experimenting_fair_coin

May 23, 2025

1 Experimenting & Plotting Relative Frequencies of Heads & Tails with respect to Number of Tosses for a Fair Coin with help of pseudo-random choosing.

```
[1]: # installing required modules
     %pip install matplotlib
     %pip install seaborn
    Requirement already satisfied: matplotlib in
    d:\projects\dwaidatta-10\.venv\lib\site-packages (3.10.3)
    Requirement already satisfied: contourpy>=1.0.1 in
    d:\projects\dwaidatta-10\.venv\lib\site-packages (from matplotlib) (1.3.2)
    Requirement already satisfied: cycler>=0.10 in
    d:\projects\dwaidatta-10\.venv\lib\site-packages (from matplotlib) (0.12.1)
    Requirement already satisfied: fonttools>=4.22.0 in
    d:\projects\dwaidatta-10\.venv\lib\site-packages (from matplotlib) (4.57.0)
    Requirement already satisfied: kiwisolver>=1.3.1 in
    d:\projects\dwaidatta-10\.venv\lib\site-packages (from matplotlib) (1.4.8)
    Requirement already satisfied: numpy>=1.23 in
    d:\projects\dwaidatta-10\.venv\lib\site-packages (from matplotlib) (2.2.5)
    Requirement already satisfied: packaging>=20.0 in
    d:\projects\dwaidatta-10\.venv\lib\site-packages (from matplotlib) (25.0)
    Requirement already satisfied: pillow>=8 in
    d:\projects\dwaidatta-10\.venv\lib\site-packages (from matplotlib) (11.2.1)
    Requirement already satisfied: pyparsing>=2.3.1 in
    d:\projects\dwaidatta-10\.venv\lib\site-packages (from matplotlib) (3.2.3)
    Requirement already satisfied: python-dateutil>=2.7 in
    d:\projects\dwaidatta-10\.venv\lib\site-packages (from matplotlib) (2.9.0.post0)
    Requirement already satisfied: six>=1.5 in
    d:\projects\dwaidatta-10\.venv\lib\site-packages (from python-
    dateutil>=2.7->matplotlib) (1.17.0)
    Note: you may need to restart the kernel to use updated packages.
    Requirement already satisfied: seaborn in
    d:\projects\dwaidatta-10\.venv\lib\site-packages (0.13.2)
    Requirement already satisfied: numpy!=1.24.0,>=1.20 in
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d:\projects\dwaidatta-10\.venv\lib\site-packages (from seaborn) (2.2.5)

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d:\projects\dwaidatta-10\.venv\lib\site-packages (from seaborn) (2.2.3)
    Requirement already satisfied: matplotlib!=3.6.1,>=3.4 in
    d:\projects\dwaidatta-10\.venv\lib\site-packages (from seaborn) (3.10.3)
    Requirement already satisfied: contourpy>=1.0.1 in
    d:\projects\dwaidatta-10\.venv\lib\site-packages (from
    matplotlib!=3.6.1,>=3.4->seaborn) (1.3.2)
    Requirement already satisfied: cycler>=0.10 in
    d:\projects\dwaidatta-10\.venv\lib\site-packages (from
    matplotlib!=3.6.1,>=3.4->seaborn) (0.12.1)
    Requirement already satisfied: fonttools>=4.22.0 in
    d:\projects\dwaidatta-10\.venv\lib\site-packages (from
    matplotlib!=3.6.1,>=3.4->seaborn) (4.57.0)
    Requirement already satisfied: kiwisolver>=1.3.1 in
    d:\projects\dwaidatta-10\.venv\lib\site-packages (from
    matplotlib!=3.6.1,>=3.4->seaborn) (1.4.8)
    Requirement already satisfied: packaging>=20.0 in
    d:\projects\dwaidatta-10\.venv\lib\site-packages (from
    matplotlib!=3.6.1,>=3.4->seaborn) (25.0)
    Requirement already satisfied: pillow>=8 in
    d:\projects\dwaidatta-10\.venv\lib\site-packages (from
    matplotlib!=3.6.1,>=3.4->seaborn) (11.2.1)
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    d:\projects\dwaidatta-10\.venv\lib\site-packages (from
    matplotlib!=3.6.1,>=3.4->seaborn) (3.2.3)
    Requirement already satisfied: python-dateutil>=2.7 in
    d:\projects\dwaidatta-10\.venv\lib\site-packages (from
    matplotlib!=3.6.1,>=3.4->seaborn) (2.9.0.post0)
    Requirement already satisfied: pytz>=2020.1 in
    d:\projects\dwaidatta-10\.venv\lib\site-packages (from pandas>=1.2->seaborn)
    (2025.2)
    Requirement already satisfied: tzdata>=2022.7 in
    d:\projects\dwaidatta-10\.venv\lib\site-packages (from pandas>=1.2->seaborn)
    (2025.2)
    Requirement already satisfied: six>=1.5 in
    d:\projects\dwaidatta-10\.venv\lib\site-packages (from python-
    dateutil>=2.7->matplotlib!=3.6.1,>=3.4->seaborn) (1.17.0)
    Note: you may need to restart the kernel to use updated packages.
[2]: #importing required modules
     import matplotlib.pyplot as plt
     import random
     import seaborn as sns
[3]: def take_input():
         # function to take input the number of tosses
```

