CS 112 – Introduction to Computing II

Wayne Snyder
Computer Science Department
Boston University

Digraphs and Graphs:

Basic notions, representations, examples

Graph Search: Depth-first, breadth-first, best-first

Computer Science

Graphs: Basic Definitions

Graphs are the most basic model of collections of information, generalizing trees to allow arbitrary links between nodes. There are two flavors, Directed and Undirected.

A Directed Graph (or Digraph) is:

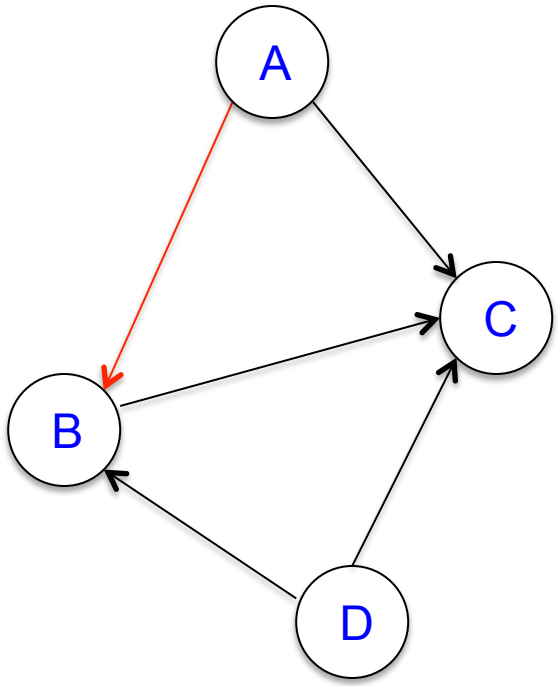
- A set V of Vertices (or: Nodes) containing (possibly):
  - A Label (1, 2, … A, B, … etc.)
  - Data fields (boolean flags, counters, etc.)
- A set E of Edges (links) connecting vertices; edges may have labels or data (e.g., weight or cost) associated with them.

E is usually expressed as a relation on V, i.e., E consists of pairs of vertices:

(source, target)

E is a subset of V x V (Cartesian Product of V)

Directed Graph G



V = { A, B, C, D }

E = { (A,B), (A, C), (B,C), (D,B), (D,C) }

Graphs: Basic Definitions

Graphs are the most basic model of collections of information, generalizing trees to allow arbitrary links between nodes.

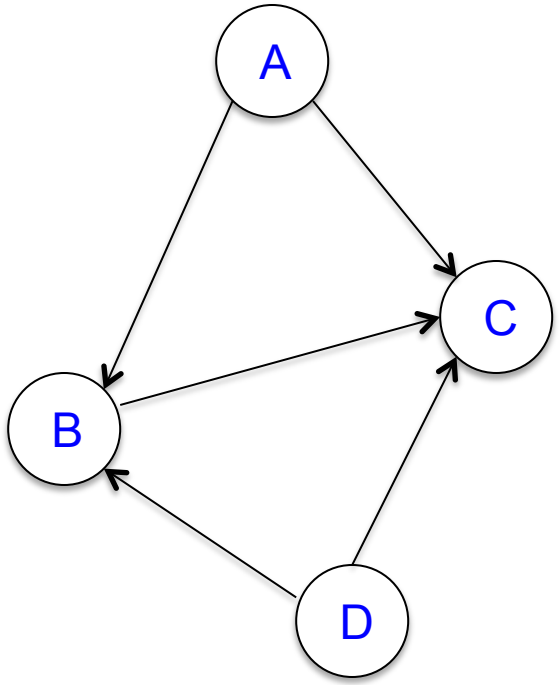
A Directed Graph (or Digraph) is:

- A set V of Vertices (or: Nodes) containing (possibly):
  - A Label (1, 2, … A, B, … etc.)
  - Data fields (boolean flags, counters, etc.)
- A set E of Edges (links) connecting vertices; edges may have labels or data (e.g., weight or cost) associated with them.

E is usually expressed as a relation on V, i.e., E consists of pairs of vertices:

(source, target)

E is a subset of V x V (Cartesian Product of V)



V = { A, B, C, D }

E = { (A,B), (A, C), (B,C), (D,B), (D,C) }

Note: Like a tree, but

- Any vertex can be connected to any other vertex;
- There is no root.

Graphs: Basic Definitions

A Directed Graph (or Digraph) is:

- A set V of Vertices (or: Nodes) containing (possibly):
  - A Label (1, 2, … A, B, … etc.)
  - Data fields (boolean flags, counters, etc.)
- A set E of Edges (links) connecting vertices; edges may have labels or data (e.g., weight or cost) associated with them.

Edges have a direction (e.g., one way streets)

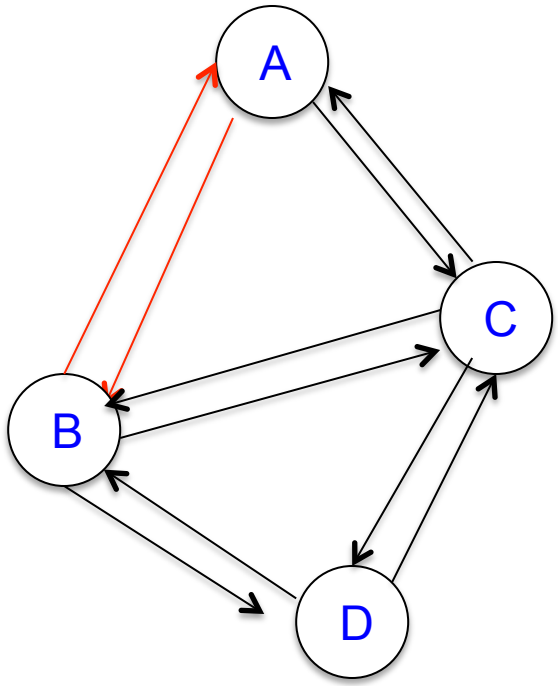
An Undirected Graph has the additional feature that all edges are two way, i.e., the relation E is symmetric:

(A, B) is in E iff (B,A) is in E

Edges do NOT have a direction (e.g., two-way streets).

"Graph" can mean either, but generally is assumed to be undirected.

Undirected Graph G



V = { A, B, C, D }

E = { (A,B), (A, C), (B,C), (D,B), (D,C),
(B,A), (C,A), (C,B), (B,D), (C,D) }

Graphs: Basic Definitions

A Directed Graph (or Digraph) is:

- A set V of Vertices (or: Nodes) containing (possibly):
  - A Label (1, 2, … A, B, … etc.)
  - Data fields (boolean flags, counters, etc.)
- A set E of Edges (links) connecting vertices; edges may have labels or data (e.g., weight or cost) associated with them.

