I. METHODS

We used the Swendsen Wang algorithm to simulate the Potts Model with $1 = 1, 2, 3, 4$ on square (dim=2) lattices of various sizes L, and with $q = 2$ in the cubic and hypercubic lattices (dim=3,4). In the largest cluster in each of $N = 10^5$ realizations, we measured the chemical distance, l, between two randomly chosen sites on the largest cluster, as well as the diameter D of the largest cluster. For $q = 1$, the standard deviation σ in the (uncorrelated) values for $\langle l \rangle$ and $\langle D \rangle$ were calculated as $\sigma_{uncorr} = \sqrt{\frac{1}{N-1}(\langle l^2 \rangle - \langle l \rangle^2)}$. For $q = 2, 3, 4$, successive measurements of l and D were not independent; each system of size L was therefore allowed to thermalize for $10^*\tau_{exp}$, where τ_{exp} is the fitted exponential correlation time for the mass of the largest cluster in the system. The standard deviation σ_{corr} was then considered to be $\sigma_{corr} = \sqrt{\frac{2\tau_{int}}{N}(\langle l^2 \rangle - \langle l \rangle^2)}$, where τ_{int} is the measured integrated correlation time for the chemical distance l. A similar analysis was used for the diameter, D.

dim	q	L	L_{min}	d_{min}	D_{min}
2	1	16,32,48,64,96,128	48	1.131(1)	1.138(1)
2	2	16,32,48,64,96,128	48	1.096(1)	1.102(1)
2	3	16,32,48,64,96,128	48	1.065(3)	1.071(1)
2	4	16,32,48,64,96,128	48	1.033(3)	1.039(1)

TABLE I: Scaling exponents d_{min} and D_{min} for $dim = 2$, $q = 1, 2, 3, 4$

dim	q	L	L_{min}	d_{min}	D_{min}
3	2	20,36,48,64,128	36	1.267(5)	na
4	2	12,24,36,48,64	24	1.485(7)	na

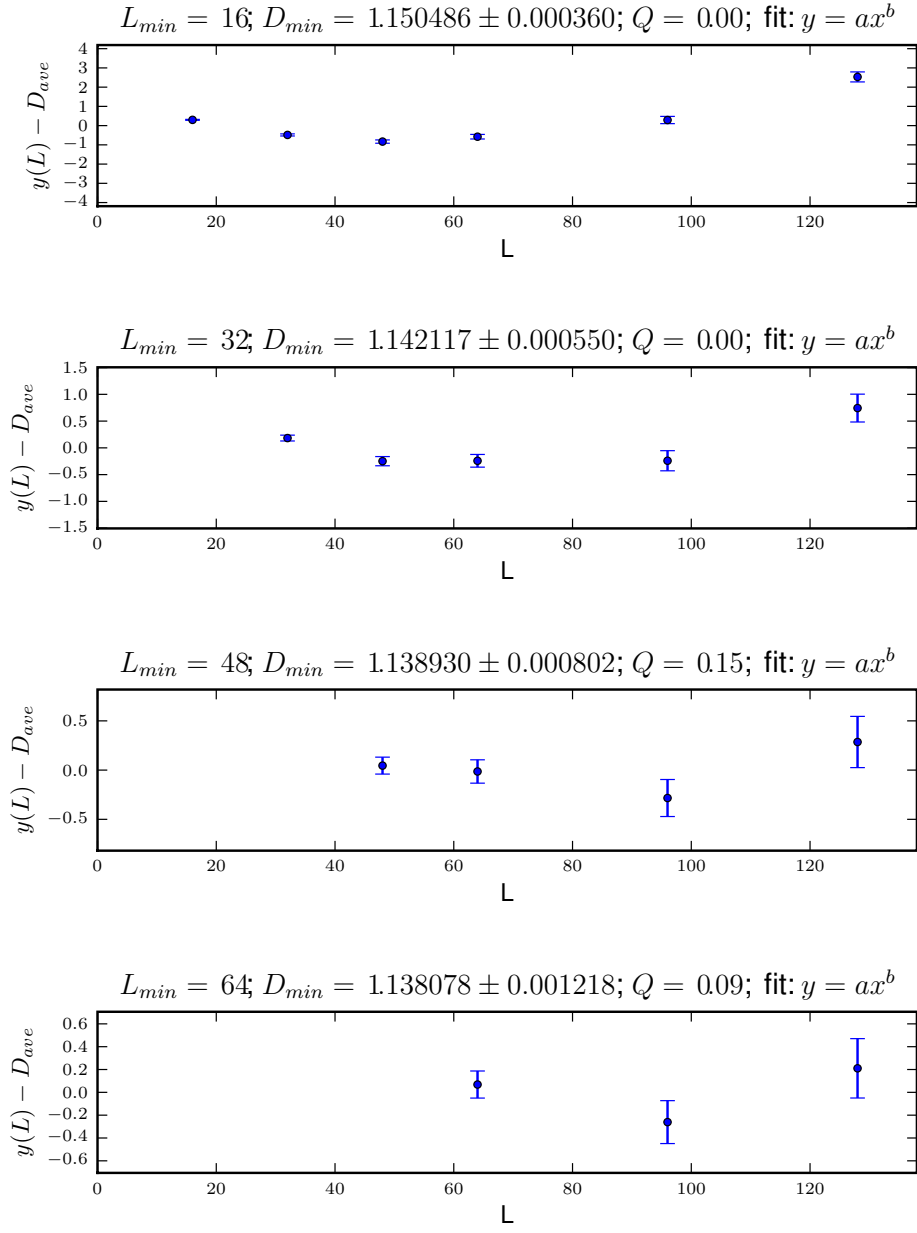TABLE II: Scaling exponents d_{min} and D_{min} for $dim = 3, 4$, $q = 2$

