

Assignment #4

Part A: Power Iteration

$$A = \begin{bmatrix} 2 & 3 & 2 \\ 10 & 3 & 4 \\ 3 & 6 & 1 \end{bmatrix} \quad x_0 = [0 \ 0 \ 1]^T = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$a.) \quad Ax_0 = \begin{bmatrix} 2 & 3 & 2 \\ 10 & 3 & 4 \\ 3 & 6 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix} = 2 \begin{bmatrix} 1 \\ 2 \\ 0.5 \end{bmatrix}$$

$$Ax_1 = \begin{bmatrix} 2 & 3 & 2 \\ 10 & 3 & 4 \\ 3 & 6 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0.5 \end{bmatrix} = \begin{bmatrix} 9 \\ 18 \\ 15.5 \end{bmatrix} = 9 \begin{bmatrix} 1 \\ 2 \\ 1.72 \end{bmatrix}$$

$$Ax_2 = \begin{bmatrix} 2 & 3 & 2 \\ 10 & 3 & 4 \\ 3 & 6 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1.72 \end{bmatrix} = \begin{bmatrix} 11.44 \\ 22.89 \\ 16.72 \end{bmatrix} = 11.44 \begin{bmatrix} 1 \\ 2 \\ 1.46 \end{bmatrix}$$

$$Ax_3 = \begin{bmatrix} 2 & 3 & 2 \\ 10 & 3 & 4 \\ 3 & 6 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1.46 \end{bmatrix} = \begin{bmatrix} 10.92 \\ 21.84 \\ 16.46 \end{bmatrix} = 10.92 \begin{bmatrix} 1 \\ 2 \\ 1.507 \end{bmatrix}$$

$$Ax_4 = \begin{bmatrix} 2 & 3 & 2 \\ 10 & 3 & 4 \\ 3 & 6 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1.507 \end{bmatrix} = \begin{bmatrix} 11.014 \\ 22.028 \\ 16.507 \end{bmatrix} = 11.014 \begin{bmatrix} 1 \\ 2 \\ 1.499 \end{bmatrix}$$

$$Ax_5 = \begin{bmatrix} 2 & 3 & 2 \\ 10 & 3 & 4 \\ 3 & 6 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1.499 \end{bmatrix} = \begin{bmatrix} 11 \\ 22 \\ 16.5 \end{bmatrix} = 11 \begin{bmatrix} 1 \\ 2 \\ 1.5 \end{bmatrix}$$

dominant eigenvalue:  $10.99 \approx 11$

$$\text{eigenvector: } \begin{bmatrix} 1 \\ 2 \\ 1.5 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 1 \\ 0.75 \end{bmatrix}$$

Part B: Deflation

$$A = \begin{bmatrix} 2 & 3 & 2 \\ 10 & 3 & 4 \\ 3 & 6 & 1 \end{bmatrix} \Rightarrow R_1 \begin{matrix} \rightarrow \\ \leftarrow \end{matrix} R_2 = \begin{bmatrix} 10 & 3 & 4 \\ 2 & 3 & 2 \\ 3 & 6 & 1 \end{bmatrix} \Rightarrow \begin{matrix} R_2 = R_2 - R_1/2 \\ R_3 = R_3 + R_1/2 \end{matrix} = \begin{bmatrix} 10 & 3 & 4 \\ -3 & 1.5 & 0 \\ 8 & 7.5 & 3 \end{bmatrix}$$
$$\Rightarrow C_1 \begin{matrix} \rightarrow \\ \leftarrow \end{matrix} C_2 = \begin{bmatrix} 3 & 10 & 4 \\ 1.5 & -3 & 0 \\ -7.5 & 8 & 3 \end{bmatrix}$$

b.) deflated matrix:  $\begin{bmatrix} -3 & 0 \\ 8 & 3 \end{bmatrix}$

2nd largest eigenvalue: -3

### Part C: Comparison

$$\begin{bmatrix} 2 & 3 & 2 \\ 10 & 3 & 4 \\ 3 & 6 & 1 \end{bmatrix}$$

Find the Eigenvalues



Find the eigenvalues.

$$\lambda = -2, -3, 11$$

Tap to view steps...

$$\begin{bmatrix} 2 & 3 & 2 \\ 10 & 3 & 4 \\ 3 & 6 & 1 \end{bmatrix}$$

Find the Eigenvectors/Eigenspace



Find the eigenvectors/eigenspace.

$$\left\{ \begin{bmatrix} -\frac{1}{5} \\ -\frac{2}{5} \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -\frac{2}{3} \\ 1 \end{bmatrix}, \begin{bmatrix} \frac{2}{3} \\ \frac{4}{3} \\ 1 \end{bmatrix} \right\}$$

Tap to view steps...

Using Mathway, an online linear algebra calculator, we get eigenvalues 11, -3, and -2 with corresponding eigenvectors  $\begin{bmatrix} 2/3 \\ 4/3 \\ 1 \end{bmatrix}$ ,  $\begin{bmatrix} 0 \\ -2/3 \\ 1 \end{bmatrix}$ , and  $\begin{bmatrix} -1/5 \\ -2/5 \\ 1 \end{bmatrix}$ .

My obtained eigenvalues for parts a and b appear to be correct; however, my obtained eigenvector differs from what is shown in Mathway.