fair coin

May 23, 2025

1 Experimenting & Plotting Relative Frequencies of Heads & Tails with respect to Number of Tosses for a Fair Coin with help of pseudo-random choosing.

```
[151]: # installing required modules
       %pip install matplotlib
      Requirement already satisfied: matplotlib in
      d:\projects\dwaidatta-10\.venv\lib\site-packages (3.10.3)
      Requirement already satisfied: contourpy>=1.0.1 in
      d:\projects\dwaidatta-10\.venv\lib\site-packages (from matplotlib) (1.3.2)
      Requirement already satisfied: cycler>=0.10 in
      d:\projects\dwaidatta-10\.venv\lib\site-packages (from matplotlib) (0.12.1)
      Requirement already satisfied: fonttools>=4.22.0 in
      d:\projects\dwaidatta-10\.venv\lib\site-packages (from matplotlib) (4.57.0)
      Requirement already satisfied: kiwisolver>=1.3.1 in
      d:\projects\dwaidatta-10\.venv\lib\site-packages (from matplotlib) (1.4.8)
      Requirement already satisfied: numpy>=1.23 in
      d:\projects\dwaidatta-10\.venv\lib\site-packages (from matplotlib) (2.2.5)
      Requirement already satisfied: packaging>=20.0 in
      d:\projects\dwaidatta-10\.venv\lib\site-packages (from matplotlib) (25.0)
      Requirement already satisfied: pillow>=8 in
      d:\projects\dwaidatta-10\.venv\lib\site-packages (from matplotlib) (11.2.1)
      Requirement already satisfied: pyparsing>=2.3.1 in
      d:\projects\dwaidatta-10\.venv\lib\site-packages (from matplotlib) (3.2.3)
      Requirement already satisfied: python-dateutil>=2.7 in
      d:\projects\dwaidatta-10\.venv\lib\site-packages (from matplotlib) (2.9.0.post0)
      Requirement already satisfied: six>=1.5 in
      d:\projects\dwaidatta-10\.venv\lib\site-packages (from python-
      dateutil>=2.7->matplotlib) (1.17.0)
      Note: you may need to restart the kernel to use updated packages.
[152]: #importing required modules
       import matplotlib.pyplot as plt
       import random
```

```
[153]: def take_input():
           # function to take input the number of tosses
           n = int(input("Enter number of tosses: "))
           return n
[154]: def calculate():
           no_of_tosses = take_input()
           print("Number of tosses:", no_of_tosses)
           toss_result = None # will store H or T
           tosses = [] # stores the results as list
           possible_outcomes = ['H', 'T'] # possible outcomes
           count_H = 0 # counter for H
           count_T = 0 # counter for T
           rel_freq_H = 0.0 # relative frequency for H
           rel_freq_T = 0.0 # relative frequency for H
           rel_freq_list_H = [] # relative frequency list for H
           rel_freq_list_T = [] # relative frequency list for T
           for toss_number in range(1, (no_of_tosses + 1)): # range: [1, total number_
        ⇔of tosses entered]
               toss_result = random.choice(possible_outcomes) # randomly selects from
        \hookrightarrow H or T
               tosses.append(toss_result) # stores the result into list
               # obvious logic implementation below (^ ^)
               if (toss_result == 'H'):
                   count_H += 1
               else:
                   count_T += 1
               rel_freq_H = round(float(count_H/toss_number), 3)
               rel_freq_T = round(float(count_T/toss_number), 3)
               rel_freq_list_H.append(rel_freq_H)
               rel_freq_list_T.append(rel_freq_T)
           # printing to see the values
```

```
print("Tosses list:")
print(tosses)

print("Relative frequency list of Heads H:")
print(rel_freq_list_H)

print("Relative frequency list of Tails T:")
print(rel_freq_list_T)

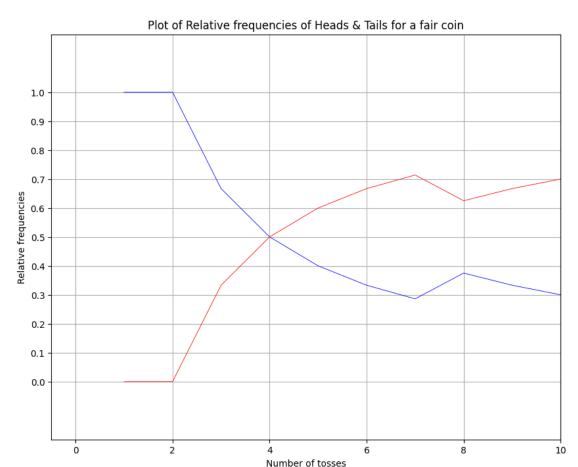
return(no_of_tosses, rel_freq_list_H, rel_freq_list_T)
```

```
[]: def plotting():
         no_of_tosses, rel_freq_list_H, rel_freq_list_T = calculate() # receive the_u
      ⇔values from the function
         x_values = list(range(1, no_of_tosses+1)) # X axis range: [1, total number_
      ⇔of tosses entered]
         # plotting customization
        plt.figure(figsize=(10, 8))
         plt.plot(x_values, rel_freq_list_H, linestyle = '-', color = 'blue',_
      →linewidth=0.6)
         plt.plot(x_values, rel_freq_list_T, linestyle = '-', color = 'red', __
      ⇒linewidth=0.6)
         plt.yticks([0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0])
         plt.xlim(-0.5, no_of_tosses)
         plt.ylim(-0.2, 1.2)
         plt.xlabel("Number of tosses")
         plt.ylabel("Relative frequencies")
         plt.title("Plot of Relative frequencies of Heads & Tails for a fair coin")
         plt.grid(True)
         plt.show()
```