$L_{min} = 16; D_{min} = 1.150486 \pm 0.000360; Q = 0.00;$ fit: $y = ax^b$

$L_{min} = 32; D_{min} = 1.142117 \pm 0.000550; Q = 0.00;$ fit: $y = ax^b$

$L_{min} = 48; D_{min} = 1.138930 \pm 0.000802; Q = 0.15;$ fit: $y = ax^b$

$L_{min} = 64; D_{min} = 1.138078 \pm 0.001218; Q = 0.09;$ fit: $y = ax^b$

FIG. 1: d_{min} for D=2, q=1.

FIG. 2: D_{min} for D=2, q=1.

$L_{min} = 16$; $D_{min} = 1.112084 \pm 0.000510$; $Q = 0.00$; fit: $y = ax^b$

$L_{min} = 32$; $D_{min} = 1.104291 \pm 0.000784$; $Q = 0.04$; fit: $y = ax^b$

$L_{min} = 48$; $D_{min} = 1.102055 \pm 0.001130$; $Q = 0.78$; fit: $y = ax^b$

$L_{min} = 64$; $D_{min} = 1.101146 \pm 0.001716$; $Q = 0.92$; fit: $y = ax^b$
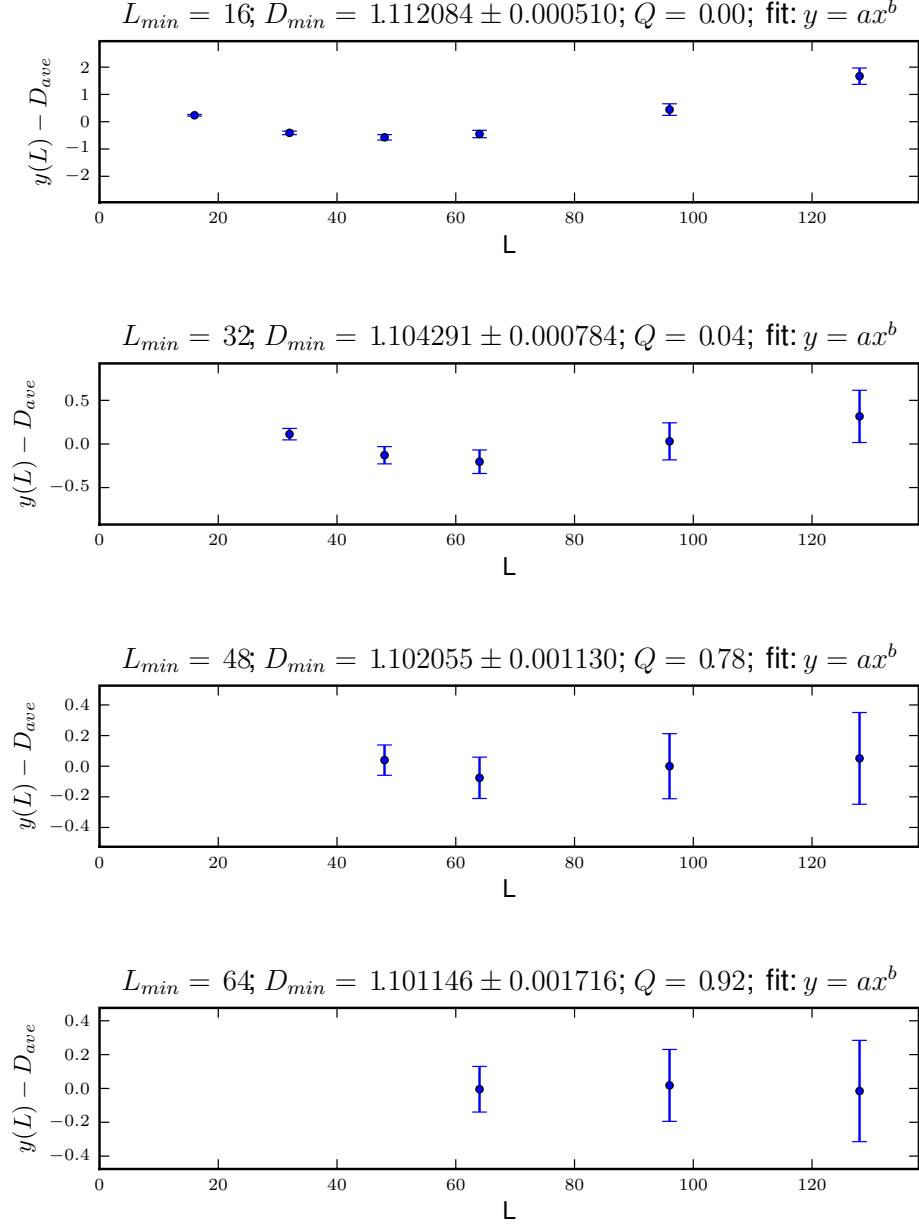
FIG. 3: d_{min} for D=2, q=2.