Edges have a direction (e.g., one way streets)

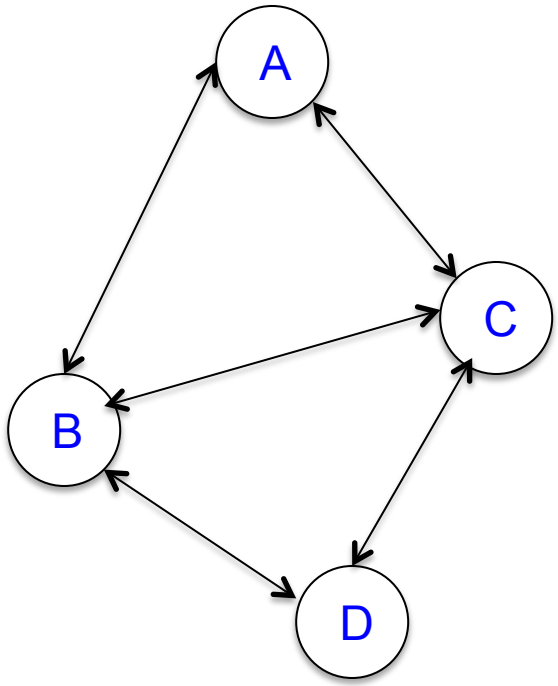
An Undirected Graph has the additional feature that all edges are two way, i.e., the relation E is symmetric:

(A, B) is in E iff (B,A) is in E

Edges do NOT have a direction (e.g., two-way streets).

"Graph" can mean either, but generally is assumed to be undirected.

Undirected Graph G



Edges in an undirected graph may have two arrows

V = { A, B, C, D }

E = { (A,B), (A, C), (B,C), (D,B), (D,C),
(B,A), (C,A), (C,B), (B,D), (C,D) }

Graphs: Basic Definitions

A Directed Graph (or Digraph) is:

- A set V of Vertices (or: Nodes) containing (possibly):
  - A Label (1, 2, … A, B, … etc.)
  - Data fields (boolean flags, counters, etc.)
- A set E of Edges (links) connecting vertices; edges may have labels or data (e.g., weight or cost) associated with them.

Edges have a direction (e.g., one way streets)

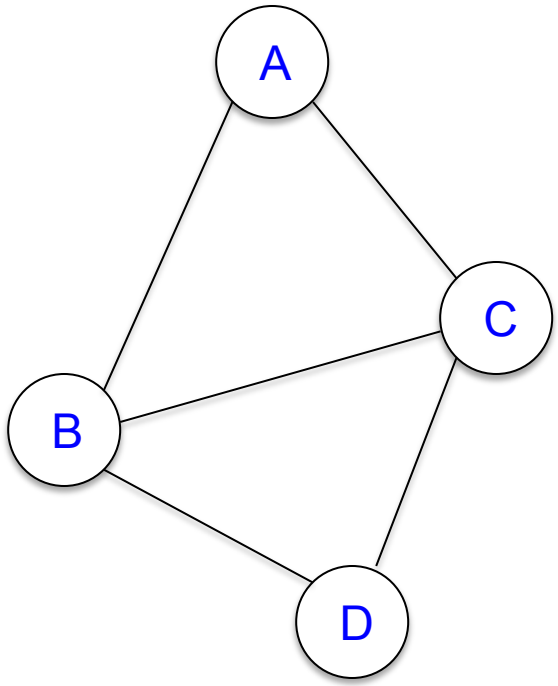
An Undirected Graph has the additional feature that all edges are two way, i.e., the relation E is symmetric:

(A, B) is in E iff (B,A) is in E

Edges do NOT have a direction (e.g., two-way streets).

"Graph" can mean either, but generally is assumed to be undirected.

Undirected Graph G



But generally are drawn without arrow-heads.

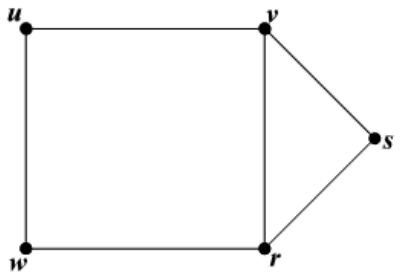
V = { A, B, C, D }

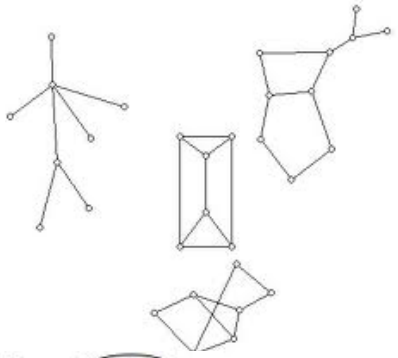
E = { (A,B), (A, C), (B,C), (D,B), (D,C),
(B,A), (C,A), (C,B), (B,D), (C,D) }

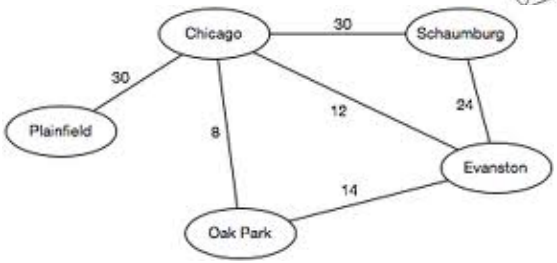
Graphs: Basic Definitions

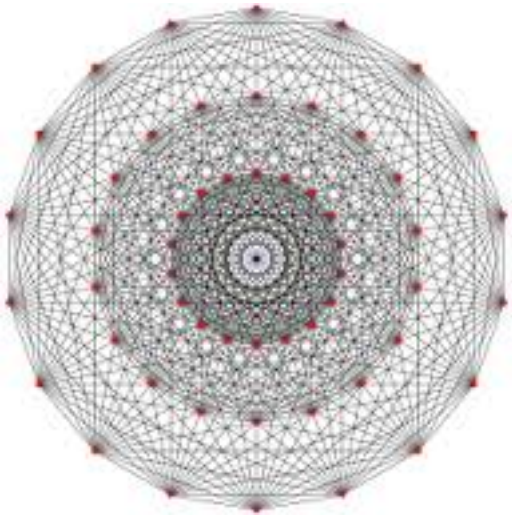
We will focus on Undirected Graphs for this lecture.

