gradient_descent_1

July 31, 2025

1 Gradient Descent

Fundamental optimization algorithm to minimize the cost/loss function which measures how well a model fits to the given data.

[10]:
```
# importing modules
import random
import matplotlib.pyplot as plt
```

Let us have a function: $f(x) = y = 20x + 5, -10 \leq x \leq 10 \therefore y_i = 20x_i + 5 \forall x_i \in \{-10, -9, ..., 9, 10\}$

[11]:
```
x_values = [x for x in range(-10, 11)] # values for x
y_values = [(20*x+5) for x in x_values] # true y for values of x

n = len(x_values)
```

[12]:
```
step = 0.001 # learning rate
epochs = 5000

# randomly select w and b
w = random.uniform(-5, 5)
b = random.uniform(-5, 5)

w_values = []
b_values = []

w_values.append(w)
b_values.append(b)

cost_values = []
```

Let predicted value of y be: $\hat{y}_i = w_i x + b_i, -10 \leq x \leq 10$

$$\therefore \text{The cost/loss function is: } J(w, b) = \frac{1}{n} \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

$$\text{Now, } \frac{\partial J(w,b)}{\partial w} = \frac{-2}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)(x_i)$$

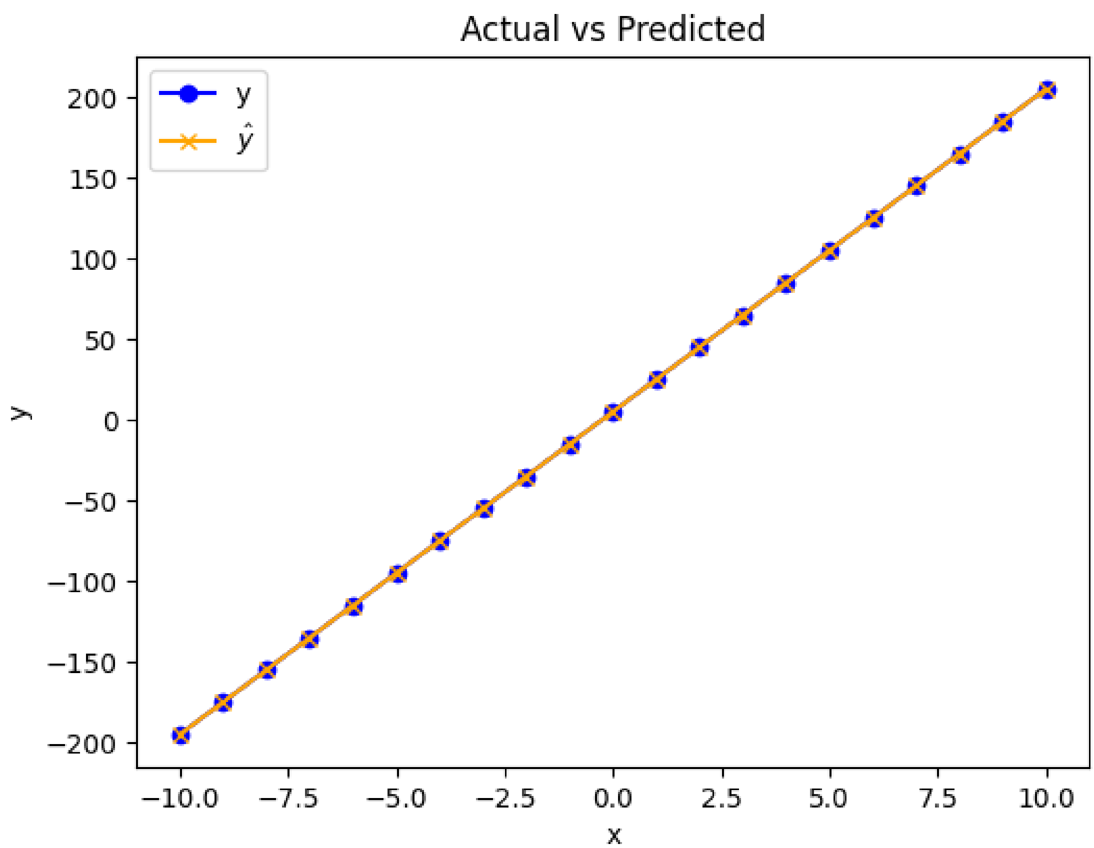
$$\text{and, } \frac{\partial J(w,b)}{\partial b} = \frac{-2}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)$$