FIG. 4: D_{min} for D=2, q=2.

3
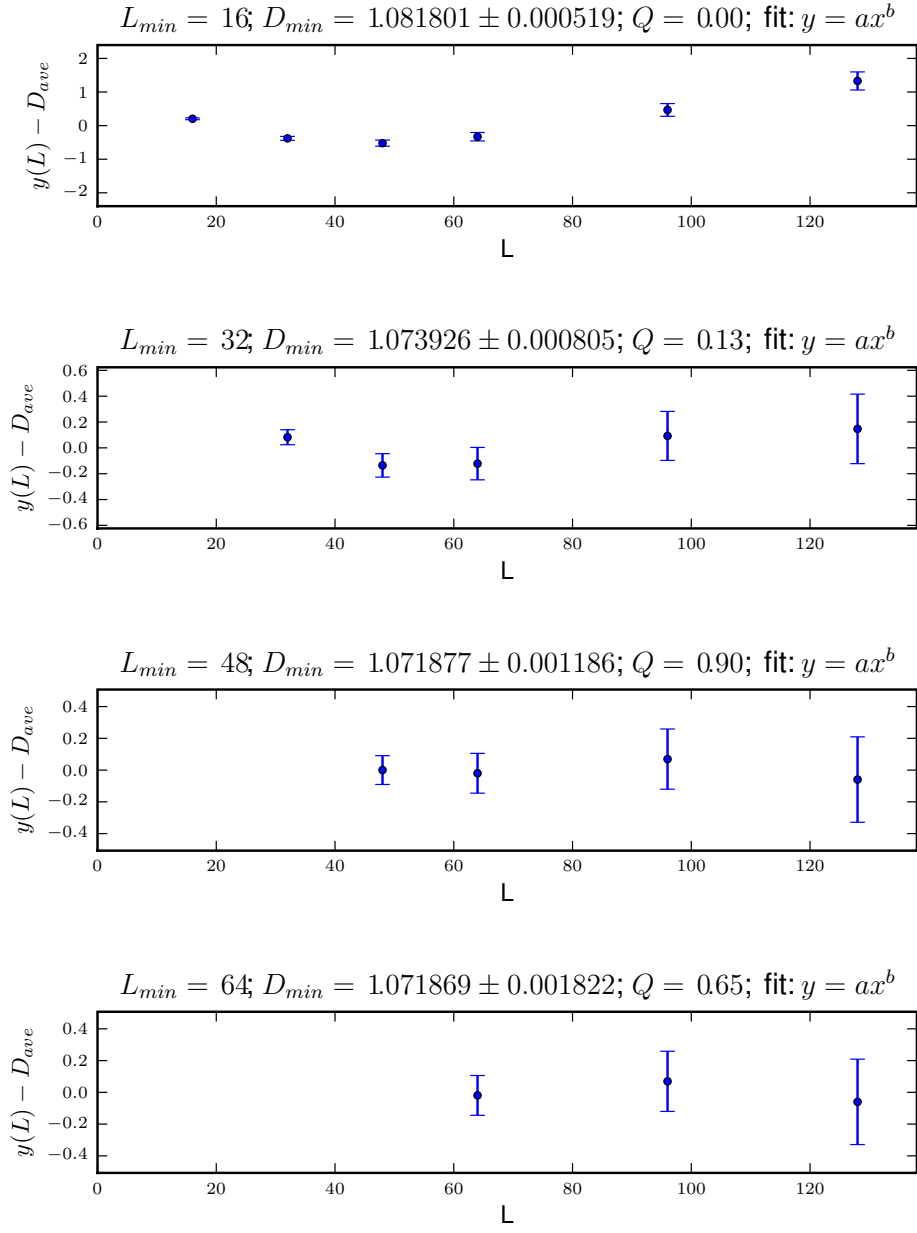
FIG. 5: d_{min} for D=2, q=3.

