CSE 3350

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Assignment 10

(4.1) a.) $e^{-2(t-1)}u(t-1)$

$$X(j\omega)=\int_{-\infty}^{\infty}x(t)e^{-j\omega t}dt=\int_{-\infty}^{\infty}e^{-2(t-1)}u(t-1)e^{-j\omega t}dt$$

$$=\int_{1}^{\infty}e^{-2(t-1)}e^{-j\omega t}dt=\int_{1}^{\infty}e^{-(2+j\omega)t}e^{2}dt$$

$$=e^{2}\frac{e^{-(j\omega+2)t}}{-(j\omega+2)}\Bigg|_{1}^{\infty}=\boxed{\frac{e^{-j\omega}}{j\omega+2}}$$

b.) $e^{-2|t-1|}$

$$X(j\omega)=\int_{-\infty}^{\infty}e^{-2|t-1|}e^{-j\omega t}dt$$

$$=e^{-2}\int_{-\infty}^{1}e^{t(2-j\omega)}dt+e^{2}\int_{1}^{\infty}e^{-t(2+j\omega)}dt$$

$$=e^{-2}\left(\frac{e^{t(2-j\omega)}}{(2-j\omega)}\Bigg|_{-\infty}^{1}\right)+e^{2}\left(\frac{e^{-t(2+j\omega)}}{-(2+j\omega)}\Bigg|_{1}^{\infty}\right)$$

$$=\frac{e^{-j\omega}}{2-j\omega}+\frac{e^{-j\omega}}{2+j\omega}=e^{-j\omega}\left[\frac{2+j\omega+2-j\omega}{4-(j\omega)^2}\right]$$

$$=e^{-j\omega}\left[\frac{4}{4+\omega^2}\right]=\boxed{\frac{4e^{-j\omega}}{4+\omega^2}}$$

(4.2) a.) $\delta(t+1)+\delta(t-1)\rightarrow X(j\omega)=\int_{-\infty}^{\infty}[\delta(t+1)+\delta(t-1)]e^{-j\omega t}dt$

$$=\int_{-\infty}^{\infty}\delta(t+1)e^{-j\omega t}dt+\int_{-\infty}^{\infty}\delta(t-1)e^{-j\omega t}dt$$

$$=e^{-j\omega(-1)}+e^{-j\omega(1)}=2\left(\frac{e^{j\omega}+e^{-j\omega}}{2}\right)=\boxed{2\cos\omega}$$

b.) $\frac{d}{dt}\{u(-2-t)+u(t-2)\} = \delta(t-2)-\delta(t+2)$

$$X(j\omega) = \int_{-\infty}^{\infty} [\delta(t-2)-\delta(t+2)]e^{-j\omega t}\,dt$$

$$= e^{-j\omega(2)} - e^{-j\omega(-2)} = -2j\left(\frac{-e^{-j2\omega}+e^{j2\omega}}{2j}\right)$$

$$= \boxed{-2j\sin 2\omega}$$

4.3 a.) $\sin(2\pi t+\frac{\pi}{4}) \rightarrow \underbrace{\frac{1}{2j}[e^{j\pi/4}e^{j2\pi t} - e^{-j\frac{\pi}{4}}e^{-j2\pi t}]}$

$$X(j\omega) = \int_{-\infty}^{\infty}\{\quad\}e^{-j\omega t}\,dt$$

$$= \frac{1}{2j}(e^{j\pi/4})\int_{-\infty}^{\infty}(e^{j2\pi t})e^{-j\omega t}\,dt - \frac{1}{2j}(e^{-j\pi/4})\int_{-\infty}^{\infty}(e^{-j2\pi t})e^{-j\omega t}\,dt$$

$$e^{j\omega t} \xleftrightarrow{FT} 2\pi\delta(\omega-\omega_0)$$

$$\Rightarrow \boxed{(\pi/j)\{e^{j\pi/4}\delta(\omega-2\pi) - e^{-j\pi/4}\delta(\omega+2\pi)\}}$$

b.) $1+\cos(6\pi t+\pi/8) \rightarrow \underbrace{1+\frac{1}{2}e^{j\pi/8}e^{j6\pi t}+\frac{1}{2}e^{-\frac{j\pi}{8}}e^{-j6\pi t}}$

$$X(j\omega) = \int_{-\infty}^{\infty}[\quad]e^{-j\omega t}\,dt$$

$$= \int_{-\infty}^{\infty}e^{-j\omega t}\,dt + \frac{1}{2}e^{j\pi/8}\int_{-\infty}^{\infty}(e^{j6\pi t})e^{-j\omega t}\,dt + \frac{1}{2}e^{-\frac{j\pi}{8}}\int_{-\infty}^{\infty}(e^{-j6\pi t})e^{-j\omega t}\,dt$$

$$e^{j\omega_0 t} \xleftrightarrow{FT} 2\pi\delta(\omega-\omega_0)$$

$$= \boxed{2\pi\delta(\omega) + \pi e^{j\pi/8}\delta(\omega-6\pi) + \pi e^{-j\pi/8}\delta(\omega+6\pi)}$$