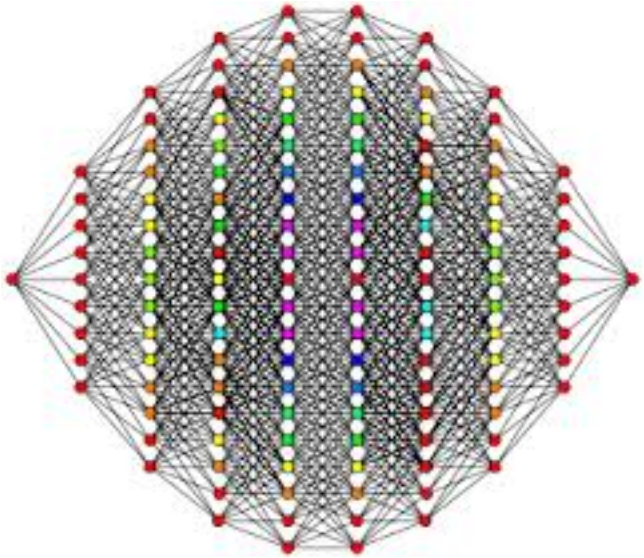
Graphs are EVERYWHERE in computer science and mathematics, and you have been looking at them for years……

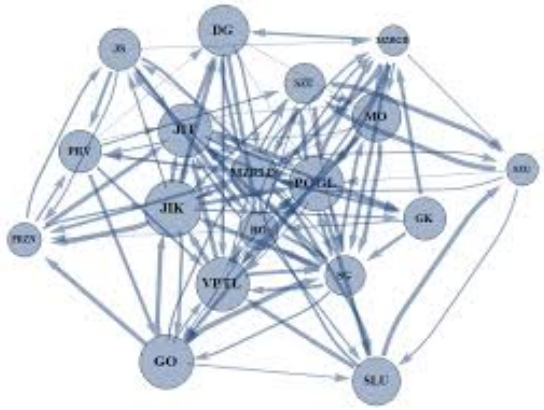






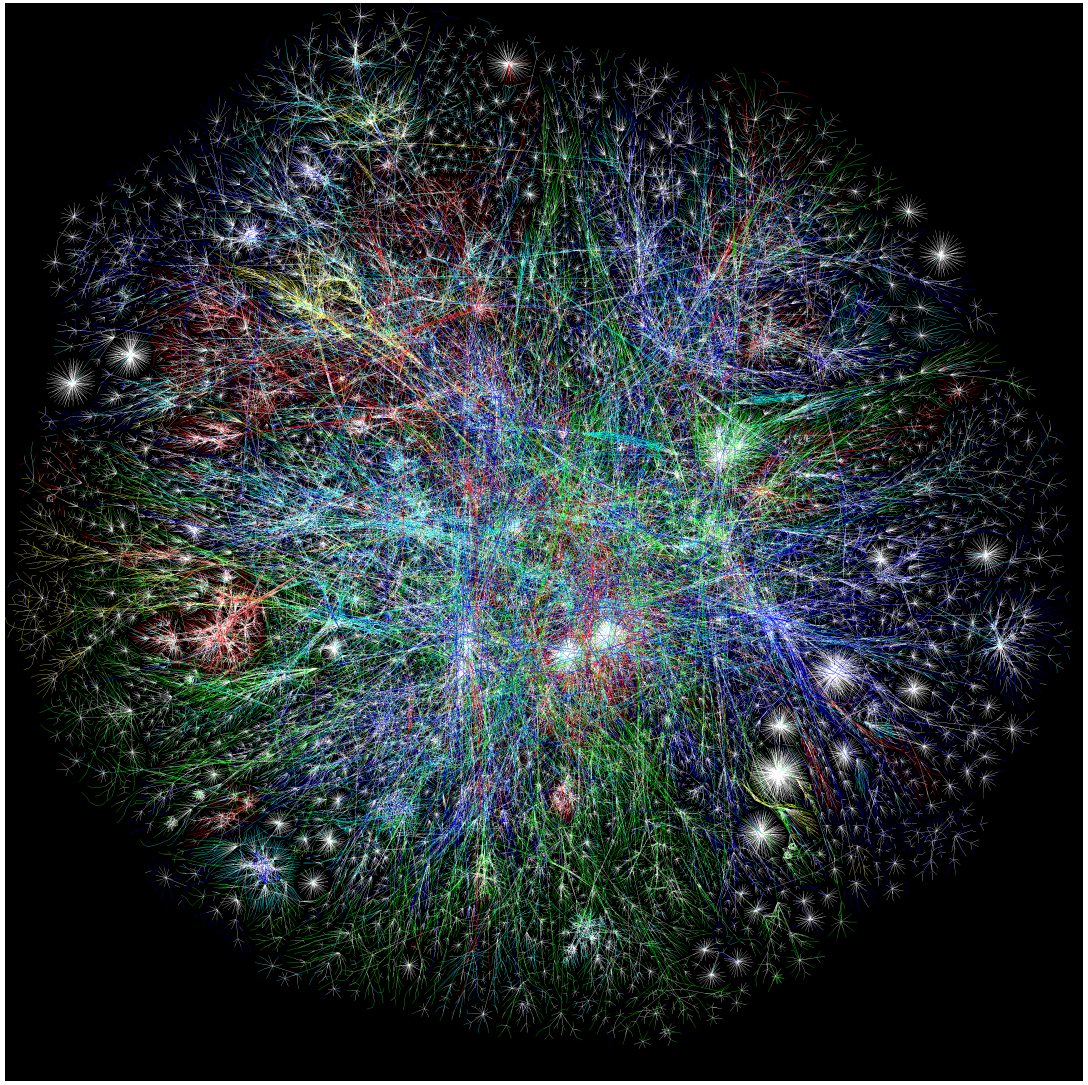






Graphs: Basic Definitions

And you've been using graphs for years……



Graphs: Basic Definitions

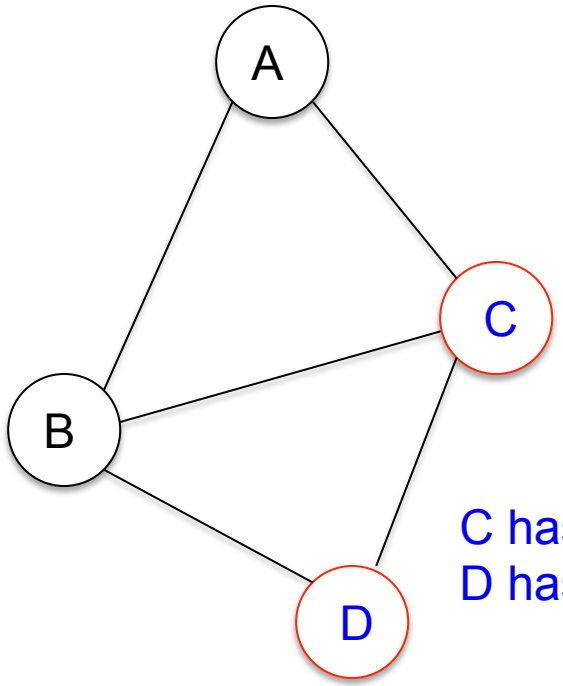
Basic Notions of Graphs:

Vertex (Vertex set V)

Edge (Edge set E)

The degree of a vertex is the number of edges it participates in .

