# Simulation of road traffic

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## Abstract

In this paper we propose a stochastic cellular automaton model to simulate road traffic flow for straight and circular roads. We present a fundamental layout of the proposed model, and others with additional features of multiple lanes and traffic lights. For our analysis, we study the behaviour of traffic over time, and measure flow rates with respect to parameters such as car density, speed limits, and traffic light change frequency. We demonstrate that our model is useful for predicting real world traffic flow behaviour, and how a maximum ideal flow rate can be achieved given a combination of fixed and variable parameters.

## 1. Introduction

A cellular automaton is a computational system composed of discrete elements called cells, forming an n-dimensional lattice. Rules that are uniformly applied to all cells determine the evolution of the system over discrete time steps. Each cell's "state" (usually given by a one or a zero) changes according to the previous state and the rules, which often involve the cell's closest neighbours known as a Neighbourhood [1].

Cellular automata were invented in the 1940s by the scientists Stanislaw Ulam and John von Neumann for researching the growth of crystals and a world of self-replicating robots, respectively. The simplicity of the concept and code behind most cellular automata made these devices relevant for running simulations in various fields of study [2].

The most popular version of a cellular automaton is John Horton Conway's Game of Life. The Game of Life is a zero-player game (meaning that the only input required is the initial state of the grid) used mainly in studies of biology and sociology.

Another relevant application for cellular automata is the modelling of traffic flow. In particular, the application of non-linear dynamics to traffic models highlighting phase transitions from laminar flow to waves that start and stop, as traffic density increases. As a result, better traffic control systems have already been installed in Germany [3].

In this project a zero-player 1D cellular automaton is used to simulate traffic flow with the intention of maximizing traffic flow for a given car density, finding optimal speed limits, and comparing the efficiency of traffic light switching for maintaining steady circulation.

## 2. Theory

We can assume road traffic is theoretically deterministic since drivers want to maintain the highest possible speed while abiding by the speed limit, and proximity to their nearest neighbouring vehicles. These conditions therefore allow us to approximately model a distribution of cars on a road via cellular automaton and a simple ruleset and analyse its evolution through time.

Our simplest system model consists of a single lane road populated with identical cars. We use the Nagel-Schreckenberg (NaSch) model (2), since it is simple yet nontrivial. The road is defined by a discrete one-dimensional array of length $L$ with open or periodic boundary conditions. Each element may be occupied by one car with integer velocity $v$ where $0 \leq v \leq v_{max}$, or may be empty. Distance is measured as the number of elements between two points. An arbitrary configuration of these variables represents one state of the system. The next state is calculated from 3 steps performed for all cars in parallel. For the $i$th car:

Step 1 states that if $v_i \leq v_{max}$ and the distance $d$ to the next occupied element satisfies $d > v_i + 1$, $v_i \rightarrow v_i + 1$. This is equivalent to accelerating when there is enough space ahead.

Step 2 states that if $v_i > 0$ and $d \leq v_i$, $v_i \rightarrow d - 1$. This is equivalent to deceleration when there is insufficient space ahead.

Step 3 states that if $v_i > 0$, $v \rightarrow v_i - 1$ with probability $p_{slow}$. This is equivalent to random deceleration at any given time. This is to simulate imperfect driving stochastically, introducing randomness and realism into an otherwise deterministic process.

Step 4 states that the car moves ahead by $v_i$ elements. This completes one step cycle.
Time $t \rightarrow t + 1$.

The cycle continues until $t = T$, the total time period. The boundary conditions define the type of road and the variable model parameters. Both have domain $road[0] \leq x \leq road[L-1]$, $x \in \mathbb{Z}^+$, where $road[i]$ gives the array element and $x$ is the position. These will be discussed in later sections. The key value we want to find for each model however is the time averaged flow rate $\bar{q}$ , the number of cars passing a certain point $i$ averaged over a simulation time period $T$. This is given by

$$\bar{q} = \frac{1}{T}\sum_{t=1}^{t=T} n_i(t), \tag{1}$$

where $n_i(t)$ is the number of cars that pass $road[i]$ for a given $t$. For our models, we take $i$ to be the end of the array ($road[L-1]$) and look to maximise $\bar{q}$ with respect to their parameters. The general parameters are $v_{max}$, $p_{slow}$ and $T$. The model specific parameters will be discussed next.

