CSE 3350

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Assignment 1

$(x+jy)$ cartesian form

1.1) $\frac{1}{2}e^{j\pi} \Rightarrow \underline{\frac{1}{2}(\cos(\pi)+j\sin(\pi))} = -\frac{1}{2}+(0)j$

$\frac{1}{2}e^{-j\pi} \Rightarrow \underline{\frac{1}{2}(\cos(-\pi)+j\sin(-\pi))} = -\frac{1}{2}+(0)j$

$e^{j\frac{\pi}{2}} \Rightarrow \underline{\cos(\frac{\pi}{2})+j\sin(\frac{\pi}{2})} = 0+j$

$e^{-j\frac{\pi}{2}} \Rightarrow \underline{\cos(\frac{-\pi}{2})+j(\frac{-\pi}{2})} = 0-j$

$e^{j5\pi/2} \Rightarrow \underline{\cos(\frac{5\pi}{2})+j\sin(\frac{5\pi}{2})} = 0+j$

$\sqrt{2}e^{j\pi/4} \Rightarrow \underline{\sqrt{2}(\cos(\frac{\pi}{4})+j\sin(\pi/4))} = 1+j$

$\sqrt{2}e^{j9\pi/4} \Rightarrow \underline{\sqrt{2}(\cos(9\pi/4)+j\sin(9\pi/4))} = 1+j$

$\sqrt{2}e^{-j9\pi/4} \Rightarrow \underline{\sqrt{2}(\cos(\frac{-9\pi}{4})+j\sin(-9\pi/4))} = 1-j$

$\sqrt{2}e^{-j\pi/4} \Rightarrow \underline{\sqrt{2}(\cos(-\pi/4)+j\sin(-\pi/4))} = 1-j$

1.2) ($re^{j\theta}$, with $-\pi < \theta < \pi$) polar form

$5 = 5+(0)j \Rightarrow \boxed{5e^{(0)j}}$

$-2 = -2+(0)j \Rightarrow \boxed{2e^{(\pi)j}}$

$-3j = 3[0-1j] \Rightarrow \boxed{3e^{-j\pi/2}}$

$\frac{1}{2}-j\frac{\sqrt{3}}{2} = \cos(\frac{-\pi}{3})+j\sin(\frac{-\pi}{3}) \Rightarrow \boxed{e^{\frac{-\pi j}{3}}}$

$1+j = \sqrt{2}[\frac{1}{\sqrt{2}}+j\frac{1}{\sqrt{2}}] \Rightarrow \boxed{\sqrt{2}e^{j\pi/4}}$

$$(1-j)^2 = 1-2j+j^2 = 2[0-j] \Rightarrow \boxed{2e^{-\pi j/2}}$$

$$j(1-j) = j-j^2 = j+1 = \sqrt{2}\left[\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right] \Rightarrow \boxed{\sqrt{2}\, e^{\frac{j\pi}{4}}}$$

$$(1+j)(1-j) = \frac{(1+j)(1+j)}{(1-j)(1+j)} = \frac{1+2j+j^2}{1-j^2} = \frac{1+2j-1}{1+1}$$

$$= j \Rightarrow \boxed{e^{j\pi/2}}$$

$$\frac{(\sqrt{2}+j\sqrt{2})}{(1+j\sqrt{3})} = \frac{\sqrt{2}(1+j)(1-j\sqrt{3})}{(1+j\sqrt{3})(1-j\sqrt{3})} = \frac{\sqrt{2}(1+(1-\sqrt{3})j+\sqrt{3})}{1+3}$$

$$= \frac{\sqrt{2}}{4}\left[(\sqrt{3}+1)+j(1-\sqrt{3})\right] = \left(\frac{\sqrt{6}+\sqrt{2}}{4}\right)+j\left(\frac{\sqrt{2}-\sqrt{6}}{4}\right)$$

$$= \cos\left(\frac{-\pi}{12}\right) + j\sin\left(\frac{-\pi}{12}\right) \Rightarrow \boxed{e^{-j\pi/12}}$$

1.3) c.) $x_3(t) = \cos(t)$

$$E_\infty = \int_{-\infty}^{\infty} |x_3(t)|^2 dt = \int_{-\infty}^{\infty} \cos^2(t)\, dt = \boxed{\infty}$$