Requirement already satisfied: pandas>=1.2 in

```
n = int(input("Enter number of tosses: "))
return n
```

```
[]: def calculate():
         no_of_tosses = take_input()
         print("Number of tosses:", no_of_tosses)
         toss_result = None # will store H or T
         tosses = [] # stores the results as list
         possible_outcomes = ['H', 'T'] # possible outcomes
         count_H = 0 # counter for H
         count_T = 0 # counter for T
         rel_freq_H = 0.0 # relative frequency for H
         rel_freq_T = 0.0 # relative frequency for T
         rel_freq_list_H = [] # relative frequency list for H
         rel_freq_list_T = [] # relative frequency list for T
         estimated_rel_freq = 0.500
         deviation_percent_list_H = []
         deviation_percent_list_T = []
         for toss_number in range(1, (no_of_tosses + 1)): # range: [1, total number_
      ⇔of tosses entered]
             toss_result = random.choice(possible_outcomes) # randomly selects from
      \hookrightarrow H or T
             tosses.append(toss_result) # stores the result into list
             # obvious logic implementation below (^ ^)
             if (toss_result == 'H'):
                 count_H += 1
             else:
                 count_T += 1
             rel_freq_H = round(float(count_H/toss_number), 3)
             rel_freq_T = round(float(count_T/toss_number), 3)
             rel_freq_list_H.append(rel_freq_H)
             rel_freq_list_T.append(rel_freq_T)
```

```
deviation_percent_list_H.append(round((abs(estimated_rel_freq -u
-rel_freq_H) * 100/estimated_rel_freq), 3))
    deviation_percent_list_T.append(round((abs(estimated_rel_freq -u
-rel_freq_T) * 100/estimated_rel_freq), 3))

# printing to see the values

print("Tosses list:")
print(tosses)

print("Relative frequency list of Heads H:")
print(rel_freq_list_H)

print("Relative frequency list of Tails T:")
print(rel_freq_list_T)

return(no_of_tosses, rel_freq_list_H, rel_freq_list_T, u
-deviation_percent_list_H, deviation_percent_list_T)
```

```
[5]: def plotting():
        no_of_tosses, rel_freq_list_H, rel_freq_list_T, deviation_percent_list_H,__
      deviation_percent_list_T = calculate() # receive the values from the function
        x values = list(range(1, no of tosses+1)) # X axis range: [1, total number]
      →of tosses entered]
         # plotting customization
        sns.set(style="whitegrid")
        plt.figure(figsize=(10, 8), dpi = 200)
        plt.plot(x_values, rel_freq_list_H, linestyle = '--', color = 'blue',
      ⇔linewidth=1, alpha=0.8, label='Heads')
        plt.plot(x_values, rel_freq_list_T, linestyle = '--', color = 'red',_
      ⇔linewidth=1, alpha=0.8, label='Tails')
        plt.grid(True, linestyle='--', alpha=0.4)
        plt.yticks([0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0])
        plt.xlim(-0.5, no_of_tosses)
        plt.ylim(-0.2, 1.2)
        plt.xlabel("Number of tosses")
        plt.ylabel("Relative frequencies")
```

```
plt.title("Plot of Relative frequencies of Heads & Tails for a Fair Coin")
  plt.legend(fancybox=True)
  plt.tight_layout()
  plt.show()
  # another plot for Deviation from converging value of 0.500 vs No. of Tosses
  # plotting customization
  sns.set(style="whitegrid")
  plt.figure(figsize=(10, 8), dpi = 200)
  plt.plot(x values, deviation percent_list_H, linestyle = '-', color =__
plt.plot(x_values, deviation_percent_list_T, linestyle = '-', color = __
plt.grid(True, linestyle='--', alpha=0.4)
  plt.yticks([0, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100])
  plt.xlim(-0.5, no_of_tosses)
  plt.ylim(-5, 105)
  plt.xlabel("Number of tosses")
  plt.ylabel("Deviation Percentage from Estimated Relative Frequency")
  plt.title("Plot of Deviation Percentage of Relative Frequency of Heads \&
→Tails for a Fair Coin")
  plt.legend(fancybox=True)
  plt.tight_layout()
  plt.show()
```

