MT1 Exam 'Cheat Sheet'

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$f_3 = n^{\log n}$
$f_4 = n^{\log 100n}$
$f_2 = 2^n$
$f_1 = e^n$

$f_7 = \log\log(n^2)$
$f_4 = (\log\log n)^2$
$f_2 = \log(100n)$
$f_3 = (\log n)^2$
$f_5 = n\log(100n)$
$f_6 = \log(n^{(n^2)})$
$f_1 = 4n^3$

$T(n) = T(\frac{n}{3}) + T(\frac{n}{6}) + \Theta(n)$
$T(n) \le 2T(\frac{n}{3}) + \Theta(n)$.
$\Theta(n)$

$T(n) = 2T(n/2) + O(1)$
$O(n)$.

$f_2 = \frac{n}{10^{1000}}$

$f_5 = \frac{\binom{n}{5}}{n^3}$

$f_4 = \binom{n}{5}$

$f_3 = n!$
f_3 is $O(n^n\sqrt{n})$

$f_1 = 2^{n^2}$

$T_2(n) = 2T_2(n/2) + 2T_1(n/2) + \Theta(1)$
$T_1(n/2) = \Theta(n)$,

$T_2(n) = 2T_2(n/2) + \Theta(n)$
Master Theorem (case 2)
$\Theta(n\log n)$. $O(n\log n)$ notation also suffices.

$T(n) = 2T(\sqrt{n}) + \Theta(\log n)$

m as $n = 2^m$.

$$T(2^m) = 2T(\sqrt{2^m}) + \Theta(\log 2^m)$$
$$= 2T(2^{(m/2)}) + \Theta(m)$$

$S(m) = T(2^m) = T(n)$. This gives us:

$$S(m) = T(2^m) = 2T(2^{(m/2)}) + \Theta(m)$$
$$= 2S(m/2) + \Theta(m)$$

$S(m) = m\log m$.
$S(m) = \Theta(m\log m)$
$T(n) = \Theta(\log n\log\log n)$

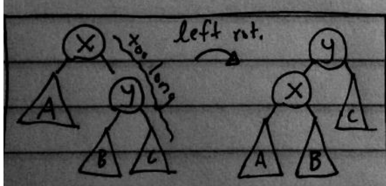
	Time complexity								Space complexity
	Average				Worst				Worst
	Find Min	Search	Insert	Delete	Find Min	Search	Insert	Delete	
Array	$O(n)$	$O(n)$	$O(1)$	$O(n)$	$O(n)$	$O(n)$	$O(1)$	$O(n)$	$O(n)$
(Min/max) Heap	$O(1)$	$O(n)$	$O(1)$	$O(\log n)$	$O(1)$	$O(n)$	$O(\log n)$	$O(\log n)$	$O(n)$
BST tree	$O(\log n)$	$O(\log n)$	$O(\log n)$	$O(\log n)$	$O(n)$	$O(n)$	$O(n)$	$O(n)$	$O(n)$
AVL tree	$O(\log n)$	$O(\log n)$	$O(\log n)$	$O(\log n)$	$O(\log n)$	$O(\log n)$	$O(\log n)$	$O(\log n)$	$O(n)$

Master Thm
$T(n) = aT(n/b) + f(n)$
a = # sub-probs; b = size of each sub-prob; $f(n)$ = time to ÷ into sub prob & combine results
case 1: if $f(n) \in O(n^c)$ where $c < \log_b a$
then: $T(n) \in \Theta(n^{\log_b a})$
case 2: if $f(n) = \Theta(n^c \log^k n)$ where $c = \log_b a$
then: $T(n) = \Theta(n^c \log^{k+1} n)$; where $k \ge 0$
case 3: if $f(n) = \Omega(n^c)$ where $c > \log_b a$
and: $af(\frac{n}{b}) \le k f(n)$ for $k < 1$ & large n
then: $T(n) = \Theta(f(n))$

$\binom{n}{k} = \frac{n!}{k!(n-k)!}$

$n! \sim (\frac{n}{e})^n \sqrt{2\pi n} = O(n^2\sqrt{n})$
$\log(n!) = n\log n$

$O(f(n))$	$\Theta(f(n))$	$\Omega(f(n))$
upper	tight	lower
$\le$	$=$	$\ge$



Heap Operations:
- max_heapify: $O(\log n) \rightarrow$ fix node
- extract_max: $O(\log n)$
- build_max_heap: $O(n)$
- heapsort: $O(n\log n)$

(a) $T(n) = 16T(n/2) + \Theta(n^3)$

Solution: $T(n) = \Theta(n^4)$.
Apply Case 1 of the Master Theorem.

(b) $T(n) = T(n-2) + \Theta(\log n)$

Solution: $\Theta(n \log n)$.
Expanding the recurrence, we get:

$$\begin{aligned} T(n) &= T(n-2) + c\log n \\ &= T(n-4) + c\log(n-2) + c\log n \\ &= T(n-6) + c\log(n-4) + c\log(n-2) + c\log n \\ &= \sum_{i=1}^{n/2} c\log(2i) \\ &= c\log\left(\prod_{i=1}^{n/2} 2i\right) = c\log\left(2^{n/2}\left(\tfrac{n}{2}\right)!\right). \end{aligned}$$

Using Stirling's Approximation for $n!$, we have:

$$\begin{aligned} T(n) &= c\log\left(2^{n/2}\sqrt{2\pi\left(\tfrac{n}{2}\right)}\left(\frac{n/2}{e}\right)^n\right) \\ &= c\left(\frac{n}{2}\log 2 + \log(\sqrt{\pi n}) + \frac{n}{2}\log n - \frac{n}{2}\log e\right) \\ &= \Theta(n\log n). \end{aligned}$$

(c) $T(n) = 4T(n/2) + \Theta(n^{2+10^{-5}})$

Solution: $T(n) = \Theta(n^{2+10^{-5}})$.
Apply Case 3 of the Master Theorem.

(d) $T(n,m) = mT(n/m, m) + O(mn)$,
where $m \geq 1$ and $T(n,m) = 1$ for $n \leq 1$.

Solution: $T(n,m) = \Theta(mn\log_m n)$.
Note that the Master Theorem cannot be used to solve this problem since m is not a constant. Expanding the recurrence, we get:

$$\begin{aligned} T(n,m) &= mT(n/m,m) + c\cdot mn \\ &= m(mT(n/m^2,m) + c\cdot nm/m) + c\cdot mn \\ &= m^2T(n/m^2,m) + 2c\cdot mn \\ &= m^2(mT(n/m^3,m) + c\cdot nm/m^2) + 2c\cdot mn \\ &= m^3T(n/m^3,m) + 3c\cdot mn = \ldots \\ &= m^{\log_m n}T(1,m) + \log_m n\cdot c\cdot mn \\ &= \Theta(mn\log_m n). \end{aligned}$$

(e) $T(n) = 8T(n-3)$

Solution: $T(n) = \Theta(2^n)$.
By expanding the recurrence, we get:

$$\begin{aligned} T(n) &= 8T(n-3) \\ &= 8\cdot 8\cdot T(n-6) \\ &= 8\cdot 8\cdot 8T(n-9) = \ldots \\ &= 8^{n/3}\cdot T(1) = (8^{1/3})^n\cdot T(1) \\ &= \Theta(2^n). \end{aligned}$$