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |x_3(t)|^2 dt = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} \cos^2(t)\, dt$$

$$= \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T}\left(\frac{1+\cos(2t)}{2}\right) dt = \boxed{2}$$

f.) $x_3[n] = \cos\left(\frac{\pi}{4}n\right)$

$$E_\infty = \sum_{n=-\infty}^{\infty} |x_3[n]|^2 = \sum_{n=-\infty}^{\infty} \cos^2\left(\frac{\pi}{4}n\right) = \boxed{\infty}$$

$$P_\infty = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x_3[n]|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \cos^2\left(\frac{\pi}{4} n\right)$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \left(\frac{1+\cos(\frac{\pi}{2} n)}{2}\right) = \boxed{\frac{1}{2}}$$

1.4) a.) $x[n-3]$, $x[n]=0 \mid n<-2 \ \& \ n>4$

$n-3<-2 \ \& \ n-3>4 \Rightarrow \boxed{n<1 \ \& \ n>7}$

c.) $x[-n]$, $x[n]=0 \mid n<-2 \ \& \ n>4$

$-n<-2 \ \& \ -n>4 \Rightarrow \boxed{n>2 \ \& \ n<-4}$

d.) $x[-n+2]$, $x[n]=0 \mid n<-2 \ \& \ n>4$

$-n+2<-2 \ \& \ -n+2>4 \Rightarrow -n<-4 \ \& \ -n>2$

$\Rightarrow \boxed{n<-2 \ \& \ n>4}$

1.5) a.) $x(1-t)$, $x(t)=0 \mid t<3$

$1-t<3 \Rightarrow -t<2 \Rightarrow \boxed{t>-2}$

b.) $x(1-t)+x(2-t)$, $x(t)=0 \mid t<3$

$\cancel{1-t<3 \Rightarrow t>-2}$

$2-t<3 \Rightarrow t>-1$

$\Rightarrow \boxed{t>-1}$

c.) $x(1-t)\,x(2-t)$, $x(t)=0 \mid t<3$

$\boxed{t>-2}$ since $2-(-2)=0$

d.) $x(3t)$, $x(t)=0 \mid t<3$

$3t<3 \Rightarrow \boxed{t<1}$

1.6) a.) $x_1(t)=2e^{j(t+\pi/4)}\,u(t)$

$\boxed{\text{No}}$ since $x_1(t)=0$ for $t<0$

1.7) b.) $x_2(t)=\sin(\frac{1}{2}t)$

$$X_{even}(t)=\frac{1}{2}\left[\sin(\tfrac{1}{2}t)+\sin(-\tfrac{1}{2}t)\right]$$

$$=\frac{1}{2}\left[\sin(\tfrac{1}{2}t)-\sin(\tfrac{1}{2}t)\right]=0$$

$\Rightarrow x_2(t)=0$ for $\boxed{\text{all } t}$

1.8) a.) $x_1(t)=-2$, $Ae^{-at}\cos(\omega t+\phi)$ | $A>0$ | $-\pi<\phi\leq\pi$

$\rightarrow x_1(t)=2\times(-1)=2\cos(\pi)$ $\{A=2\ \ \omega=0\ \ \phi=\pi$

$\Rightarrow \boxed{x_1(t)=2e^{-0t}\cos(\pi)}\ \boxed{A=2,\ a=0,\ \omega=0,\ \phi=\pi}$

b.) $x_2(t)=\sqrt{2}\,e^{j\pi/4}\cos(3t+2\pi)$

$$x_2(t)=\sqrt{2}\left(\cos(\tfrac{\pi}{4})+j\sin(\tfrac{\pi}{4})\right)\left(\cos(3t+2\pi)\right)$$

