HW 7

Derek Walker

1:

(a):

want $p(x) = c_n + c_{n-1}x + c_{n-2}x^2 + \cdots$

$$V \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix} = \begin{bmatrix} c_n + c_{n-1}x_1 + c_{n-2}x_1^2 + \cdots \\ \vdots \end{bmatrix}$$

$$\boxed{V = \begin{bmatrix} x_1^{n-1} & x_1^{n-2} & \cdots & x_1 & 1 \\ \vdots & & \ddots & \vdots & \vdots \\ x_n^{n-1} & \cdots & & x_n & 1 \end{bmatrix}}$$

Check!

$$Vc = \begin{bmatrix} x_1^{n-1} & \cdots & x_1 & 1 \\ \vdots & \ddots & \vdots & \vdots \\ x_n^{n-1} & \cdots & x_n & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix}$$

$$= \begin{bmatrix} c_1 x_1^{n-1} + \cdots c_{n-1}x_1 + c_n \\ \vdots \\ c_1 x_n^{n-1} + \cdots c_{n-1}x_n + c_n \end{bmatrix}$$

$\Rightarrow$ V correct form if you want the coefs to go in reverse order like $p(x)$

(b):

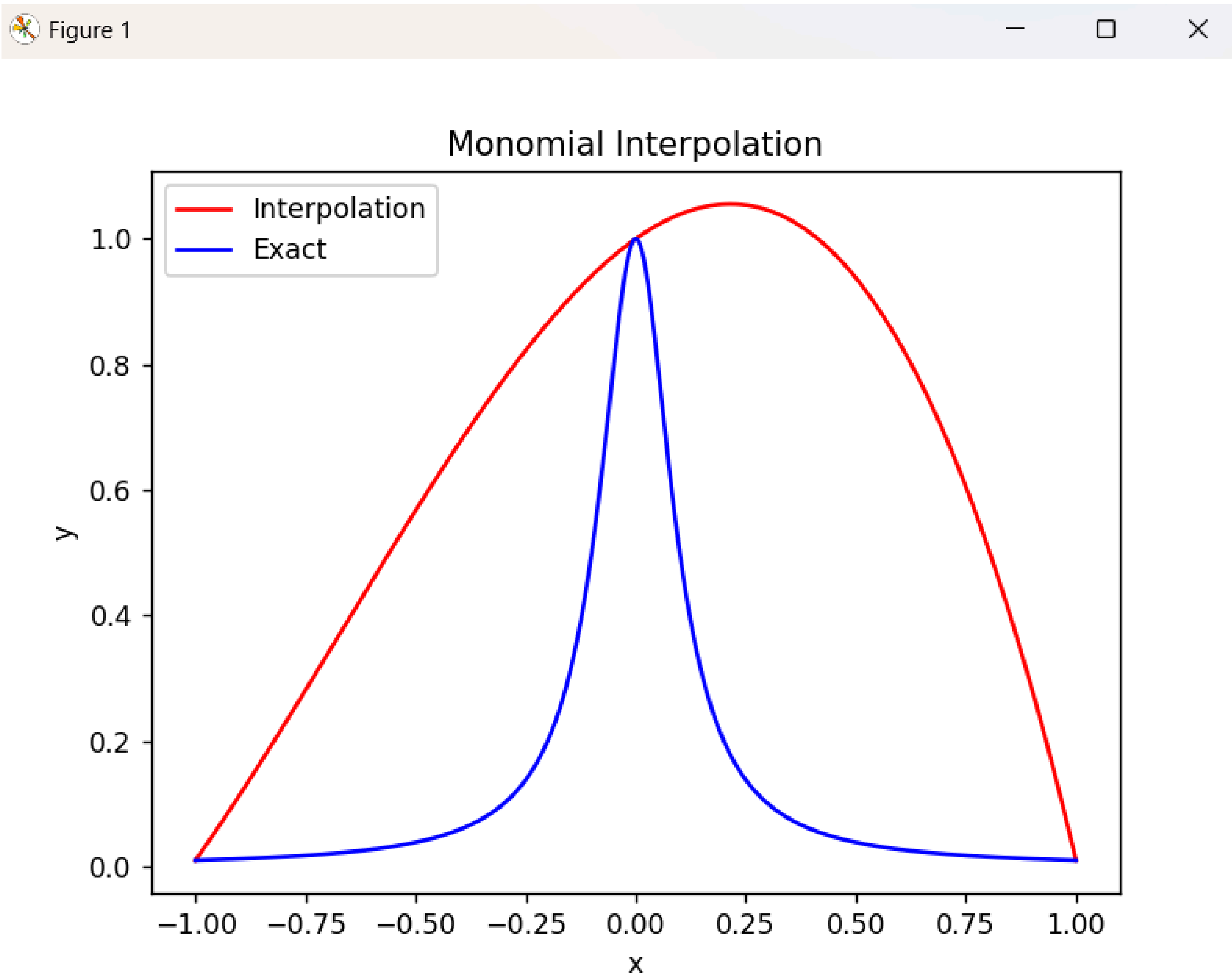
Figure 1
Monomial Interpolation
Interpolation
Exact
1.0
0.8
0.6
0.4
0.2
0.0
y
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00
x

