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TITLE: Packing Squares in a Torus

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Referee report

The authors study the possible densest packing of squares in a torus, i.e. in a bigger square with periodic boundary conditions. The same problem without periodic boundary conditions has a long history and it involves considerable interests in combinatorics. It is highly non-trivial problem even for relatively small systems. However, in my opinion, this problem has much less interest in physics, because of the finite and small system size. The problem is more easy, however - in many cases - not trivial with periodic boundary conditions discussed in the paper. The results could be interesting for physicists but I feel that the motivation of the work is a little bit poor.

The exact results presented in subsection II/A/1 is almost trivial. All the other packing have found by the authors are based on NPT Monte Carlo simulations and only conjectured that they are densest packings. In spite of this, the non-Bravais lattice packings are interesting, I feel that this part gives the scientific value of the paper.

I have found some minor errors, see my notes 2, 4 and 5. Apart from this, the manuscript is clearly written, its length as well as the role and the number of figures is appropriate. If the authors correct the minor errors and improve the paper taking into account my notes I suggest for publication.

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My notes:

- 1) I suggest to improve the motivation of the work in the Introduction.
- 2) In the first line of II/A/2 the authors write about the case " $N=n_2^2+n_4^2$ ", however, in the previous subsection they have proved that this is the case of perfect packing. I think this is a mistake, they should write " $N=n_2^2+n_4^2-k$ ".
- 3) Moreover, I feel that the title of the subsection II/A/2 is ambiguous, at the first sight the reader does not understand how can be a packing is density-one if it contains vacancies. I suggest the simple title "vacancies" or "vacancies in density-one packings" or something similar.
- 4) In the first line of II/A/3 the authors write " $d<1$ ". It is a mistake, correctly it would be $d>1$.
- 5) The reference [22] is incomplete, this famous article has two authors, R. L. Graham beside of Paul Erdős.

My question.

Q1) In II/A/2 the authors speak about density-one packings with a vacancy. It has no figure about this packing but the text suggests that the unit squares are situated precisely at the lattice points of a Bravais lattice except of one lattice point at which the vacancy is situated. Why do not the nearest neighbours of the vacancie move partially into the empty place? I feel that the empty space covered by the vacancie should be distributed randomly along a line (or column), breaking the translational invariance of the Bravais lattice. Am I right or not? I suggest to clarify this issue in the text.