II. FIGURES

FIG. 6: D_{min} for D=2, q=3.
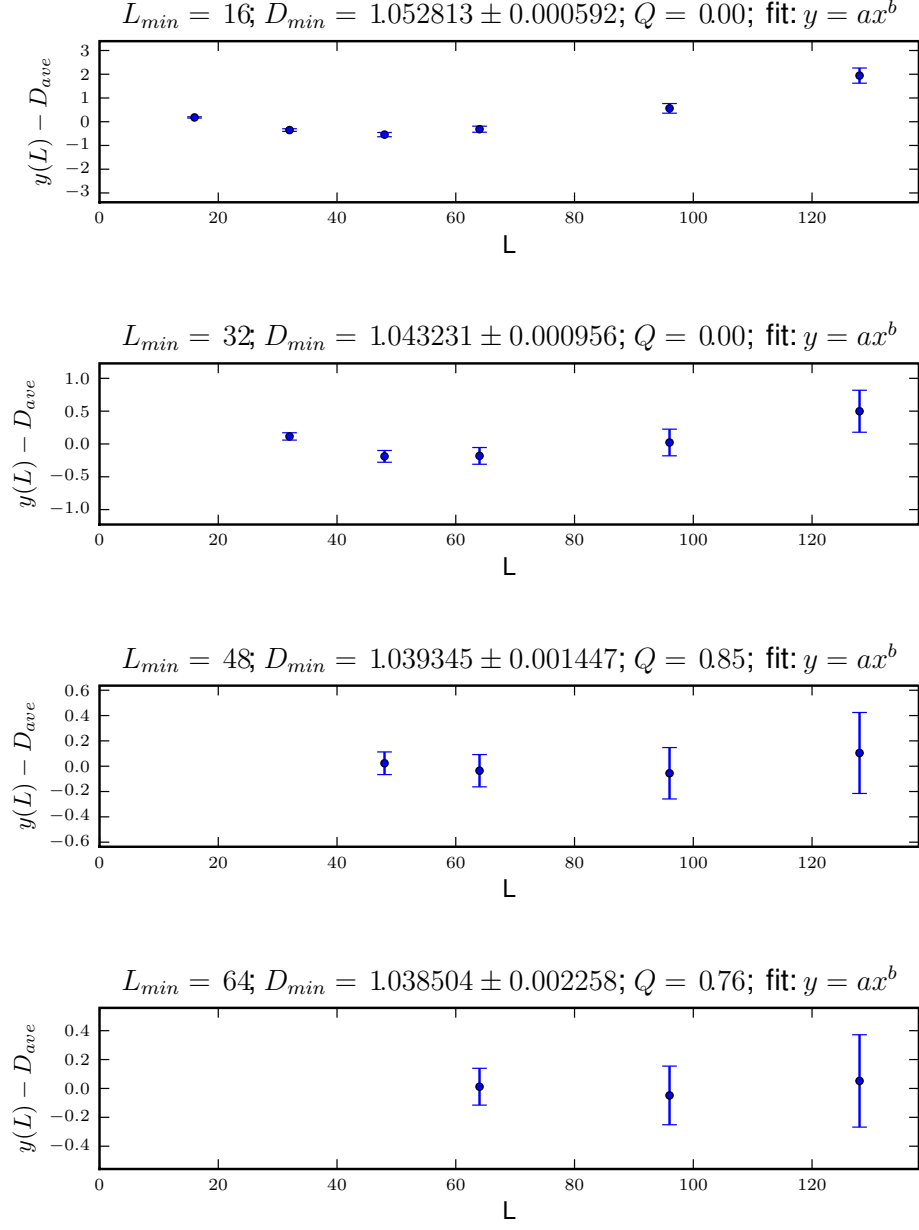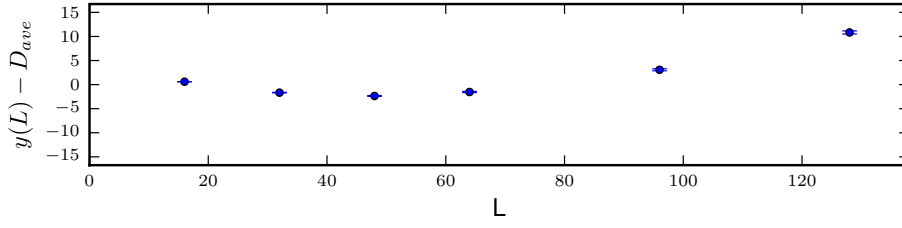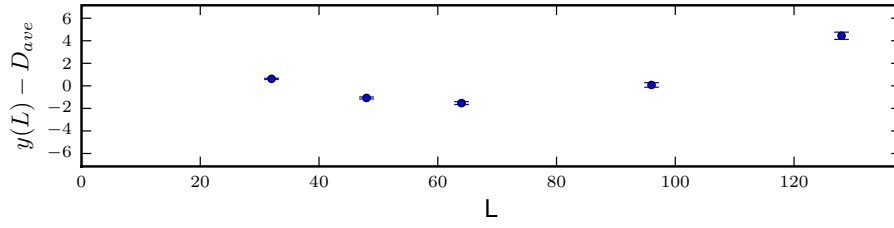
$L_{min} = 16;\ D_{min} = 1.052813 \pm 0.000592;\ Q = 0.00;$ fit: $y = ax^b$

