracy or ex ante information coefficient of the active strategy.

Example: An actively managed portfolio has an information ratio of 0.50 and active risk of 7.5% and the benchmark portfolio has a Sharpe ratio of 0.60 and total risk of 18.0%, then according to above equation,

Optimal amount of aggressiveness in the actively managed portfolio = (0.50/0.60)18.0% = 15.0%.

If the actively managed portfolio is constructed with this amount of active risk, the Sharpe ratio will be $(0.60^2 + 0.50^2)^{1/2} = 0.70$

Expected Active Return of Actively Managed Portfolio = (0.5) × 15% = 7.50%

Total expected excess return = (0.6 × 18%) + 7.50% = 18.0%

Where, 0.6 × 18% = benchmark portfolio Sharpe ratio × Total risk of benchmark portfolio

Total risk of the actively managed portfolio = Benchmark return variance + Active return variance

Total Portfolio Risk = $(18.0^2 + 15.0^2)^{0.5}$ = 23.43%

This implies that the **Maximum possible Sharpe ratio = 18.0/23.43 = 0.77**

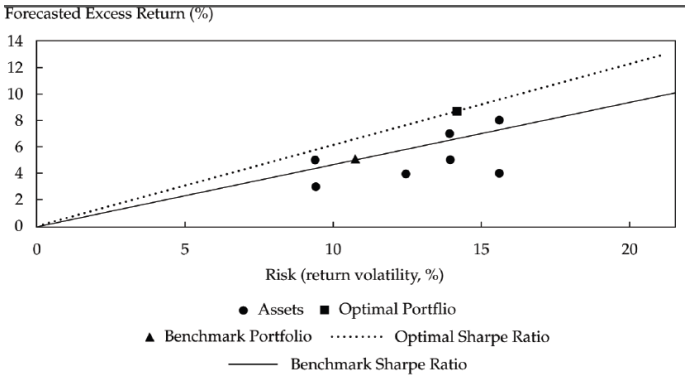
The initial actively managed portfolio has active risk of only 7.5%, whereas the optimal amount required under the assumed information ratio needed to maximize the Sharpe ratio is 15.0%.

The active risk can be increased either by managing the portfolio more aggressively or by short-selling the

benchmark and using those proceeds to invest in actively managed fund.
The proportion required to be invested in the actively managed fund = 15.0/7.50 = 2 times while shorting the benchmark by 1 times to fund the increase.

Since the benchmark is diversified, the Sharpe ratio of the benchmark portfolio is higher than those of most of the individual assets. However, the benchmark portfolio does not have the highest possible Sharpe ratio of all portfolios that can be constructed from these assets.

The optimal portfolio (i.e., mean–variance efficient frontier portfolio with the highest possible Sharpe ratio) as shown below in the dotted line, has the Sharpe ratio of 8.7%/14.2% = 0.61. The risk of the optimal portfolio can be reduced along the dotted line to the benchmark portfolio risk of 10.8% with an expected excess return of 0.61(10.8%) = 6.6%, compared with the benchmark expected excess return of 5.0%.



Practice: Example 3, Curriculum, Reading 50.

4. THE FUNDAMENTAL LAW OF ACTIVE MANAGEMENT

4.1 Active Security Returns

The active security return as the residual return in a single-factor statistical model can be calculated as follows:

$$R_{Ai} = R_i - \beta i R_B$$

Where, βi is the sensitivity of the security return to the benchmark return.

- In the above equation, the benchmark return may or may not be the market return.

The individual security active return can also be estimated as the residual return in a multi-factor statistical model as follows:

$$R_{Ai} = R_i - \sum_{j=1}^{K} \beta_{j,i} R_j$$

- Signal quality is measured by the correlation between the forecasted active returns, μi, and the realized active returns, RAi (commonly called the information coefficient (IC)).
- Investors with higher IC, or ability to forecast returns, will add more value over time, but only to the extent that those forecasts are exploited while managing a portfolio.

Transfer coefficient (TC): It is the cross-sectional correlation between the forecasted active security returns (μ_i) and actual active weights (Δw_i). It measures the degree to which the investor's forecasts are translated into active weights. It also measures the extent to which constraints reduce the expected value added of the investor's forecasting ability

Mean–variance-optimal active security weights for uncorrelated active returns, subject to a limit on active portfolio risk, are given by

$$\Delta w_i^* = \frac{\mu_i}{\sigma_i^2} \frac{\sigma_A}{\sqrt{\sum_{i=1}^{N} \frac{\mu_i^2}{\sigma_i^2}}}$$

Where,
σA = Active portfolio risk; STD(RA) in the prior notation.
σi = Forecasted volatility of the active return on security i.

- The desired deviations (positive or negative) from the benchmark weight for security 'i' are higher for larger values of the forecasted active return, μi, but

are reduced by forecasted volatility, σi.
- The active weights vary depending on the active risk of the portfolio, σA; i.e. in order to increase active portfolio risk, the individual active weights need to be increased.
- Active return forecasts are scaled prior to optimization using the following rule (called **Grinold rule**).

$$\mu_i = IC\sigma_i S_i$$

Where,
IC = Expected information coefficient
S_i = Set of standardized forecasts of expected returns across securities, sometime called "scores."

- If the assumed IC value is low, then the cross-sectional variation of the expected active returns in above rule will be low.

