ISDA609 Week 1 Homework

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Annuity

- Interest Rate: 1%
- Withdrawl: \$1000/month
- Current Value: \$50,000

The dynamical system can be modeled using the following equation:

$$a_{n+1} = a_n + 0.01a_n - 1000$$
$$a_0 = 50000$$

The `annuityModel` function, below, defines the basic dynamical system:

```
annuityModel <- function(a_n, i, w)
{
  a_next <- a_n + (a_n * i) - w

  return (a_next)
}
```

If we run the model through some iterations, what happens?

```
# Setup variables related to the annuity
a <- 50000
rate <- 0.01
withdrawl <- 1000
# Store results in a data frame.
result <- data.frame(month=c(0), value=c(a))
# Loop through time
for(n in 1:100)
{
  a <- annuityModel(a, rate, withdrawl)

  result <- rbind(result, c(n, a))

  if(a < 0)
  {
    # End when a_n is less than zero
    break
  }
}

# Update data frame names to be user friendly.
```

```r
colnames(result) <- c("month", "value")

# show some raw data
head(result)
```

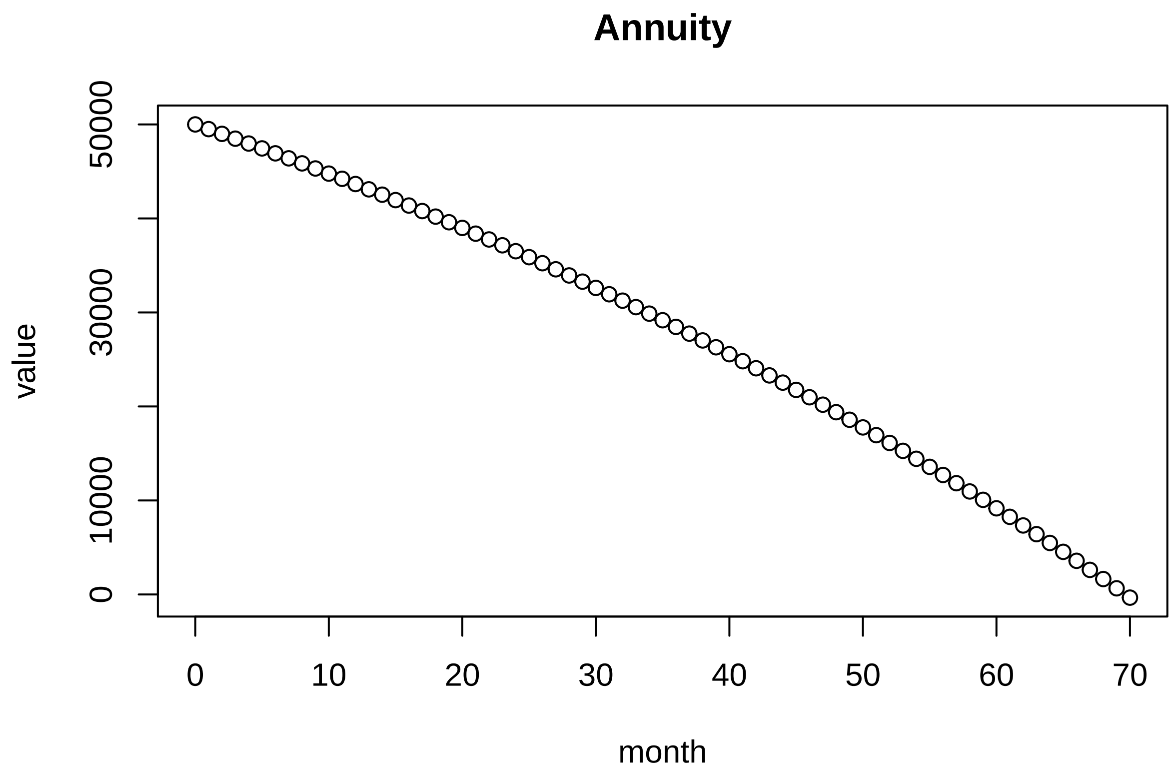
```
##   month    value
## 1     0 50000.00
## 2     1 49500.00
## 3     2 48995.00
## 4     3 48484.95
## 5     4 47969.80
## 6     5 47449.50
```

```r
tail(result)
```

```
##    month     value
## 66    65 4531.6756
## 67    66 3576.9923
## 68    67 2612.7623
## 69    68 1638.8899
## 70    69  655.2788
## 71    70 -338.1684
```

The annuity will run out of money after 70 months. When the annuity is depleted, the value of a_n would be -338.1684198 if the full withdrawal were allowed. Otherwise a_n will be 0.

