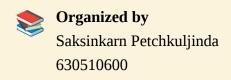
Summary for 204381: Chapter 5



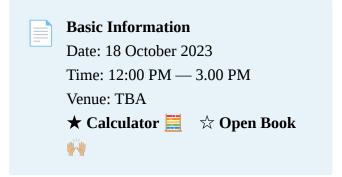


Table of Contents

Table of Contents

System of Linear Equations \rightarrow Matrix

Ways to find $ec{x}$

Summary for 204381: Chapter 5

Inversion

Gaussian Elimination

Best way to select pivot rows

Gauss-Seidel Method

System of Linear Equations → **Matrix**

Given a system of n equations with n variables:

$$egin{aligned} a_{11}x_1+a_{12}x_2+...+a_{1n}x_n&=b_1\ a_{21}x_1+a_{22}x_2+...+a_{2n}x_n&=b_2\ &&\cdots\ a_{n1}x_1+a_{n2}x_2+...+a_{nn}x_n&=b_n \end{aligned}$$

We can transform the problem into a linear equation, as

$$egin{bmatrix} a_{11} & a_{12} & ... & a_{1n} \ a_{21} & a_{22} & ... & a_{2n} \ a_{n1} & a_{n2} & ... & a_{nn} \end{bmatrix} egin{bmatrix} x_1 \ x_2 \ ... \ x_n \end{bmatrix} = egin{bmatrix} b_1 \ b_2 \ ... \ b_n \end{bmatrix}$$
 $A\vec{x} = B$

Ways to find $ec{x}$

Inversion

Because

$$A\vec{x} = B$$
$$\vec{x} = A^{-1}B$$

This can be done easily on scientific calculators, or with any techniques you like. However, the computation cost for large matrices is considerable.

Gaussian Elimination

Perform elemental row operations onto A and B until A is in the form of

$$A' = egin{bmatrix} imes & imes &$$

where \times is any non-zero number. Note that it does not always required to be perfectly align diagonally, but the zero terms should form like that.

then suppose B is transformed into

$$B' = egin{bmatrix} b_1' \ b_2' \ ... \ b_n' \end{bmatrix}$$

and a'_{ij} is a term corresponding to A above, then the solution for $ec{x}$ is

$$ec{x} = egin{bmatrix} rac{a'_{11}}{b'_1} \ rac{a'_{22}}{b'_2} \ rac{a'_{nn}}{b'_n} \end{bmatrix}$$

To sum up, a_{ij}^\prime and b_i^\prime can be evaluated for any pivot row k as follows:

$$a_{ij}' = egin{cases} a_{ij} - rac{a_{ik}}{a_{kk}} a_{kj} & ext{for } k < i, j \leq n \ 0 & ext{for } k < i \leq n, j = k \ a_{ij} & ext{else} \end{cases}$$

$$b_i' = egin{cases} b_i - rac{a_{ik}}{a_{kk}} b_k & ext{for } k < i \leq n \ b_i & ext{else} \end{cases}$$

Best way to select pivot rows

Always choose the row with greatest ratio, for example:

$$egin{bmatrix} 1 & -1 & 2 & 1 \ 3 & 2 & 1 & 4 \ 5 & 8 & 6 & 3 \ 4 & 2 & 5 & 3 \end{bmatrix} egin{bmatrix} x_1 \ x_2 \ x_3 \ x_4 \end{bmatrix} = egin{bmatrix} 1 \ 1 \ 1 \ -1 \end{bmatrix}$$

Calculate a ratio r_i for each row i, for pivot row k we get

$$egin{aligned} r_{k,i} &= rac{a_{i,k}}{\max_{1 \leq j \leq n} a_{i,j}} \ & \ r_{1,1} &= rac{1}{2}, r_{1,2} = rac{3}{4}, r_{1,3} = rac{5}{8}, r_{1,4} = rac{4}{5} \end{aligned}$$

In this case, row 4 has the greatest ratio, so we should perform the elimination using row 4 as a pivot row.

Repeat this step then we will get A^\prime as intended.

Gauss-Seidel Method

- 1. Initiate a **random** real vector \vec{x}
- 2. For each equation i in the system of equations,

$$a_{i,1}x_1 + a_{i,2}x_2 + ... + a_{i,i}x_i + ... + a_{i,n}x_n = b_i \ x_i^{ ext{new}} = rac{b_i - a_{i,1}x_1 - a_{i,2}x_2 - ... - a_{i,i-1}x_{i-1} - a_{i,i+1}x_{i+1} - ... - a_{i,n}x_n}{a_{i,i}}$$

3. Repeat until the relative change is below a threshold ϵ that

$$\Big|rac{x_i^{ ext{new}}-x_i^{ ext{old}}}{x_i^{ ext{new}}}\Big| imes 100<\epsilon$$