Using the **Grinold rule**, the mean–variance optimal active weights are calculated as follows:

$$\Delta w_i^* = \frac{\mu_i}{\sigma_i^2}\frac{\sigma_A}{IC\sqrt{BR}}$$

Where,
IC = Information coefficient. It is a measure of our level of skill, our ability to forecast each asset's residual return.
BR = Breadth, which is number of independent decisions made per year by the investor in constructing the portfolio and is equal to the number of securities only if the active returns are cross-sectionally uncorrelated.

- Breadth can be higher (lower) than the number of securities if factors in the risk model suggest that their active returns are negatively (positively) correlated. In complicated cases, breadth will be a non-integer number.
- Similarly, if some characteristic of a security is fairly constant over time and the investor makes decisions about expected active return based on that characteristic, then breadth over time is lower.
- Breadth increases with the number of rebalancing periods only if the active returns are uncorrelated over time.
- If the investor makes quarterly or monthly forecasts about a security that are truly independent over time, then

Breadth = Number of securities × Number of rebalancing periods per year

IC is the ex-ante (i.e., anticipated) cross-sectional correlation between the N forecasted active returns, μi, and the N realized active returns, RAi. IC ranges from -1.00 to +1.00.

$$IC = COR\left(\frac{R_{Ai}}{\sigma_i}, \frac{\mu_i}{\sigma_i}\right)$$

Where,
COR(·) = Correlation

- In order to do pursue active management, the ex-ante, or anticipated, IC must be positive.
- Lower information coefficient reduces the information ratio and the expected active return.

Practice: Example 4, Curriculum, Reading 50.

4.2 The Basic Fundamental Law

The anticipated value added for an actively managed portfolio, or expected active portfolio return = Sum product of active security weights and forecasted active security returns as shown below:

$$E(R_A) = \sum_{i=1}^{N} \Delta w_i \mu_i$$

The basic fundamental law of active management:
Using, optimal active weights and forecasted active security returns, the expected active portfolio return is calculated as follows:

Optimal Expected Active Return = Information Coefficient × Square root of Breadth × Portfolio Active risk

$$E(R_A)^* = IC\sqrt{BR}\sigma_A$$

Information ratio of the unconstrained optimal portfolio, E(RA)*/σA, can be calculated as the product of just two terms:

$$IR^* = IC\sqrt{BR}$$

Important to Note: The fundamental law separates the expected value added, or portfolio return relative to the benchmark return, into following basic elements of the strategy:
a) Skill, which is measured by the information coefficient.
b) Structuring of the portfolio as measured by the transfer coefficient.
c) Breadth of the strategy as measured by the number of independent decisions per year, and
d) Aggressiveness, which is measured by the benchmark tracking risk.

The aggressiveness, breadth, and structuring are generally beyond the control of the investor if they are specified by investment policy or constrained by regulation.

Practice: Example 5, Curriculum, Reading 50.

4.3 The Full Fundamental Law

For a single-factor risk model, transfer coefficient (TC) is the following risk-weighted correlation:

$$TC = COR(\mu_i/\sigma_i, \Delta w_i \sigma_i)$$

Where, Δwi (without an *) represent the actual active security weights for a constrained portfolio.

The transfer coefficient (TC) can also be expressed as the risk-weighted correlation between the optimal active weights and the actual active weights,

$$TC = COR\left(\Delta w_i^* \sigma_i, \Delta w_i \sigma_i\right)$$

As a correlation coefficient, TC can take on values anywhere from –1.00 to +1.00, although

- TC values are typically positive and range from about 0.20 to 0.90.
- When TC = 0.00, there is no correlation between the active return forecasts and active weights taken, implying no expectation of value added from active management.
- When TC = 1.00 (no binding constraints) there is a perfect correspondence between active weights taken and forecasted active returns. In this case, full expected value added can be generated from active management.
- When TC < 0.00, this implies that relative weights are negatively correlated with current expected returns because the portfolio needs rebalancing.
- A low TC results from the formal or informal constraints imposed on the structure of the portfolio.

According to Fundamental law,
Expected active return, E(RA) = Transfer coefficient (TC) × Assumed information coefficient (IC) × Square root of breadth (BR) × Portfolio active risk (σA)

- The portfolio's information ratio, E(RA)/σA, is calculated as follows:

$$IR = (TC)(IC)\sqrt{BR}$$

Portfolio active risk (σA) can be expressed as follows:

$$\sigma_A = TC\frac{IR^*}{SR_B}\sigma_B$$

- When a transfer coefficient is 0.00, the optimal amount of active risk is zero- implying that the investor should just invest in the benchmark portfolio.

Where, IR* is the information ratio of an otherwise unconstrained portfolio.

Maximum possible value of the constrained portfolio's squared Sharpe ratio =

$$SR_P^2 = SR_B^2 + (TC)^2(IR^*)^2$$

Practice: Example 6, Curriculum, Reading 50.

4.4 Ex Post Performance Measurement

Expected value added conditional on the realized information coefficient, ICR, is

$$E(R_A \mid IC_R) = (TC)(IC_R)\sqrt{BR}\sigma_A$$

Using the above equation, **portfolio's active return variance** can be decomposed on ex-post (i.e., realized) basis into two parts:

1) Variation due to the realized information coefficient.
2) Variation due to constraint-induced noise.

- The two parts of the realized variance are proportional to TC^2 and $1 - TC^2$.

