determining_pi_chudnovsky_algorithm

May 29, 2025

1 Chudnovsky Algorithm

The **Chudnovsky algorithm** is a very fast method for calculating the digits of (pi), based on Ramanujan's formulae. It was published by the Chudnovsky brothers in 1988 and has been used to set numerous world records for the most digits of ever computed.

1.0.1 *Working Formula:*

$$\frac{1}{\pi} = 12 \sum_{k=0}^{\infty} \frac{(-1)^k \cdot (6k)! \cdot (13591409 + 545140134k)}{(3k)! \cdot (k!)^3 \cdot 640320^{3k+3/2}}$$

Purpose:

The Chudnovsky algorithm is designed for extremely rapid computation of to millions, billions, or even trillions of digits.

How it works:

It uses a special infinite series (a generalized hypergeometric series) where each term is constructed from large factorials, specific constants, and powers. The denominator grows so fast that each term contributes about 14 new correct digits of , making the series converge extremely quickly.

[1]:
```
import math # importing required modules
from decimal import Decimal, getcontext

getcontext().prec = 50 # set precision to 50 decimal places
```

[2]:
```
def f(x): # defining a factorial function for cleaner code
    return math.factorial(x)
```

[3]:
```
sum = Decimal(0) # Decimal() sets the precision of these variables
term = None
n = 10 # summation upto n terms

for k in range(0, n+1):

    numerator = Decimal(((-1)**k) * (f(6*k)) * (13591409 + 545140134*k)) #␣
 ↪numerator term
```

```
    denominator = Decimal((f(3*k)) * ((f(k))**3) * ((640320)**(3*k + 1.5))) # denominator term

    term = numerator / denominator

    sum += term

const = Decimal(12)
one_by_pi = const * sum # multiply with 12 as per formula

est_pi = 1 / one_by_pi # inverse
actual_pi = Decimal('3.14159265358979323846264338327950288419716939937510') # actual PI upto 50 digits
diff = abs(actual_pi - est_pi) # checking difference from actual PI value

print(f"Actual PI =\t{actual_pi}")
print(f"Estimated PI =\t{est_pi}")
print(f"Difference =\t{diff:.50f}")
```

```
Actual PI =     3.14159265358979323846264338327950288419716939937510
Estimated PI =  3.14159265358979334233705724217276509644728677748 37
Difference =    0.00000000000000010387441385889326221225011737810860
```