CSE3350

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Assignment 6

2.13 $h[n] = \left(\frac{1}{5}\right)^n u[n]$

a.) Find A; $h[n] - A h[n-1] = \delta[n]$

$\rightarrow h[n-1] = \left(\frac{1}{5}\right)^{n-1} u(n-1)$

$\Rightarrow \left(\frac{1}{5}\right)^n u[n] - A\left(\frac{1}{5}\right)^{n-1} u(n-1) = \delta[n]$

If $n=1$; $\left(\frac{1}{5}\right)^1 u[1] - A\left(\frac{1}{5}\right)^{1-1} u[1-1] = \delta[1]$

$\Rightarrow u[1]=1,\ u[0]=1,\ \&\ \delta[0]=1\ \&\ \delta[n]=0$ for any $n \neq 0$

$\longrightarrow \left(\frac{1}{5}\right)(1) - A\left(\frac{1}{5}\right)^0(1) = 0 \Rightarrow \frac{1}{5} - A = 0$

$\Rightarrow \boxed{A = \frac{1}{5}}$

b.) $h[n] - \frac{1}{5}h[n-1] = \delta[n]$

Convolution: $h[n] * \{\delta[n - \frac{1}{5}\delta[n-1]\} = \delta[n]$

inverse def: $h[n] * g[n] = \delta[n]$

$\Rightarrow \boxed{g[n] = \delta[n] - \frac{1}{5}\delta[n-1]}$

2.14 a.) $h_1(t) = e^{-(1-2j)t} u(t) \rightarrow \int_{-\infty}^{\infty} |h_1(t)|\, dt < \infty$

$\Rightarrow \int_{-\infty}^{\infty} |e^{-(1-2j)t} u(t)|\, dt \leq \int_{-\infty}^{\infty} |e^{-(1-2j)t}|\,|u(t)|\, dt$

$\int_{-\infty}^{\infty} |e^{-t}|\,|e^{-j2t}|\, dt$

$\rightarrow |e^{j2t}| = \sqrt{\cos(2t) + j\sin(2t)} = 1$

$\Rightarrow \int_{-\infty}^{\infty} |e^{-(1-2j)t} u(t)|\, dt \leq \int_{0}^{\infty} |e^{-t}|(1)\, dt$

$\int_{0}^{\infty} e^{-t} dt \rightarrow -e^{-t}\Big|_0^{\infty}$

$\rightarrow -(e^{-\infty} - e^{-0})$

$\Rightarrow \int_{-\infty}^{\infty} |e^{-(1-2j)t} u(t)|\, dt \leq -(0-1)$

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Since $\int_{-\infty}^{\infty} |h_1(t)|\, dt \leq \infty$, $h_1(t)$ is $\boxed{\text{stable}}$

b.) $h_2(t) = e^{-t}\cos(2t)\,u(t) \rightarrow \int_{-\infty}^{\infty} |h_2(t)|\, dt$

$= \int_{-\infty}^{\infty} |e^{-t}\cos(2t)\,u(t)|\, dt$

$\Rightarrow \int_{-\infty}^{\infty} |e^{-t}\cos(2t)\,u(t)|\, dt \leq \int_{-\infty} |e^{-t}||\cos(2t)||u(t)|\, dt$

$\cos 2t = \dfrac{e^{j2t} + e^{-j2t}}{2} = \frac{1}{2}e^{j2t} + \frac{1}{2}e^{-j2t}$

$\int_{0}^{\infty} |e^{-t}||\cos(2t)|\, dt$

$\Rightarrow |\cos(2t)| = |\frac{1}{2}e^{j2t} + \frac{1}{2}e^{-j2t}| = \frac{1}{2} + \frac{1}{2} = \underline{1}$

$\rightarrow \int_{-\infty}^{\infty} |e^{-t}\cos(2t)\,u(t)|\, dt \leq \int_{0}^{\infty} |e^{-t}|(1)\, dt$

$-e^{-t}\Big|_0^{\infty} \rightarrow = \underline{(e^{-\infty} - e^{0})}$

$\Rightarrow \int_{-\infty}^{\infty} |e^{-t}\cos(2t)\,u(t)|\, d(t) \leq -(0-1)$

$\underline{1}$

Since $\int_{-\infty}^{\infty} |h_2(t)|\, dt < \infty$, $h_2(t)$ is $\boxed{\text{stable}}$