Example: Suppose, a TC is 0.40, this implies that TC^2 = 16% of the realized variation in performance is attributed to variation in the realized information coefficient, and $1 - TC^2$ = 84% is attributed to constraint-induced noise.

Important to Note:

- The highest value added achievable is proportional to the squared information ratio.
- The information ratio measures the active management opportunities, whereas, the squared information ratio indicates active manager's ability to add value.

The realized value added of an actively managed portfolio can be divided into two parts.

$$R_A = E(R_A \mid IC_R) + Noise$$

1) First term is Expected value added given the realized skill of the investor that period.
2) Second term is Any noise resulting from constraints that intrudes the optimal portfolio structure.

Practice: Example 7, Curriculum, Reading 50.

5. APPLICATIONS OF THE FUNDAMENTAL LAW

There are following three specific applications of active portfolio management:

1) Selecting country equity markets in a global equity fund with different sets of active return forecasts and constraints;
2) Timing of credit and duration exposures in a fixed-income fund.

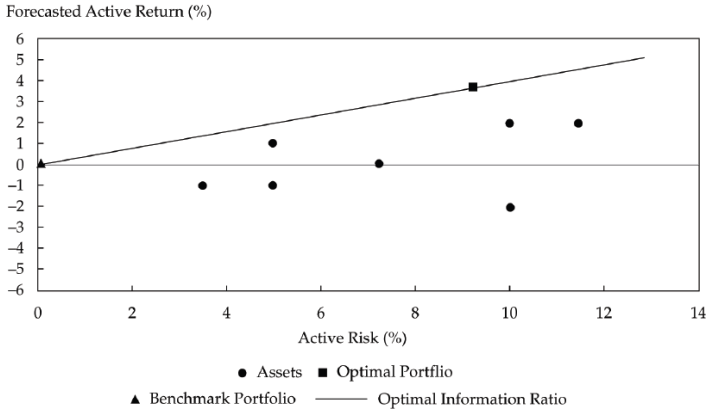
The long-only and maximum over- or underweight constraints substantially reduce the transfer of active return forecasts into active weights.

- If active weights are constrained to sum to zero, then active weights will not be perfectly proportional to the forecasted active returns.
- If sum of active weights can be non-zero, implying that one can use risk-free cash or leverage, the transfer coefficient would be equal to 1.0.
- Typically, when active risk is high, the constraints become more binding because it results in more variation in unconstrained active weights.

Key Concepts:
The decision of how aggressively to do active management depends on the constraints that are imposed on the portfolio.

- An unconstrained IR is unaffected by the level of active risk.
- A constrained IR generally decreases with the aggressiveness of the strategy.

In the figure below, as TC decreases, the dark line for a constrained portfolio would curve downward from left to right.



Practice: Example 8, Curriculum, Reading 50.

5.2 Fixed-Income Strategies

Suppose, the quarterly return volatility of the Investment-grade asset is 2.84%, and the quarterly return volatility of the high-yield asset is 4.64%, with an estimated correlation between the two of 0.575.

Active risk of this decision is the volatility of the differential returns between the two bond portfolios calculated as follows:

Active Risk = $[(2.84)^2 – 2(2.84)(4.64)(0.575) + (4.64)^2]^{1/2}$ = 3.80%

The active investor assigns a "score" of either +1.0 or –1.0 on credit exposure each quarter.

Annualized active risk = $3.80 \times (4)^{1/2}$ = 7.60%

Suppose the fixed-income investor expects to call the market correctly 55% of the time (i.e., 11 out of 20 quarters).

If the investor makes the correct call 55% of the time and an incorrect call 45% of the time, then the

Time-series information coefficient = 0.55 – 0.45 = 0.10

Without a limit on active risk, the expected active return can be calculated as follows:

Expected Active Return = 0.55(3.80) + 0.45(–3.80) = 38 bps per quarter

Using the Grinold rule of "alpha equals IC times volatility times score":

0.10 × (3.80) × (1.0) = 38 bps

The investor decides to limit the annual active risk to 2.00% and thus sets the active weight (i.e., deviation from the 70/30 benchmark weights) as follows:

Target Active Risk / Annualized Actual Active risk = 2.00/7.60 = 26.3%

Breadth of this strategy: Assuming that active returns are ***uncorrelated over time***, the **breadth of this strategy is 4.0**, the four quarterly rebalancing decisions made each year.

During quarters when the investor believes credit risk will add value,

Managed portfolio is invested = 70.0% – 26.3% = 43.7% in investment-grade bonds and 30% + 26.3% = 56.3% in high-yield bonds

During quarters when the investor believes credit risk not add value,

Managed portfolio is invested = 70.0% + 26.3% = 96.3% in investment-grade bonds and only 30% – 26.3% = 3.7% in high-yield bonds

According to the fundamental law, Expected annualized active return to this strategy is = 0.10 × $(4.0)^{1/2}$ × 2.00 = 40 bps a year, or 40/4 = 10 bps per quarter.

Annual information ratio = 0.10 × $(4.0)^{1/2}$ = 0.20

Rule of thumb:

Breadth is $BR = N/[1 + (N - 1)\rho]$

Where,

- N is the number of securities
- ρ is the average correlation between the active security returns

In the above example, ρ = 0.0, so breadth is BR = 4.0.