1.0.1 Plotting with three different values for total number of tosses.

```
[156]: plotting()

Number of tosses: 10
Tosses list:
```



[157]: plotting()

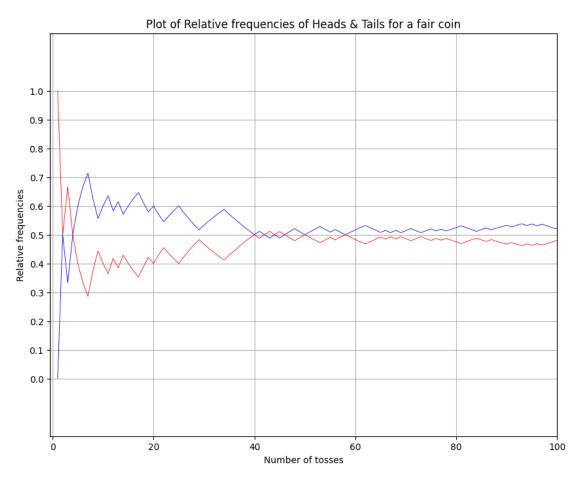
Number of tosses: 100

Tosses list:

Relative frequency list of Heads H:

[0.0, 0.5, 0.333, 0.5, 0.6, 0.667, 0.714, 0.625, 0.556, 0.6, 0.636, 0.583,

0.615, 0.571, 0.6, 0.625, 0.647, 0.611, 0.579, 0.6, 0.571, 0.545, 0.565, 0.583, 0.6, 0.577, 0.556, 0.536, 0.517, 0.533, 0.548, 0.562, 0.576, 0.588, 0.571, 0.556, 0.541, 0.526, 0.513, 0.5, 0.512, 0.5, 0.488, 0.5, 0.489, 0.5, 0.511, 0.521, 0.51, 0.5, 0.51, 0.519, 0.528, 0.519, 0.509, 0.518, 0.509, 0.5, 0.508, 0.517, 0.525, 0.532, 0.524, 0.516, 0.508, 0.515, 0.507, 0.515, 0.507, 0.514, 0.521, 0.514, 0.507, 0.514, 0.52, 0.513, 0.519, 0.513, 0.519, 0.525, 0.531, 0.524, 0.518, 0.512, 0.518, 0.523, 0.517, 0.523, 0.528, 0.533, 0.527, 0.533, 0.538, 0.532, 0.537, 0.531, 0.536, 0.531, 0.525, 0.52] Relative frequency list of Tails T: [1.0, 0.5, 0.667, 0.5, 0.4, 0.333, 0.286, 0.375, 0.444, 0.4, 0.364, 0.417,0.385, 0.429, 0.4, 0.375, 0.353, 0.389, 0.421, 0.4, 0.429, 0.455, 0.435, 0.417, 0.4, 0.423, 0.444, 0.464, 0.483, 0.467, 0.452, 0.438, 0.424, 0.412, 0.429,0.444, 0.459, 0.474, 0.487, 0.5, 0.488, 0.5, 0.512, 0.5, 0.511, 0.5, 0.489, 0.479, 0.49, 0.5, 0.49, 0.481, 0.472, 0.481, 0.491, 0.482, 0.491, 0.5, 0.492,0.483, 0.475, 0.468, 0.476, 0.484, 0.492, 0.485, 0.493, 0.485, 0.493, 0.486, 0.479, 0.486, 0.493, 0.486, 0.48, 0.487, 0.481, 0.487, 0.481, 0.475, 0.469, 0.476, 0.482, 0.488, 0.482, 0.477, 0.483, 0.477, 0.472, 0.467, 0.473, 0.467, 0.462, 0.468, 0.463, 0.469, 0.464, 0.469, 0.475, 0.48