Graph G



C has degree 3;
D has degree 2

V = { A, B, C, D }

E = { (A,B), (A, C), (B,C), (D,B), (D,C),
(B,A), (C,A), (C,B), (B,D), (C,D) }

Graphs: Basic Definitions

Basic Notions of Graphs:

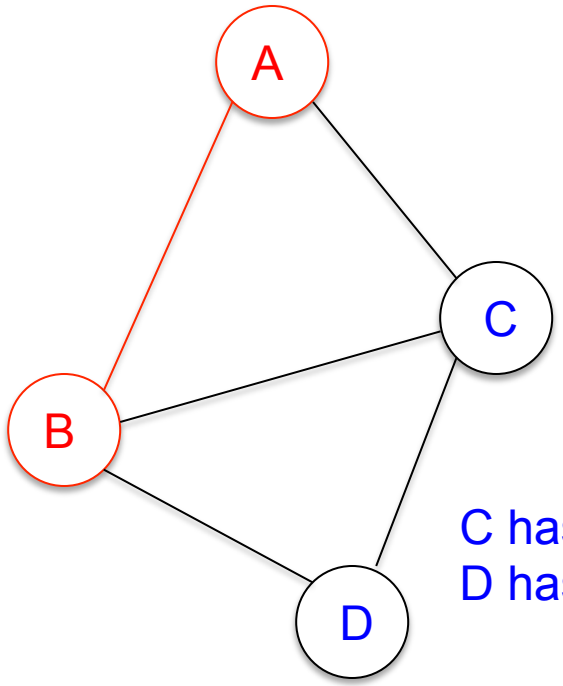
Vertex (Vertex set V)

Edge (Edge set E)

The degree of a vertex is the number of edges it participates in .

Two vertices are adjacent if there is an edge between them.

Graph G



C has degree 3;
D has degree 2

V = { A, B, C, D }

E = { (A,B), (A, C), (B,C), (D,B), (D,C),
(B,A), (C,A), (C,B), (B,D), (C,D) }

Graphs: Basic Definitions

Basic Notions of Graphs:

Vertex (Vertex set V)

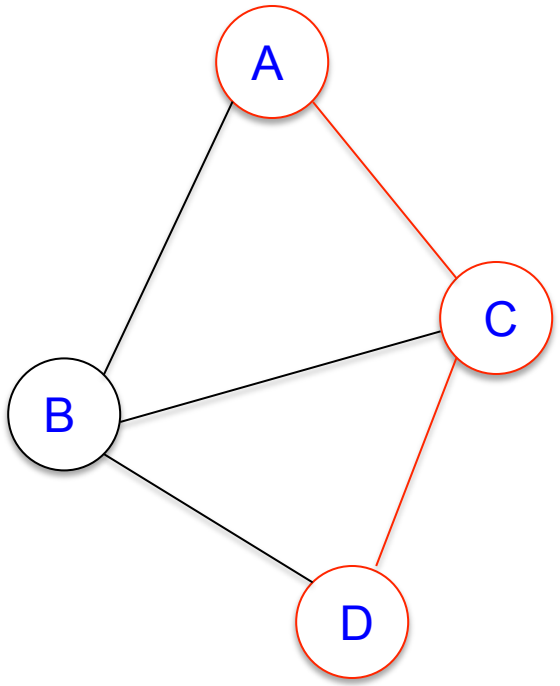
Edge (Edge set E)

The degree of a vertex is the number of edges it participates in .

Two vertices are adjacent if there is an edge between them.

A path is a sequence of adjacent edges.

Graph G



Path: A, C, D

V = { A, B, C, D }

E = { (A,B), (A, C), (B,C), (D,B), (D,C),
(B,A), (C,A), (C,B), (B,D), (C,D) }

Graphs: Basic Definitions

Basic Notions of Graphs:

Vertex (Vertex set V)

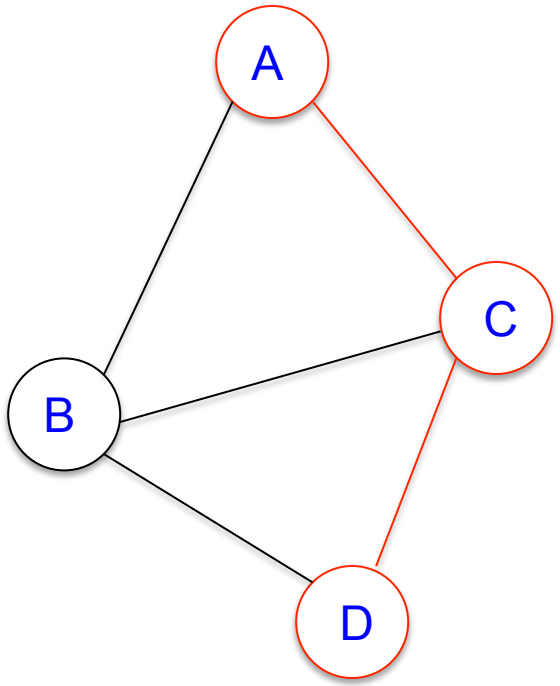
Edge (Edge set E)

The degree of a vertex is the number of edges it participates in .

Two vertices are adjacent if there is an edge between them.

A path is a sequence of adjacent edges. The length of a path is the number of edges.

Graph G



Path A, C, D has length 2.

V = { A, B, C, D }

E = { (A,B), (A, C), (B,C), (D,B), (D,C),
(B,A), (C,A), (C,B), (B,D), (C,D) }

Graphs: Basic Definitions

Basic Notions of Graphs:

Vertex (Vertex set V)

Edge (Edge set E)

The degree of a vertex is the number of edges it participates in .

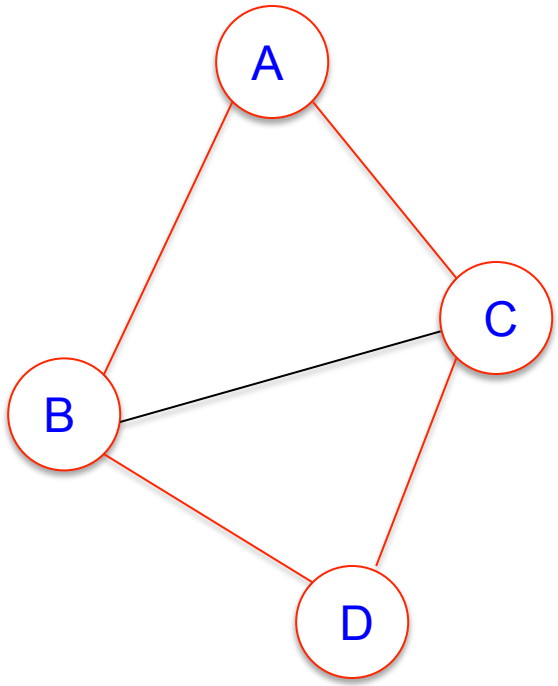
Two vertices are adjacent if there is an edge between them.

A path is a sequence of adjacent edges.

A cycle (or loop) is a path that begins and ends on the same vertex.

A, C, D, B, A

Graph G



V = { A, B, C, D }

E = { (A,B), (A, C), (B,C), (D,B), (D,C),
(B,A), (C,A), (C,B), (B,D), (C,D) }

Graphs: Basic Definitions

Basic Notions of Graphs:

Vertex (Vertex set V)

Edge (Edge set E)

The degree of a vertex is the number of edges it participates in .

Two vertices are adjacent if there is an edge between them.

A path is a sequence of adjacent edges.

