A. Methods

We used the Swendsen Wang algorithm to simulate the Potts Model for $q = 1, 2, 3, 4$ on 2D square lattices of various sizes L, and for $q = 2$ in 3D and 4D cubic and hypercubic lattices. In 2D, L ranged from 16 to 128; in 3D, from 20 to 128; and in 4D, from 12 to 64. For 2D, $q = 1$, the chemical distance l and the diameter D were measured after every Monte Carlo sweep, for a total of $N = 10^5$ measurements. For 2D $q > 1$, the systems were allowed to equilibrate: 100 initial sweeps of the lattice were discarded at the beginning of each simulation, and an additional $10 * \tau_{exp}(m)$ sweeps were discarded after data was collected, where τ_{exp} was the measured exponential correlation time of the mass of the largest cluster in each lattice. In order to further reduce correlations in the data for $q > 1$ in 2D, an interval of 10 sweeps separated each measurement during the simulation; this interval was always greater than $2\tau_{int}(y)$, where $\tau_{int}(y)$ was the measured value of the integrated correlation time of y ($y = l$, or D). The total number of measurements made in this manner for 2D, $q > 1$ was $N = 10^5$; for the 2D, $q = 4$, $L = 128$ lattice, this amounted to a simulation time of approximately $3.1 \times 10^5 \tau_{int}(D)$. In 3D and 4D, measurements were made every Monte Carlo sweep for a total of $N = 10^5$ measurements. For all lattices in 2D, 3D, and 4D with $q > 1$, the estimated standard deviation in the averaged values of l and D was considered to be $\sigma_{corr} = \sqrt{\frac{2\tau_{int}(y)}{N}(\langle y^2 \rangle - \langle y \rangle^2)}$. For $q = 1$, the standard deviation was calculated as $\sigma_{uncorr} = \sqrt{\frac{1}{N-1}(\langle y^2 \rangle - \langle y \rangle^2)}$.

In order to determine the value of B in the scaling Ansatz $y = AL^B$ (where y is equal to l or D, and B is equal to, respectively, d_{min} or D_{min}), we performed a weighted least-squares fit using the Levenberg-Marquardt [REF] that minimized $((y - data)/\sigma))^2$, where σ was defined as above. The resultant fits are displayed in Figures 1 through 15 below. A summary of the fit results for the scaling exponents $B = l_{min}$ and D_{min} is reported in Tables I and II as $B \pm \sqrt{\nu_B}$, where ν_B is the diagonal element of the covariance matrix corresponding to parameter B.

To account for corrections to scaling, we performed fits on subsets of the data with a variable lower L cutoff of L_{min}, and chose to report the value of B that resulted from including the smallest L_{min} where the goodness of fit $Q > .2$; Q is the incomplete gamma function $Q(p/2, \chi^2/2)$, defined [REF Numerical Recipes (6.2.3)] as $\frac{1}{\Gamma(p)} \int_x^\infty e^{-t} t^{p-1} dt$, with p being the number of degrees of freedom in the fit.

For $q = 4$ in 2D, we also attempted a fit of the form $y = AL \log L(1 + B/L)$, which yielded Q values much lower than those resulting from the corresponding $y = AL^B$ fits.

I. TABLES

dim	q	L	L_{min}	d_{min}	D_{min}
2	1	16,32,48,64,96,128	48	1.131(1)	1.138(1)
2	2	16,32,48,64,96,128	48	1.096(1)	1.102(1)
2	3	16,32,48,64,96,128	48	1.065(3)	1.071(1)
2	4	16,32,48,64,96,128	48	1.033(3)	1.039(1)

TABLE I: Scaling exponents d_{min} and D_{min} for $dim = 2$, $q = 1, 2, 3, 4$

dim	q	L	L_{min}	d_{min}	D_{min}
3	2	20,36,48,64,128	36	1.267(5)	na
4	2	12,24,36,48,64	24	1.485(7)	na

TABLE II: Scaling exponents d_{min} and D_{min} for $dim = 3, 4$, $q = 2$

II. FIGURES

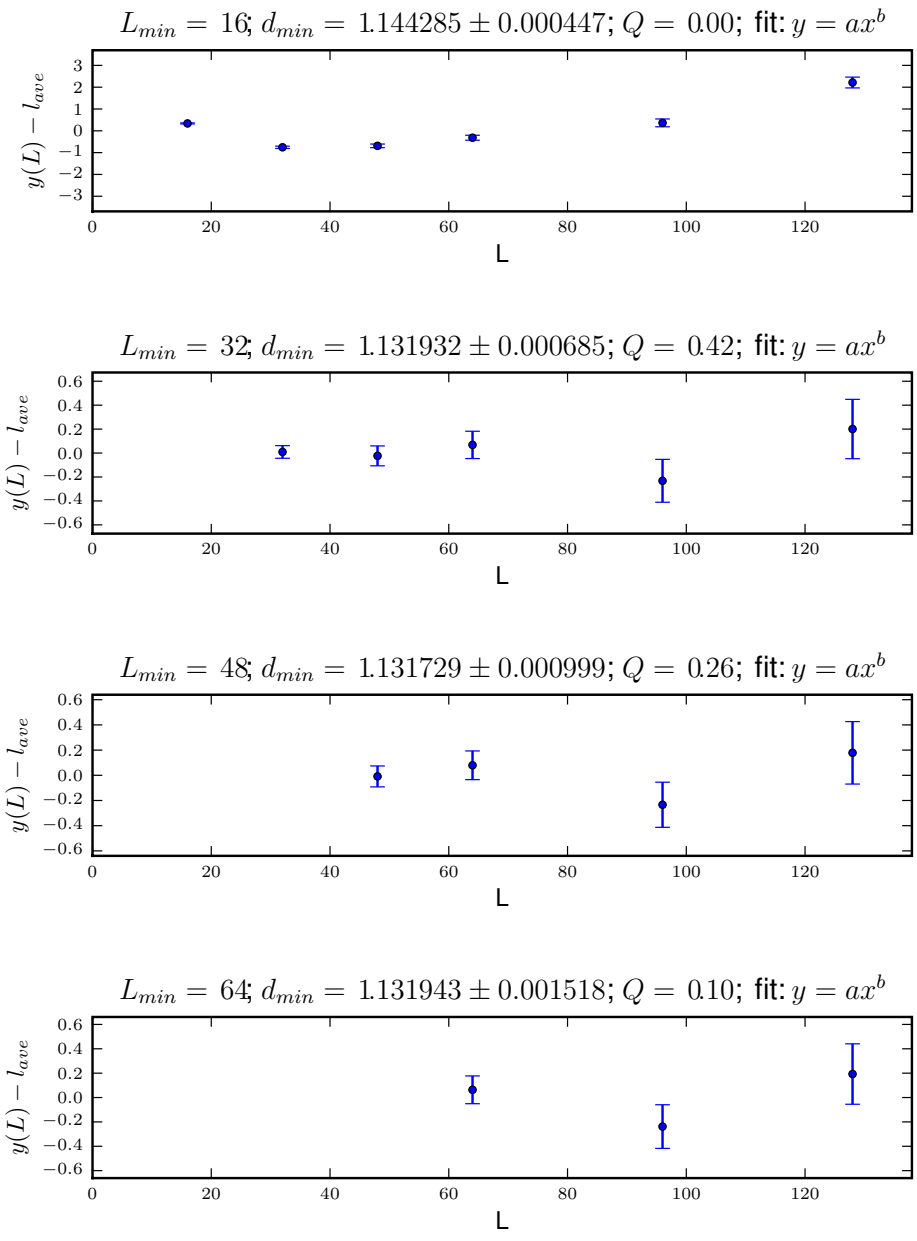

FIG. 1: The difference between the fit, $y(L) = cL^{d_{min}}$, and the average chemical distance $\langle l \rangle$ for dim=2, q=1.

