CSE 3350

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Assignment 5

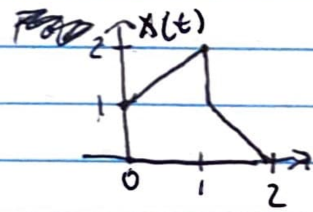
2.8. $x(t) \begin{cases} t+1, & 0 \le t \le 1 \\ 2-t, & 1 \le t \le 2 \\ 0, & \text{elsewhere} \end{cases}$ $\qquad h(t) = \delta(t+2) + 2\delta(t+1)$

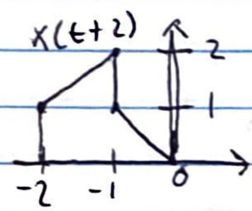
$y(t) = x(t) * h(t) = x(t) * [\delta(t+2) + 2\delta(t+1)]$

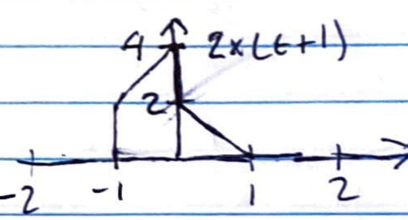
$= x(t) * \delta(t+t_0) = x(t+t_0) = \underline{x(t+2) + 2x(t+1)}$

For $0 \le t \le 1$: $x(0) = 1$ & $x(1) = 2$

For $1 < t \le 2$: $x(1) = 1$ & $x(2) = 0$



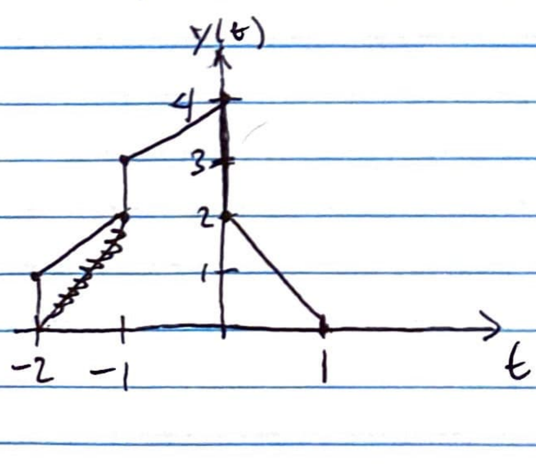




If $x(t+2) \begin{cases} t+3, & -2 \le t \le -1 \\ 2-(t+2), & 1 < t+2 \le 2 \\ 0, & \text{elsewhere} \end{cases}$

& $2x(t+1) = \begin{cases} 2t+4, & -1 \le t \le 0 \\ 2-2t, & 0 < t \le 1 \\ 0, & \text{elswhere} \end{cases}$

Then $Y(t) = x(t+2) + 2x(t+1) = \boxed{\begin{cases} t+3, & -2 \le t \le -1 \\ t+4, & -1 \le t \le 0 \\ 2-2t, & 0 < t \le 1 \\ 0, & \text{elswhere} \end{cases}}$.



(2.9) $h(t) = e^{2t}u(-t+4) + e^{-2t}u(t-5)$

Determine A & B such that

$$h(t-\tau) = \begin{cases} e^{-2(t-\tau)}, & \tau < A \\ 0, & A < \tau < B \\ e^{2(t-\tau)}, & B < \tau \end{cases}$$

Signal $u(-t+4)$: $= 1$ when $t < 4$
$= 0$ when $t > 4$

signal $u(t-5)$: $= 1$ when $t > 5$
$= 0$ when $t < 5$

$$\Rightarrow h(t) = \begin{cases} e^{2t}, & t < 4 \\ 0, & 4 < t < 5 \\ e^{-2t}, & t > 5 \end{cases}$$

$$\rightarrow h(t-\tau) = \begin{cases} e^{2(t-\tau)}, & t-\tau < 4 \rightarrow t-4 < \tau \\ 0, & 4 < t-\tau < 5 \rightarrow 4-t < -\tau < 5-t \\ e^{-2(t-\tau)}, & t-\tau > 5 \rightarrow t-5 > \tau \end{cases}$$

$\rightarrow t-4 < \tau$

$\rightarrow t-5 < \tau < t-4$

$\rightarrow \tau < t-5$

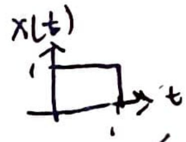
$\rightarrow \tau < t-5 \rightarrow \tau < A$

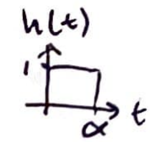
$\rightarrow t-5 < \tau < t-4 \rightarrow A < \tau < B$

$\rightarrow t-4 < \tau \rightarrow B < \tau$

Therefore

$$\boxed{\begin{aligned} A &= t-5 \\ B &= t-4 \end{aligned}}$$

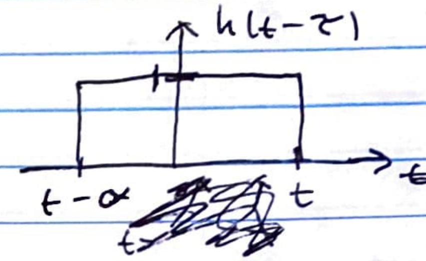




(2.10) $x(t) = \begin{cases} 1, & 0 \le t \le 1 \\ 0, & \text{elsewhere} \end{cases}$ & $h(t) = x(t/\alpha)$, where $0 < \alpha \le 1$

a.) $y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$

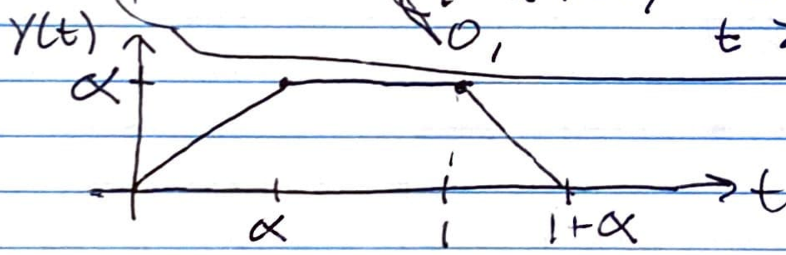
$h(t-\tau)$: $t = 0;\ \begin{cases} t - \tau = 0 \\ \tau = t \end{cases}$ $t = \alpha;\ \begin{cases} t - \tau = \alpha \\ \tau = t - \alpha \end{cases}$