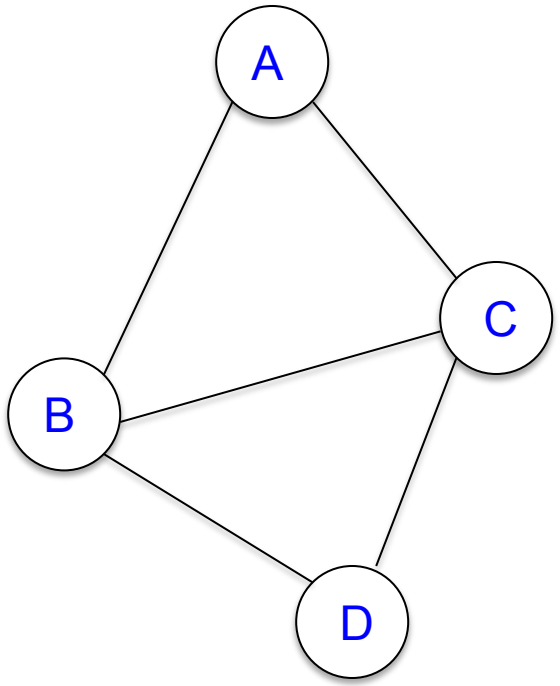
A cycle (or loop) is a path that begins and ends on the same vertex.

A, C, D, B, A

A simple cycle has no other repeated vertices (i.e., does not cross itself).

A, C, B, A, C, D, B, A is not simple.

Graph G



V = { A, B, C, D }

E = { (A,B), (A, C), (B,C), (D,B), (D,C),
(B,A), (C,A), (C,B), (B,D), (C,D) }

Graphs: Basic Definitions

Basic Notions of Graphs:

Vertex (Vertex set V); Edge (Edge set E)

The degree of a vertex is the number of edges it participates in .

Two vertices are adjacent if there is an edge between them.

A path is a sequence of adjacent edges.

A cycle (or loop) is a path that begins and ends on the same vertex.

A, C, D, B, A

A simple cycle has no other repeated vertices (i.e., does not cross itself).

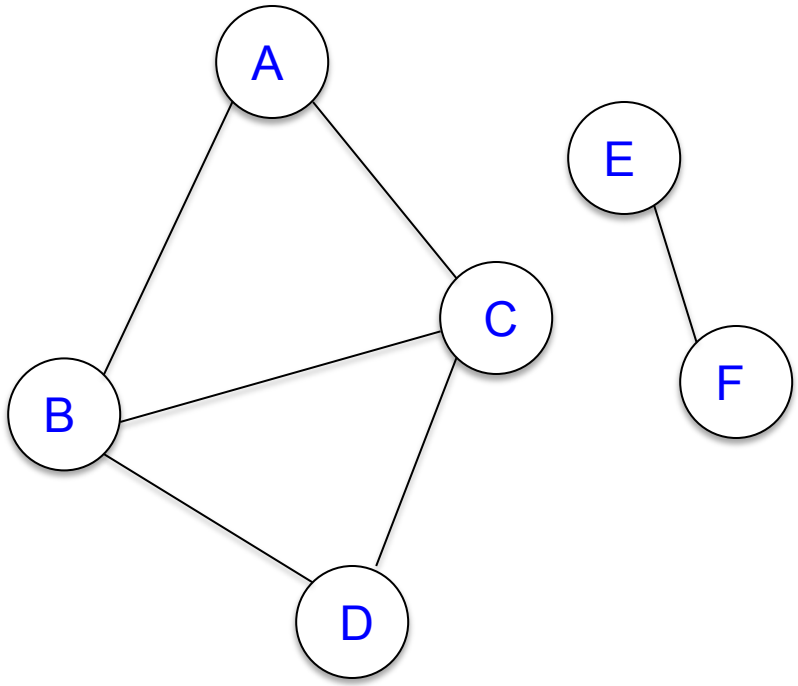
A, C, B, A, C, D, B, A is not simple.

Two vertices are connected if there is a path between them.

A set of vertices is connected if there is a path between each pair; a graph is connected if there is a path between any two nodes.

{ A, B, C, D } is connected; so is {E, F}

Graph G



Graphs: Basic Definitions

Basic Notions of Graphs:

Vertex (Vertex set V); Edge (Edge set E)

Two vertices are adjacent if there is an edge between them.

A path is a sequence of adjacent edges.

A cycle (or loop) is a path that begins and ends on the same vertex.

A, C, D, B, A

A simple cycle has no other repeated vertices (i.e., does not cross itself).

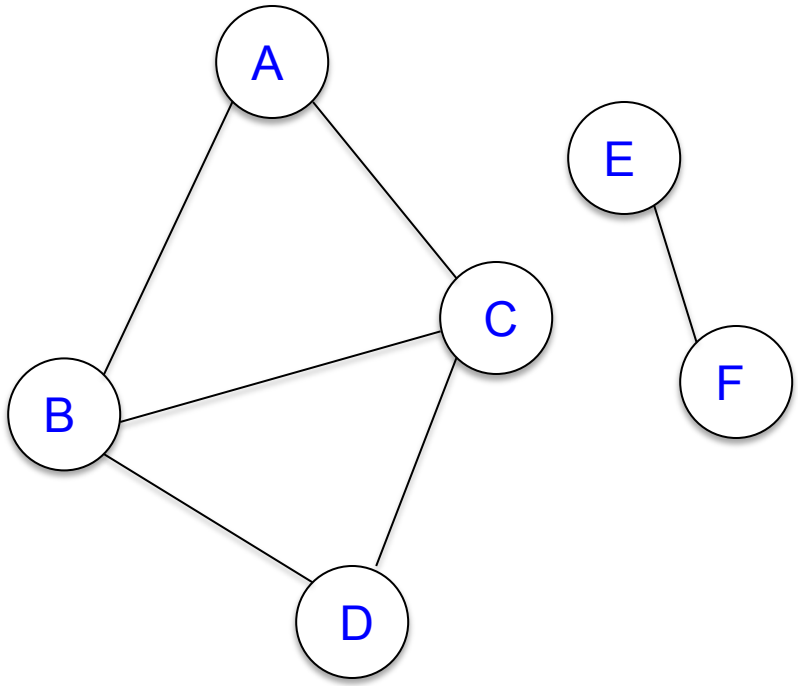
A, C, B, A, C, D, B, A is not simple.

Two vertices are connected if there is a path between them.

A set of vertices is connected if there is a path between each pair; a graph is connected if there is a path between any two nodes.

The set of connected components of G is the collection of maximal connected subsets of vertices. The connected components of G are { A, B, C, D } and {E, F}

Graph G



Graphs: Basic Definitions

Basic Notions of Graphs:

Vertex (Vertex set V); Edge (Edge set E)

Two vertices are adjacent if there is an edge between them.

A path is a sequence of adjacent edges.

A cycle (or loop) is a path that begins and ends on the same vertex.

A, C, D, B, A

A simple cycle has no other repeated vertices (i.e., does not cross itself).