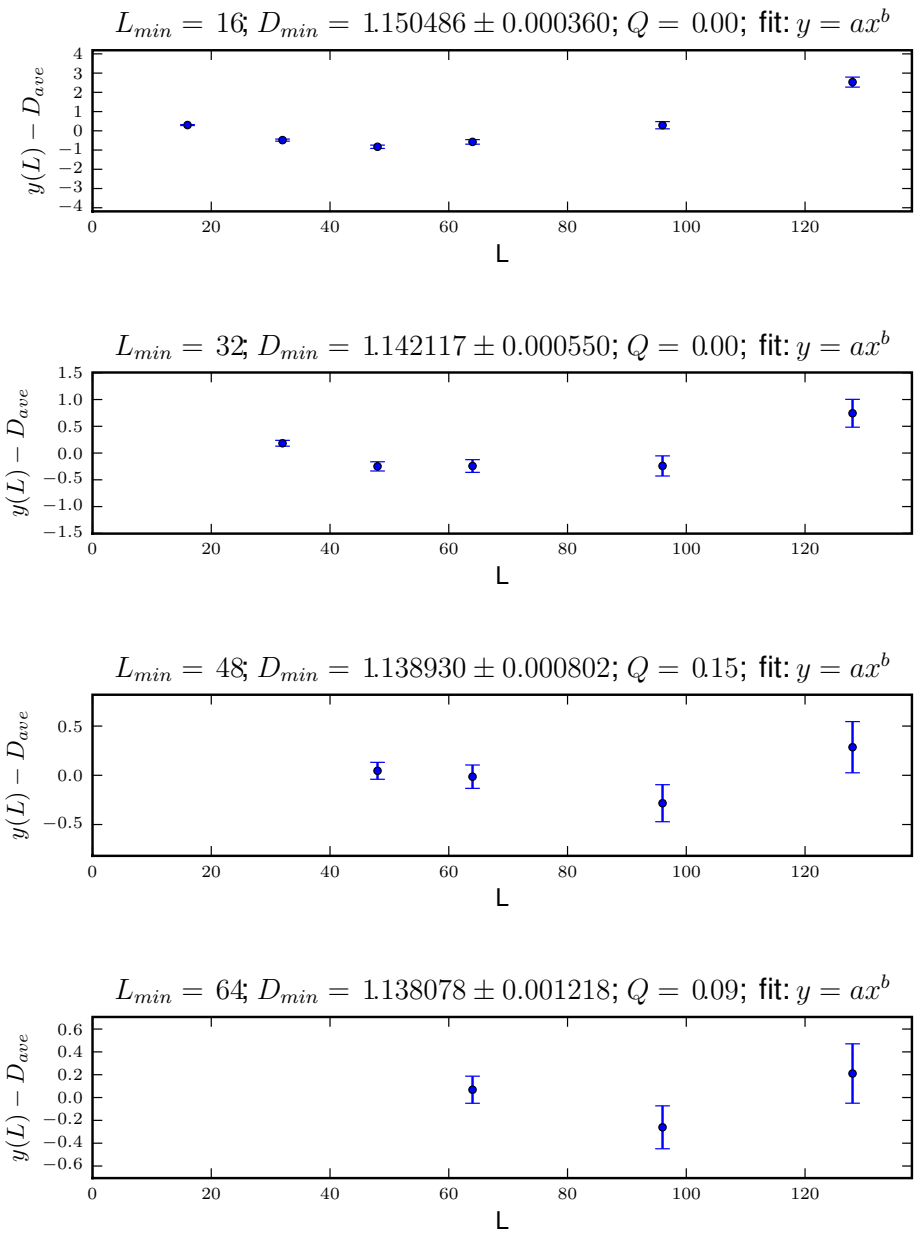

FIG. 2: The difference between the fit, $y(L) = cL^{D_{min}}$, and the average diameter $\langle D \rangle$ for dim=2, q=1.

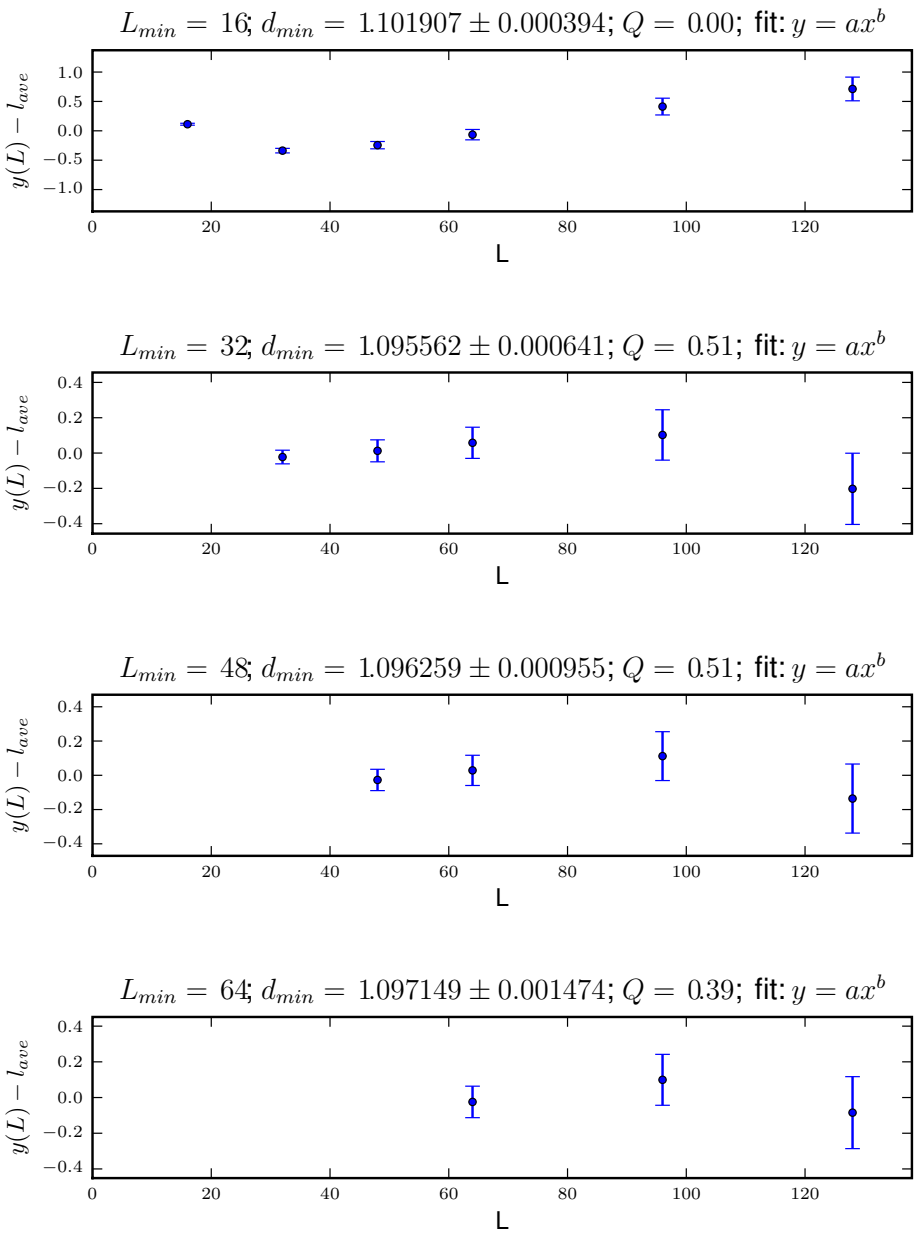

FIG. 3: The difference between the fit, $y(L) = cL^{d_{min}}$, and the average chemical distance $\langle l \rangle$ for dim=2, q=2.

