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Abstract

A relatively unexplored geometric property of Potts models clusters is their "diameter", D – the longest shortest path between any two points on the cluster. We report numerical results for the fractal dimension of the diameter, D_{min} and the fractal dimension of the chemical distance, d_{min}, for 2D critical Potts clusters with $q = 1, 2, 3, 4, 5$. We find that $D_{min} = d_{min}$ within numerical error.

I. INTRODUCTION

II. POTTS MODEL SIMULATIONS

Topics: Swedsen Wang; Determining the chemical distance; determining the diameter; autocorrelation time; scaling

A. The Potts Model

B. Swendsen Wang Algorithm

We performed Monte Carlo simulations of critical q-state Potts model clusters in 2D and 3D using the Swendsen-Wang algorithm (SW) [**?** **?**].

The SW algorithm, which is itself based on the work of Fortuin and Kasteleyn [**?**], works by first introducing bonds between neighboring spins, with probability

$$p(\sigma_i, \sigma_j) = \delta_{\sigma_i,\sigma_j}(1 - e^{-K}), \tag{1}$$

thus creating clusters of bonded spins. All clusters thus formed are then, with probability $1/2$, flipped by choosing a random spin value from the q possible values, and assigning this value to all sites in the cluster. Such cluster-flipping algorithms dramatically reduce critical slowing down in computer simulations of spin models, as compared with algorithms that flip each spin individually [**?**] (e.g. the Metropolis algorithm [**?**]).

C. Algorithms for determining the chemical distance and the diameter

The average chemical distance $\langle l \rangle$ for each lattice size L was determined in the following manner. The largest cluster in the lattice was identified, and a randomly chosen site A on this cluster was used as the initial seed for a Leath growth (CITE) process, which amounts to a breadth-first growth along cluster bonds. Each iteration i of the Leath growth process covers all sites at chemical shell i; once the Leath process has covered all N sites in the

cluster, the chemical distance l between the seed site, A, and any other site, B, is thus equal to the chemical shell on which site B resides. We chose B at random from the sites on the largest possible chemical shell reached from site A on the cluster. The chemical distance between A and B chosen in this manner was then averaged over the largest cluster in all of the lattice realizations considered.

D. Details of simulation

1. autocorrelation time & independence

We measured the autocorrelation time for each L and q. We then chose intervals such that the samples were deemed to be statistically independent. (As a check, we also performed the block method technique.)

2. scaling methods, detail

We chose the fit with the lowest L (max no. of points) and highest Q value for each fit. (Alternative methods? See Sokal)

III. RESULTS AND DISCUSSION

A. Results

Using these methods we were able to determine through numerical simulations the scaling exponents for the chemical distance d_{min} and for the diameter D_{min} for system sizes $L \times L$, $4 \leq L \leq 128$ in 2D and $4 \leq L \leq X$ in 3D. The results of these simulations (see Tables I and **??**) indicate that d_{min} and D_{min} are equivalent to within error .

[Mean field limit?]

IV. BIBLIOGRAPHY

q	1	2	3	4	5
d_{min}	1.127(3)	1.0911(2)	1.063(1)	1.023(7)	(1.000)
D_{min}	1.129(2)	1.087(8)	1.060(2)	1.025(2)	(1.000)

TABLE I: **Results for 2D Potts Model.** Scaling exponent for the chemical distance (d_{min}) and for the diameter (D_{min}) for the 2D Potts model with various values of q, with system size L=4, 8, 16, 32, 48, 64, 96, 128.