

## \* Abstract

A relatively unexplored geometric property of Potts models clusters is their "diameter",  $D$  -- the longest shortest path between any two points on the cluster. We report numerical results for the fractal dimension of the diameter,  $D_{\min}$  and the fractal dimension of the chemical distance,  $d_{\min}$ , for 2D critical Potts clusters with  $q=1,2,3,4,5$ . We find that  $D_{\min} = d_{\min}$  within numerical error.

## \* Intro

### \* Motivation

- \* Determines efficiency of cluster algorithms
- \* Unexplored geometric property of random clusters
- \* Places bounds on communication of processes in a network
- \* Comparison to previous studies of the chemical distance

### \* Potts Model

- \* The Model (refer to Sokal)
- \* Connection to Random Cluster Model

### \* The chemical distance and the diameter

- \* Definitions
- \* [FIGURE] illustrating typical configuration
- \* Mean field expectations
  - \* Upper critical dimensions for percolation, Ising, three-state (with refs)
  - \* Work of Nachmias and Peres
- \*  $d_{\min} = 1.0$  for  $q \geq 5$  (rationale??)

### \* Swendsen Wang Algorithm

- \* See Sokal's description

### \* Measuring the Chemical Distance

- \* Methods in the literature
  - \* Bus bar method
  - \* average over various clusters
  - \* Grassberger growth method
- \* Our methods
  - \* Average over various clusters and euclidean distances
  - \* "Random perimeter point" method
  - \* Expectation that they will be the same

### \* Measuring the Diameter

- \* Our method

### \* Simulation Details

- \* Range of  $L$ , 16 to 128
- \* Initial configuration random; discard first  $10^5$  iterations
- \* Compare discard interval with autocorrelation time
- \* Total run length for all systems
- \* Each data set consists of several runs combined
- \* CPU time required in units of  $L^2$  microseconds / iteration, type of processor
- \* run the system for shorter times at higher dimensions (??)

### \* Data Analysis

- \* Fit to power-law Ansatz  $D = AL^p$  using the standard weighted least-squares method
- \* Fit points with lower cutoff of  $L \geq L_{\min}$  to minimize corrections to scaling
- \* Choose fit with smallest  $L_{\min}$  for which goodness of fit is reasonable (Q value within a certain range)
- \* Error analysis
  - \* Data is taken at intervals assumed to be statistically independent
    - \*  $\sigma =$  [fill in]
  - \* blocking method is used for comparison
  - \* brief blocking method description, references

## \* Results

### \* Results for $D=2$ , $q=1,2,3,4,5$

- \* Table
- \* [FIGURE]: difference, fit and data for various  $q$

### \* Table of results for $D=3$ , $q=1,2,3,4,5$

- \* Preliminary results for  $D=6,4,8$  for  $q=1,2,3$  respectively
- \* comparison with mean field expectations