Important to Note: The expected information ratio can be increased by switching more frequently, e.g. say, monthly, weekly or daily (i.e. 250 trading days a year). E.g. if an investor made daily decisions, truly independent, and were still correct 55% of the time, the

Expected information ratio could potentially increase to= (IC) × $(BR)^{1/2}$ = 0.10 × $(250)^{1/2}$ = 1.58

- The high 1.58 information ratio indicates that the investor could earn an expected active return of 3.16% with active risk of only 2.00%.
- If expected return is doubled i.e. 2 × 3.16% = 6.32% and active risk is doubled i.e. 2 × 2.00% = 4.00%.
- At higher active risk (4.00%), the required active weights would be plus and minus new active risk / old annualized active risk = 4.00/7.60 = 52.6%.
- This implies that portfolio will now be invested 70% + 52.6% = 122.6% in investment-grade bonds by taking short position of 22.6% in high-yield bonds.
- For a negative credit signal of -0.57, active weight will be –0.57(4.0)/7.6 = – 30.0%, implying 100% position in investment-grade bonds and no position in high-yield bonds.
- For a positive credit signal of 0.32, the required active weight would be 1.32(4.00)/7.60 = 69.5%, implying 100% in high-yield bonds and almost no position in investment-grade bonds.
- For an active risk of 4.00%, the expected active return = 0.98 × 4.00% = 3.92%, not 6.32%.

Practice: Example 9, Curriculum, Reading 50.

6. PRACTICAL LIMITATIONS

Limitations of the fundamental law:

1) Ex Ante Measurement of Skill
2) Assumptions of independence in forecasts across assets and over time

6.1 Ex Ante Measurement of Skill

Ex-ante measurement of skill means that there is uncertainty about the ex ante information coefficient. Due to uncertainty of skill, actual information ratios are substantially lower than predicted by an objective application of the original form of the fundamental law. The higher the uncertainty about forecasting ability, the smaller the expected value added is likely to be.

A more accurate representation of the basic fundamental law can be expressed as follows:

$$E(R_A) = \frac{IC}{\sigma_{IC}}\sigma_A$$

Realized active portfolio risk, σ_A can be expressed as follows:

$$\sigma_A = \sigma_{IC}\sqrt{N}\sigma_{RM}$$

- σRM = Benchmark tracking risk predicted by the risk model
- σIC = Additional risk induced by the uncertainty of the information coefficient

6.2 Independence of Investment Decisions

The second limitation of fundamental law is that there is issue with the conceptual definition of breadth as the number of independent decisions by the investor.

When the active returns between individual assets are correlated and forecasts are not independent from period to period, then number of individual assets, N, is not an adequate measure of strategy breadth, BR. E.g. overweighting all the stocks in a given industry or all the countries in a given region is not independent decision, hence, breadth would be lower than the number of assets.
Breadth is higher than the number of assets when investor employs hedging strategies using derivatives or other forms of arbitrage.

$$\text{BR} = \frac{N}{1 + (N - 1)\rho}$$

Where,

- ρ = correlation coefficient.

The limitation of independent decisions within the fundamental law also affects time-series implementation. This implies that if the rebalancing frequency is increased, the realized information ratio can be increased only to the extent that sequential active return forecasts are not correlated.

Lack of decision independence in active management of fixed-income portfolio:

Another limitation of the fundamental law is that there is lack of decision independence in the active management of fixed-income portfolios because all bonds' returns are highly correlated as they represent some form of duration risk, credit risk and optionality.

In contrast, in equities, risk of equity securities is decomposed into systematic and idiosyncratic factors. Once the systematic risk factors are removed, the active asset returns (defined as the returns on idiosyncratic risks) are essentially independent. Hence, breadth can be easily determined.

Note: Time-series dependence means that decisions on any particular stock may be correlated from month to month.

Practice: Example 10, Curriculum, Reading 50.

End of Reading Practice Problems:

1. INTRODUCTION

Algorithmic trading involves use of programs and computers to generate and execute (large) orders in markets with electronic access. Algorithmic trading is used by institutional investors, hedge funds and Wall Street trading desks. Algorithmic trading is instrumental in increasing the liquidity on the exchanges. The main objective of algorithmic trading is to reduce execution costs and market risk by making the execution more efficient. Majority of the volumes on the leading exchanges these days happens via algorithmic trading. In US, nearly 75% of stock trades are placed by computer algorithms rather than humans. Algorithmic trading is used for almost every asset classes, i.e. stocks, futures, foreign exchange (FX), bonds, energy, and so on.

2. THE BASICS OF ALGORITHMIC TRADING

An algorithm is "a sequence of steps to achieve a goal," and algorithmic trading involves use of programs and computers to automate a trading strategy. There are two types of trading algorithms: algorithms for execution and algorithms for high-frequency trading (HFT).

2.1 Execution Algorithms

Execution algorithms are used to split a large order into several small slices and execute them over a period of time. The smaller trades are executed at irregular intervals in order to keep the trading strategy being detected by other market participants. The execution algorithms help in minimizing the market impact and help in achieving a benchmarked price.

Examples of execution algorithms:

- ***Volume-weighted average price (VWAP):*** The VWAP strategy uses historical market volume patterns and the volume patterns of the individual stock to generate a robust volume pattern and trajectory. The order is then divided into slices proportioned to this distribution. This is an attractive strategy for those who seek to minimize risk toward the volume-weighted average price.