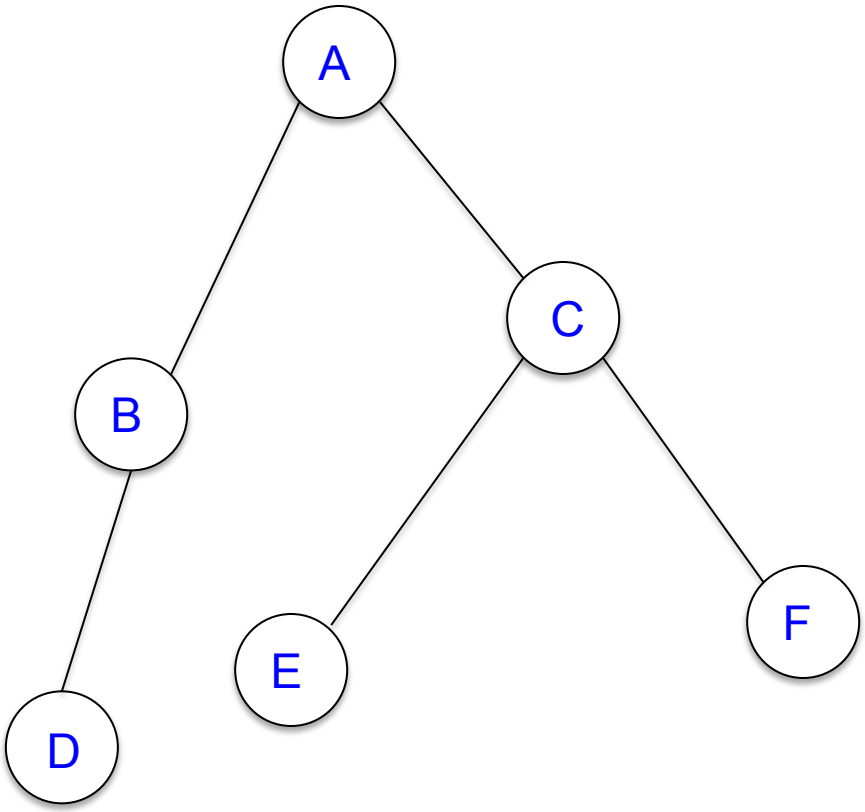
A, C, B, A, C, D, B, A is not simple.

Two vertices are connected if there is a path between them.

A set of vertices is connected if there is a path between each pair; a graph is connected if there is a path between any two nodes.

A tree is a acyclic, connected graph.

Graph G



Graphs: Basic Definitions

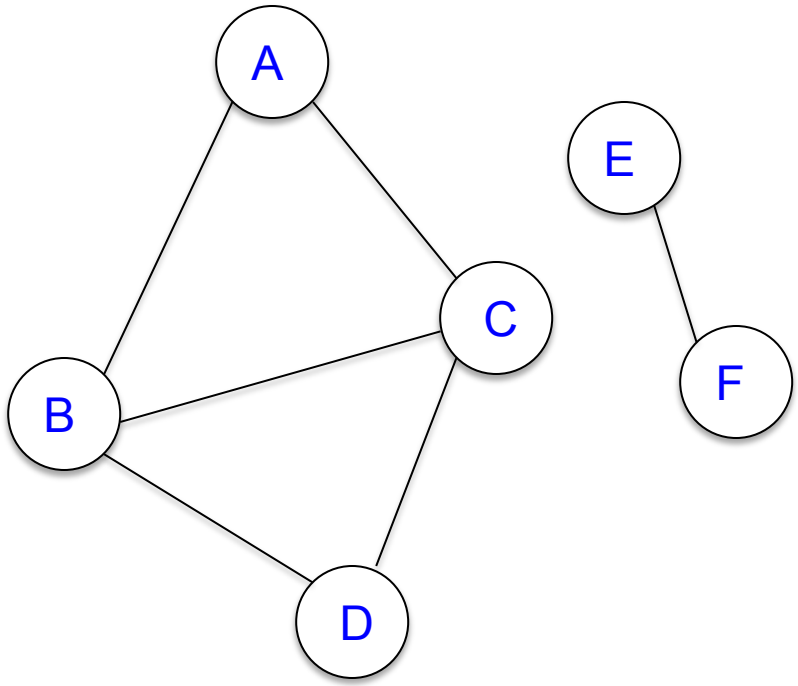
Implementing Graphs.

Vertex set V is a list of vertices:

V = (A, B, C, D, E, F)

Edge set is a boolean array:

	A	B	C	D	E	F
A		T	T			
B	T		T	T		
C	T	T		T		
D		T	T			
E						T
F					T	



Graphs: Basic Definitions

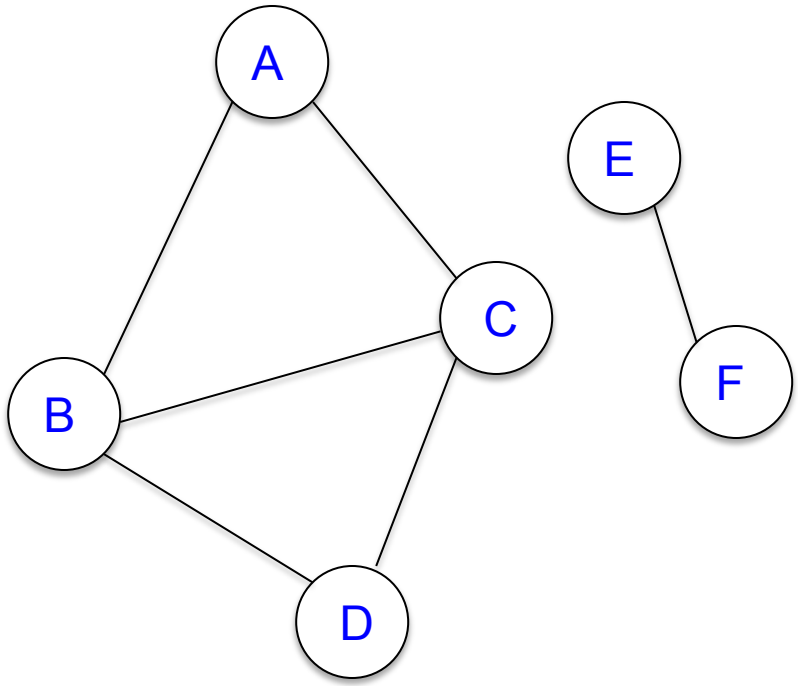
Implementing Graphs.

Vertex set V is a list of vertices:

V = (A, B, C, D, E, F)

Edge set is a boolean array: Note the symmetry!