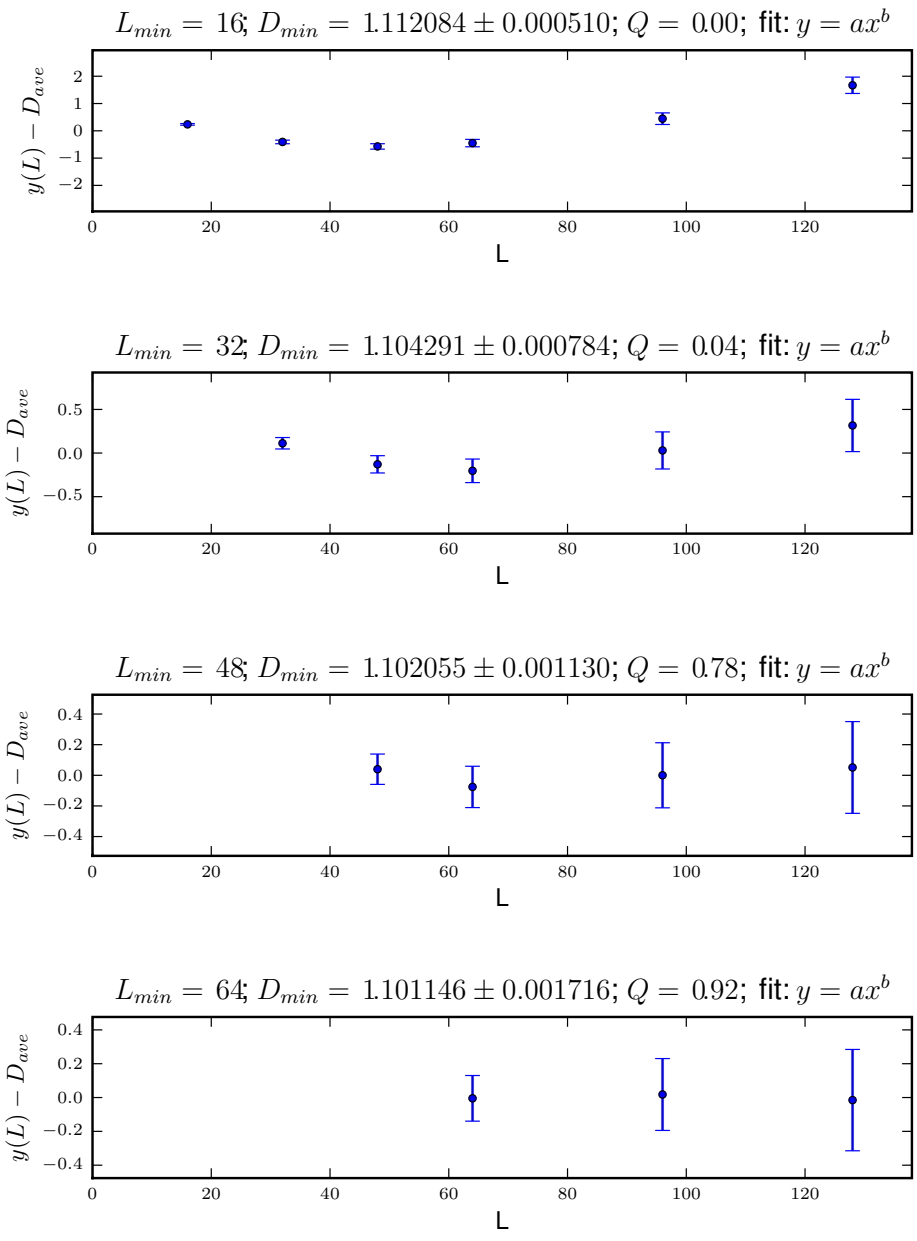

FIG. 4: The difference between the fit, $y(L) = cL^{D_{min}}$, and the average diameter $\langle D \rangle$ for dim=2, q=2.

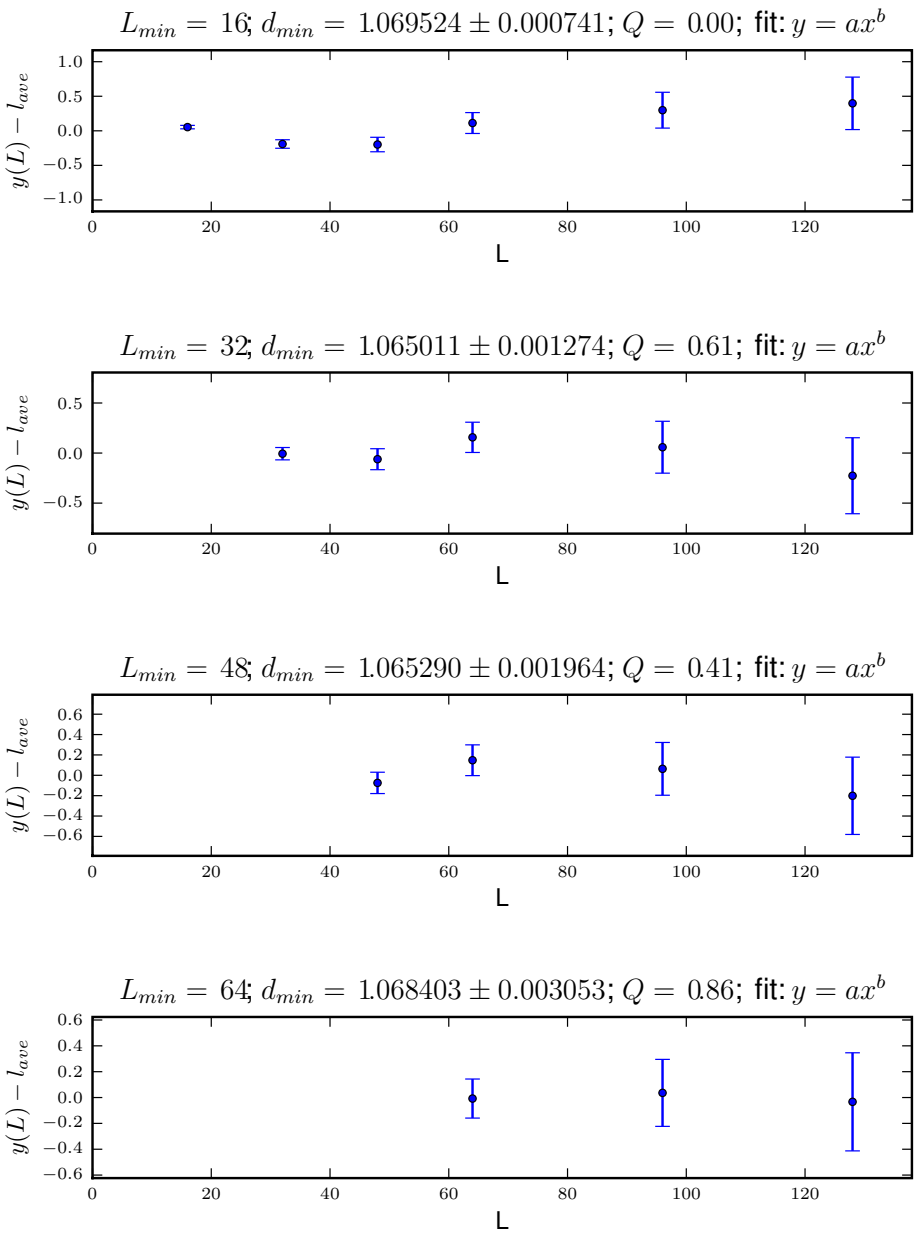

FIG. 5: The difference between the fit, $y(L) = cL^{d_{min}}$, and the average chemical distance $\langle l \rangle$ for dim=2, q=3.