# 3. Proposed models

## 3.1 Straight and circular roads

Boundary conditions define the type of road our model simulates. Open boundaries (OBs) define a section of straight road, allowing cars to leave and new cars to join. The initial road state is empty, and populating the road happens while the simulation runs. Cars are spawned at $road[0]$, if the site is vacant, with a certain velocity $v_{spawn}$, where $0 \leq v_{spawn} \leq v_{max}$, and leave once their index is greater than $road[L-1]$. The spawn rate is given by $r_{spawn}$.

Periodic boundaries (PBs) define a closed circular road with the condition $road[L+i] = road[i]$, allowing cars to loop around the array. Since no new cars can join or leave, the initial road state is populated randomly following a uniform distribution with density $\rho$, defined as

$$\rho = \frac{N}{L} = \frac{number\ of\ cars\ on\ road}{length\ of\ road}. \tag{2}$$

Each car is given a random initial velocity $v_{init}$ where $0 \leq v_{init} \leq v_{max}$

## 3.2 Traffic lights and multiple lanes

Traffic lights can be placed in series at road sites for a straight or circular road. They have an on-off light switch rate $r_{light}$. When a certain traffic light is off, or 'green', cars pass its site as usual. When on, or 'red', cars will not pass its site, and react like it's a stationary car. Multiple traffic lights can run in-phase or out of phase, and their direction of propagation may be with or against the flow of traffic.

For a straight 3-lane model, we create three adjacent single lane models and introduce new traffic laws. Cars only overtake from the left, and switch lanes if the car ahead is too close and if the adjacent lane is free.

By combining these features and road types, we can observe many scenarios. We will select those which have practical purposes on roads, such as increasing traffic flow, identifying causes of congestion. We will also see if any physical phenomena evolve out of our models.

## 4. Results and analysis

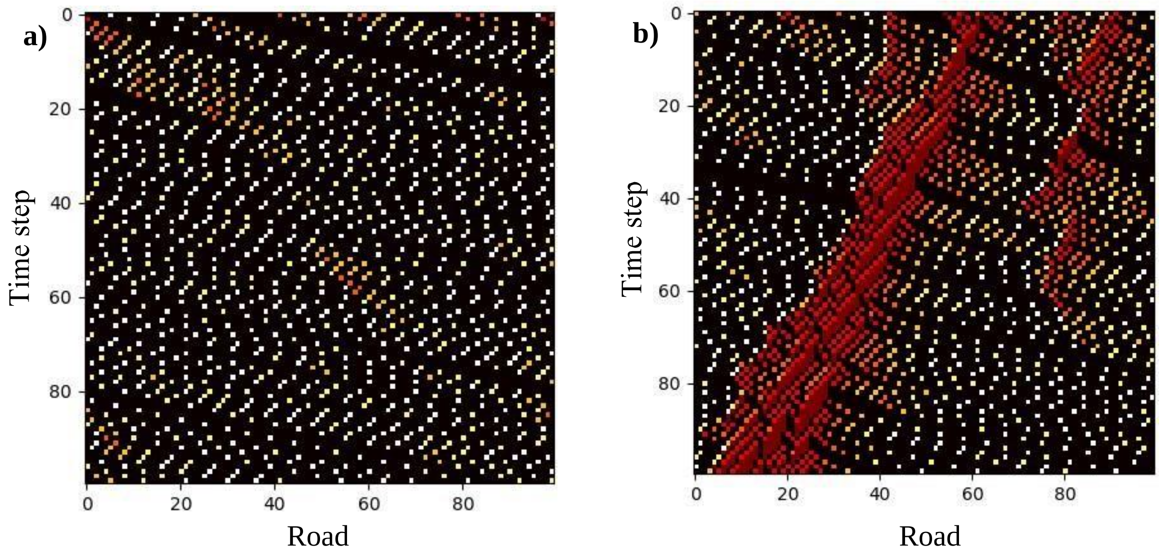


**Figure 1**: Heat maps depicting distributions of cars along a periodic lane of length 100, over 100 consecutive time steps. The darker the element, the slower the speed (white element is a car at $v_{max}$, darkest red is a car at $v = 0$). A black element is a vacant element. For a), $\rho = 0.1$. For b), $\rho = 0.2$.