Figure 1: N = 3

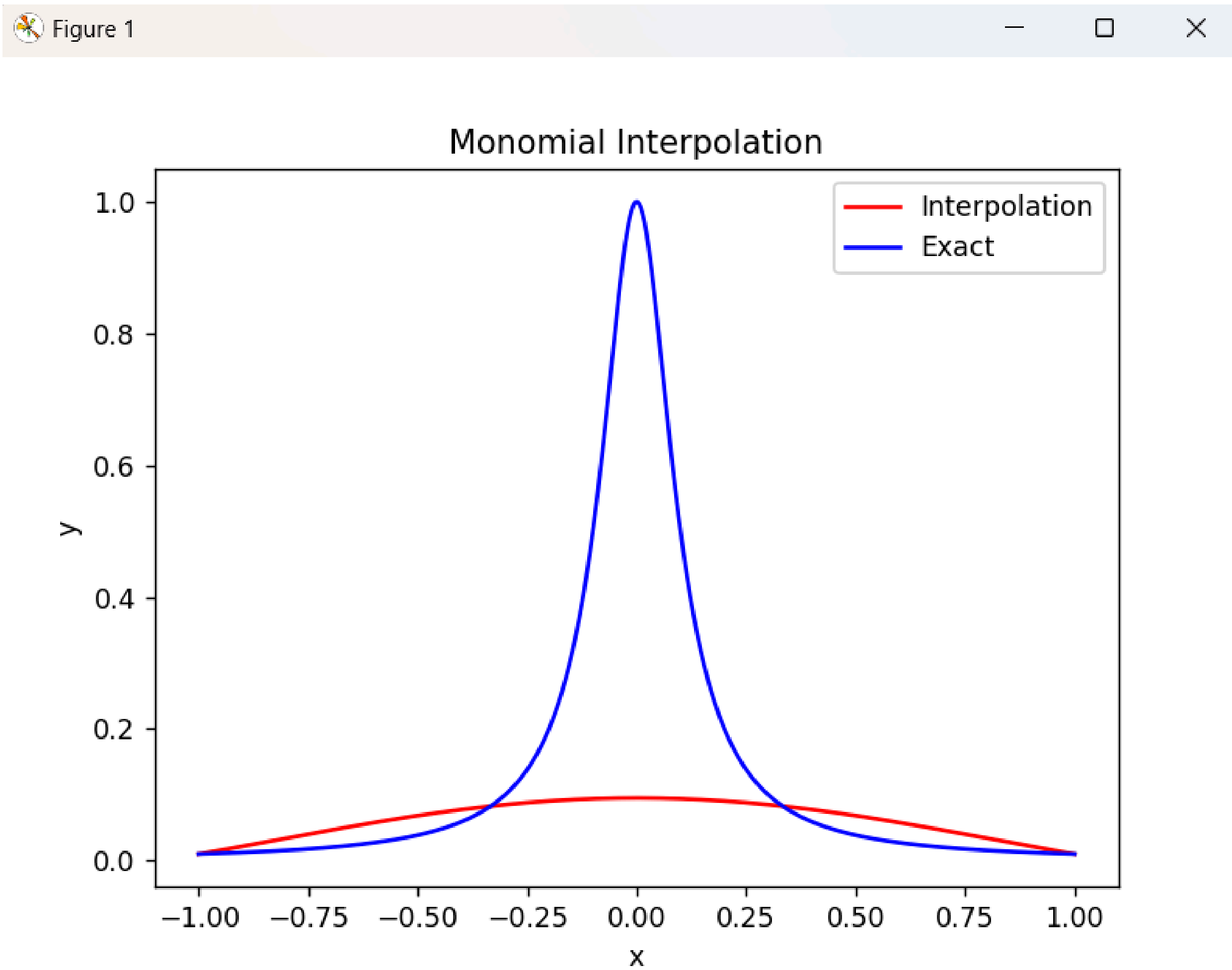

Figure 1
Monomial Interpolation
1.0
0.8
0.6
0.4
0.2
0.0
y
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00
x
Interpolation
Exact

