CSE3350

Daniel Delgado Acosta

3-7-22

Assignment 7

(3.1) If $T=8$ & $a_1 = a_{-1} = 2,\ a_{-3} = 4j$

$\omega = \frac{2\pi}{T} = \frac{2\pi}{8} = \frac{\pi}{4}$

$$x(t) = \sum_{k=0}^{\infty} A_k \cos(\omega_k t + \phi_k) \Leftrightarrow x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega t}$$

$$= a_{-3} e^{-j3\left(\frac{2\pi}{T}\right)t} + a_{-1} e^{-j\left(\frac{2\pi}{T}\right)t} + a_1 e^{j\left(\frac{2\pi}{T}\right)t} + a_3 e^{j3\left(\frac{2\pi}{T}\right)t}$$

$$= -4j e^{-j3\left(\frac{\pi}{4}\right)t} + 2e^{-j\left(\frac{\pi}{4}\right)t} + 2e^{j\left(\frac{\pi}{4}\right)t} + 4j e^{j3\left(\frac{\pi}{4}\right)t}$$

$$= 2\left(e^{j\left(\frac{\pi}{4}\right)t} + e^{-j\left(\frac{\pi}{4}\right)t}\right) + 4j\left(e^{j\left(\frac{3\pi}{4}\right)t} - e^{-j\left(\frac{3\pi}{4}\right)t}\right)$$

$$= 2\left(2\cos\frac{\pi}{4}t\right) + 4j\left(2j\sin\frac{3\pi}{4}t\right)$$

$$= 4\cos\frac{\pi}{4}t - 8\sin\frac{3\pi}{4}t = \underline{4\cos\frac{\pi}{4}t + 8\cos\left(\frac{3\pi}{4}t + \frac{\pi}{2}\right)}$$

$$\rightarrow \boxed{x(t) = \sum_{k=1,3} A_k \cos(\omega_k t + \phi_k) \quad \text{Where } A_1 = 4,\ A_3 = 8,\ \omega_1 = \frac{\pi}{4},\ \omega_3 = \frac{3\pi}{4},\ \phi_1 = 0,\ \&\ \phi_3 = \frac{\pi}{2}}$$

(3.3) $x(t) = 2 + \cos\left(\frac{2\pi}{3}t\right) + 4\sin\left(\frac{5\pi}{3}t\right)$

determine ω_0 & the Fourier series coefficients a_k

s.t. $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$

$$\rightarrow x(t) = 2 + \left(\frac{e^{j\frac{2\pi}{3}t} + e^{-j\frac{2\pi}{3}t}}{2}\right) + 4\left(\frac{e^{j\frac{5\pi}{3}t} - e^{-j\frac{5\pi}{3}t}}{2j}\right)$$

$$= \underbrace{2}_{a_0} + \underbrace{\tfrac{1}{2}}_{a_2} e^{j\overset{k}{2}\underbrace{\left(\frac{\pi}{3}\right)}_{\omega_0}t} + \underbrace{\tfrac{1}{2}}_{a_{-2}} e^{-j\overset{k}{2}\underbrace{\left(\frac{\pi}{3}\right)}_{\omega_0}t} \underbrace{- 2j}_{a_5} e^{j\overset{k}{5}\underbrace{\left(\frac{\pi}{3}\right)}_{\omega_0}t} + \underbrace{2j}_{a_{-5}} e^{-j\overset{k}{5}\underbrace{\left(\frac{\pi}{3}\right)}_{\omega_0}t}$$

Therefore $\boxed{\omega_0 = \frac{\pi}{3}}$

$\rightarrow \boxed{a_5 = -2j,\ a_{-5} = 2j,\ a_2 = \frac{1}{2},\ a_{-2} = \frac{1}{2},\ a_0 = 2}$

3.4 find a_k: $x(t) = \begin{cases} 1.5, & 0 \le t < 1 \\ -1.5, & 1 \le t < 2 \end{cases}$ w/ $\omega_0 = \pi$

Fourier: $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$

$\rightarrow a_k = \frac{1}{T}\int_T x(t) e^{-j\omega_0 k t} dt$, $k=0$

$\Rightarrow a_0 = \frac{1}{T}\int_T x(t) dt$, $\omega = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi}{\omega} = \frac{2\pi}{\pi} = 2$

$\rightarrow a_0 = \frac{1}{2}\int_0^2 x(t) dt = \frac{1}{2}\left(\int_0^1 1.5 dt + \int_1^2 -1.5 dt\right.$

$= \frac{1}{2}\left(1.5(t|_0^1) - 1.5(t|_1^2)\right) = \frac{1}{2}\left((1.5) - (1.5)(1)\right) = 0$

$a_k = \frac{1}{2}\int_0^2 x(t) e^{-jk\pi t} dt = \frac{1}{2}\left(\int_0^1 (1.5) e^{-jk\pi t} dt + \int_1^2 (-1.5) e^{-jk\pi t} dt\right.$

$= \frac{3}{4(-jk\pi)}\left((e^{-jk\pi} - 1) - (e^{-j2k\pi} - e^{-jk\pi})\right)$

$= \frac{3}{4\pi kj}\left(1 + e^{-j2k\pi} - 2e^{-j2k\pi}\right) = \frac{3}{2jk\pi}\left(1 - e^{-jk\pi}\right)$

$= \frac{3}{2jk\pi}\left(e^{j0} - e^{-jk\pi}\right) = \frac{3e^{-jk\frac{\pi}{2}}}{k\pi}\left(\frac{e^{jk\frac{\pi}{2}} - e^{-jk\frac{\pi}{2}}}{2j}\right)$

$= \frac{3e^{-jk\frac{\pi}{2}}}{k\pi} \sin\left(\frac{k\pi}{2}\right)$

$\rightarrow \boxed{a_k = \begin{cases} 0, & k = 0 \\ e^{-j\frac{k\pi}{2}}\left(\frac{3\sin(\frac{k\pi}{2})}{k\pi}\right), & k \neq 0 \end{cases}}$