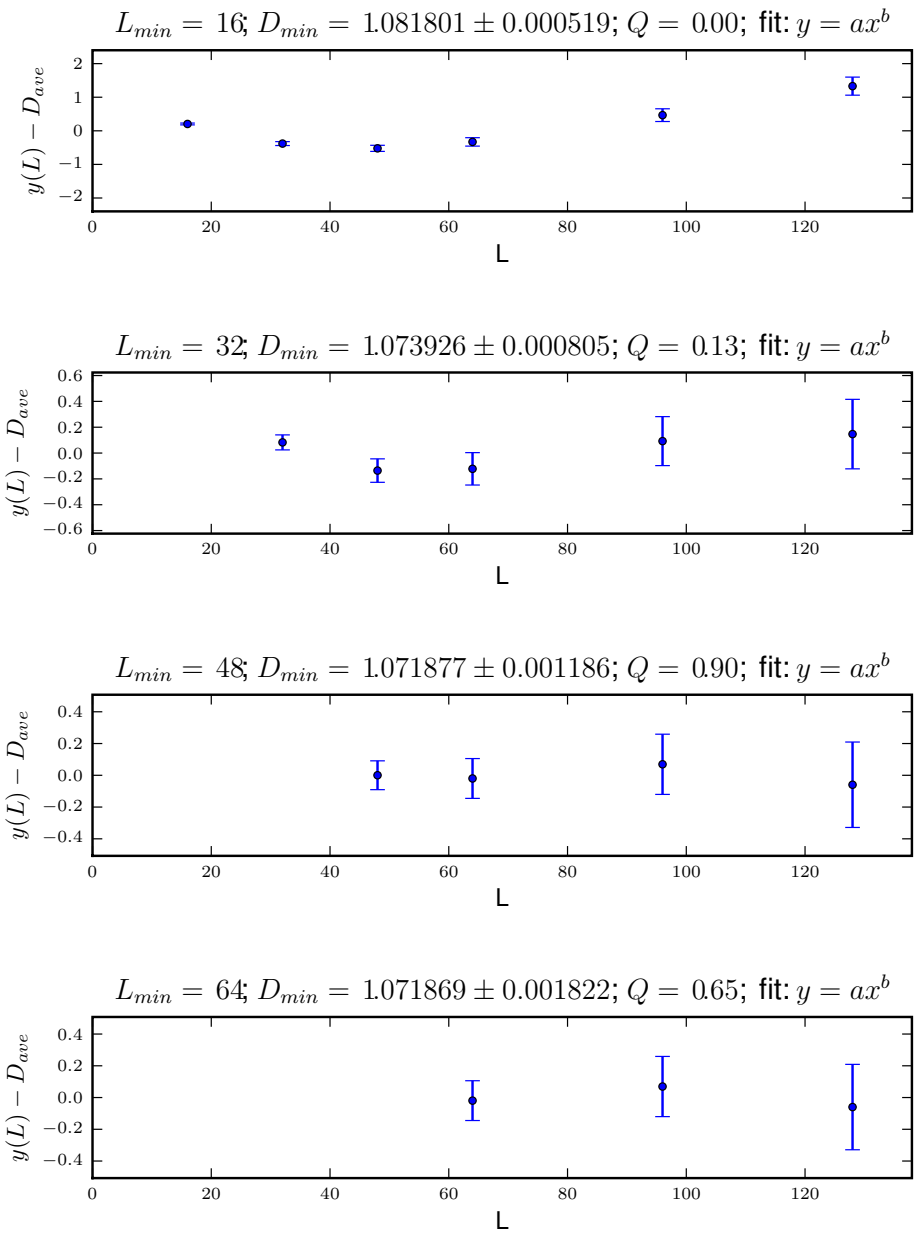

FIG. 6: The difference between the fit, $y(L) = cL^{D_{min}}$, and the average diameter $\langle D \rangle$ for dim=2, q=3.

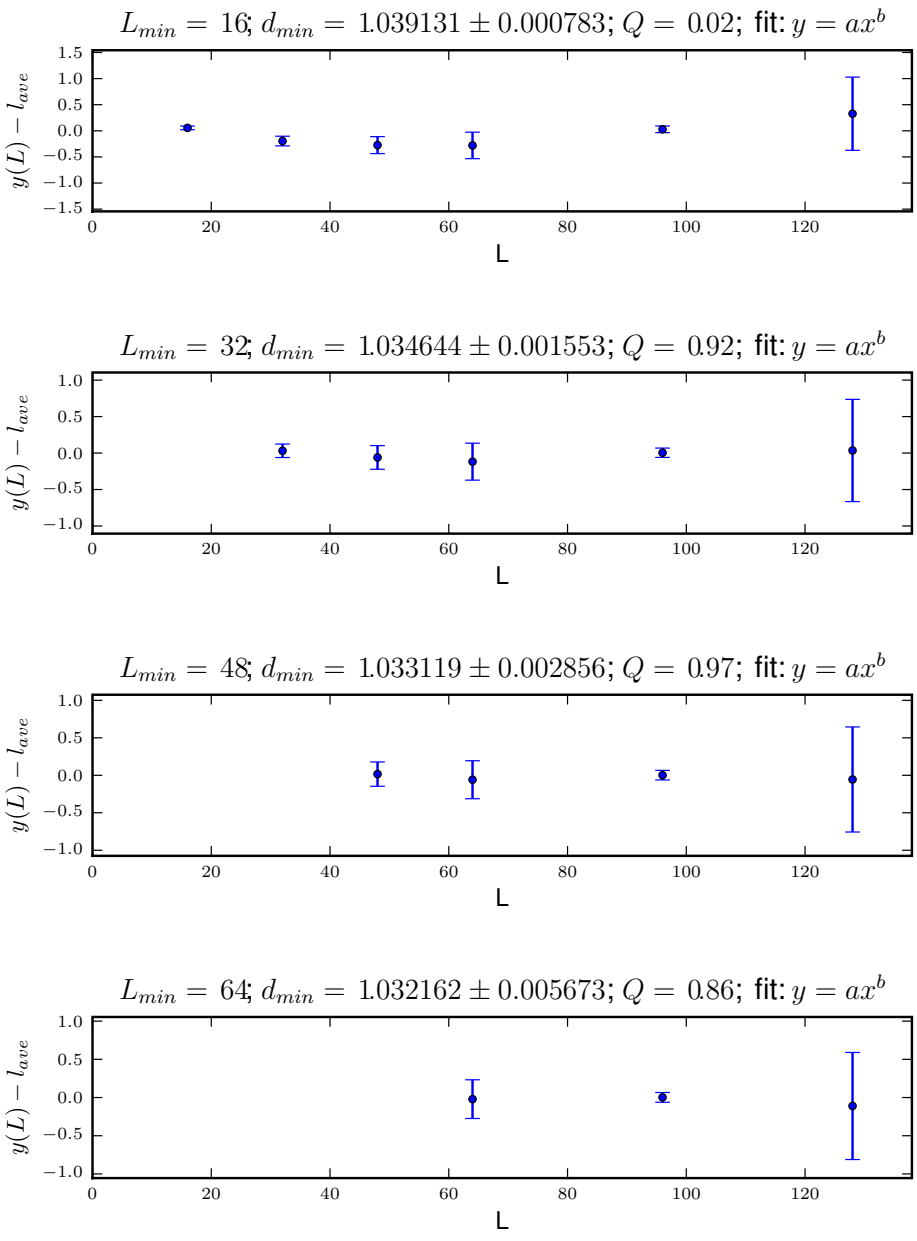

FIG. 7: The difference between the fit, $y(L) = cL^{d_{min}}$, and the average chemical distance $\langle l \rangle$ for dim=2, q=4.