  Example: For example, assume that one has a period of three hours in which to trade. During that time horizon, the volume distribution is one million shares, as follows:

  - Average volume per hour:
    Hour 1: 300,000
    Hour 2: 400,000
    Hour 3: 300,000
    Suppose that one has a trade for 100,000 shares. To reduce the market impact, one intuitively wants to minimize the demand/supply impact at any point. To do that we will have to trade as follows:
    Hour 1: 30,000 Demand/Supply: 10%
    Hour 2: 40,000 Demand/Supply: 10%
    Hour 3: 30,000 Demand/Supply: 10%

- ***Implementation shortfall:*** Implementation shortfall represents a purely cost driven algorithm. It seeks to minimize the shortfall between the average trade price and the assigned benchmark, which should reflect the investor's decision price. The strategy will increase the targeted participation rate when the stock price moves favorably and decrease it when the stock price moves adversely. Strategy parameters are given below.

  - Start Time: Any orders are not sent to market even if the decision logic generates buy/sell signal before Start Time.
  - End Time: All open orders are cancelled and no fresh signals accepted after End Time
  - %Volume: The strategy automatically adjusts the participation rate to limit it to the percentage of stocks total traded volume. Example, if the stock trades 100,000 shares in one minute and %Volume is 10, the strategy will trade 10,000 shares in the same minute.
  - Price Brand: The desired price band for the average traded price. If market moves beyond the price band limit, the order will not be completely executed
  - Reference Price: A given price which the strategy will try to better in execution

- ***Market participation algorithms:*** The Participate algorithm trades in proportion to actual market activity. The algorithm targets a user-defined percentage rate of the traded volume in the market. A participation algorithm is similar to a VWAP algorithm except that it uses a constant participation rate. So, if a trader believes that he or she can live with the impact that will be caused by 10% participation, then he or she can simply use a 10% participation algorithm. E.g. if a 10% participation algorithm is used instead of VWAP, trading three million shares (with 30 million shares

average daily volume) will have results similar to using a VWAP algorithm. But, in contrast, 30000 shares would be traded in a few minutes, with little deviation likely from the arrival price.

Use of Execution algorithm: Execution algorithm can be used by a buy-side participant, such as a mutual fund, pension fund, or hedge fund. The buy-side participant who wants to execute a large order (such as building a new portfolio position or selling an entire position) and wants to achieve a benchmarked price, can minimize the cost of execution, and minimize the likelihood of other market participants front running the order, by using a broker algorithm (a trading algorithm managed by a broker rather than the buy-side participant). The order can be placed either by phone or in an automatic way from a buy-side execution management system (EMS) as a FIX (financial information exchange) order.

Parent Order: It is the information (including specific instrument, whether the order is a buy or a sell order, the quantity, and the algorithm to use) provided by buy-side participant. Once parent order is given by the buy-side participant, execution algorithm is then created and started by the broker to execute the order.

- These algorithms can be run within the buy side by simply sending the child orders straight to the market through direct market access (DMA). DMA enables the buy-side participant to trade directly using the exchange membership of a sell-side firm (its brokerage firm).

2.1 High-Frequency Trading Algorithms

High frequency trading (HFT) is a subset of algorithmic trading where a large number of small-in-size orders are sent into the market at high speed, with round-trip execution times usually measured in milliseconds. The assets that are traded are usually held for short periods of time typically in seconds or even less at times.

Key difference between Execution Algorithm and High-Frequency Trading algorithm:

- Execution algorithms are used to automate "how to trade" whereas high-frequency trading algorithms are used to automate "when to trade" and even sometimes "what to trade."
- Execution algorithms aims at minimizing market impact and trying to ensure a fair price, whereas HFT algorithms aims at maximizing profit.

In HFT, algorithms gather data from feeds stream directly from trading venues (i.e. stock or futures exchanges, foreign exchange markets, or bond markets) and look for patterns that indicate interesting trading opportunities.. The data streams are made up of events, i.e. quote events, trade events, and news events.

- **Quote event**: A quote event is a new bid or offer in the market for a certain instrument at a certain price level and with a certain available quantity (volume).
- **Trade event**: A trade event refers to a new trade that has taken place at a certain price and a certain volume.
- **News event**: A news event refers to a news related to particular instruments or economic indicators. The news event reflecting "surprises" tend to have greater impact on the market than the news event which merely confirms pre-existing expectations.

Types of HFT Strategies:

A. Statistical arbitrage (or "stat arb"): In Stat arb algorithms, traders look to correlate prices between securities in some way and trade-off the imbalances in those correlations that indicate trading opportunities. For example, consider the relationship (called the delta 1:1) between a bond, such as the 10-year government bond and a derivative of it on some exchange. These instruments tend to move together, but if that relationship breaks for a few milliseconds, there is an opportunity to buy one and sell the other at a profit. Different types of HFT algorithms for stat arb trading are as follows:

a) Pairs trading: Pair Trading is a market neutral strategy where two highly co-related instruments are bought and sold together when there is a certain degree of deviation in their co-relation. Usually the stock or commodities selected for Pair Trading are from the same sector and moves together during most of the market events. Creating a new pairs trade is called ***instantiating a new instance of an algorithm***. A set of key parameters given in the new trading strategy instance include the instruments and the level of correlation deviation should one be bought and the other sold.

b) Index arbitrage: Index Arbitrage is trading an index derivative such as the S&P500 futures against a basket of stocks. When the futures and the stocks diverge too much (e.g. futures go up, stocks go down), then an arbitrage opportunity occurs (sell the futures, buy the stocks).

c) Basket trading: This strategy involves buying a group of securities all at once when some conditions are met. The value of the basket is constantly recalculated by weighting the relative prices and holdings of each instrument in the basket and calculating an overall basket value as if it were a single instrument. This basket can then be used as part of a more complex strategy, such as pairs trading or index arbitrage.

d) Spread trading: A spread is defined as the sale of one or more futures contracts and the purchase of one or more offsetting futures contracts. A spread tracks the difference between the price of whatever it is you are long and whatever it is you are short. This strategy is particularly popular in the futures market. In spread trading, traders often use specialized tools called spreaders to model, implement, and manage spread trading.