2.15 a.) $h_1[n] = n\cos(\pi/4(n))\,u[n]$

$$\rightarrow \sum_{k=-\infty}^{\infty} |h[k]| < \infty \Rightarrow \sum_{k=-\infty}^{\infty} |h_1[k]| = \sum_{k=-\infty}^{\infty} \left|k\cos\left(\tfrac{\pi}{4}k\right)u[k]\right|$$

$$\Rightarrow \sum_{k=-\infty}^{\infty} h_1[k] \le \sum_{k=-\infty}^{\infty} |k|\left|\cos\left(\tfrac{\pi}{4}k\right)\right||u[k]|$$

$$\le \sum_{k=0}^{\infty} k\cos\left(\tfrac{\pi}{4}k\right)$$

$$\rightarrow \sum_{k=-\infty}^{\infty} h_1[k] = \infty \text{, therefore } \boxed{\text{not stable}}$$

b.) $h_2[n] = 3^n u[-n+10] \rightarrow \sum_{k=-\infty}^{\infty} |h[k]| < \infty$

$$\Rightarrow \sum_{k=-\infty}^{\infty} |3^k||u[-k+10]| \ge \sum_{k=-\infty}^{\infty} |3^k u[-k+10]|$$

$$\Rightarrow \sum_{k=-\infty}^{\infty} h_2[k] \le \sum_{k=-\infty}^{10} |3^k| \rightarrow \frac{3^{-\infty} - 3^{11}}{3^{-\infty} - 3} = \frac{3^{11}}{2} \text{ finite}$$

since $\sum_{k=-\infty}^{\infty} |h_2[k]| < \infty$; the system is $\boxed{\text{stable}}$

2.16 a.) If $x[n]=0$ for $n<N_1$ & $h[n]=0$ for $n<N_2$, then $x[n] * h[n] = 0$ for $n < N_1 + N_2$

$$\rightarrow x[n]*h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] \text{, where } k \ge N_1 \text{ \& } n-k \ge N_2$$

$$\rightarrow n-k \ge N_2 \Rightarrow n-N_2 \ge k \Rightarrow k \le n-N_2$$

$$\rightarrow N_1 \le k \le n-N_2 \rightarrow N_1+N_2 \le k+N_2 \le n$$

Convolution is 0 for $n < N_1+N_2$, therefore $\boxed{\text{true}}$

b.) If $y[n] = x[n] * h[n]$, then $y[n-1] = x[n-1] * h[n-1]$

$$\rightarrow y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

$$y[n-1] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-1-k] = x[n] * h[n-1]$$

therefore, the statement is $\boxed{\text{false}}$

c.) If $y(t) = x(t) * h(t)$, then $y(-t) = x(-t) * h(-t)$

$$\rightarrow y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$\Rightarrow y(-t) = \int_{-\infty}^{\infty} x(\tau)\, h(-t-\tau)\, d\tau$$

$$\Rightarrow \int_{-\infty}^{\infty} x(-\tau)\, h(-t-(-\tau))\,(-d\tau)$$

$$= \int_{-\infty}^{\infty} x(-\tau)\, h(-t-(-\tau))\, d\tau$$

$$\rightarrow \int_{-\infty}^{\infty} f(t)\, dt = \int_{\infty}^{-\infty} f(t)\, dt$$

$$\Rightarrow y(-t) = \int_{-\infty}^{\infty} x(-\tau)\, h(-t-(-\tau))\, d\tau = x(t) * h(-t)$$

Therefore, the statement is $\boxed{\text{true}}$

d.) If $x(t)=0$ for $t>T_1$ & $h(t)=0$ for $t>T_2$,
then $x(t) * h(t)=0$ for $t>T_1+T_2$

$$y(t)=x(t) * h(t)=\int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau$$

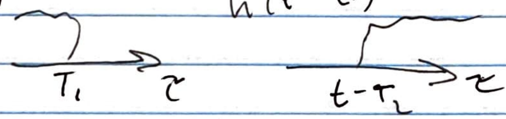
$\rightarrow x(\tau)h(t-\tau)=0$

if $x(\tau)=0$ for $\tau>T_1$ & $h(t-\tau)=0$ for $t-\tau>T_2$

$\downarrow$

$h(t-\tau)=0$ for $\tau<t-T_2$

If $x(\tau)$ & $h(t-\tau)$



$\rightarrow x(\tau)h(t-\tau)=0$ for $t-T_2>T_1$

$\rightarrow x(\tau)h(t-\tau)=0$ for $t-T_2>T_1$

$x(\tau)h(t-\tau)=0$ for $t>T_2+T_1$

~~$\rightarrow x(\tau)h(t-\tau)=0$ for t~~

$\Rightarrow x(t) * h(t)=0$ for $t>T_1+T_2$

therefore the statement is $\boxed{\text{true}}$