4.4 a.) $X_1(j\omega) = \underbrace{2\pi\delta(\omega) + \pi(\omega - 4\pi) + \pi\delta(\omega + 4\pi)}$,

$$x_1(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}[\quad]e^{j\omega t}d\omega$$

$$= \frac{1}{2\pi}\left\{\int_{-\infty}^{\infty}2\pi\delta(\omega)e^{j\omega t}d\omega + \pi\int_{-\infty}^{\infty}\delta(\omega - 4\pi)e^{j\omega t}d\omega + \pi\int_{-\infty}^{\infty}\delta(\omega+4\pi)e^{j\omega t}d\omega\right\}$$

$$= \frac{1}{2\pi}\left[2\pi e^{jt(0)} + \pi e^{j(4\pi)t} + \pi e^{j(-4\pi)t}\right]$$

$$= 1 + \frac{e^{j4\pi t} + e^{-j4\pi t}}{2} = \boxed{1 + \cos 4\pi t}$$

b.) $X_2(j\omega) = \begin{cases} 2, & 0 \le \omega \le 2 \\ -2, & -2 \le \omega < 0 \\ 0, & |\omega| > 2 \end{cases} \rightarrow \frac{1}{2\pi}\int_{-\infty}^{\infty}X_2(j\omega)e^{j\omega t}d\omega$

$$x_2 = \frac{1}{2\pi}\left[\int_{-2}^{0}(-2)e^{j\omega t}d\omega + \int_{0}^{2}(2)e^{j\omega t}d\omega\right]$$

$$= \frac{-1}{\pi}\left(\frac{1 - e^{-2jt}}{jt}\right) + \frac{1}{\pi}\left(\frac{e^{j2t} - 1}{jt}\right)$$

$$= \frac{e^{j2t} + e^{-j2t} - 2}{\pi jt} = \frac{2\cos(2t) - 2}{\pi jt} = \boxed{\frac{4j\sin^2 t}{\pi t}}$$

4.6 a.) $x_1(t) = x(1-t) + x(-1-t)$, $\quad x(-t-1) \xleftrightarrow{FT} e^{-j\omega}X(-j\omega)$

$$F\{x_1(t)\} = F\{x(1-t) + x(-1-t)\}$$

$$\Rightarrow X_1(j\omega) = e^{j\omega}X(-j\omega) + e^{-j\omega}X(-j\omega) = [e^{j\omega} + e^{-j\omega}]X(-j\omega)$$

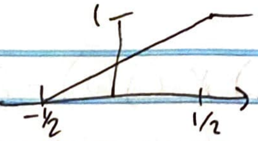
$$= \boxed{(2\cos\omega)X(-j\omega)}$$

b.) $x_2(t) = x(3t-6)$, $\quad x(t - t_0) \xleftrightarrow{FT} e^{-j\omega t}X(j\omega)$

$F\{x_2(t)\} = F\{x(3(t-2))\} \Rightarrow X_2(j\omega) = e^{-j2\omega}\left(\frac{1}{3}X(j\omega/3)\right)$

$$= \boxed{\left(\frac{e^{-j2\omega}}{3}\right)X(j\omega/3)} \rightarrow \underline{x_2(t) \xleftrightarrow{FT} \frac{1}{3}e^{-2j\omega}X(j\omega/3)}$$

4.8 a.) $x(t) = \begin{cases} 0, & t<-\frac{1}{2} \\ t+\frac{1}{2}, & -\frac{1}{2} \le t \le \frac{1}{2} \\ 1, & t>\frac{1}{2} \end{cases}$



$= \begin{cases} 1, & |t|<\frac{1}{2} \\ 0, & |t|>\frac{1}{2} \end{cases} \rightarrow Y(j\omega) = \frac{2\sin(\omega/2)}{\omega}$ $\quad x(t)=\int_{-\infty}^{t} [y(\tau)]\,dt$

$$X(j\omega) = \frac{1}{j\omega}Y(j\omega) + \pi Y(0)\delta(\omega) = \frac{1}{j\omega}\left(\frac{2\sin(\frac{\omega}{2})}{\omega}\right) + \pi\left(\frac{\sin(\frac{0}{2})}{(\frac{0}{2})}\right)\delta(\omega)$$

$$= \boxed{\left(\frac{2\sin(\frac{\omega}{2})}{j\omega^2}\right) + \pi\delta(\omega)}$$

b.) $g(t) = x(t) - \frac{1}{2} \rightarrow G(j\omega) = X(j\omega) - \frac{1}{2}(2\pi\delta(\omega)) = X(j\omega) - (\pi\delta(\omega))$

$$= \frac{2\sin(\omega/2)}{j\omega^2} + \pi\delta(\omega) - \pi\delta(\omega) = \boxed{\frac{2\sin(\omega/2)}{j\omega^2}}$$

4.11 A & B are $\boxed{\frac{1}{3} \,\&\, 3}$

4.17 a.) False

b.) ~~P~~ True

4.19 $H(j\omega) = \frac{1}{j\omega+3}$, $y(t) = \overbrace{e^{-3t}u(t) - e^{-4t}u(t)}^{[e^{-3t}-e^{-4t}]u(t)}$

$$\Rightarrow Y(j\omega) = \int_{-\infty}^{\infty} y(t)e^{-j\omega t}\,dt = \int_{-\infty}^{\infty} [e^{-3t} - e^{-4t}]u(t)\}e^{-j\omega t}\,dt$$

$$= \int_{0}^{\infty} [e^{-3t}e^{-j\omega t} - e^{-4t}e^{-j\omega t}]\,dt$$

$$= \frac{e^{-j(j\omega+3)}}{-(j\omega+3)}\Big|_0^{\infty} - \frac{e^{-j(j\omega+4)}}{-(j\omega+4)}\Big|_0^{\infty}$$

$$= \frac{-1}{-(j\omega+3)} - \frac{-1}{-(j\omega+4)} = \frac{1}{(j\omega+3)(j\omega+4)}$$

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} \Rightarrow X(j\omega) = \frac{Y(j\omega)}{H(j\omega)} = \frac{\left(\frac{1}{(j\omega+3)(j\omega+4)}\right)}{\left(\frac{1}{j\omega+3}\right)}$$

$$= \frac{1}{j\omega+4} \Rightarrow \boxed{x(t) = e^{-4t}u(t)}$$