Our first set of simulations were to test traffic behaviour on periodic roads. We initially fixed $v_{max} = 5$, $p_{slow} = 0.3$, and varied the site density $\rho$ per simulation. As shown in Figure 1 a), we found that traffic flow is laminar and congestion free at a low $\rho$. Since distance between neighbours is large, cars could maintain $v_{max}$ with occasional fluctuations due to $p_{slow}$. However, as $\rho$ increased, congestion clusters formed randomly from the velocity fluctuations. These small jams could quickly dissipate, as seen on the far right of road b) between time steps 0 and 20, since the rate of cars leaving the jam was faster than the rate of approaching cars from behind. If their rates were equal or otherwise, we saw a persistent traffic jam that moves backwards over time, as seen in b). This caused all cars to approach the jam at near top speed, become stuck, and accelerate back to top speed once it had left. Therefore, this cluster represented a start-stop-wave typically found in real freeway traffic, where many independent moving traffic jams are present. We could see that the jam in our simulation seemed to also increase in breadth over time as well, representing the idea that even one small incident can have a cascading knock-on effect that results in long, static traffic jams on fast moving motorways.

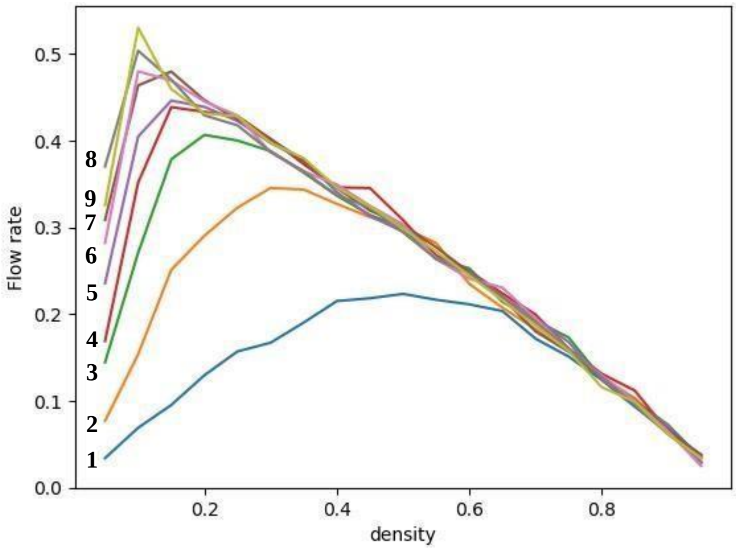


**Figure 2**: Fundamental diagram depicting the relationship between the flow rate $\bar{q}$ and density $\rho$ for a series of different $v_{max}$, given by the numbers at the start of their respective curves.

For our analytical treatment of the simulations, we obtained the curves shown in Figure 2. For each data point, we ran the simulation 20 times and took an average $\bar{q}$ for accuracy. One trivial observation was that $\bar{q}$ generally increased with $v_{max}$ up till a certain point for a given $\rho$, since faster cars meant a quicker rate of rotation around the road. This represented the difference between motorways, Aroad and B-road, with the faster speed limits allowing a greater rate of traffic flow. We could also see that the $\bar{q}$ peak shifted toward higher $\rho$ as $v_{max}$ decreased, showing that roads with lower speed limits rely on a greater road density to ensure optimum traffic flow, while faster roads like motorways rely on speed. As $\bar{q}$ approached its max, the road still showed signs of laminar flow and large gaps, and as $\bar{q}$ left its max, over-congestion resulted in frequent jams. We can see the curve for $v_{max} = 5$ peaked at around $\rho = 0.2$, which were the variables used for Figure 1.

A key aspect of Figure 2 was the convergence of flow rates along the diagonal line on the graph. For a given $\rho > 0.15$, $v_{max}$ didn't seem to affect $\bar{q}$ differently above a certain point. At around $\rho = 0.8$, $\bar{q}$ for all $v_{max}$ varied equivalently with $\rho$. This behaviour demonstrates why speed limits are often reduced on motorways when there is heavy congestion. Traffic flow will not be affected, and more laminar flow will resurface to avoid needless start-stop-waves, thus reducing chances of accidents and future jams.