Types of spread trading:

i. ***Intra-market spread:*** Intra-market spreads are created only as calendar spreads. You are long and short futures in the ***same*** market, but in ***different*** months. An example of an Intra-market spread is that you are Long July Corn and simultaneously Short December Corn.

ii. ***Intermarket spread***: An Intermarket spread can be created by going long futures in one market, and short futures of the same month in another market. For example: Short May Wheat and Long May Soybeans. Intermarket spreads can become calendar spreads by using long and short futures in different markets and in different months.

iii. ***Inter-exchange spread***: This strategy involves creating spreads via the use of contracts in similar markets, but on different exchanges. These spreads can be calendar spreads using different months, or they can be spreads in which the same month is used. Although the markets are similar, because the contracts occur on different exchanges they are able to be spread. An example of an Inter-exchange calendar spread would be simultaneously Long July Chicago Board of Trade (CBOT) Wheat, and Short an equal amount of May Kansas City Board of Trade (KCBOT) Wheat. More complex inter-exchange multi-legged spreads include following:

- **Crack spreads:** It involves trading the differential between the price of crude oil and petroleum products.
- ***Spark spreads:*** *It involves trading the theoretical gross margin of a gas-fired power plant derived from selling a unit of electricity against the price of the fuel required to produce this unit of electricity, including all other costs of operation, maintenance, and capital and other financial costs.*
- **Crush spreads:** It involves purchase of soybean futures and the sale of soybean oil and soybean meal futures.

e) Mean reversion: This strategy seeks to generate returns by looking for gap between the current price and the expected price. It is based on the underlying assumption that if an instrument moves too far from its average price over some recent time period, it will trade back toward that average (revert to the mean). Mean reversion algorithms use real-time analytics to spot these buying and selling opportunities.

f) Delta neutral strategies: A delta-neutral strategy aims to make a profit regardless of the price moves of the underlying asset. For example, a trading strategy that uses gold derivatives (gold futures, gold options, gold variance swaps etc.) would be a delta-neutral strategy if its success or failure was independent of the actual price of gold. For example, a trader may own a call with a 20% delta and also own a put with a -20% delta. This is known as a strangle and in this case it is delta-neutral (because the deltas from the call and the put cancel one another out). If say the put only had a -12% delta, the strangle as a whole would have an 8% delta [20% + (-12%) = 8%]. To make this strangle delta-neutral, the trader needs to sell the underlying product in the correct ratio. Or he needs to hedge in some other way, perhaps by selling call options or buying more put options.

Uses of HFT algorithms: HFT algorithms are typically used in bank proprietary trading groups, hedge funds, and proprietary trading firms for following purposes.

- **Liquidity aggregation and smart order routing**: As market fragmentation continues, algorithmic techniques have been employed to aggregate liquidity and use smart order routing to send orders to the venues with the best price and liquidity. These techniques can be used to by high frequency algorithms to operate more effectively in a fragmented market on the sell side. In a fragmented market, there are at least 100 trading venues meaning, if you want to sell Apple stock, you can go to 100 different electronic trading networks which are collectively known as Alternative Trading Systems. Among the Alternative Trading Systems is the so-called dark liquidity pool (also known as a crossing network). It is called a dark pool because the order book is hidden from the participants (bid/ask in the pool is unknown); this allows traders to park huge blocks of shares in the order book without having to worry about sending out a signal to market participants. The dark pool matches limit orders and executes them at the midpoint of the bid/ask price quoted in the normal exchange. There are many different dark liquidity pools in the US, and

in order to "source liquidity" (get your order filled) you have to go around "pinging" these dark pools, until you hit a match. There are algorithms which perform this sweeping in an intelligent manner by estimating the probability of hitting a match in a given dark pool. This dark pool sweeping algorithms is integrated into smart order routing systems. Smart order routing refers to a technique used to get the best price in the least amount of time while incurring the least amount of exchange-related costs.

- **Real-time pricing of instruments**. Algorithmic techniques can be used in the real-time pricing of such instruments as bonds, options, and foreign exchange. Unlike traditional pricing techniques which use slower-moving pricing analytics and fundamentals to price instruments, higher-frequency algorithmic techniques use information on what is happening in the aggregated market (how can we make money by increasing the spread on liquidity available) and the type of customer the price is being published for).
- **Trading on news.** Traders can use HFT trading strategies to trade automatically on news before a human trader can react to market variables, such as economic releases, announcement of a war, or unexpected weather events, before a human trader can react. News providers, such as Thomson Reuters and Bloomberg are including tags in the feeds that enable algorithms to quickly extract information such as data associated with an economic release.
- **Genetic tuning.** This involves thousands of permutations & computations of algorithms which run in parallel and fed with real market data for trading in the market.