Figure 2: N = 4

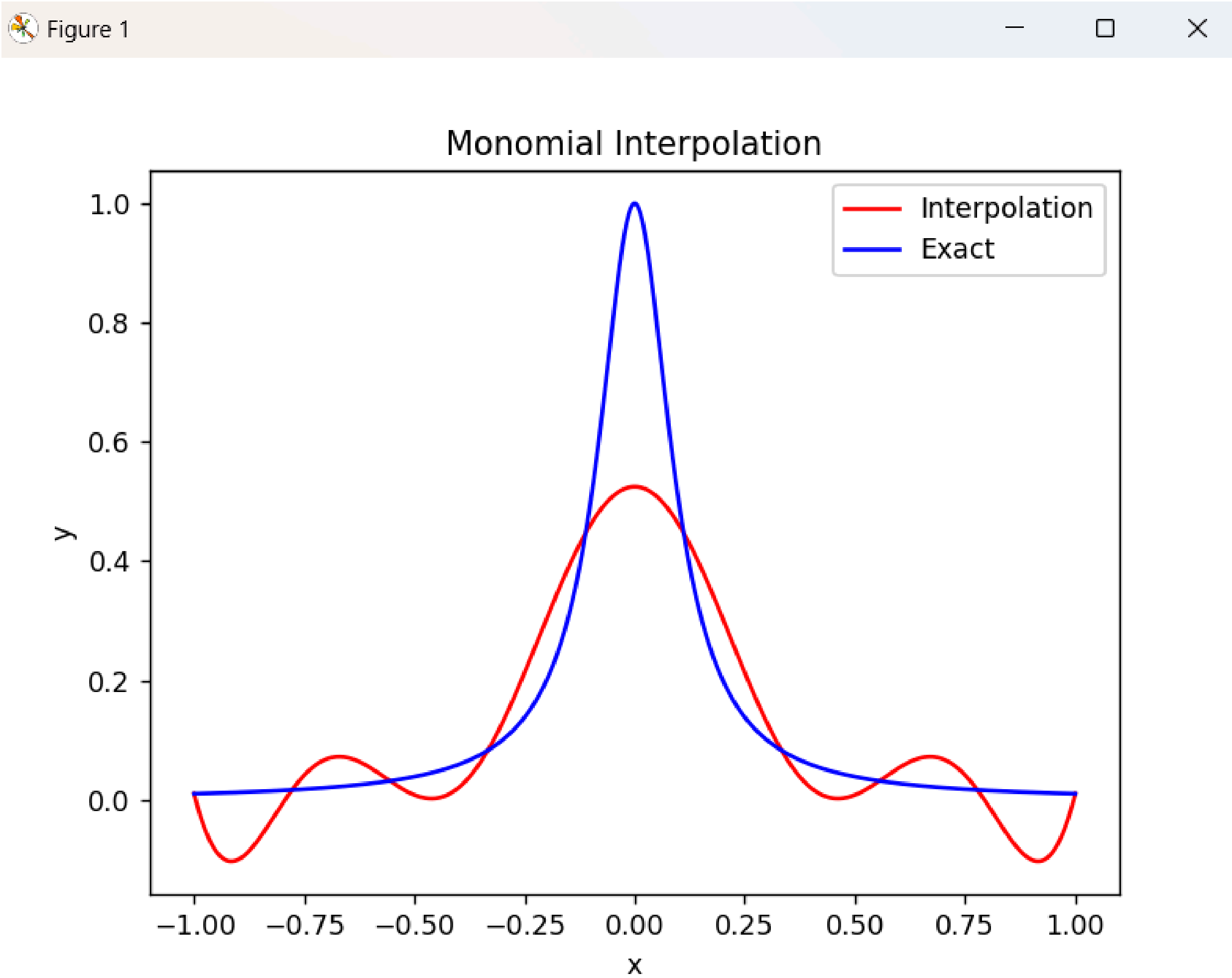

Figure 1
Monomial Interpolation
Interpolation
Exact
1.0
0.8
0.6
0.4
0.2
0.0
y
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00
x