$\Rightarrow y(t) = \int_{t-\alpha}^{t} x(\tau)\, h(t-\tau)\, d\tau = t - t + \alpha = \alpha,\ \alpha \le t \le 1$

$y(t) = \int_{t-\alpha}^{1} x(\tau) \cdot h(t-\tau)\, dt = 1 - t + \alpha,\ 1 \le t \le 1+\alpha$

$$\Rightarrow \boxed{y(t) = \begin{cases} t, & 0 < t \le \alpha \\ \alpha, & \alpha \le t < 1 \\ 1 - t + \alpha, & 1 \le t \le 1+\alpha \\ 0, & t > 1+\alpha \end{cases}}$$



b.) If $dy(t)/dt$ contains only 3 discontinuties, what is the value of α?

If $y(t) = t\,u(t) - (t-\alpha)u(t-\alpha) - (t-1)u(t-1)$
$\dots + (t-1-\alpha)u(t-1-\alpha) \rightarrow dy(t)/dt \rightarrow$

$\rightarrow y(t) = t\,\delta(t) - (t-\alpha)\,\delta(t-\alpha) - (t-\alpha) - (t-1)\,\delta(t-1)$
$\dots + (t-1-\alpha)\,\delta(t-1-\alpha)$

discontinuties @ $t = 0,\ \alpha,\ 1,\ \&\ 1+\alpha$
$\rightarrow t = 0, 1, 1, \&\ 2 \rightarrow \boxed{\alpha = 1}$

2.11 $x(t) = u(t-3) - u(t-5)$ & $h(t) = e^{-3t}u(t)$

a.) $y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(t-\tau)h(\tau)d\tau$

$$= \int_{-\infty}^{\infty} \{u[t-\tau-3] - u[t-\tau-5]\} e^{-3\tau} u(\tau) d\tau$$

$u(\tau)$ @ $t=0$: $y(t) = \int_{0}^{\infty} \{u[u[t-3\tau] - u[t\ 5-\tau]\} e^{-3\tau} d\tau$

$\rightarrow y(t) = \int_{0}^{t-3} (1) e^{-3\tau} d\tau = \frac{1}{3}[1 - e^{-3(t-3)}]$ @ $3 \leq t < 5$

$\rightarrow y(t) = \int_{t-5}^{t-3} e^{-3t} d\tau = \frac{1}{3}[e^{-3(t-5)} - e^{-3(t-3)}]$

$$\rightarrow \boxed{y(t) = \begin{cases} 0, & -\infty < t < 3 \\ \frac{1}{3}[1 - e^{-3(t-3)}], & 3 \leq t \leq 5 \\ \frac{1}{3}[e^{-3(t-5)} - e^{-3(t-3)}], & t \geq 5 \end{cases}}$$

b.) $g(t) = (dx(t)/dt) * h(t)$; $\frac{d}{dt}u(t) = \delta(t)$

$\rightarrow g(t) = \{\frac{d}{dt}[u(t-3) - u(t-5)]\} * e^{-3t}u(t)$

$$= \{\delta(t-3) - \delta(t-5)\} * e^{-3\tau} u(t)$$

$= \int_{-\infty}^{\infty} \{\delta(t-\tau-3) - \delta(t-\tau-5)\} e^{-3\tau} u(\tau) d\tau$

$= \int_{-\infty}^{\infty} \delta(t-\tau-3) e^{-3\tau} u(\tau) d\tau - \int_{-\infty}^{\infty} \delta(t-\tau-5) e^{-3\tau} u(\tau) d\tau$

If $\tau = t-3$ & $\tau = t-5$ $\rightarrow$ $g(t) = e^{-3\tau}u(\tau) | - e^{-3\tau}u(\tau)$

$\Rightarrow \boxed{g(t) = e^{-3(t-3)}u(t-3) - e^{-3(t-5)}u(t-5)}$

c.) $g(t) = \frac{d}{dt} y(t)$

(2.12) $y(t) = e^{-t} u(t) * \sum_{K=-\infty}^{\infty} \delta(t-3K)$

Show that $y(t) = Ae^{-t}$ for $0 \le t \le 3$, & determine A.

~~[illegible]~~

$$\Rightarrow y(t) = \sum_{K=-\infty}^{\infty} e^{-t} u(t) * \delta(t-3K)$$

If $x(t) \delta(t-t_0) = x(t-t_0)$

$$\rightarrow y(t) = \sum_{K=-\infty}^{\infty} e^{-(t-3K)} u(t-3K)$$

for $0 \le t \le 3$: $y(t) = \{ \ldots + e^{-(t+12)} u(t+12) + e^{-(t+9)} u(t+9) + \ldots \}$

$$\rightarrow y(t) = e^{-t}\left[1 + e^{-3} + (e^{-3})^2 + (e^{-3})^3 + \ldots\right.$$

$$\Rightarrow y(t) = e^{-t}\left[\frac{1}{1-e^{-3}}\right] e^{-t} \; ; \; 0 \le t < 3$$

$$\rightarrow \boxed{A = \frac{1}{1-e^{-3}}}$$