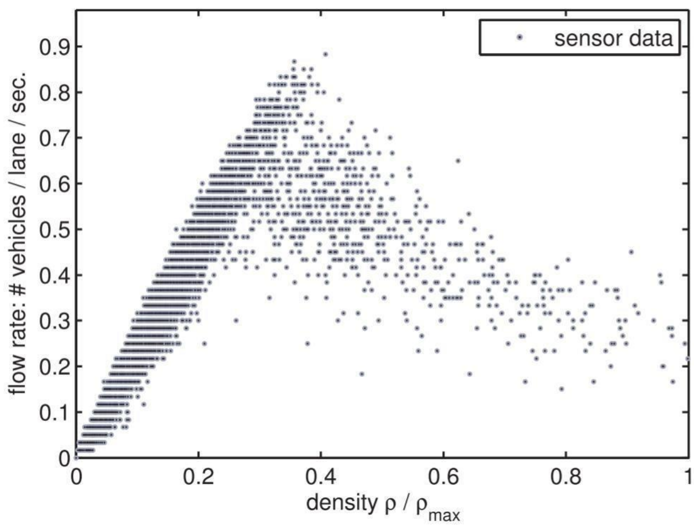


**Figure 3**: Fundamental diagram obtained from real traffic data, reproduced from [4]

Figure 3 shows a fundamental diagram generated from real sensor measurement data that was aggregated over 30-second intervals. It successfully compared with the averaged data obtained in Figure 2, showing a steep climb toward a max, and a steady drop off. The region for low $\rho$ is characterized by a relatively well defined single valued curve aside from some noise, which compared with the laminar undisrupted flow map in Figure 1 a). In the region for medium to high $\rho$ a multi valued flow region emerges, which compared with the congested, fluctuating flow map in Figure 1 b). The first region can be defined as the free flow phase, and the second as the synchronized flow phase for medium densities, and wide moving jams phase for high densities. This satisfies the three-phase traffic theory [5] and confirms our model to be valid for real world traffic predictions.

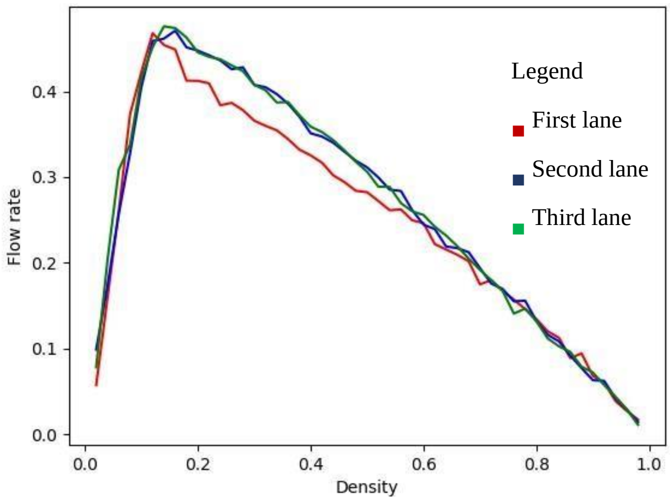


**Figure 4:** Line graphs depicting relationship between the flow rate $\bar{q}$ and initial road density $\rho$ for 3 adjacent lanes with unidirectional traffic, with lane changing allowed. $v_{max} = 5$, $p_{slow} = 0.3$.

To test the behaviour of multi-lane highways, we coupled 3 identical road models to form a 3-lane road, with the lane switching rules specified in the theory section. Our simulations yielded Figure 4, showing that the cumulative flow rate was obviously higher than a single lane. The lanes also peaked higher than a single lane, meaning that adding more lanes increased the flow rate beyond simply adding flow rates linearly for single lanes. Switching lanes to free up space and avoid congestion clearly had a positive impact.

Lanes 2 and 3 were almost identical in their flow rates, while lane 1 was lower for medium densities. Since cars in lane 1 couldn't needlessly change lanes, multiple breaks in laminar flow occurred from cars joining from lane 2. Lane 2 had constant exchange between its adjacent lanes keeping its traffic flow steady, while cars in the least dense lane 3 could attain high speeds, ensuring both lanes had high flow rates.