Figure 3: N = 10

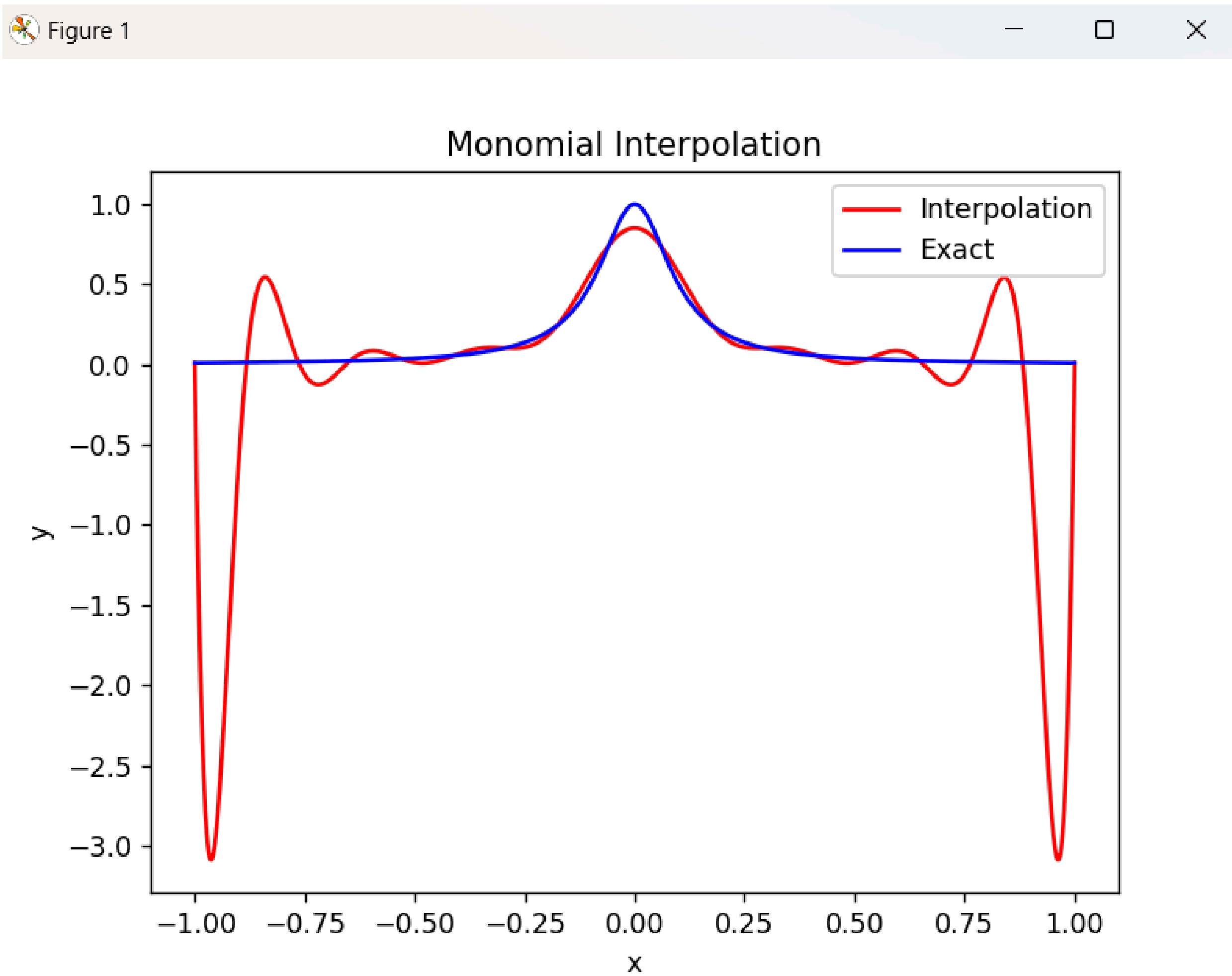

Figure 4: N = 18

The interpolant better and better assumes the form of the original function. In other words, the interpolation becomes smoother due to the increase in data. Runge's phenomenon can be seen at the endpoints clearly with N = 18, but not so much with lower N values.

2:

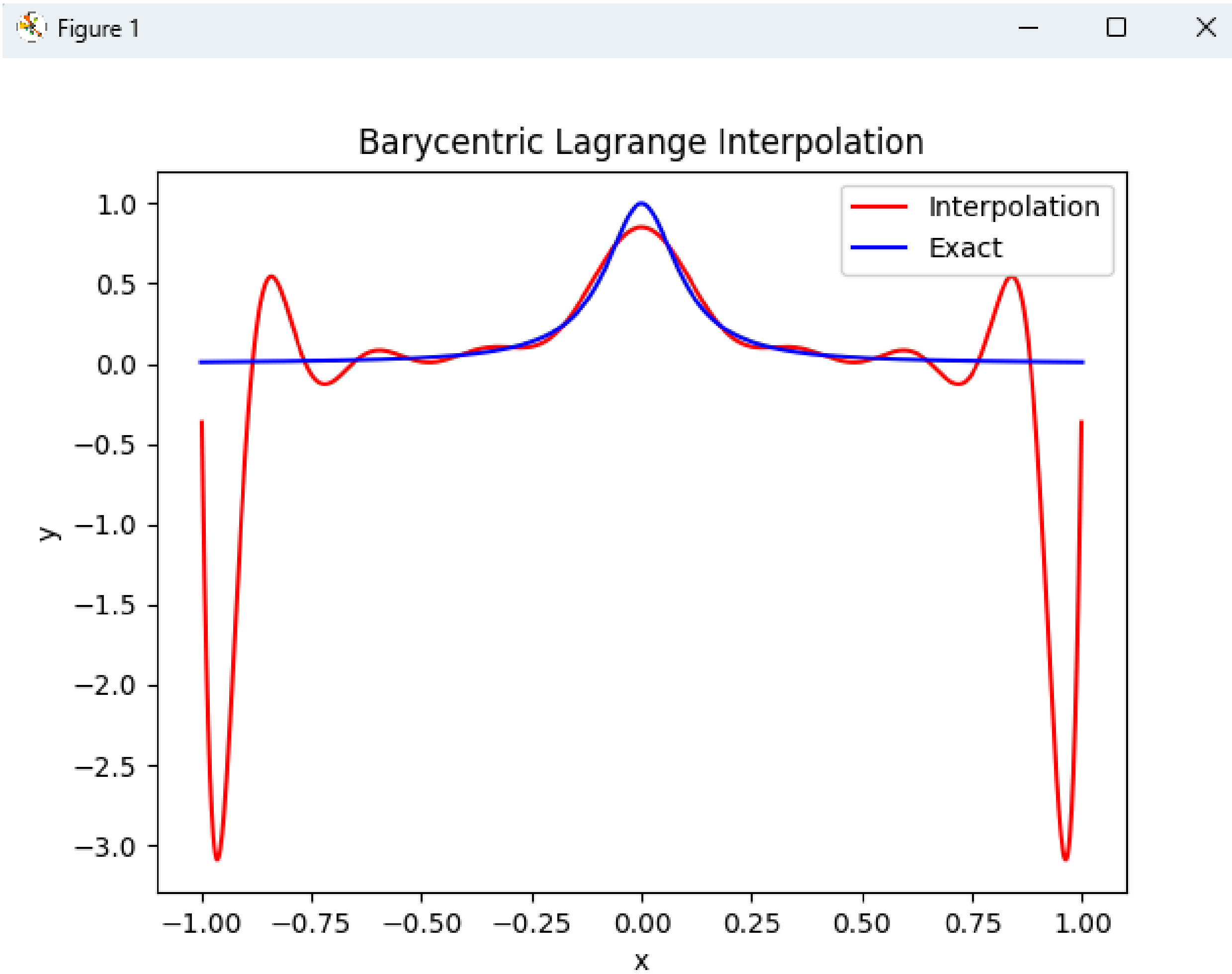

Figure 5: Barycentric Lagrange interpolation with N = 18

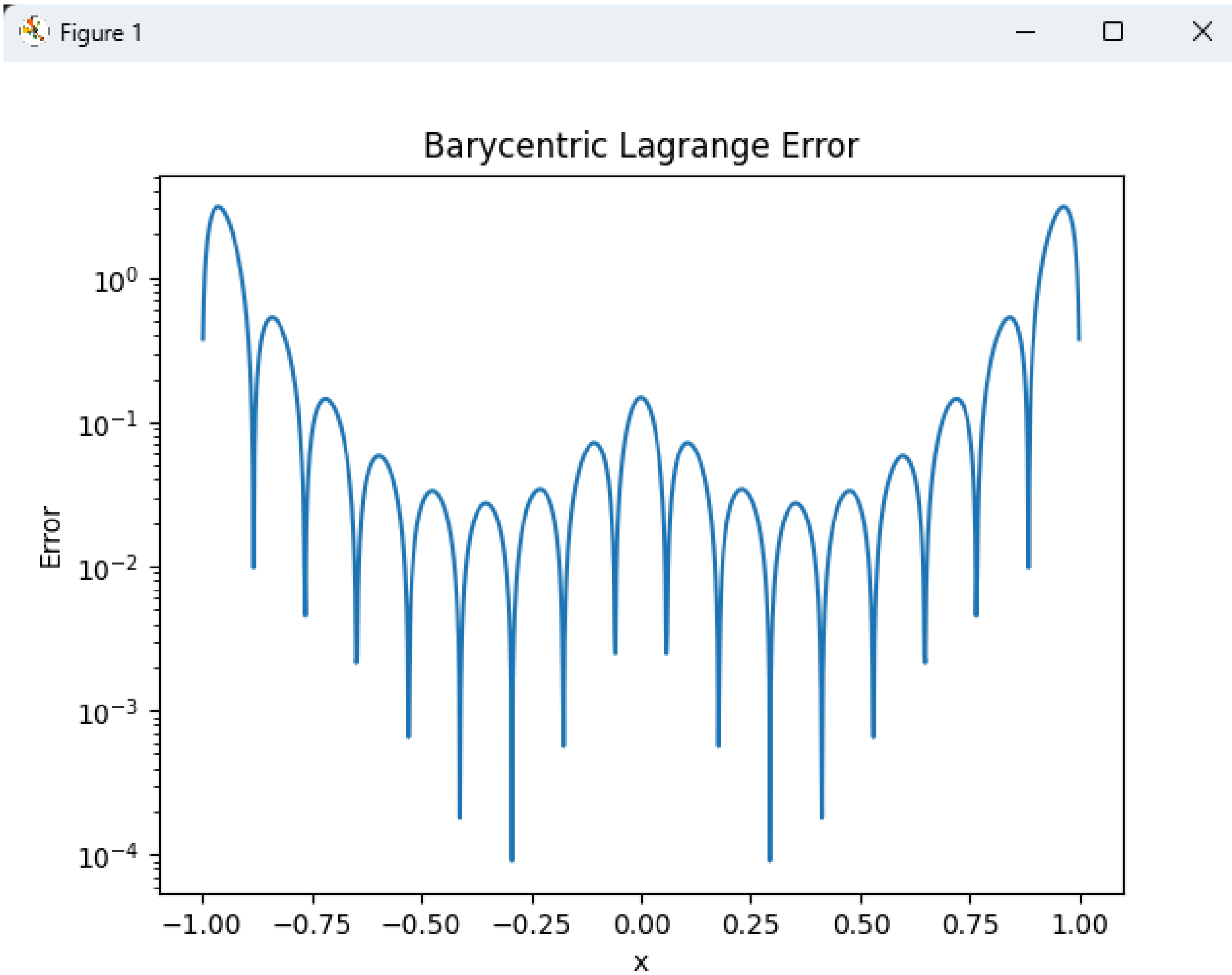

Figure 6: Barycentric Lagrange error with N = 18

For small x and large N, the interpolation is well behaved similar to monomial expansion. But now condition number is less.

