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Given a cubic spline interpolation:

$$S(x) = \begin{cases} S_0(x) = 1 + 2x - x^3 & ; 0 \leq x < 1 \\ S_1(x) = 2 + b(x-1) + c(x-1)^2 + d(x-1)^3 & ; 1 \leq x < 2 \end{cases},$$

determine constants b, c, and d so that all conditions for a natural cubic spline hold.

$S(x)$ must follow the propositions so that $S(x)$ is a natural cubic spline for given data $(x_0, y_0), (x_1, y_1), \dots, (x_{n-1}, y_{n-1}), (x_n, y_n)$

- $S_0''(x_0) = S_{n-1}''(x_n)$
- For $i \in [1, n-1]$; $S_i'(x_i) = S_{i+1}'(x_i)$
- For $i \in [1, n-1]$; $S_i''(x_1) = S_i''(x_1)$

i.) consider $S_0''(0) = S_1''(2) = 0$

$S_0'(x) = 2 - 3x^2$ ——— (1)

$S_0''(x) = 6x$ ——— (2)

$S_0''(0) = 0$ is already true.

$S_1'(x) = b + 2C(x-1) + 3d(x-1)^2$ ——— (3)

$S_1''(x) = 2C + 6d(x-1)$ ——— (4)

(4); $S_1''(0) = 2c - 6d = 0$ ——— (5)

ii.) consider $S_0'(1) = S_1'(1)$

(1); $S_0'(1) = -1$ ——— (6)

(2); $S_1'(1) = b$ ——— (7)

(6) = (7); $b = -1$ ——— (8)

iii.) Consider $S_0''(1) = S_1''(1)$

(2); $S_0''(1) = 6$ ——— (9)

(4); $S_1''(1) = 2C$ ——— (10)

(9) = (10); $6 = 2C$

$C = 3$ ——— (11)

iv.) substitute $b = -1$ (8) and $c = 3$ — (11) in (5); $2(3) - 6d = 0$

$d = 1$ ——— (12)

From (8), (11), and (12);

$\therefore$ $b = -1$, $C = 3$, and $d = 1$ are constants so that $S(x)$ be a natural cubic spline. ☐