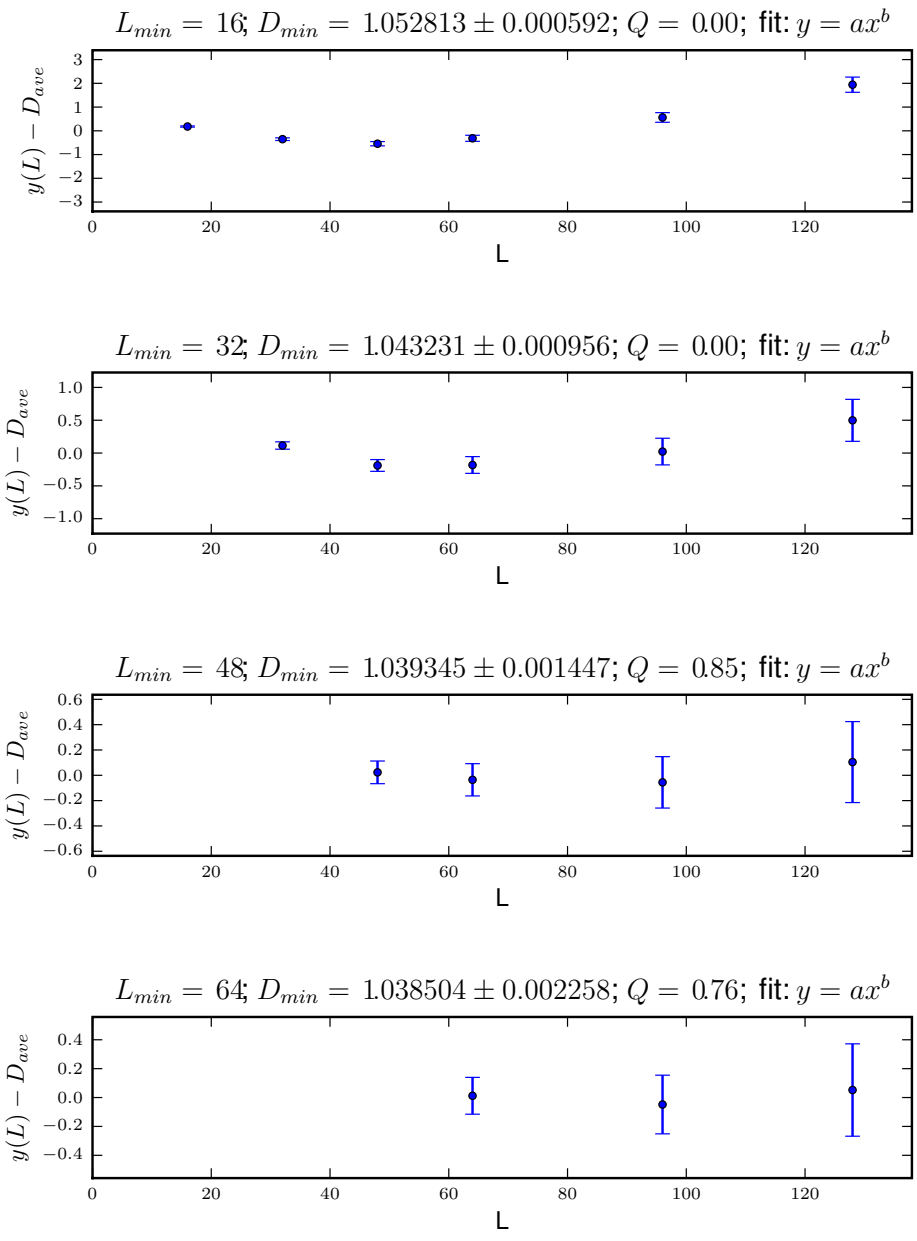

FIG. 8: The difference between the fit, $y(L) = cL^{D_{min}}$, and the average diameter $\langle D \rangle$ for dim=2, q=4.