1.0.1 Plotting with three different values for total number of tosses.

In the absolute deviation plot for a fair coin, the lines for heads and tails perfectly overlap because, at every toss, the deviation of each from the expected probability (0.5) is always equal in magnitude. Since the relative frequencies of heads and tails always sum to 1, their absolute deviations from 0.5 are identical, resulting in a single overlapping curve on the graph.

[6]: plotting()

Number of tosses: 10

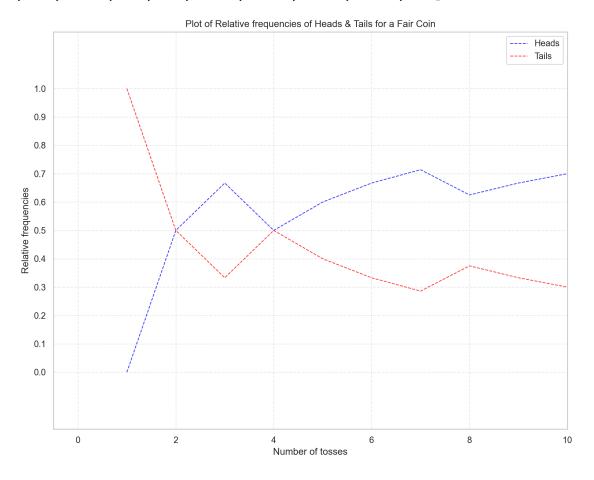
Tosses list:

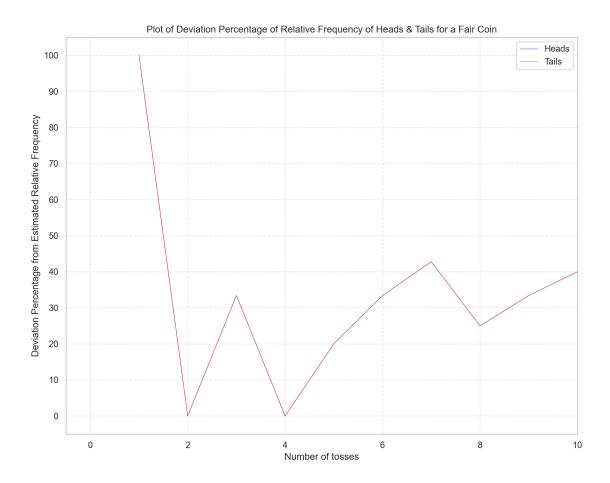
Relative frequency list of Heads H:

[0.0, 0.5, 0.667, 0.5, 0.6, 0.667, 0.714, 0.625, 0.667, 0.7]

Relative frequency list of Tails T:

[1.0, 0.5, 0.333, 0.5, 0.4, 0.333, 0.286, 0.375, 0.333, 0.3]



