E9 222 Signal Processing in Practice
Assignment 6

February 16, 2026

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1. Constant k – High Boost Filtering

Ans:

Given an image $f(m,n)$, the high-boost sharpened image is

$$g(m,n) \;=\; f(m,n) \;+\; k\left[f(m,n) * h(m,n)\right],$$

where h is the Laplacian high-pass kernel

$$h = \begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix},$$

and k is a constant gain factor. The output is clipped to $[0, 255]$ to handle saturation.

Three constant values of k were tested: $k \in \{1, 2, 4\}$. As k increases, edges become progressively more emphasized. However, a large constant k also amplifies noise in smooth/flat regions of the image, since the Laplacian response in those regions – though small – gets scaled up uniformly.

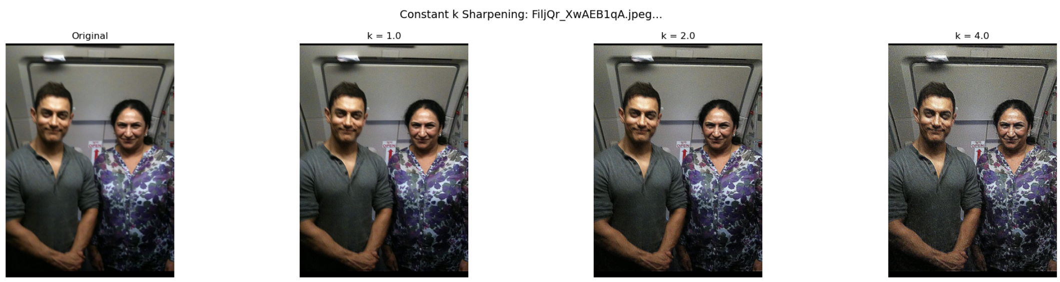

Figure 1: Constant k sharpening on Image 1 (portrait). From left to right: original, $k = 1$, $k = 2$, $k = 4$.

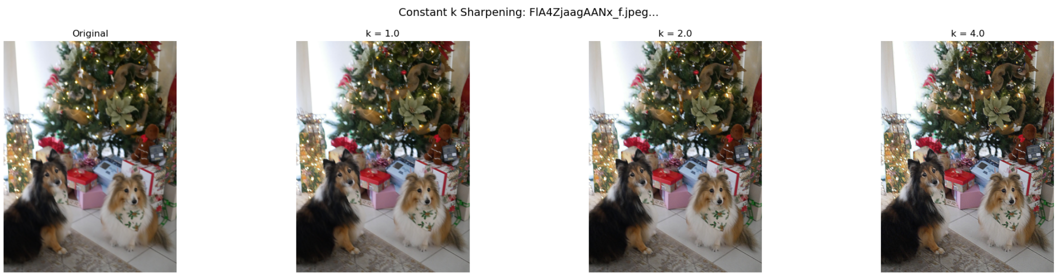

Figure 2: Constant k sharpening on Image 2 (dog with Christmas tree). From left to right: original, $k = 1$, $k = 2$, $k = 4$.

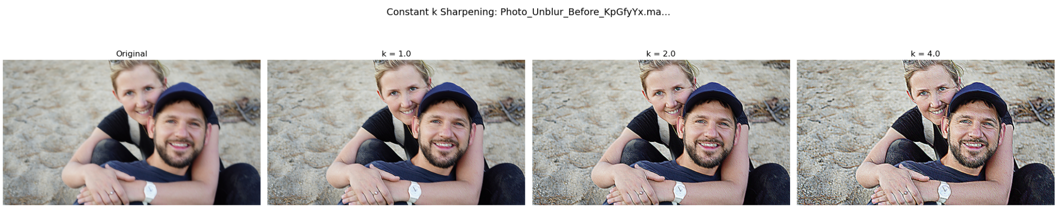

Figure 3: Constant k sharpening on Image 3 (couple on beach). From left to right: original, $k = 1$, $k = 2$, $k = 4$.

As seen in the figures, increasing k sharpens edges more aggressively, but at $k = 4$ significant artifacts and noise amplification become visible, particularly in smooth regions such as skin and sky.

2. Spatially Varying $k(m, n)$ – Adaptive High Boost Filtering

Ans:

Motivation

A constant k applies the same gain everywhere. This is sub-optimal because:

- **Flat / smooth regions** (very small $|f * h|$): The Laplacian response here is predominantly noise. Amplifying it with a large k would boost noise without improving perceptual sharpness.

- **Moderate-gradient regions** (medium $|f * h|$): These correspond to weak or blurred edges – precisely the regions that benefit most from sharpening. Here, k should be large to enhance edge contrast.

- **Strong edges** (large $|f * h|$): These edges are already perceptually sharp. Applying a large k here risks overshooting – creating halo artifacts and ringing around edges.

We therefore need a spatially varying gain $k(m, n)$ that is *small for noise*, *large for weak edges*, and *moderate-to-small for strong edges*.

Designed k Function

Let $L(m, n) = f(m, n) * h(m, n)$ denote the Laplacian output. The adaptive gain is defined as:

$$\boxed{k(m,n) \;=\; k_{\max} \cdot \underbrace{\frac{|L(m,n)|}{|L(m,n)| + T_1}}_{\text{noise suppression}} \cdot \underbrace{\exp\!\left(-\frac{|L(m,n)|}{T_2}\right)}_{\text{strong-edge roll-off}}}$$

This function is a product of two factors:

1. **Sigmoid-like noise gate** $\dfrac{|L|}{|L| + T_1}$: This factor is ≈ 0 when $|L| \ll T_1$ (noise regime) and saturates to ≈ 1 when $|L| \gg T_1$. It effectively suppresses gain in flat regions where the Laplacian magnitude is dominated by noise, preventing noise amplification.