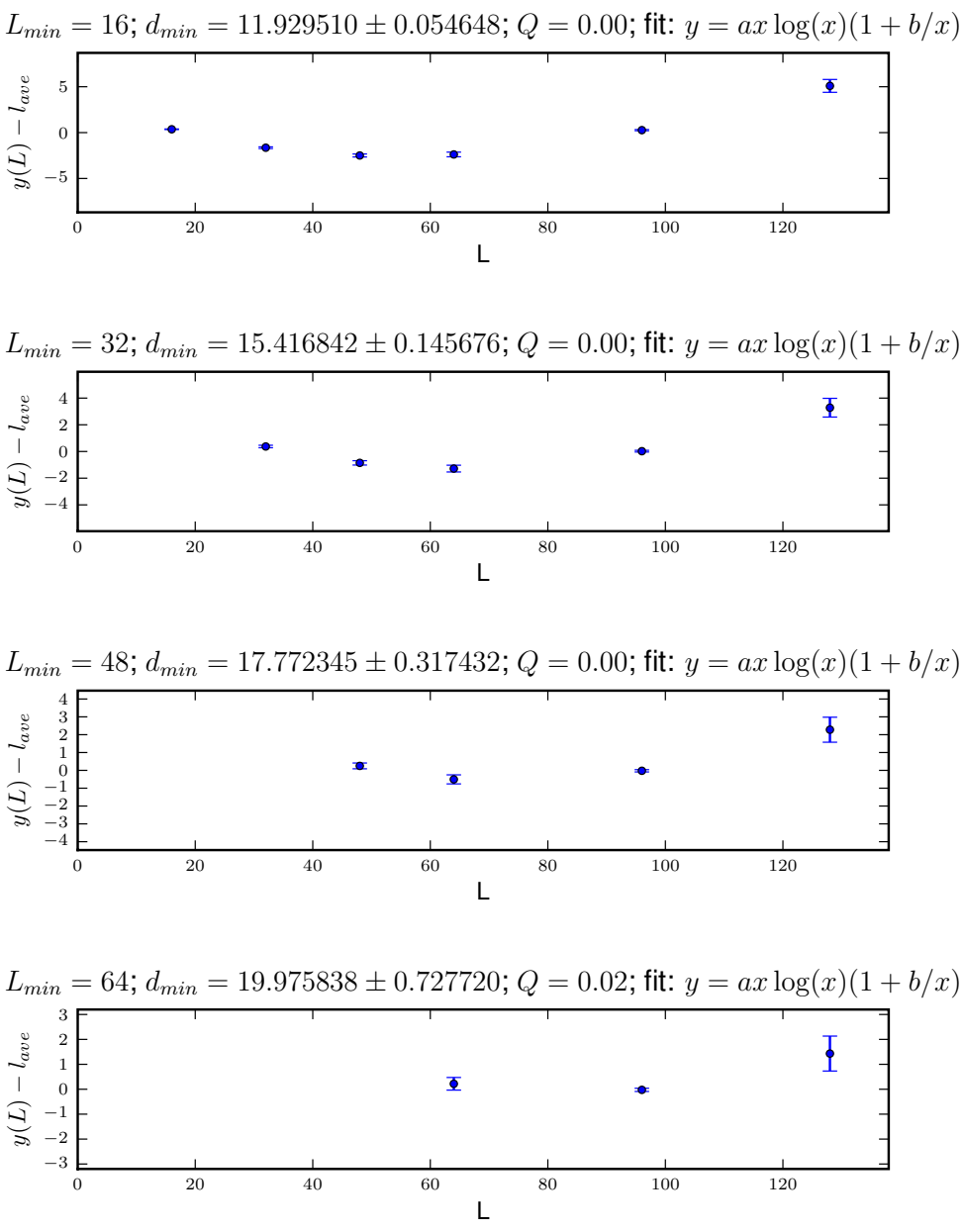

FIG. 9: The difference between the fit, $y = AL\log L(1 + B/L)$ and the average chemical distance, $\langle l \rangle$ for dim=2, q=4.

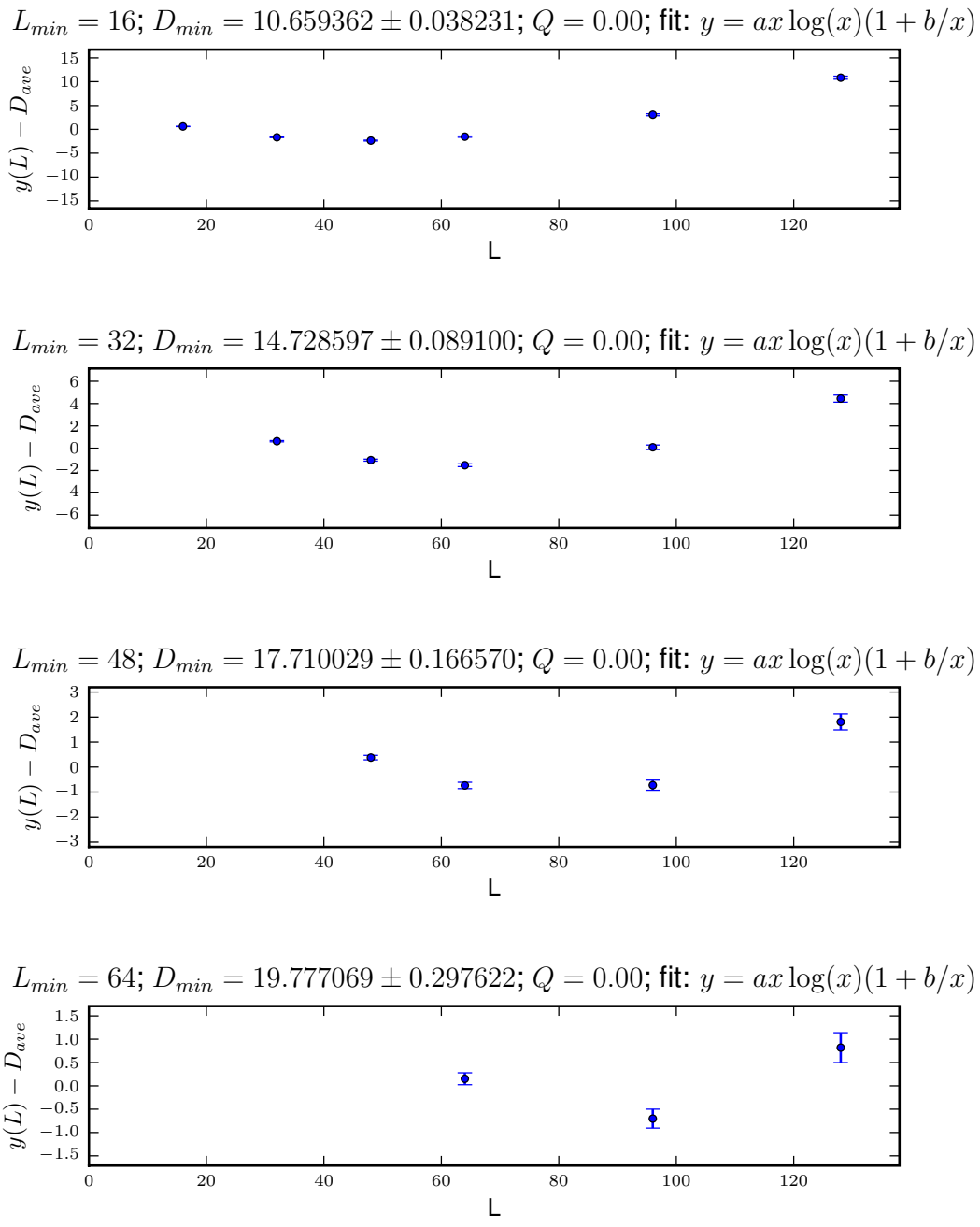

FIG. 10: The difference between the fit, $y = AL\log L(1 + B/L)$ and the average diameter, $\langle D\rangle$ for dim=2, q=4.