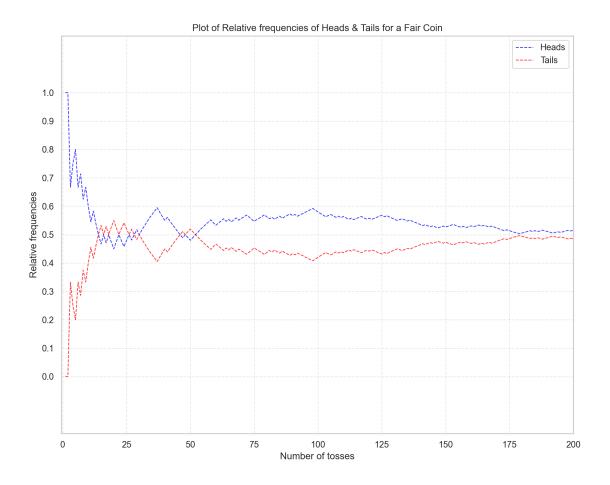


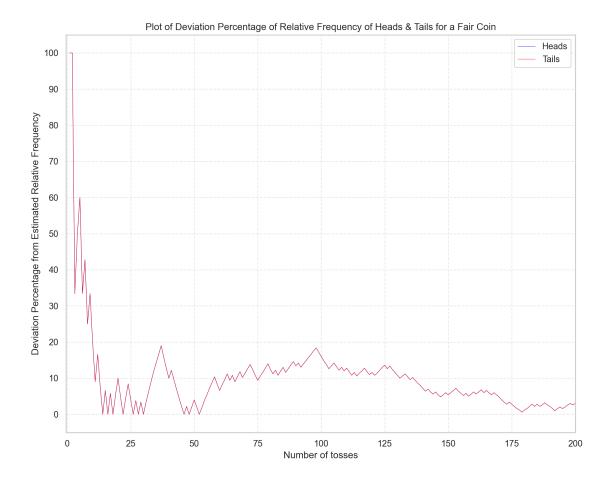
[7]: plotting()

```
Number of tosses: 200
Tosses list:
'H', 'H', 'T', 'H', 'H', 'H', 'T', 'H']
Relative frequency list of Heads H:
[1.0, 1.0, 0.667, 0.75, 0.8, 0.667, 0.714, 0.625, 0.667, 0.6, 0.545, 0.583,
```

0.538, 0.5, 0.467, 0.5, 0.471, 0.5, 0.474, 0.45, 0.476, 0.5, 0.478, 0.458, 0.48,

```
0.5, 0.481, 0.5, 0.517, 0.5, 0.516, 0.531, 0.545, 0.559, 0.571, 0.583, 0.595,
0.579, 0.564, 0.55, 0.561, 0.548, 0.535, 0.523, 0.511, 0.5, 0.489, 0.5, 0.49,
0.48, 0.49, 0.5, 0.509, 0.519, 0.527, 0.536, 0.544, 0.552, 0.542, 0.533, 0.541,
0.548, 0.556, 0.547, 0.554, 0.545, 0.552, 0.559, 0.551, 0.557, 0.563, 0.569,
0.562, 0.554, 0.547, 0.553, 0.558, 0.564, 0.57, 0.562, 0.556, 0.561, 0.554,
0.56, 0.565, 0.558, 0.563, 0.568, 0.573, 0.567, 0.571, 0.565, 0.57, 0.574,
0.579, 0.583, 0.588, 0.592, 0.586, 0.58, 0.574, 0.569, 0.563, 0.567, 0.571,
0.566, 0.561, 0.565, 0.56, 0.564, 0.559, 0.554, 0.558, 0.553, 0.557, 0.56,
0.564, 0.559, 0.555, 0.558, 0.554, 0.557, 0.561, 0.565, 0.568, 0.563, 0.567,
0.562, 0.558, 0.554, 0.55, 0.553, 0.556, 0.552, 0.548, 0.551, 0.547, 0.543,
0.54, 0.536, 0.532, 0.535, 0.531, 0.528, 0.531, 0.527, 0.524, 0.527, 0.53,
0.527, 0.53, 0.533, 0.536, 0.532, 0.529, 0.526, 0.529, 0.525, 0.528, 0.531,
0.528, 0.531, 0.534, 0.53, 0.533, 0.53, 0.527, 0.53, 0.527, 0.524, 0.52, 0.517,
0.514, 0.517, 0.514, 0.511, 0.508, 0.506, 0.503, 0.506, 0.508, 0.511, 0.514,
0.511, 0.514, 0.511, 0.513, 0.516, 0.513, 0.511, 0.508, 0.505, 0.508, 0.51,
0.508, 0.51, 0.513, 0.515, 0.513, 0.515]
Relative frequency list of Tails T:
[0.0, 0.0, 0.333, 0.25, 0.2, 0.333, 0.286, 0.375, 0.333, 0.4, 0.455, 0.417,
0.462, 0.5, 0.533, 0.5, 0.529, 0.5, 0.526, 0.55, 0.524, 0.5, 0.522, 0.542, 0.52,
0.5, 0.519, 0.5, 0.483, 0.5, 0.484, 0.469, 0.455, 0.441, 0.429, 0.417, 0.405,
0.421, 0.436, 0.45, 0.439, 0.452, 0.465, 0.477, 0.489, 0.5, 0.511, 0.5, 0.51
0.52, 0.51, 0.5, 0.491, 0.481, 0.473, 0.464, 0.456, 0.448, 0.458, 0.467, 0.459,
0.452, 0.444, 0.453, 0.446, 0.455, 0.448, 0.441, 0.449, 0.443, 0.437, 0.431,
0.438, 0.446, 0.453, 0.447, 0.442, 0.436, 0.43, 0.438, 0.444, 0.439, 0.446,
0.44, 0.435, 0.442, 0.437, 0.432, 0.427, 0.433, 0.429, 0.435, 0.43, 0.426,
0.421, 0.417, 0.412, 0.408, 0.414, 0.42, 0.426, 0.431, 0.437, 0.433, 0.429,
0.434, 0.439, 0.435, 0.44, 0.436, 0.441, 0.446, 0.442, 0.447, 0.443, 0.44,
0.436, 0.441, 0.445, 0.442, 0.446, 0.443, 0.439, 0.435, 0.432, 0.437, 0.433,
0.438, 0.442, 0.446, 0.45, 0.447, 0.444, 0.448, 0.452, 0.449, 0.453, 0.457,
0.46, 0.464, 0.468, 0.465, 0.469, 0.472, 0.469, 0.473, 0.476, 0.473, 0.47
0.473, 0.47, 0.467, 0.464, 0.468, 0.471, 0.474, 0.471, 0.475, 0.472, 0.469,
0.472, 0.469, 0.466, 0.47, 0.467, 0.47, 0.473, 0.47, 0.473, 0.476, 0.48, 0.483,
0.486, 0.483, 0.486, 0.489, 0.492, 0.494, 0.497, 0.494, 0.492, 0.489, 0.486,
0.489, 0.486, 0.489, 0.487, 0.484, 0.487, 0.489, 0.492, 0.495, 0.492, 0.49,
0.492, 0.49, 0.487, 0.485, 0.487, 0.485]
```