2. **Exponential decay** $\exp\big(-|L|/T_2\big)$: This factor is ≈ 1 for small-to-moderate $|L|$ and decays toward 0 for large $|L|$. It reduces the gain at strong edges, preventing overshoot and halo artifacts.

The combined curve rises from 0 (noise), peaks at moderate Laplacian magnitudes (weak/blurred edges), and then decays for strong edges – exactly the desired behaviour. The parameter $k_{\max}$ controls the overall maximum gain (set to 7.5 in our experiments).

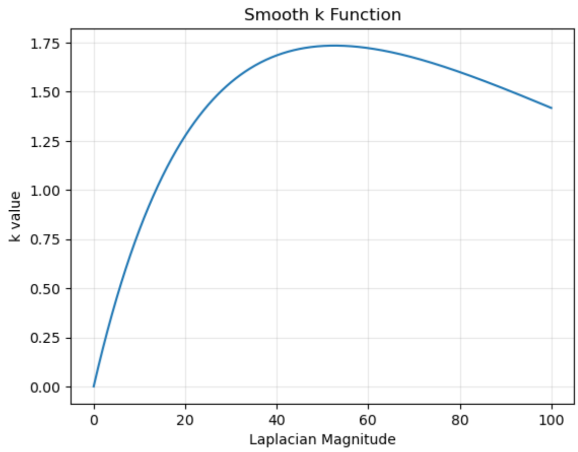

Figure 4: The designed k as a function of $|L|$. The curve rises from zero (suppressing noise), peaks at moderate Laplacian magnitudes (boosting weak edges), and decays for large magnitudes (avoiding overshoot at strong edges).

Selection of T_1 and T_2

Both thresholds are estimated adaptively from each image (per channel):

- $T_1 = \hat{\sigma} = \dfrac{\text{median}(|L|)}{0.6745}$: This is the **Median Absolute Deviation (MAD)** estimator of the noise standard deviation. The factor 0.6745 normalizes the median of $|L|$ so that $\hat{\sigma}$ is a consistent estimator of σ for Gaussian noise. Setting $T_1 = \hat{\sigma}$ means the sigmoid gate transitions around the noise level – Laplacian magnitudes below the noise floor get suppressed, while those above it pass through. This is a robust, data-driven noise threshold.

- $T_2 = P_{90}(|L|)$ (the 90^{th} percentile of $|L|$): This sets the decay scale at the strong-edge boundary. Only the top 10% of Laplacian magnitudes (the strongest edges) experience significant decay. This ensures that moderate edges – the primary targets of sharpening – receive full gain, while only the most prominent edges are attenuated to avoid artifacts. Using a percentile makes T_2 adaptive to image content.

Results: Variable k vs. Constant k

Figures 5–7 compare the adaptive variable-k result ($k_{\max} = 7.5$) against a constant-k result ($k = 2$) for each image. The $k(m, n)$ heatmap is also shown, averaged over RGB channels.

Figure 5: Image 1 (portrait): Original, variable-k result, $k(m,n)$ heatmap, constant $k = 2$.

Figure 6: Image 2 (dog with Christmas tree): Original, variable-k result, $k(m,n)$ heatmap, constant $k = 2$.

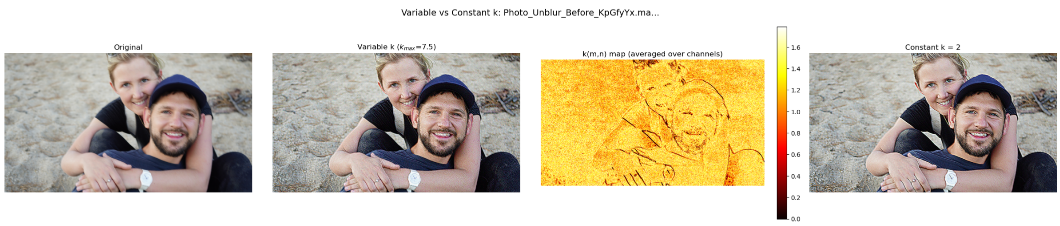

Figure 7: Image 3 (couple on beach): Original, variable-k result, $k(m,n)$ heatmap, constant $k = 2$.

Interpreting the $k(m,n)$ Heatmap

The $k(m,n)$ heatmaps confirm the intended behaviour of the adaptive gain:

- **Dark regions** (low k) appear in smooth/flat areas (e.g., background walls, sky, skin) where the Laplacian response is primarily noise. The noise-gate term suppresses the gain here, preventing noise amplification.

- **Bright regions** (high k) appear along moderate-contrast edges and textured regions (e.g., hair, clothing folds, facial features, foliage). These are the weak/blurred edges that benefit most from sharpening.
- **Strong edges** (e.g., sharp object boundaries) have moderate k values – not the highest – because the exponential decay term attenuates the gain there, preventing halo artifacts.

Comparison

The variable-k sharpened images exhibit cleaner smooth regions (less noise than constant k) while still achieving strong edge enhancement. In contrast, the constant-k result ($k = 2$) either under-sharpens weak edges (if k is too low) or amplifies noise in flat regions (if k is too high). The spatially adaptive approach provides a better trade-off: it can use a higher effective $k_{\max}$ where needed (at weak edges) without the penalty of global noise amplification.