$$\text{Let step } = \alpha w_{new} = w - \alpha\frac{\partial J(w,b)}{\partial w} b_{new} = b - \alpha\frac{\partial J(w,b)}{\partial b}$$

[13]:
```
for i in range(epochs):
  y_pred = [w*x+b for x in x_values]
  cost = sum(((y_i - y_pred_i)**2
              for y_i, y_pred_i in zip(y_values, y_pred))) / n
  cost_values.append(cost)

  dw = -2 * sum((y_i - y_pred_i)*x_i
        for y_i, y_pred_i, x_i in zip(y_values, y_pred, x_values)) / n
        # partial differentiation of J wrt w
  db = -2 * sum((y_i - y_pred_i)
        for y_i, y_pred_i in zip(y_values, y_pred)) / n
        # partial differentiation of J wrt b

  w = w - step*dw
  b = b - step*db

  w_values.append(w)
  b_values.append(b)
```

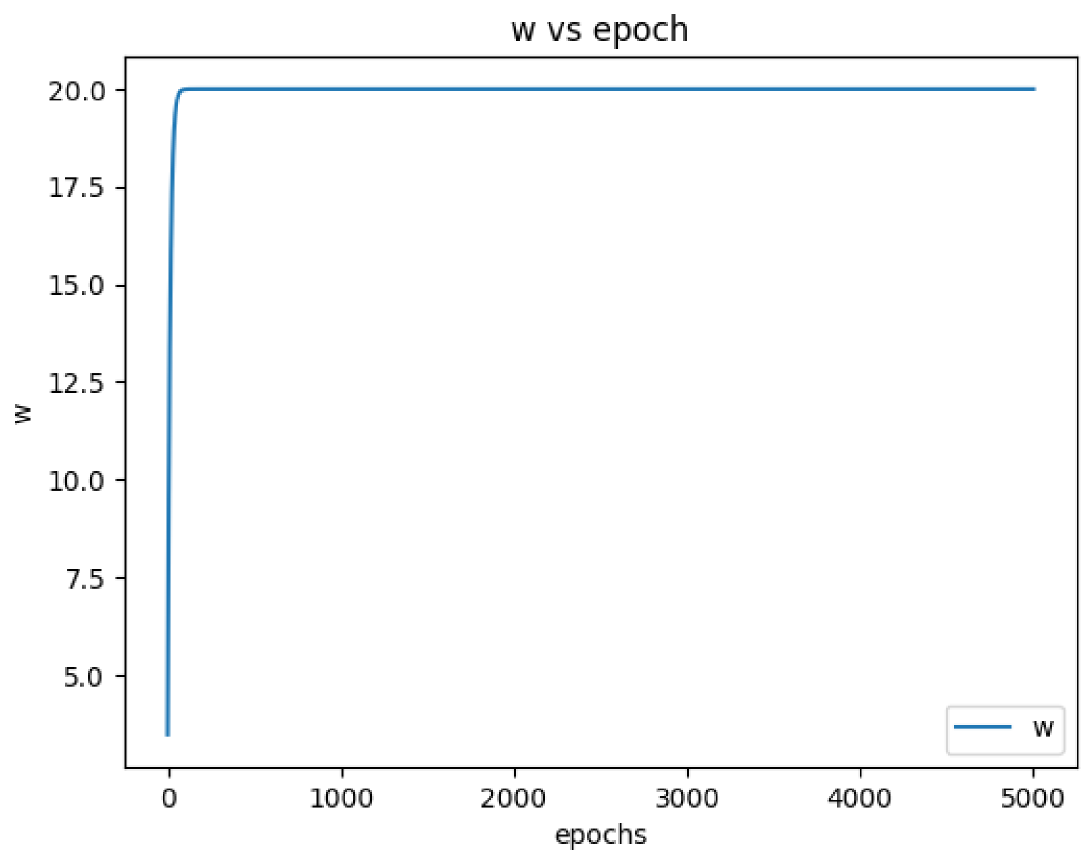
[14]:
```
y_pred_final = [w*x+b for x in x_values] # predicted y values based on final w
                                         # and b values
```