Significance of Low Latency in HFT Strategies: Low-latency refers to the ability to quickly route and execute orders irrespective of their position-holding time, whereas high-frequency refers to the fast turnover of capital that may require low-latency execution capability. In HFT strategies, low latency is very important.

- Low latency is needed to get first mover advantage.
- Low latency decision making is highly important in multi-legged trade, wherein, multiple trades are placed as part of a stat arb strategy and each trade is considered as a leg.

Components in the low latency value chain:

- **Market data:** Market Data intermediaries can add significant latency and firms focused on HFT are interested in connecting directly to the trading venues through their market data APIs (Application Programming Interfaces).
- **Algorithmic and high-frequency trading engine.** These days, due to the requirement for quicker time to market of new algorithms, new technologies with low latency response to complex patterns in market have become popular.
- **Order execution**: In order to obtain order execution latency (which is the time between the receipt of an order arriving at the edge of your systems, and the order confirmation leaving your systems), many trading venues have adopted the FIX protocol as the standard way to place orders. This enables them to connect directly to the venues and place and manage orders over FIX.
- **Physical connection:** To reduce physics of reducing latency, which refers to making the wire connection over which market data and orders are transmitted as short as possible, traders can use a number of suppliers that can provide a dedicated network that is already wired into trading venues around the world.
- **Co-location.** Co-location (co-lo) involves installing algorithms next to or in the facilities of a trading venue. Several companies have built businesses around providing hosting platforms to allow trading firms to install their software in these co-lo facilities. The colocation is difficult to achieve for firms that run cross-market, cross-asset, or cross border algorithms that might involve trading with multiple venues that are not geographically co-located.

3. THE EVOLUTION OF ALGORITHMIC AND HIGH-FREQUENCY TRADING

Key drivers in the evolution of algorithmic trading and HFT include the following:

A. **Market fragmentation:** With the rapid increase in the number of trading venues, trading in any given instrument has been split (or fragmented) across multiple venues. Resultantly, the available liquidity on any one exchange represents just a small portion of the aggregate liquidity for that instrument. Algorithmic techniques, such as liquidity aggregation and smart order routing (as discussed above) can be used to exploit the challenges and opportunities presented by fragmentation.

B. **Opportunities in new asset classes:** Besides exchange-traded equities and futures markets, the foreign exchange and bond markets have also become increasingly electronic, open, and fragmented. As a result, demand for algorithmic

trading and HFT have been created in those markets as well.

C. **Opportunities in cross-asset class trading:** Owing to opportunities available in cross-asset class trading, algorithmic trading can be used both for profit opportunities and hedging purposes. An example of a cross-asset, profit-oriented trading opportunity is statistical arbitrage across futures and bonds.

D. **Opportunities in new geographies:** With the development of new markets, algorithmic trading and HFT are now spread across geographically over time.

E. **Opportunities in cross-border trading:** As a result of listing of instruments in multiple countries, statistical arbitrage strategies have become popular to capitalize on any pricing disparities that arise as a result.

4. ALGORITHMIC AND HIGH-FREQUENCY TRADING PLATFORMS AND TECHNOLOGIES

Key technologies integral to algorithmic trading include the following:

A. **Execution management systems**: For algorithmic trading, participants need front-end trading systems that allow access to broker algorithms as well as access to custom algorithms integrated with the EMS.

B. **Complex event processing**: CEP platform is a platform that is specifically designed for complex analysis and response to high-frequency data. CEP platforms incorporate graphical modeling tools that can rapidly capture and customize strategies. It also facilitate connecting of a trading engine to any combination of cross-asset market data and trading venues. CEP is used commonly for algorithmic trading, HFT, liquidity aggregation, smart order routing, pre-trade risk analysis, and market surveillance.

C. **Tick database**: Tick database is a real-time time-series database designed to capture and store high-frequency market data for analysis and back testing.

5. USE OF ALGORITHMIC TECHNIQUES AS A SAFETY NET

Use of algorithmic trading techniques involve a number of risks, particularly market, technology, and compliance risks. The algorithms that control such techniques are based on a number of assumed market conditions, and any change in the market can have an unexpected impact on the outcome. Also, given the high dependence on technology and the IT infrastructure, there is always a technological risk, such as an element of the infrastructure going down. There is also a compliance risk that firms need to closely monitor.

5.1 Risk Management Uses of Trading Algorithms

Following are the two approaches used to mitigate trading risk associated with trading algorithms:

1) **Real-time pre-trade risk firewall**: This approach involves placing a firewall that blocks a trade from going to market if it breaches a pre-established risk threshold. Real-time pre-trade risk firewall can also be used to monitor and block erroneous trades, i.e. Fat finger trades that is, buying 1 share at $1,000 instead of 1,000 shares at $1. Using the latest technology platforms, such as CEP, pre-trade checks can be performed with minimal latency.

2) **Back testing and market simulation:** Trading risk associated with trading algorithms can be mitigated by using a variety of real historical, pre-planned scenarios, and back-testing.

5.2 Regulatory Oversight: Real-Time Market Monitoring and Surveillance

The goal of real-time monitoring and surveillance is to detect anomalous market movements as well as market abuse to avoid potential market problems and to respond efficiently. Algorithms can be used to avoid these market abuses.