[8]: plotting()

Number of tosses: 1000 Tosses list:

```
'T', 'T', 'T', 'T', 'H', 'H', 'H', 'T',
 'H', 'H', 'T',
             'H'.
              'H'.
               'H'.
 'T', 'T',
'T', 'T',
 'H'. 'T'.
 'T', 'T',
              'T', 'H',
 'T',
        'T',
              'T',
  'H',
   'H',
     'H', 'T', 'T',
         'H', 'H', 'T',
             'H',
 'T', 'H', 'H', 'H',
'T', 'H', 'T', 'T', 'T', 'T', 'T', 'H', 'T',
            'H', 'T',
              'H', 'H',
'T', 'T', 'H', 'T', 'H', 'H', 'T', 'T',
              'T',
 'T', 'H', 'T',
    'T',
               'H'.
 'T', 'T',
'H', 'H', 'T', 'T', 'T', 'H', 'T', 'H', 'T',
            'H', 'T', 'H', 'T',
 'T', 'H', 'H', 'H', 'T', 'H',
            'T',
               'T',
'H', 'H', 'H',
    'H',
             'T',
              'T',
 'T', 'T',
'H', 'H',
              'T', 'H',
 'T', 'H', 'H', 'H', 'H', 'T', 'H', 'T',
            'H', 'T',
              'T',
 'H', 'T',
               'H',
 'H', 'H',
'T', 'H', 'T', 'T', 'T', 'H', 'H', 'T', 'H', 'T', 'H',
              'H', 'H',
'T', 'H', 'H', 'H',
'H', 'T', 'H', 'T', 'T', 'H', 'T']
```

Relative frequency list of Heads H:

```
[0.0, 0.0, 0.0, 0.0, 0.0, 0.167, 0.143, 0.125, 0.222, 0.3, 0.364, 0.417, 0.385,
0.429, 0.4, 0.438, 0.471, 0.5, 0.526, 0.55, 0.524, 0.5, 0.522, 0.5, 0.52, 0.5
0.519, 0.5, 0.517, 0.5, 0.516, 0.531, 0.545, 0.529, 0.543, 0.528, 0.541, 0.526,
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0.484, 0.492, 0.485, 0.478, 0.471, 0.464, 0.471, 0.465, 0.458, 0.466, 0.473,
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0.477, 0.482, 0.477, 0.473, 0.469, 0.465, 0.461, 0.457, 0.453, 0.449, 0.445,
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0.427, 0.426, 0.428, 0.426, 0.424, 0.423, 0.425, 0.423, 0.422, 0.42, 0.422,
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