	A	B	C	D	E	F
A		T	T			
B	T		T	T		
C	T	T		T		
D		T	T			
E						T
F					T	



Graphs: Basic Definitions

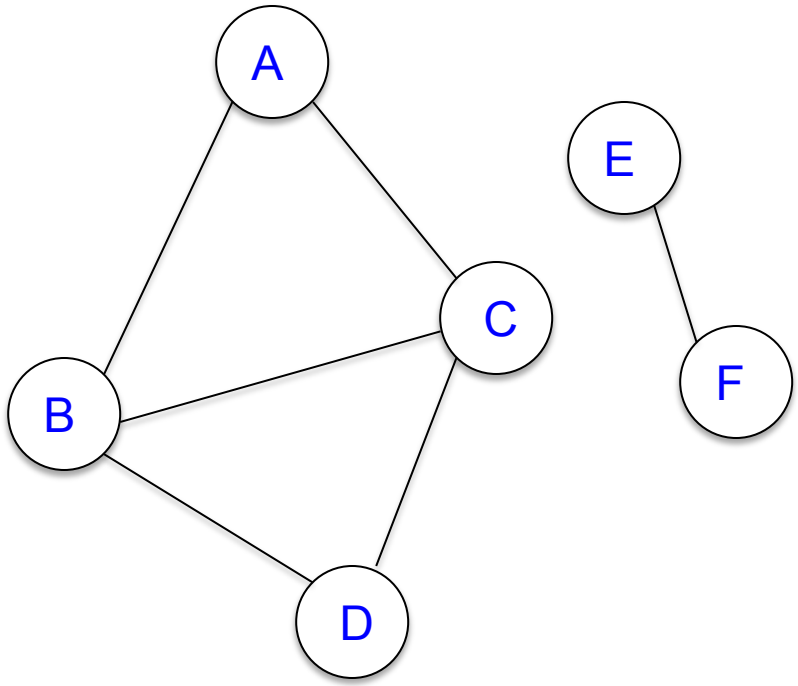
Implementing Graphs.

Vertex set V is a list of vertices:

V = (A, B, C, D, E, F)

Edge set is a boolean array; you could optimize by only storing half!

	A	B	C	D	E	F
A		T	T			
B	T		T	T		
C	T	T		T		
D		T	T			
E						T
F					T	



Graphs: Basic Definitions

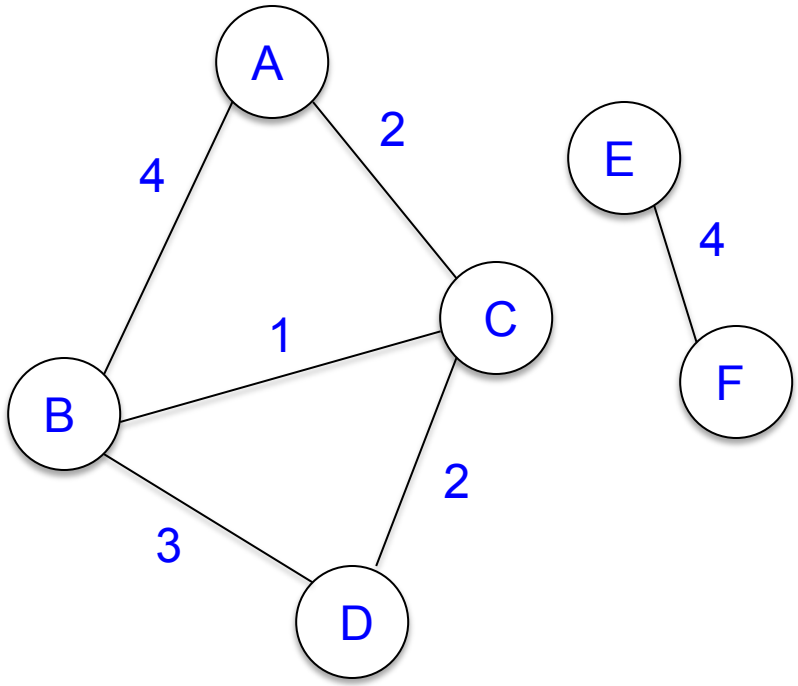
Implementing Graphs.

Vertex set V is a list of vertices:

V = (A, B, C, D, E, F)

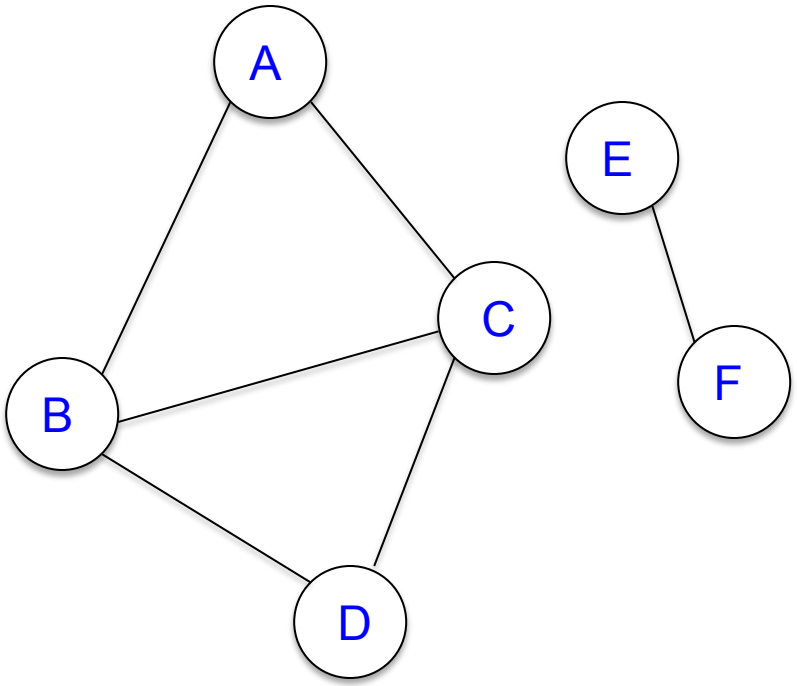
OR, edge set is an integer array giving the weights of the edges:

	A	B	C	D	E	F
A		4	2			
B	4		1	3		
C	2	1		2		
D		3	2			
E						4
F					4	



Graphs: Basic Definitions

	A	B	C	D	E	F
A		T	T			
B	T		T	T		
C	T	T		T		
D		T	T			
E						T
F					T	



You need a number of methods:

Adjacent(v) = list of vertices adjacent to v, in some order (if scan across the row for v would be in order of set V, but could be otherwise)

Degree(v) = number of T values in row for v

Etc. (more as we develop the algorithms)

Graphs: Basic Definitions

Graph Search Algorithms are all generalizations of the following algorithm for Depth-First Search