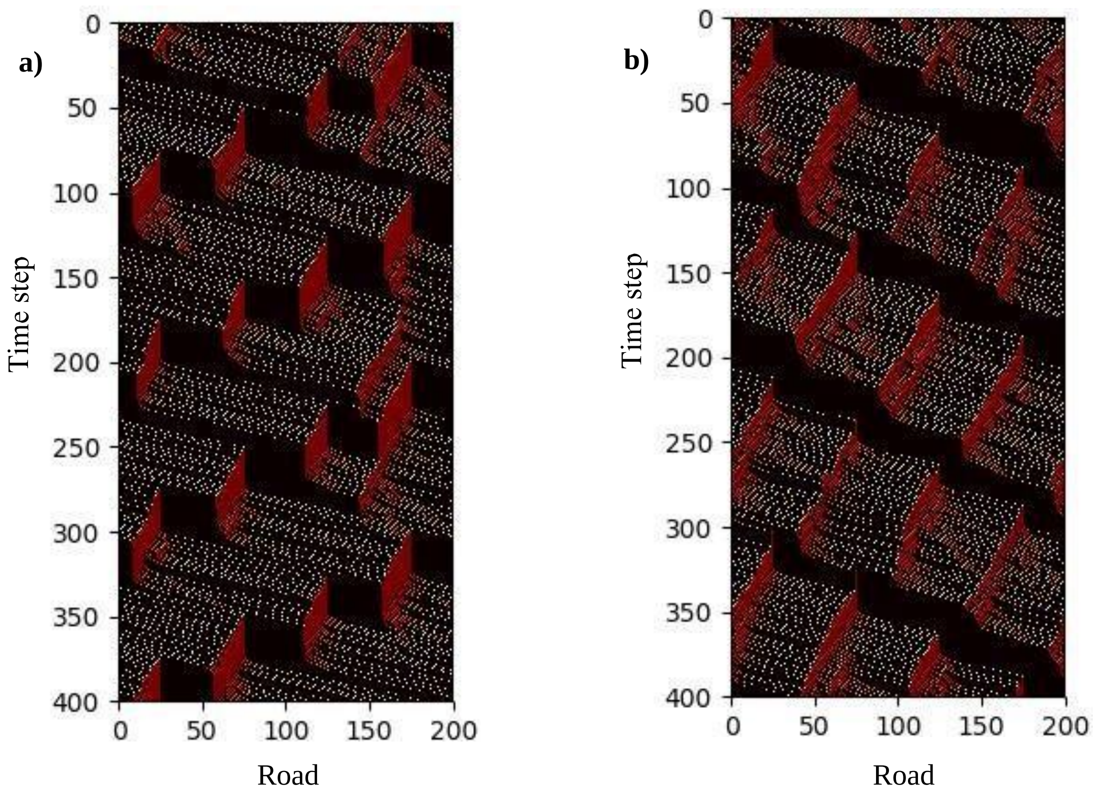


**Figure 5:** Heat maps depicting distributions of cars along periodic roads with 4 traffic lights of length 200, $T = 400$, and with $l_{step} = 100$. Map a) shows backward light rotation, and map b) shows forward light rotation. $v_{max} = 5$, $p_{slow} = 0.3$.

For our next set of simulations, we reverted to a single lane, increased the road size and simulation time, and added traffic lights at equally spaced intervals along the road (at index 25, 75, 125, 175). Their switching pattern involved one being red and the rest being green at any point in time, with the red light rotating through the lights at constant time intervals. We defined the time for one full rotation as a lightstep $l_{step}$. From Figure 5, the queue block pattern ran from right to left in a) and left to right in b) as expected. While a light rotation moving with traffic ensured smaller time frames in which no flow is occurring, rotation against traffic allowed traffic to be less congested.

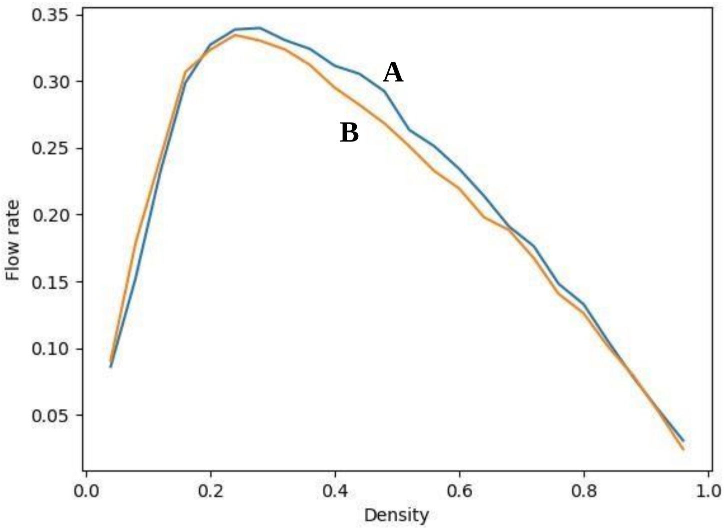


**Figure 6:** Line graphs depicting the relationship between the flow rate of a periodic road with 4 traffic lights and its density. Line A shows the result for forward rotation, line B shows the result for backward rotation.