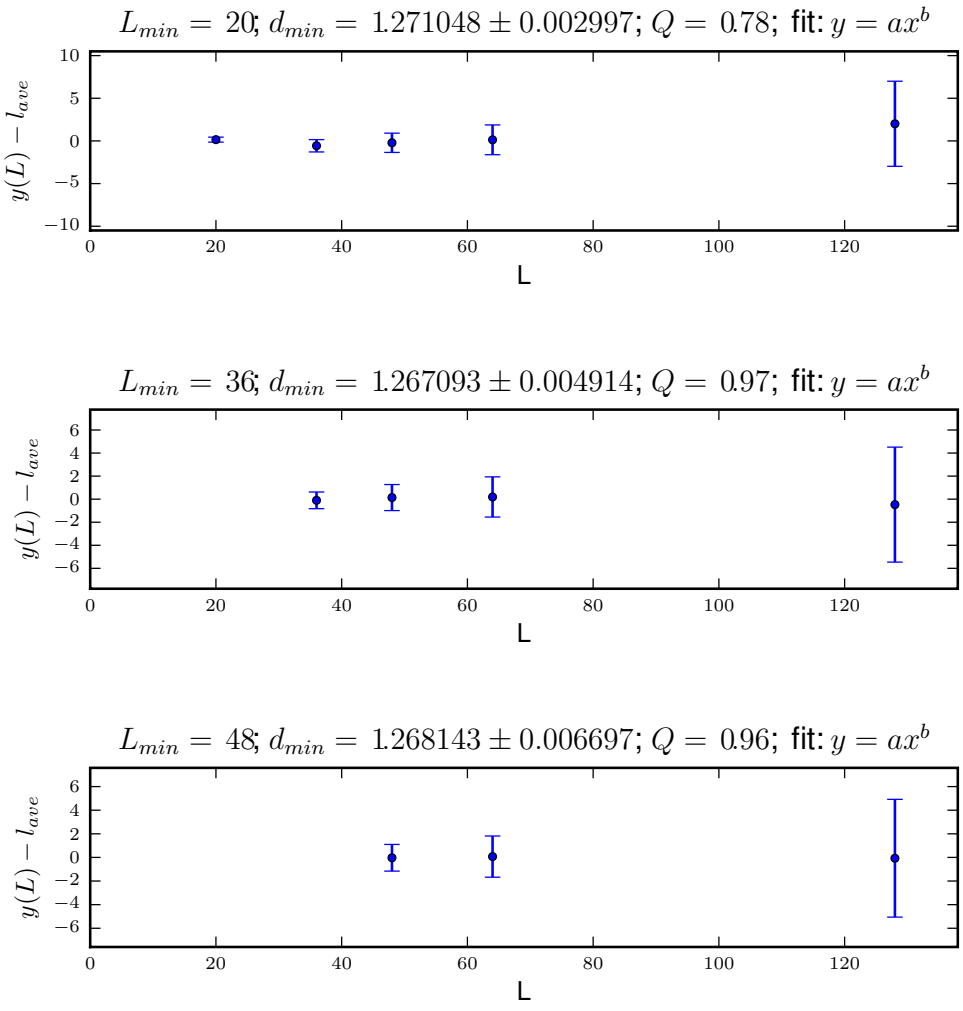

FIG. 11: The difference between the fit, $y(L) = cL^{d_{min}}$, and the average diameter $\langle l \rangle$ for dim=3, q=2.

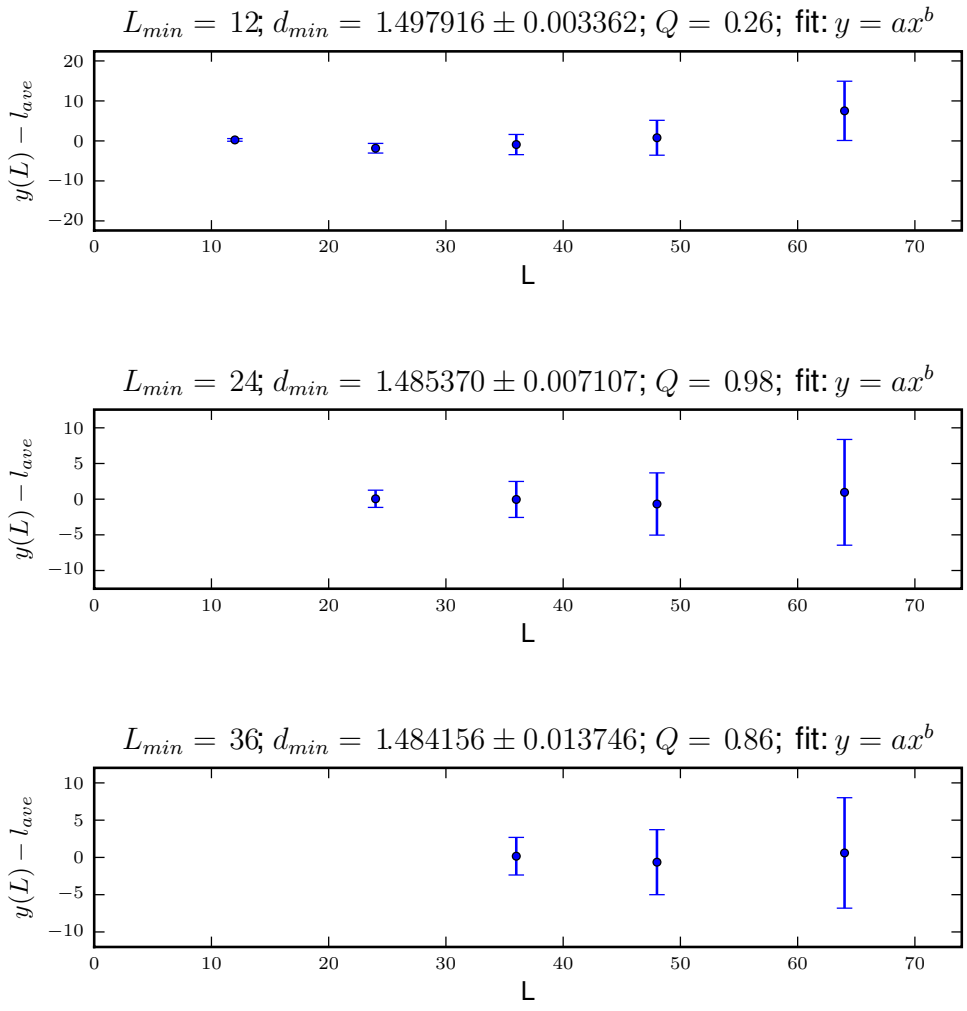

FIG. 12: The difference between the fit, $y(L) = cL^{d_{min}}$, and the average diameter $\langle l \rangle$ for dim=4, q=2.

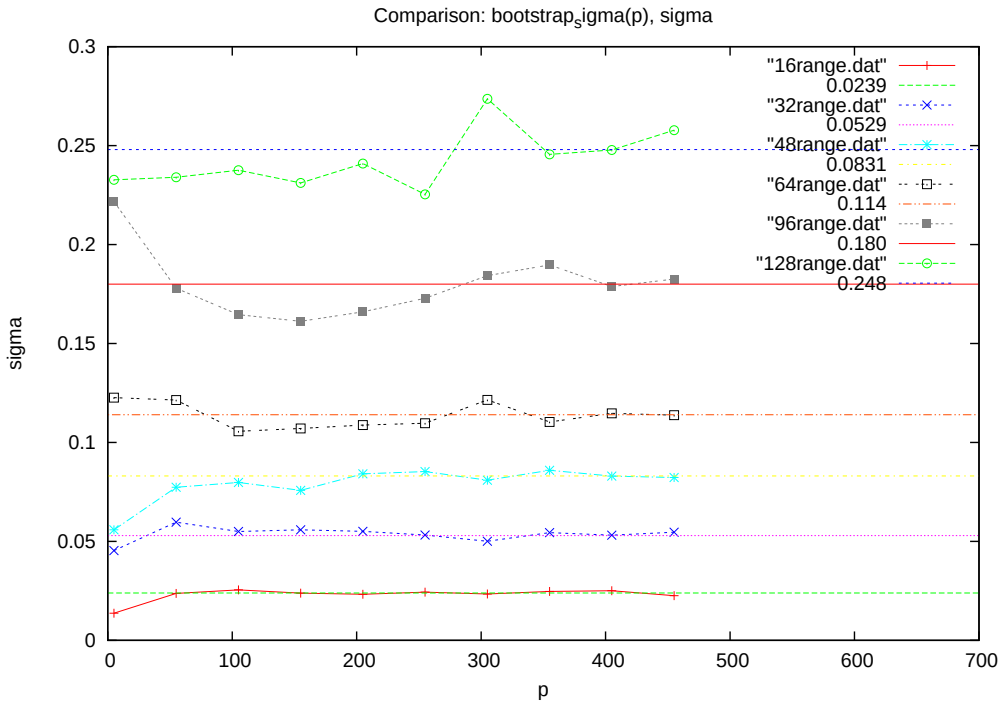

FIG. 13: $D = 2, q = 1, d_{min}$: comparison between σ calculated as per the above description "Methods", and the bootstrap method with p iterations.