Following is a list of market abuses:

i. **Insider trading.** In insider trading, traders have an access to material, non-public information. This activity can be detected by analyzing an unusually large trade by a trader who does not usually trade that particular instrument followed closely by a new.

ii. **Front running orders.** Front-running is where market makers trade on client information

before the client's trade is executed. For example, if Trader A knows Client B is about to buy Currency C, then Trader A might buy Currency C before Client B's trade is processed. If the currency moves as a result of the client transaction, Trader A cashes in. Algorithms can be used to detect unusual and coincidental orders from a proprietary trader just prior to an event that moves the market.

iii. **Painting the tape.** This involves placing multiple buy and sell orders to move artificially a stock price up and down. For example, a trader continuously takes the best offer in the market in a particular instrument to drive the price up.

iv. **Fictitious orders.** Fictitious orders involve entering orders on one side of the market, then completing orders on the other side of the market and deleting the original order after the trade occurs. The term ***quote stuffing*** refers to one such practice in which large quantities of fictitious orders are rapidly entered into the market by an algorithm and then just as quickly cancelled. These orders distract other algorithms. **Layering of bids-asks** refers to traders or brokers that stagger orders from the same client reference at different price and volume levels to give the misleading impression of greater interest in the security from a more diverse set of exchange participants, and might be viewed as being carried out for the purpose of manipulation. Spoofing is a form of market manipulation that involves actions taken by market participants to give an improper or false impression of unusual activity or price movement in a security.

v. **Wash trading.** Wash trading occurs when a trader buys and sells the same securities simultaneously. Wash trades benefit brokers who earn commissions from the trades. Wash trades can also be used to create the false impression that there is investor interest in the security.

vi. **Trader collusion.** Trader collusion occurs when traders cooperate to deliberately manipulate the market in their favor. E.g. manipulation of Libor and the foreign exchange benchmark rates through trader collusion.

For all these market abuses, we can use tick databases for keeping an audit trail of market data and potential abuse cases. Further, we can also identify new abuse patterns by building on surveillance systems in the business analytics platforms.

6. IMPACT OF ALGORITHMIC AND HIGHFREQUENCY TRADING ON THE SECURITIES MARKETS

Algorithmic and high-frequency trading can have both positive and negative impact on the markets as a whole.

Positive Impacts:

- **Minimized market impact of large trades**. Algorithmic trading helps in reducing market impact of large trades by breaking down large orders into smaller slices.
- **Lower cost of execution**. The use of algorithms helps in reducing the cost of execution for investors.
- **Improved efficiency in certain markets**. Algorithms helps in exploiting new statistical arbitrage opportunities quickly and efficiently.
- **More open and competitive trading markets**. Algorithms promote more open and competitive trading markets as small teams can use CEP and hosting environments to run an advanced quant trading operation.
- **Improved and more efficient trading venues**. Algorithms trading techniques creates improves and more efficient trading venues as they provide services, such as lower matching latency, improved order throughput, and more value added services (e.g. co-location).

Negative Impacts:

- **Fear of an unfair advantage.** Availability of algorithmic techniques to few large firms create a fear of unfair advantage as these algorithmic firms have access to order book data using which they can learn and back test strategies that is not readily available to other investors.
- **Difficult to monitor activity of HFT trades:** Due to complexity associated with algorithmic trading, there is a perception in the market that regulators are lagging in their capability to monitor the activity of high-frequency traders.
- **Acceleration and accentuation of market movements.** Algorithms have tendency to trigger and accelerate market crash because market panic in particular instruments can trigger stop-losses and a rapidly declining price, which may lead an algorithm to short those instruments to buy them back at a profit.
- **Gaming the market**. It is relatively easier to spoof the market by placing millions of anomalous quotes via algorithmic trading. It is also easier to carry out wash trades or painting the tape, because it is quite challenging to identify market abuse in a high-frequency, fragmented world.

  - **Increased risk profile**. Algorithms can lead to breach of critical exposure limits if no proper pre-trade risk precautions are taken.
  - **Algorithms can result in a large potential loss by placing an incorrect order.** In worst case, algorithms can result in a stream of spurious orders being placed. Since, algorithms are executed at very high speed, detecting such errors can be challenging.
    - Risk of placing an incorrect order can be mitigated by having human trader watching positions and the behavior of the algorithm in real time.
    - Incorrect or spurious trades can be blocked by having a "kill switch" that pull one or all algorithms from the market.
- **Potential for market denial-of-service-style attacks**. The market can experience a significant slowdown if a stream of orders are sent into the market in quick succession.
- **Additional load on trading venues.** Since algorithms respond to small changes in the markets, they continuously adjust bids and offers. This process tends to create additional load on trading venues – resulting in slow down of the markets.
- **Increased difficulty of policing the market**. Owing to multitude of high-frequency algorithms, market fragmentation, cross-asset trading, and dark pools (trading venues that do not publish their liquidity and are only available to selected clients), it is highly challenging for regulators to monitor the markets and ensure their effective functioning.

Do algorithmic and high-frequency trading disadvantage the smaller trading firms and other market participants?

The algorithmic trading techniques are quite costly to develop and run, and many investors cannot afford them. This creates a fear of unfair advantage. Hence, smaller investors are disadvantaged by this unequal access to information. Nevertheless, HFT has benefited those small investors indirectly by narrowing of bid–ask spreads, lower transaction costs, and an increase in liquidity and price efficiency—without an increase in volatility.