E9 213 Time-Frequency Analysis - Assignment 1

Submission Deadline: September 14, 2025, 11:59 PM

Instructions

- Give concise answers.
- Use either **MATLAB** or **Python** to solve the programming problems. Comment your code appropriately to enhance readability.
- Plots must be clearly labelled with titles, scale, and axes labels.
- For Python Coders: Submit a single Jupyter Notebook named `E9_213_A1_FirstNameLastName_Code.ipynb` and delineate the code for each question in separate, clearly labeled cells. Upload only the the Notebook (`.ipynb`) and report (`.pdf`) via Teams before the deadline.
- For Matlab Coders: For each problem, create a corresponding file named `run_Problem1.mat`, `run_Problem2.mat`, etc. Submit a single zipped folder named `E9_213_A1_FirstNameLastName.zip`, containing all scripts and the report (`.pdf`), via Teams before the deadline.
- Submit the report with all the results, such as images or numerical outputs, along with your assumptions, analytical computations, observations and conclusions.
- Name your report as `E9_213_A1_FirstNameLastName_Report.pdf`.
- Use of AI tools such as ChatGPT to solve this assignment will result in zero marks.
- Resorting to unfair means such as copying will result in zero marks.

1. Fundamentals of Fourier — 10 Points

Consider the function $f_1(t) = e^{-\alpha|t|}$, for $\alpha > 0, t \in \mathbb{R}$.

i) Does f_1 belong to $L^2(\mathbb{R})$? **(1 p)**

ii) Determine the Fourier Transform, $\hat{f}_1(\omega)$.

iii) Given $\hat{f}_2(\omega) = e^{-\alpha|\omega|}$, $\alpha > 0, \omega \in \mathbb{R}$, does $\hat{f}_2$ belong to $L^2(\mathbb{R})$? **(1 p)**

iv) Find $f_2(t)$, which is the Inverse Fourier Transform of $\hat{f}_2(\omega)$, using the following methods: **(4 p)**

   a) Using the Duality property.

   b) Using Fourier transform properties other than the Duality property.

v) Does $f_2(t)$ qualify to be a density function? **(2 p)**

   a) If yes, give reason. Furthermore, does it qualify as a probability density function?

   b) If not, explain why not. **(1 p)**

2. Simulation of Plots — 12 Points

Sketch the time-domain plot and the frequency spectrum (Magnitude and Phase) for the following signals. Cross-validate these plots using either Matlab or Python.

i) $\text{sinc}(2\pi t)$ **(3 p)**

ii) $(\delta(t-1) + \delta(t+1)) * e^{-at^2}$ **(3 p)**

iii) $(\delta(t-1) + \delta(t+1)) \times e^{-at^2}$ **(3 p)**

iv) $e^{-at^2}\cos(\omega_0 t)u(t)$, for $a > 0, a \in \mathbb{R}$,
$u(t)$ denotes the unit-step function. **(3 p)**

3. Discrete Fourier Transform (DFT) — 6 Points

Write your own DFT function `myDFT(x,N)`.
Compute the DFT of the following using myDFT and a library function ($n \in \mathbb{Z}$). **(6 p)**

a) $\delta[n]$

b) $x[n] = \begin{cases} 1, & |n| \leq 2 \\ 0, & \text{otherwise} \end{cases}$

c) $2^{-|n|}$

4. Learning Learning — 2 Points

Summarize your learnings from 1. to 3. in six sentences. **(2 p)**