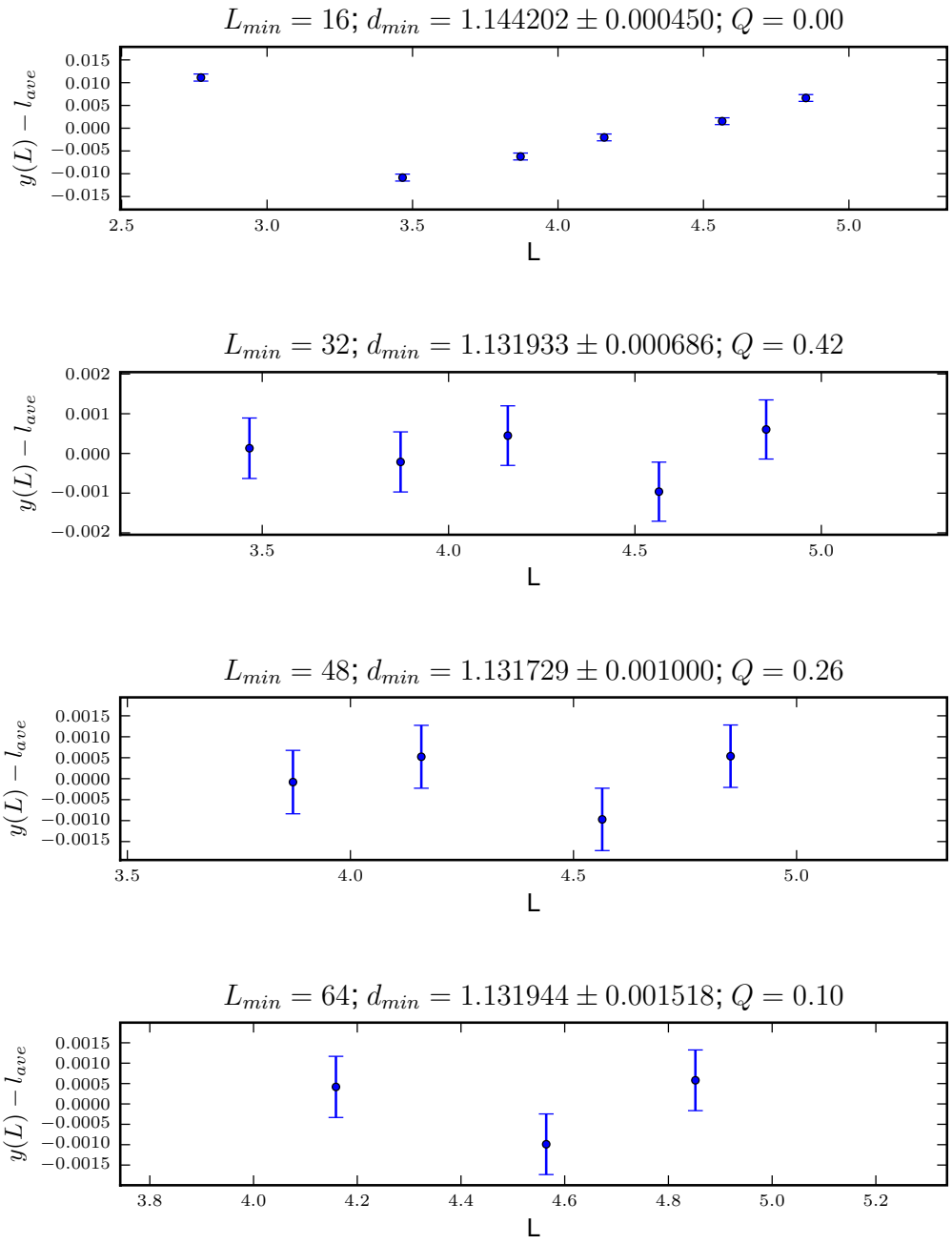

FIG. 14: $D = 2, q = 1, d_{min}$: linear fit of the "log-log" data (axes should be "log")

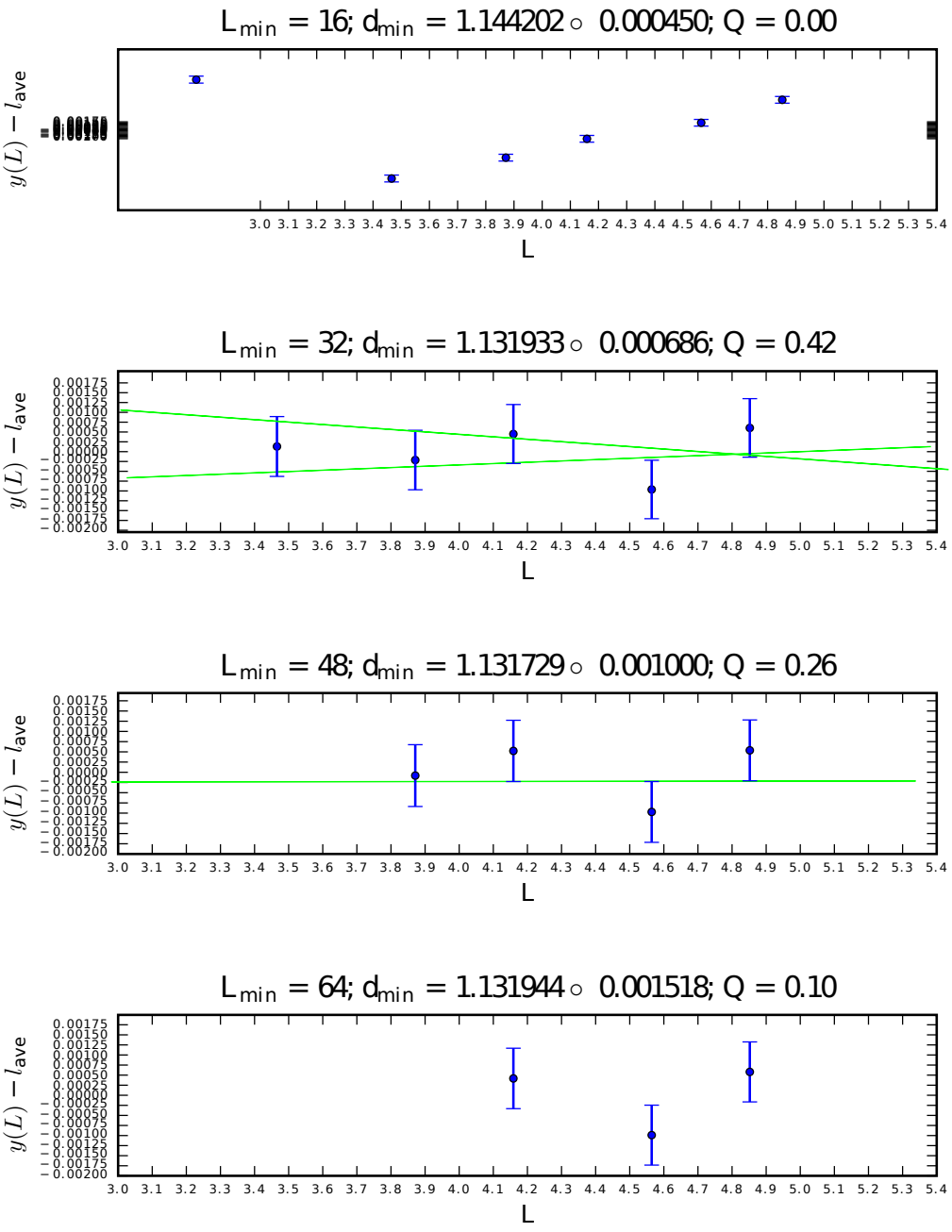

FIG. 15: $D = 2, q = 1, L_{min} = 32, d_{min}$: "Hand-estimate" of the error in the slope of the log-log data yields an error of ± 0.0010 in the value for d_{min} (axes should be "log").