3:

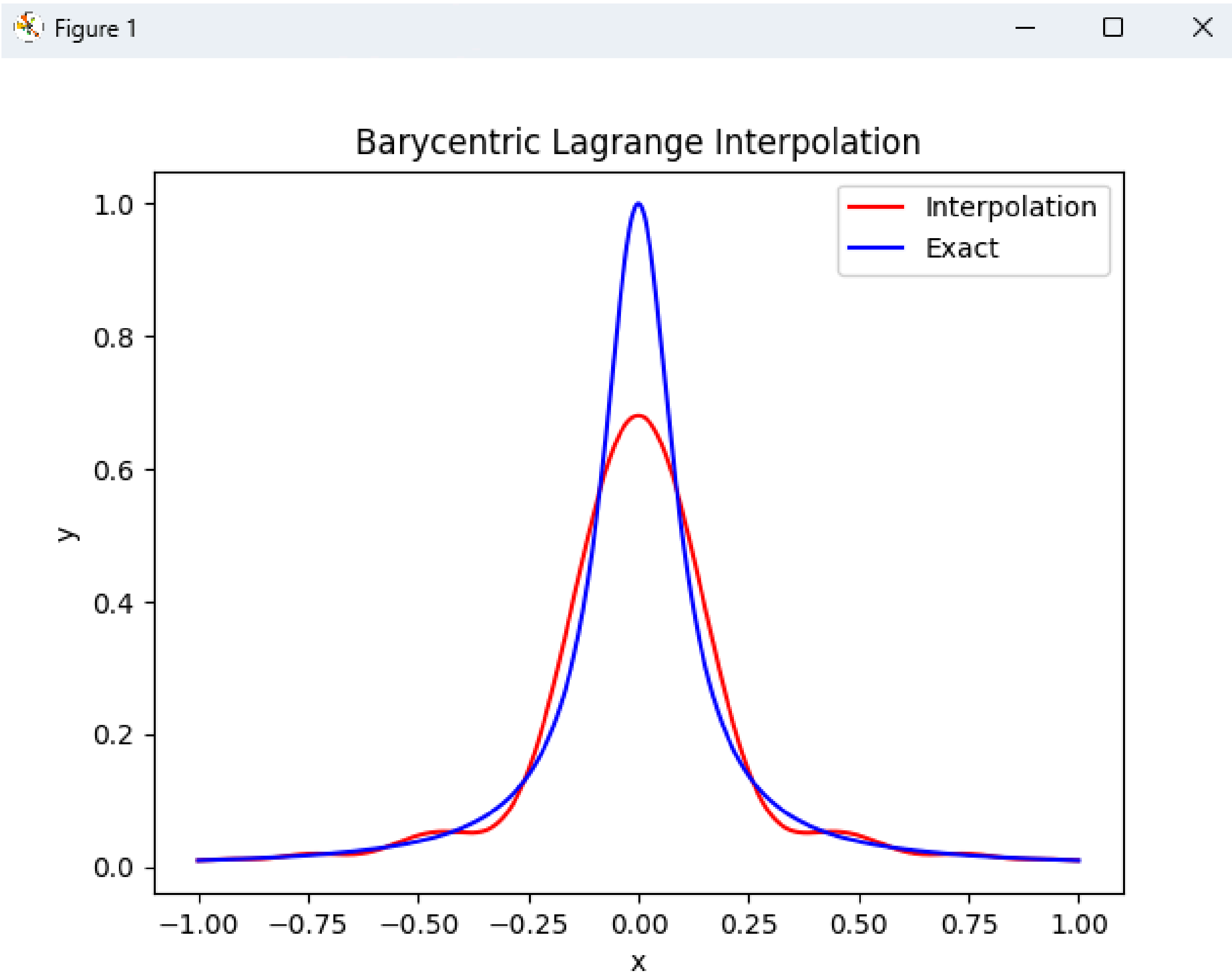

Figure 7: Barycentric Lagrange with Chebyshev nodes. N = 18

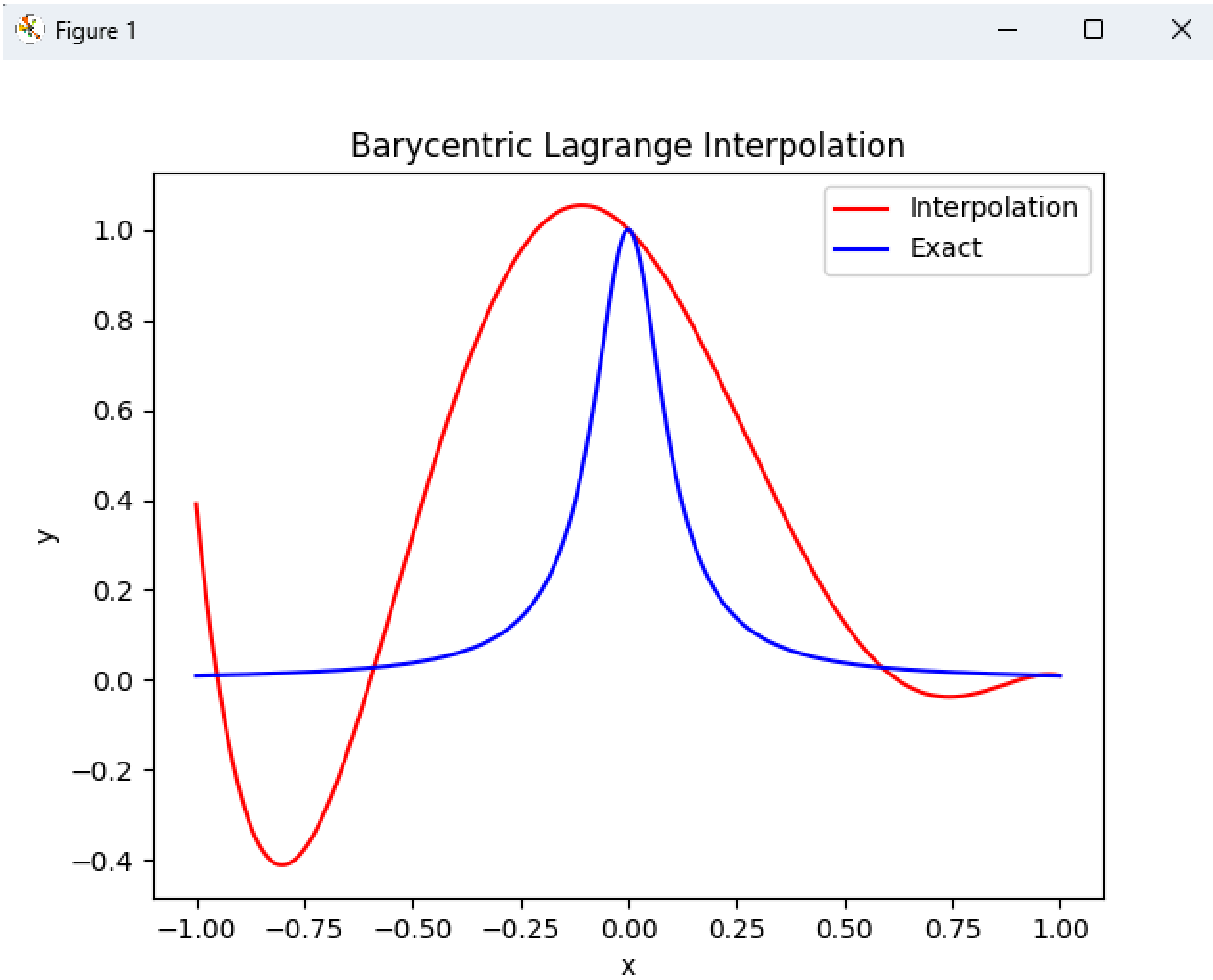

Figure 8: Barycentric Lagrange with Chebyshev nodes. N = 5

With low enough N values, the endpoints can fail. But with high N values, I cannot get it to fail. This is due to the extra nodes at the endpoints restricting the interpolation more.