[15]:
```
plt.plot(x_values, y_values, color='blue', marker='o', label='y')
plt.plot(x_values, y_pred, color='orange', marker='x', label='$\hat{y}$')
plt.xlabel('x')
plt.ylabel('y')
plt.title('Actual vs Predicted')
plt.legend()
plt.show()
```

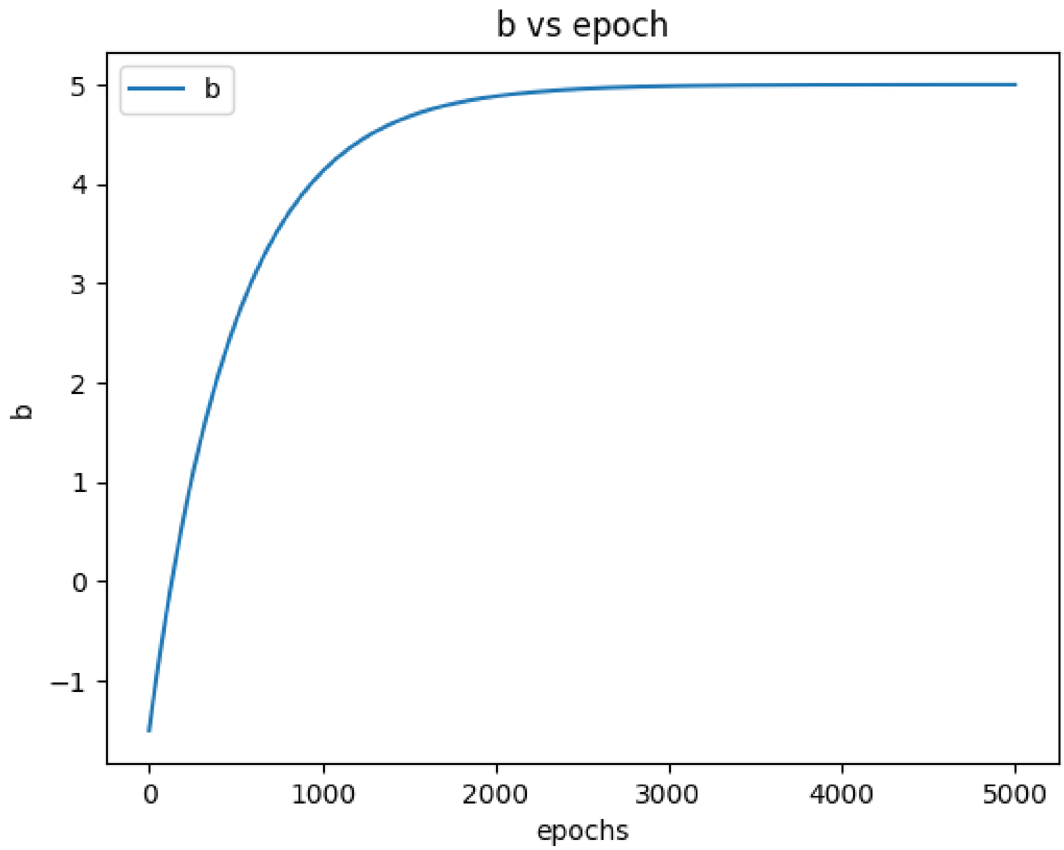


[16]:

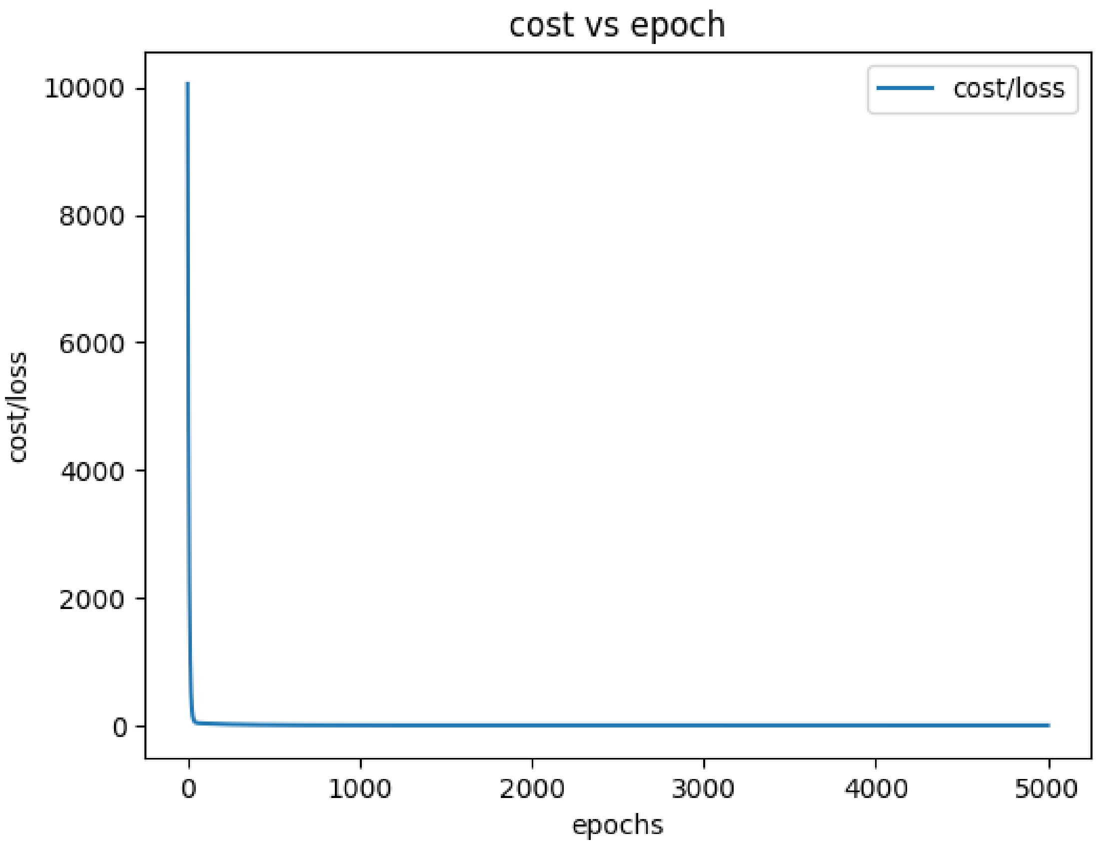
```
x_axis = range(epochs+1)
plt.plot(x_axis, w_values, label='w')
plt.xlabel('epochs')
plt.ylabel('w')
plt.title('w vs epoch')
plt.legend()
plt.show()
```



[17]:
```
x_axis = range(epochs+1)
plt.plot(x_axis, b_values, label='b')
plt.xlabel('epochs')
plt.ylabel('b')
plt.title('b vs epoch')
plt.legend()
plt.show()
```



[18]:
```
x_axis = range(epochs)
plt.plot(x_axis, cost_values, label='cost/loss')
plt.xlabel('epochs')
plt.ylabel('cost/loss')
plt.title('cost vs epoch')
plt.legend()
plt.show()
```


cost vs epoch
cost/loss
10000
8000
6000
4000
2000
0
cost/loss
0
1000
2000
3000
4000
5000
epochs