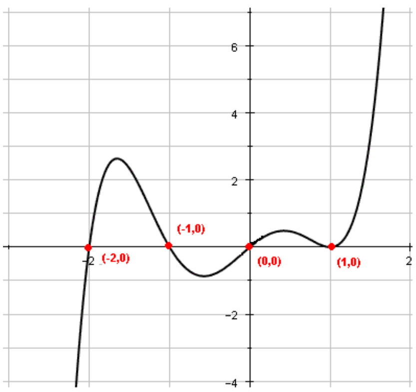
Summary for 204381: Chapter 6

📚 **Organized by**
Saksinkarn Petchkuljinda
630510600

📄 **Basic Information**
Date: 18 October 2023
Time: 12:00 PM — 3.00 PM
Venue: TBA
★ **Calculator** 🧮 ☆ **Open Book** 🙌🏽

Table of Contents

General Ways to Solve for x



Example Illustration the solutions for x that satisfy $y = f(x) = 0$

Given we have a function $y = f(x)$, we want to find x^* that $y = f(x^*) = 0$. There are numerous way this is ***useful***. For example:

- **Solving quadratic equation**: $y = f(x) = ax^2 + bx + c = 0$ leads to

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- Solving general polynomial problems: given any equations can become $y = f(x) = (x -$

$a)(x-b)\ldots$, then the solution for $y = f(x) = 0$ is $x \in \{a, b, \ldots\}$

- **Solving for optimal points:** the optimal point for $f(x)$ is $(x^*, f(x^*))$ such that $f'(x^*) = 0$

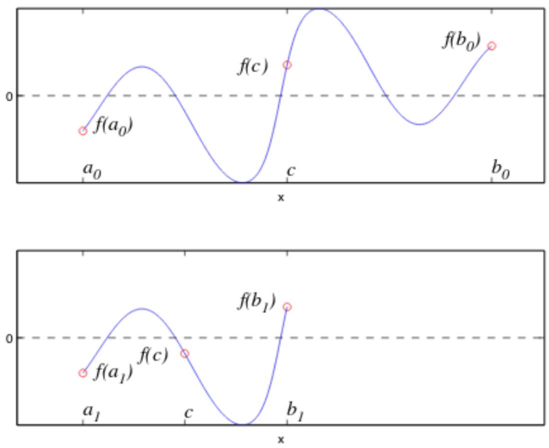
Numerical Ways to Solve for x

General Approach

1. <u>**Random**</u> the value for x^*
2. Evaluate $y \leftarrow f(x^*)$
3. Reevaluate the value of x^* until y approaches zero

Bisection Method

1. Assume that our solution is x^* within a closed interval $x^* \in [a, b]$
2. Given $m = (a+b)/2$ is the midpoint between a and b
3. Consider the following conditions:
   - If $f(a)f(m) < 0$, assign $b \leftarrow m$
   - Otherwise, assign $a \leftarrow m$
4. Repeat from step 2 until $f(m) \approx 0$



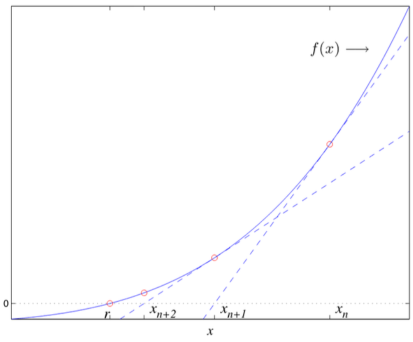
Additional Illustration to describe how step 3 works.

Given $x = r$ the true solution, and c_n is the estimated solution at the n^{th} iteration, then $|r - c_n|$ is the approximation error from the n^{th} iteration, which is

$$|r - c_n| \leq 2^{-(n+1)}(b_0 - a_0)$$

where a_0, b_0 is the minimum and the maximum of expected interval for x^* respectfully. In other words, the rate of convergence for **Bisection Method** is *linear.*

Newton's Method



The difference between each iteration

1. Given x_0 be an initial <u>**random**</u> point.
2. Any next term x_i is

$$x_i = x_{i-1} - \frac{f(x_{i-1})}{f'(x_{i-1})}$$

3. Repeat until $f(x_n) \approx 0$

c_n is the approximation error at the n^{th} iteration, then the rate of convergence is

$$e_{n+1} \approx ce_n^2$$

We can say that **Newton's method finds the solution <u>quicker</u> than Bisection method.**

In case when $f'(x)$ couldn't be determined symbolically, then

$$f'(x) \approx \frac{f(x_n) - f(x_{n-1})}{x_n - x_{n-1}}$$

Secant Method

is the modification from Newton's method. Instead of implicitly evaluate $f'(x)$, it's done better using the approximation from prior value of x, in the same way you'd approximate any slope m.

$$x_{n+1} = x_n - f(x_n)\frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})}$$