The visualization below shows the graphical representation of the dynamical system.



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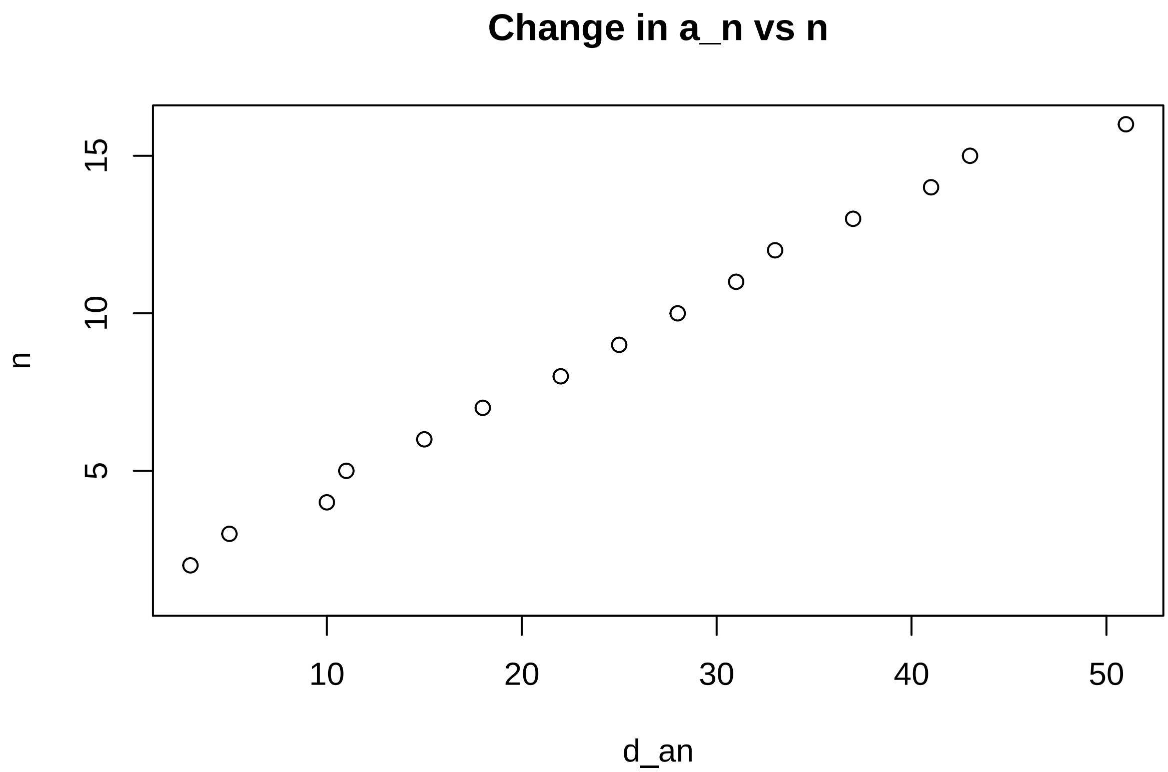
```
# Setup the vectors of data
n <- 1:16
speed <- n * 5
a_n <- c(3,6,11,21,32,47,65,87,112,140,171,204,241,282,325,376)
# Compute the delta between a_n and a_n+1
d_an <- c()
d_an[0] <- NA
for(i in 1:length(a_n))
{
  d_an[i] <- a_n[i] - a_n[i - 1]
}
# Convert to data.frame
d9 <- data.frame(n, speed, a_n, d_an)
```

(a) Calculate and plot the change Δa_n versus n. Does the graph reasonably approximate a linear relationship? The visualization below plots the change in a_n vs n. As you can see, the graph does reasonably approximate a linear relationship.




```r
# First find slope of the estimated difference line
delta_d_an <- max(d9$d_an, na.rm=TRUE)
delta_n <- max(d9$n)
r <-  delta_d_an / delta_n
r
```

(b) Based on your conclusions in part (a), find a difference equation model for the stopping distance data. Test your model by plotting the errors in the predicted values against n. Discuss the appropriateness of the model.