$$=\sqrt{2}\left(\tfrac{1}{\sqrt{2}}+j\tfrac{1}{\sqrt{2}}\right)\cos(3t+2\pi)$$

$$=(1+j)\cos(3t+2\pi)=\cos(3t+2\pi)+j\cos(3t+2\pi)$$

$\Rightarrow \boxed{A=1\quad a=0\quad \omega=3\quad \phi=0}$

c.) $x_3(t) = e^{-t} \sin(3t+\pi)$

since $\sin\theta = \cos(\pi/2 - \theta)$

$\Rightarrow \sin 3t+\pi = \cos(\pi/2 - 3t - \pi) = \cos(3t + \frac{\pi}{2})$

$\Rightarrow x_3(t) = e^{-t} \cos(3t + \pi/2)$

$= A e^{-at} \cos(\omega t + \phi)$

$\boxed{A = 1 \quad a = 1 \quad \omega = 3 \quad \phi = \pi/2}$

d.) $x_4(t) = je^{(-2+j100)t}$

$je^{-2t} \cdot e^{j100t} = je^{-2t}((\cos(100t) + j\sin(100t))$

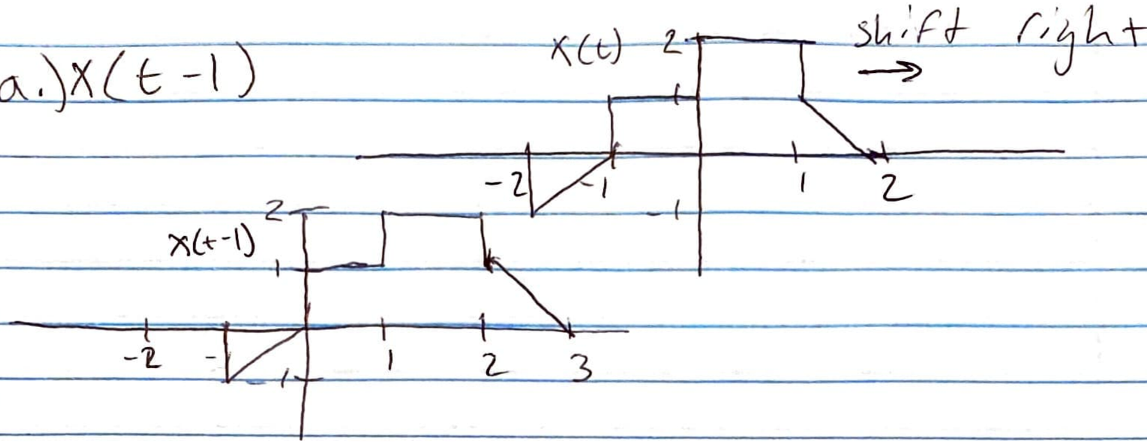
$= je^{-2t} \cos(100t) + e^{-2t} \sin(100t+\pi)$

$\Rightarrow x_4 = e^{-2t} \sin(100t+\pi) = e^{-2t} \cos(100t + \frac{\pi}{2})$

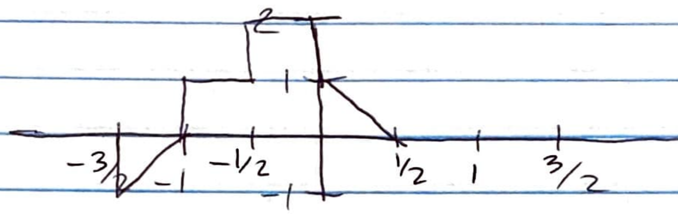
$\boxed{A = 1 \quad a = 2 \quad \omega = 100 \quad \phi = \pi/2}$

a.)