Figure 5 showed a wave-like motion in b), periodically letting many cars pass a point at once but restricting individual drivers from traversing quickly. A physical parallel can be drawn to fluid motion through a duct, where a higher cross-sectional area and lower pressure/velocity may yield a higher flow rate than a smaller area and higher pressure/velocity.

Figure 6 proved this, showing that a forward rotation gave a higher $\bar{q}$ for medium densities. Given that a series traffic light setup would be found in cities with these densities, we can argue that a forward rotation would be favourable, despite a backward rotation allowing cars to cover more distance and maintaining $v_{max}$ longer before having to stop at a light. This method of collectively shifting traffic rather than favouring individuals can be seen in city grid layouts where one may hit many red lights in a row, giving the illusion that the flow is stagnant.

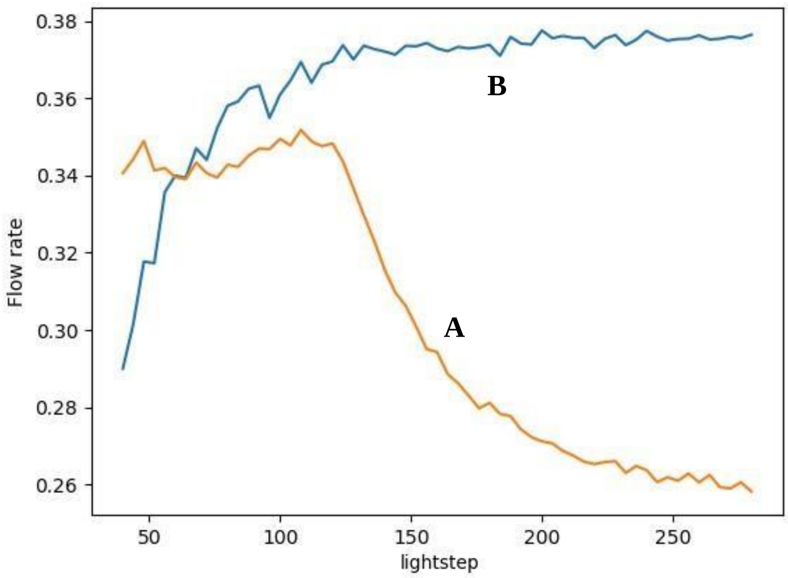


**Figure 7:** Line graphs depicting relationship between the flow rate $\bar{q}$ and light-step $l_{step}$ of a section of straight road. Line A shows the result for forward rotation, line B shows the result for backward rotation. $v_{max} = 5$, $p_{slow} = 0.2$. Simulation time $T = l_{step} \times 8$ to ensure constant number of cycles for each $l_{step}$ for fairness.

Finally, analysed the traffic light setup for a section of straight road, as shown in Figure 7. Line A showed negligible variation in $\bar{q}$ for small $l_{step}$. However, a sharp flow rate decline was evident after a critical period for switching at $l_{step} = 120$. This was due to the backward traffic jam motion initiated by a red light reaching the cars at the light behind it for $l_{step} > 120$, causing the two clusters to interact and interfere with the wave motion. As $l_{step}$ increased beyond this point, the local road density increased, further disrupting flow and creating static clusters. $\bar{q}$ eventually levelled off since further cars could not spawn in due to saturation of the road, which was a physical limitation of our sectioned model. For $l_{step} < 120$, these two clusters were independent, and thus $l_{step}$ had no bearing on the cumulative $\bar{q}$ .

Line B appeared more beneficial for $l_{step} > 68$, shown by the intersection of the two lines. Due to the backward rotation of lights, cars that finished waiting could move unhindered by the next light. This meant that cars saw fewer red lights on average as $l_{step}$ increased. As such, $\bar{q}$ increased until $l_{step}$ became so large that it was inefficient, and traffic built up at a single light, shown by the levelling of Line B.

Therefore, we can show that in order to maximise traffic flow, a rotation cycle that is inverse to the direction of traffic should be employed for stretches of high speed, low density road such as motorways.

## 5. Conclusions

In conclusion, our cellular automaton model to simulate road traffic has proved to be computationally advantageous for predicting real world traffic behaviour and finding optimal variables to maximise flow rates. Our models show interesting links between the non-linear dynamics of cellular automaton, the physics of waves, and fluid dynamics. We can use these models to assess speed limits and road layouts theoretically before implementing them. To improve our model in the future, we could add interchanges and link different roads together to create a full 2D system. We could improve on our original models by adding junctions to multi-lane roads and adding more complex cellular automaton rules such as slow-to-start [6] and slow-to-stop [7].

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