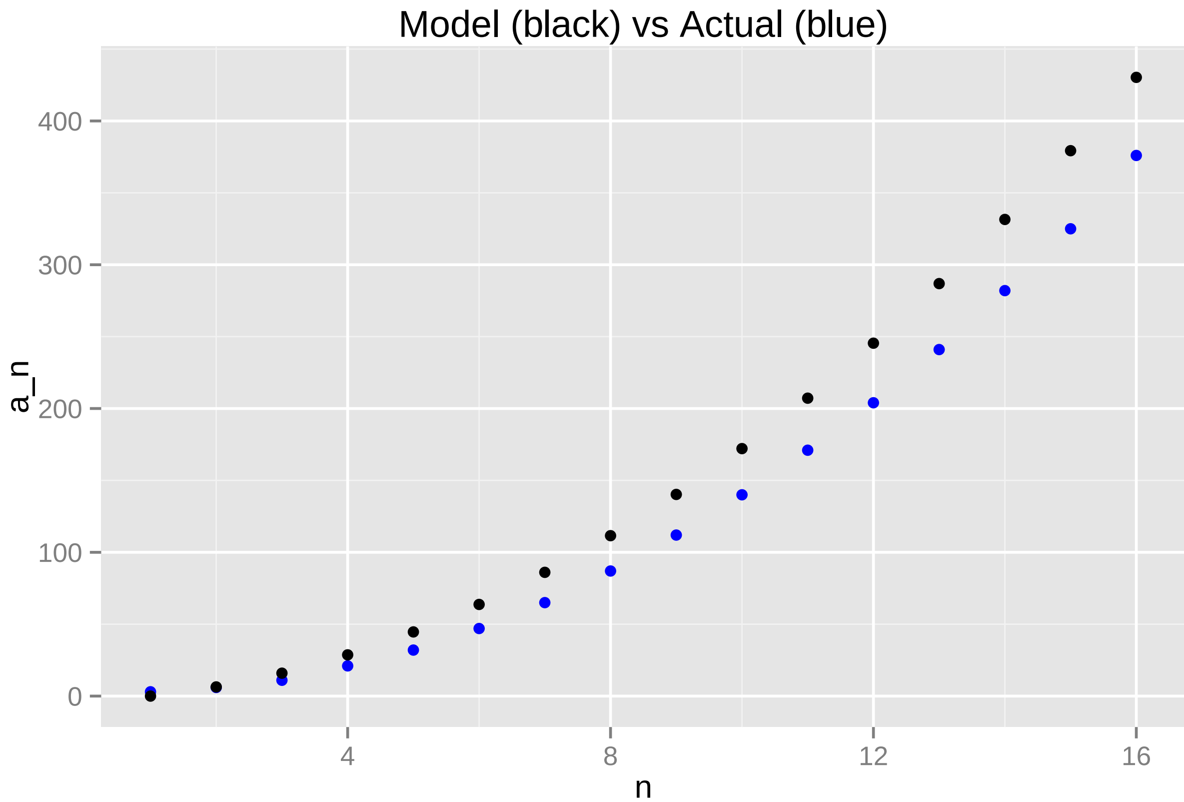
```
## [1] 3.1875
```

```r
# Define the difference equation model
stoppingDistanceModel <- function(n, a, r)
{
    an <- r * n + a
    return(an)
}

m <- c()
m[1] <- 0
for(i in 2:length(a_n))
{
```

```r
  m[i] <- stoppingDistanceModel(i, m[i-1], r)
}

d9a <- cbind(d9, m)
d9a
```

```
##      n speed a_n d_an        m
## 1    1     5   3   NA   0.0000
## 2    2    10   6    3   6.3750
## 3    3    15  11    5  15.9375
## 4    4    20  21   10  28.6875
## 5    5    25  32   11  44.6250
## 6    6    30  47   15  63.7500
## 7    7    35  65   18  86.0625
## 8    8    40  87   22 111.5625
## 9    9    45 112   25 140.2500
## 10  10    50 140   28 172.1250
## 11  11    55 171   31 207.1875
## 12  12    60 204   33 245.4375
## 13  13    65 241   37 286.8750
## 14  14    70 282   41 331.5000
## 15  15    75 325   43 379.3125
## 16  16    80 376   51 430.3125
```

```r
library(ggplot2)
d9viz <- ggplot(data=d9a, aes(x=n)) +
  geom_point(color="blue", aes(y=a_n)) +
  geom_point(aes(y=m)) +
  labs(title="Model (black) vs Actual (blue)")
d9viz
```




```r
# Plot the errors in predicted values against n
d9a$err <- d9a$a_n - d9a$m
d9ErrViz <- ggplot(data=d9a, aes(x=n)) +
  geom_point(color="red", aes(y=err)) +
  labs(title="Model Error")
d9ErrViz
```

This is a very crude linear based model. The actual change in not quite linear, which contributes to the error. As larger values of n are applied, the model error increases steadily. The model works as a rough estimator, but there is definitely room for improvement.