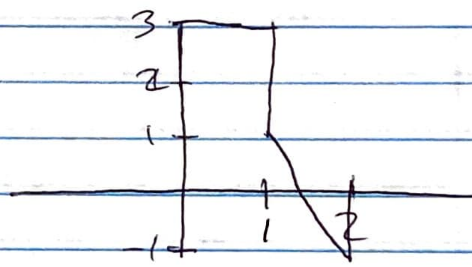
1.21) a.) $x(t-1)$



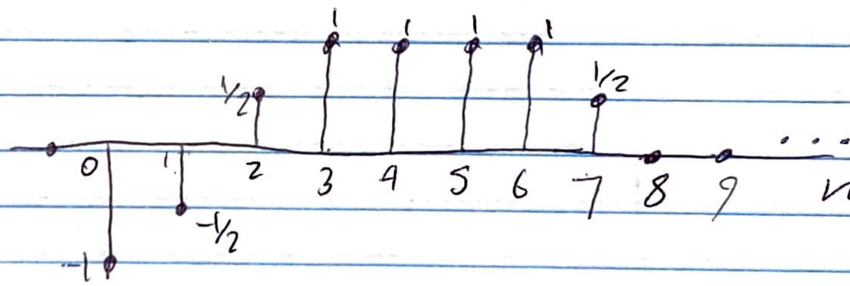
b.) $x(2t+1)$



e.) $[x[n]u[3-n]]$



1.22) a.) $x[n-4]$

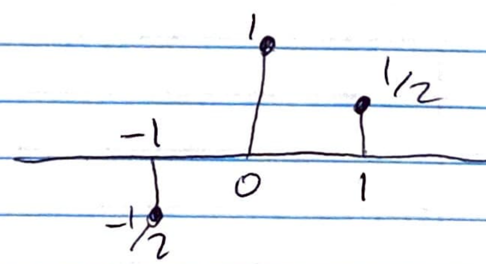


c.) $x[3n]$

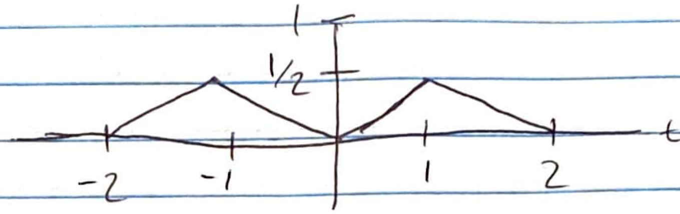
$3n=-3 \Rightarrow n=-1$

$3n=0 \Rightarrow n=0$

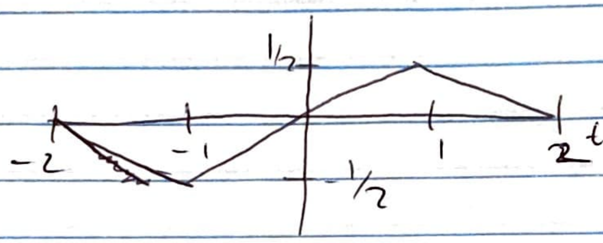
$3n=3 \Rightarrow n=1$



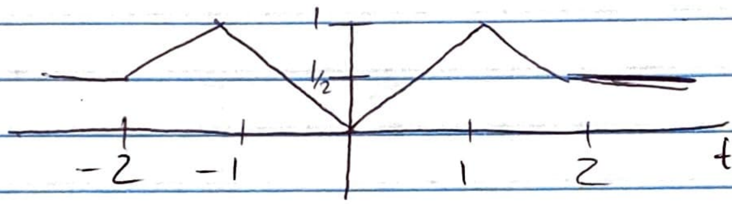
1.23) a.) even:



odd:



b.) even:



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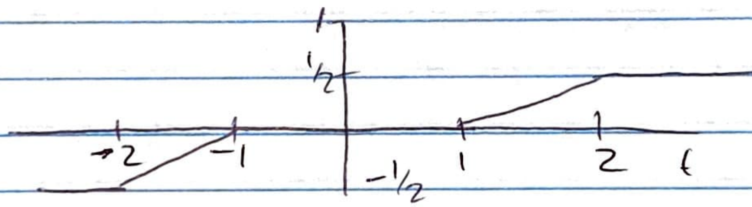
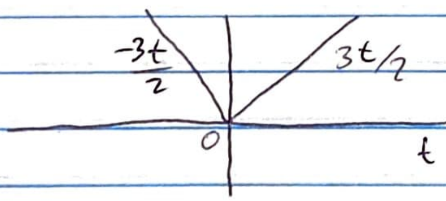


c.) even:



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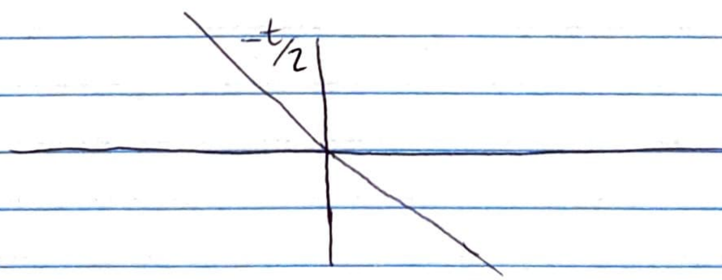
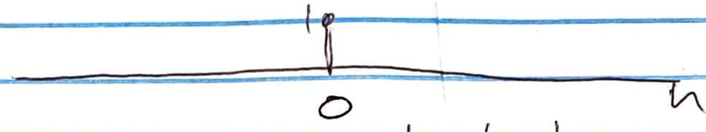


1.24) a.) even:



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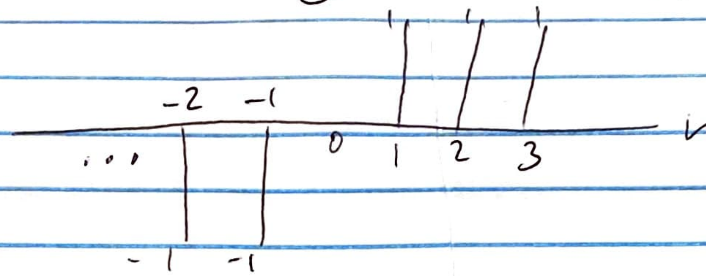
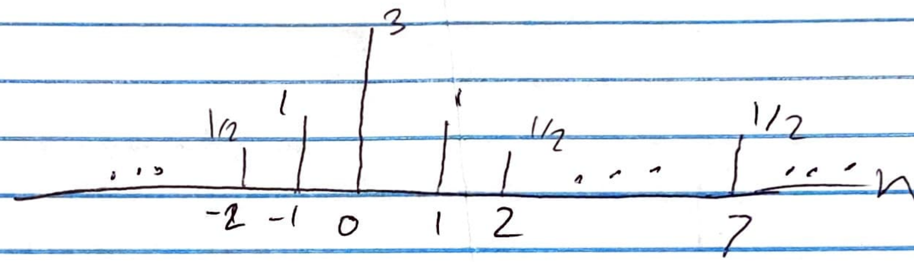


b.) even:



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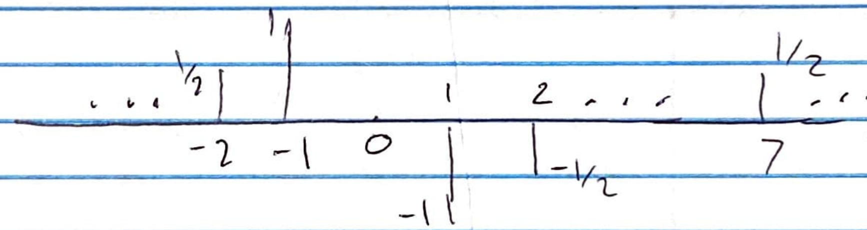
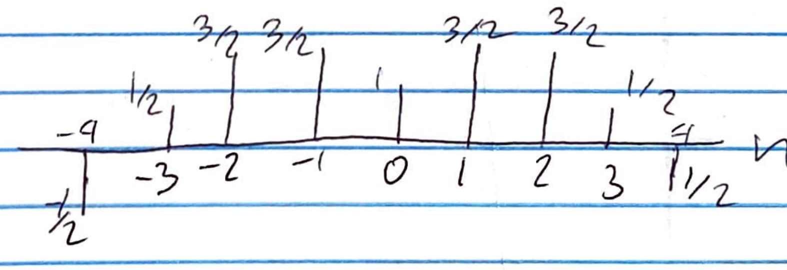


c.) even:



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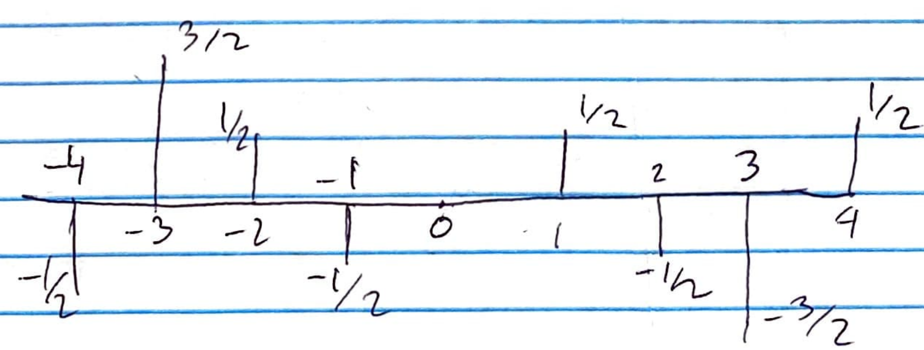

1.25) a.) $x(t) = 3\cos(4t + \pi/3)$, $\omega = 4$

$$T = \frac{2\pi}{4} = \boxed{\frac{\pi}{2} s} \quad \boxed{Yes}$$

c.) $x(t) = [\cos(2t - \pi/3)]^2$, $\omega = 2$

$$T = \frac{2\pi}{2} = \boxed{\pi s} \quad \boxed{Yes}$$

e.) $x(t) = \mathcal{E}v\{\sin(4\pi t)u(t)\}$

$\boxed{No}$

1.26) a.) $x[n] = \sin\left(\frac{6\pi n}{7} + 1\right)$, $\omega = \frac{6\pi}{7}$

$\frac{\omega}{2\pi} = \frac{6\pi/7}{2\pi} = 3/7$; rational # = periodic

$\boxed{Yes}$ $T = 3(7/3) = \boxed{7s}$

c.) $x[n] = \cos(\pi/8\, n^2)$

$\boxed{Yes}$

e.) $x[n] = 2\cos(\pi/4\, n) + \sin(\pi/8\, n) - 2\cos(\pi/2\, n + \frac{\pi}{6})$

$\omega_1 = \pi/4$, $\omega_2 = \pi/8$; $T_1 = 8$, $T_2 = 16$

$\omega_3 = \pi/2$; $T_3 = 4$ $\boxed{Yes}$