

The main contributions to the entropy of the various Bravais and non-Bravais lattice configurations for $N \leq 27$ are readily assessed. For a Bravais lattice packing, there are no non-trivial continuous symmetries and thus there is no entropy. However, one can construct density-one packings from the Bravais lattice packings by shifting rows relative to one another. For example, if N is a perfect square then each row can be arbitrarily shifted and, ignoring overall shifts of the lattice, the entropy is proportional $(\sqrt{N} - 1)$. Note that either rows or columns may be shifted but not both for density-one packings. More generally, rows are free to shift unless the constraints of periodicity forbid it. Periodicity forbids two rows from shifting relative to one another if some linear combination of torus lattice vectors connects the rows. Suppose that rows are aligned in the $\hat{\mathbf{x}}$ -direction. Then, two rows are constrained to their relative positions in the Bravais lattice configuration if there are integers a and b such that $a\mathbf{A}_1 + b\mathbf{A}_2$ connects the two rows. Thus, rows separated by $an_2 + bn_4$ are locked (see Eq. ?? and Fig. ?? (b)). Bezout's Lemma [?] states that this integer linear combination can be made equal to, but not smaller than, g , where g is the greatest common divisor of n_2 and n_4 (assuming that both n_2 and n_4 are nonzero). Thus, the set of rows can be divided into $|n_4|/g$ groups, each of which can be arbitrarily shifted, and the entropy is proportional to $|n_4|/g - 1$. Note that if $|n_2|$ and $|n_4|$ are mutually prime, all rows are locked and this contribution to the entropy of the configuration vanishes. There are two other sources of entropy for packing related to Bravais lattice packings. The gapped bricklayer configurations (Section ??) allow squares within a row to shift perpendicular to the row axis (see Figures ?? and ??), and this "poor workmanship" contribution to the entropy will be roughly proportional to the free volume of the configuration. The density-one configurations with defects $(n_2^2 + n_4^2 - k)$, discussed in Section ??, also possess finite entropy, since the hole(s) created by the k missing squares may be moved throughout the lattice, or split along a row; and for $k > 1$, holes can appear in different rows. In contrast, the unusual, non-Bravais lattice packings of the $N = 12$ and $N = 23$ exhibit no entropy.