[158]: plotting()

Number of tosses: 1000

Tosses list:

['H', 'T', 'T', 'H', 'T', 'T', 'T', 'H', 'T', 'H', 'T', 'H', 'T', 'H', 'T', 'H', 'H', 'H', 'T', 'H', 'T', 'H', 'H', 'T', 'H', 'T', 'H', 'H', 'T', 'H', 'T', 'T', 'T', 'Η', 'H', 'H', 'H', 'T', 'T', 'T', 'H', 'H', 'T', 'H', 'H', 'T', 'H', 'T', 'H', 'T', 'Τ', 'H', 'H', 'H', 'T', 'T'. 'T'. 'H'. 'H'. 'T', 'H', 'T', 'T', 'H', 'T', 'H', 'T', 'H', 'T', 'T', 'H', 'H', 'H', 'H', 'H', 'H', 'T', 'T', 'H', 'T', 'H', 'T', 'H', 'H'. 'H', 'H', 'T', 'H', 'H', 'T', 'T', 'H', 'T'. 'T'. 'H', 'T', 'T', 'T', 'H', 'T', 'H', 'T', 'T', 'H', 'H', 'T', 'Η'. 'T', 'T', 'H', 'T', 'T', 'T', 'H', 'H', 'H', 'H', 'H', 'T', 'T', 'H', 'T', 'T', 'T', 'H', 'T', 'H', 'H', 'T', 'H',

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'H', 'T', 'T',
            'T', 'H', 'T', 'T', 'H', 'T', 'H', 'T']
Relative frequency list of Heads H:
[1.0, 0.5, 0.333, 0.25, 0.4, 0.333, 0.286, 0.25, 0.333, 0.3, 0.273, 0.333,
0.308, 0.357, 0.333, 0.375, 0.353, 0.333, 0.316, 0.3, 0.286, 0.273, 0.304,
0.292, 0.28, 0.308, 0.333, 0.321, 0.31, 0.3, 0.323, 0.344, 0.364, 0.353, 0.343,
0.361, 0.378, 0.368, 0.385, 0.375, 0.39, 0.405, 0.395, 0.409, 0.4, 0.413, 0.426,
0.417, 0.429, 0.44, 0.431, 0.423, 0.415, 0.407, 0.4, 0.411, 0.404, 0.397, 0.407,
0.4, 0.41, 0.419, 0.429, 0.438, 0.446, 0.439, 0.448, 0.441, 0.435, 0.429, 0.437,
0.444, 0.452, 0.459, 0.467, 0.474, 0.481, 0.474, 0.468, 0.463, 0.457, 0.463,
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Relative frequency list of Tails T:
[0.0, 0.5, 0.667, 0.75, 0.6, 0.667, 0.714, 0.75, 0.667, 0.7, 0.727, 0.667,
0.692, 0.643, 0.667, 0.625, 0.647, 0.667, 0.684, 0.7, 0.714, 0.727, 0.696,
0.708, 0.72, 0.692, 0.667, 0.679, 0.69, 0.7, 0.677, 0.656, 0.636, 0.647, 0.657,
0.639, 0.622, 0.632, 0.615, 0.625, 0.61, 0.595, 0.605, 0.591, 0.6, 0.587, 0.574,
0.583, 0.571, 0.56, 0.569, 0.577, 0.585, 0.593, 0.6, 0.589, 0.596, 0.603, 0.593,
0.6, 0.59, 0.581, 0.571, 0.562, 0.554, 0.561, 0.552, 0.559, 0.565, 0.571, 0.563,
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