$L_{min} = 32;\ D_{min} = 1.043231 \pm 0.000956;\ Q = 0.00;$ fit: $y = ax^b$

$L_{min} = 48;\ D_{min} = 1.039345 \pm 0.001447;\ Q = 0.85;$ fit: $y = ax^b$

$L_{min} = 64;\ D_{min} = 1.038504 \pm 0.002258;\ Q = 0.76;$ fit: $y = ax^b$

FIG. 7: d_{min} for D=2, q=4.

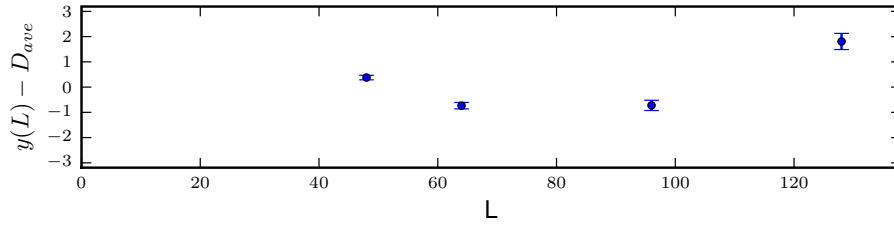FIG. 8: D_{min} for D=2, q=4.

5

$L_{min} = 16$; $D_{min} = 10.659362 \pm 0.038231$; $Q = 0.00$; fit: $y = ax\log(x)(1 + b/x)$

$L_{min} = 32$; $D_{min} = 14.728597 \pm 0.089100$; $Q = 0.00$; fit: $y = ax\log(x)(1 + b/x)$

$L_{min} = 48$; $D_{min} = 17.710029 \pm 0.166570$; $Q = 0.00$; fit: $y = ax\log(x)(1 + b/x)$

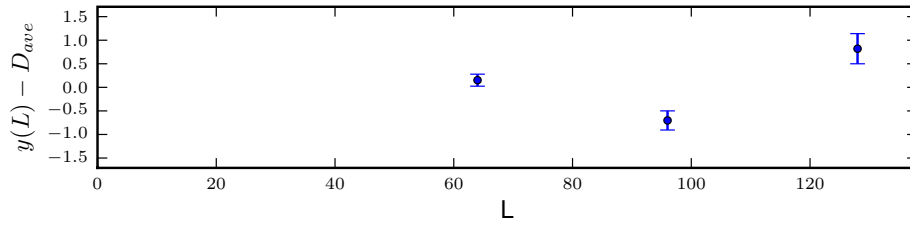$L_{min} = 64$; $D_{min} = 19.777069 \pm 0.297622$; $Q = 0.00$; fit: $y = ax\log(x)(1 + b/x)$

FIG. 9: d_{min} for D=2, q=4; log fit

FIG. 10: D_{min} for D=2, q=4; log fit