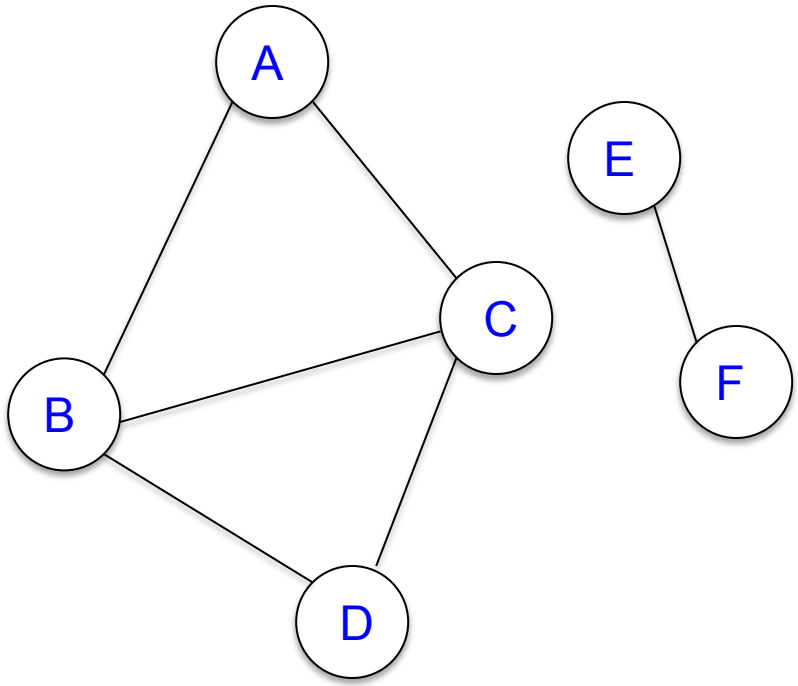
Suppose that vertices have a flag:

```
boolean visited;


searchGraph(V, E) {
    foreach( v in V )              // initialize all vertices
          v.visited = false;
    foreach( v in V )
         if(!v.visited)
           DFS(v);
}

DFS( v )  {
     if(!v.visited) {
          visit(v);                // do something, e.g. print out
          v.visited = true;
          foreach( u in Adjacent(v) ) {
              DFS( u );
          }
    }
}
```

Graph G



Graphs: Basic Definitions

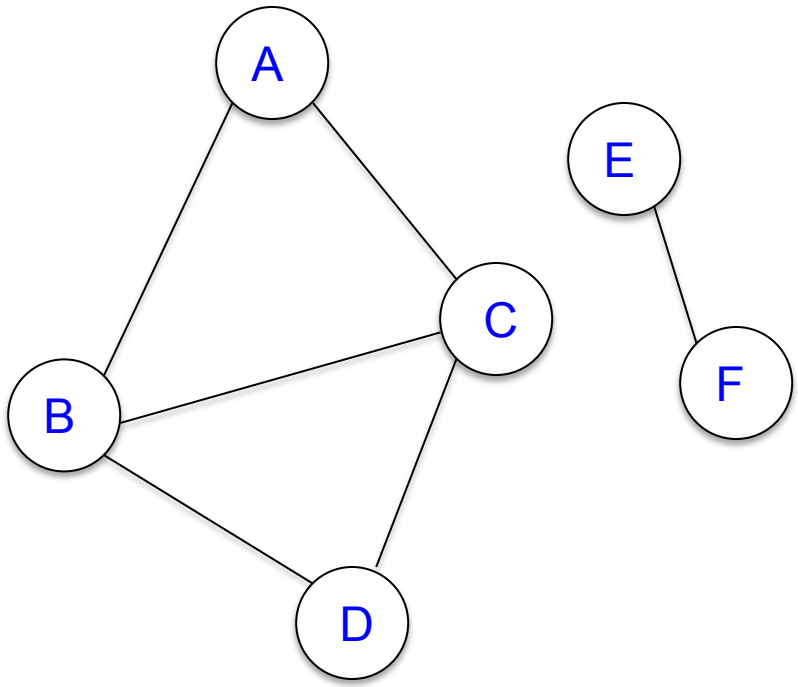
To consider the general search algorithms, we have to rephrase this as a non-recursive algorithm.

Here is depth-first search again:

```
searchGraph(V, E) {
    foreach( v in V )              // initialize all vertices
        v.visited = false;
    foreach( v in V )
        if(!v.visited)
            DFS(v);
}

DFS( v ) {
    Stack S = new Stack();
    S.push(v);
    while( ! S.isempty() ) {
        Vertex u = S.pop();
        if(!u.visited) {
            visit(u);
            u.visited = true;
            foreach( w in Adjacent(u) )
                if(!w.visited)
                    S.push(w);
        }
    }
}
```

Graph G



Graphs: Basic Definitions

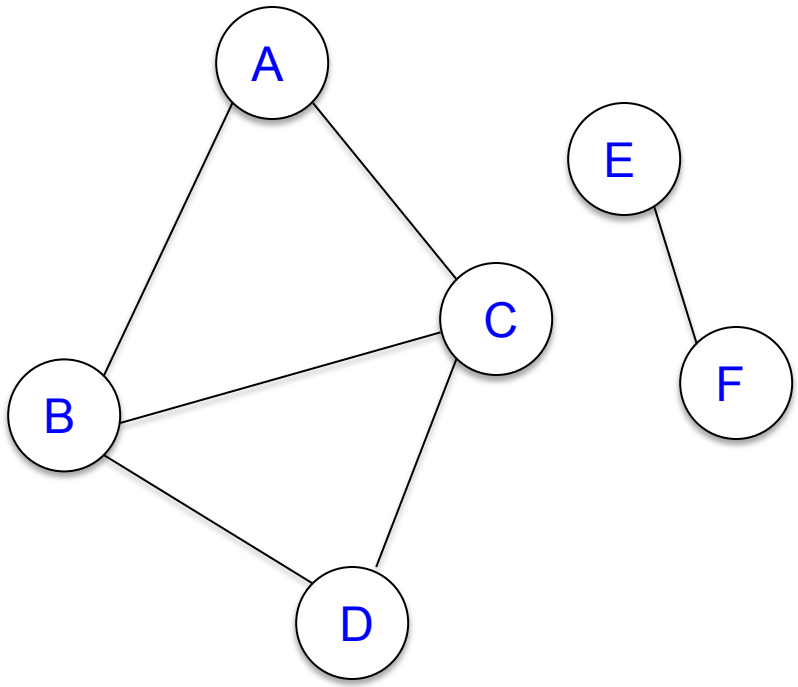
To consider the general search algorithms, we have to rephrase this as a non-recursive algorithm.

By replacing the Stack with a Queue, we have Breadth-First Search

```
boolean visited;
searchGraph(V, E) {
    foreach( v in V )              // initialize all vertices
        v.visited = false;
    foreach( v in V )
        if(!v.visited)
          BFS(v);
}

BFS( v )  {
    Queue S = new Queue();
    S.enqueue(v);
    while( ! S.isempty() ) {
        Vertex u = S.dequeue();
        if(!u.visited) {
            visit(u);
            u.visited = true;
            foreach( w in Adjacent(u) )
                if(!w.visited)
                    S.enqueue(w);
        }
    }
```

Graph G



Graphs: Basic Definitions

Optional Slide: You are not responsible for best-first search.

To c
reph

By re
Best

```
boole
sear
    fo

    fo


}

BFS
    PC
    S.
    wh
        vertex u = S.dequeue();
        if(!u.visited) {
            visit(u);
            u.visited = true;
            foreach( w in Adjacent(u) )
                if(!w.visited)
                    S.enqueue(w);
        }
    }
```