CSE 3350

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Assignment 10

5.1 a.) $\left(\frac{1}{2}\right)^{n-1} u[n-1]$, $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$

$$\Rightarrow \sum_{n=-\infty}^{\infty} \left(\frac{1}{2}\right)^{n-1} u[n-1] e^{-j\omega n} = \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{n-1} e^{-j\omega n}$$

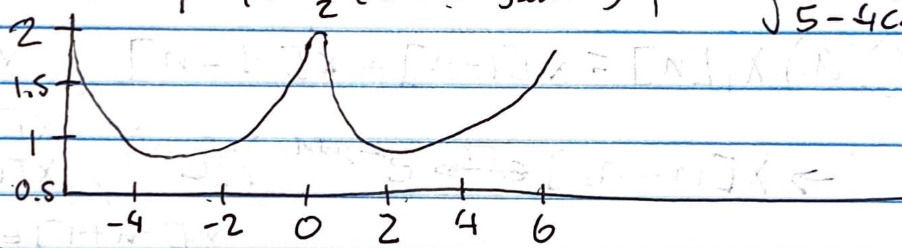
$n_1 = n-1 = 1-1 = 0$
$n_1 = \infty - 1 = \infty$
$\hat{n}$

$$\Bigg\} \rightarrow X(e^{j\omega}) = \sum_{n_1=0}^{\infty} \left(\frac{1}{2}\right)^{n_1} e^{-j\omega(n_2+1)}$$

$$= e^{-j\omega} \sum_{n_1=0}^{\infty} \left(\frac{1}{2} e^{-j\omega}\right)^{n_1} = \boxed{e^{-j\omega}\left(1 - \frac{1}{2} e^{-j\omega}\right)^{-1}}$$

$$\Rightarrow |X(e^{j\omega})| = \left| \frac{\cos\omega - j\sin\omega}{1 - \frac{1}{2}(\cos\omega - j\sin\omega)} \right| = \frac{2}{\sqrt{5 - 4\cos\omega}}$$

Sketch:



$X(e^{j\omega})$

b.) $\left(\frac{1}{2}\right)^{|n-1|} \Rightarrow \sum_{n=-\infty}^{\infty} \left(\frac{1}{2}\right)^{|n-1|} e^{-j\omega n} = \underbrace{\sum_{n=-\infty}^{0} \left(\frac{1}{2}\right)^{-(n-1)} e^{-j\omega n}}_{X_1} + \underbrace{\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{n-1} e^{-j\omega n}}_{X_2}$

$$\rightarrow X_1(e^{j\omega}) = \sum_{n=-\infty}^{0} \left(\frac{1}{2}\right)^{(-n+1)} e^{-j\omega n}$$

$n_1 = -n = -(-\infty) = \infty$
$n_1 = -n = 0$

$$\rightarrow X_1(e^{j\omega}) = \sum_{n_1=0}^{\infty} \left(\frac{1}{2}\right)^{(n_1+1)} e^{j\omega n_1} = \frac{1}{2} \sum_{n_1=0}^{\infty} \left(\frac{1}{2} e^{j\omega}\right)^{n_1}$$

$$= \underline{\frac{1}{2}\left(1 - \frac{1}{2} e^{j\omega}\right)^{-1}} \quad , \quad X_2(e^{j\omega}) = \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{n-1} e^{-j\omega n}$$

$n_1 = n-1 = 1-1 = 0$
$n_1 = n-1 = \infty - 1 = \infty$

$$\rightarrow X_2 = \sum_{n_1=0}^{\infty} \left(\frac{1}{2}\right)^{n_1} e^{-j\omega(n_1+1)} = e^{-j\omega} \sum_{n_1=0}^{\infty} \left(\frac{1}{2} e^{-j\omega}\right)^{n_1}$$

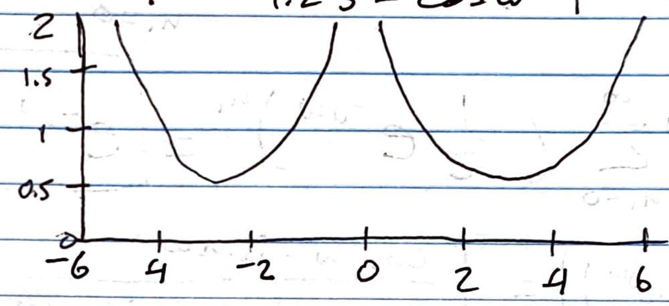
$$= \boxed{e^{-j\omega}\left(1 - \frac{1}{2} e^{-j\omega}\right)^{-1}} \Rightarrow X(e^{j\omega}) = \sum_{n=-\infty}^{0} \left(\frac{1}{2}\right)^{-(n-1)} e^{-j\omega n} + \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{n-1} e^{-j\omega n}$$

$$= \frac{1}{2}\left(1-\frac{1}{2}e^{j\omega}\right)^{-1} + e^{-j\omega}\left(1-\frac{1}{2}e^{-j\omega}\right)^{-1}$$

$$= \frac{3/4\, e^{-j\omega}}{\frac{5}{4} - \frac{e^{j\omega}+e^{-j\omega}}{2}} = \boxed{\frac{(0.75)(e^{-j\omega})}{1.25-\cos\omega}}$$

$$\to |X(e^{j\omega})| = \left|\frac{0.75(\cos\omega - j\sin\omega)}{1.25-\cos\omega}\right| = \frac{0.75}{1.25-\cos\omega}$$

Sketch:



5.6 a) $x_1[n] = x[1-n] + x[-1-n]$, $x[n] \xleftrightarrow{FT} X(e^{j\omega})$

$\to x[n-n_0] \xleftrightarrow{FT} e^{-j\omega n_0} X(e^{j\omega})$, $x[n+1] \xleftrightarrow{FT} e^{j\omega} X(e^{j\omega})$, $x[n-1] \xleftrightarrow{FT} e^{-j\omega} X(e^{j\omega})$

; $x[-n] \xleftrightarrow{FT} X(e^{-j\omega}) \Rightarrow$ $x[-n+1] \xleftrightarrow{FT} e^{-j\omega} X(e^{-j\omega})$, $x[-n-1] \xleftrightarrow{FT} e^{j\omega} X(e^{-j\omega})$

$\} \; x[n] \xleftrightarrow{FT} e^{-j\omega} X(e^{-j\omega}) + e^{j\omega} X(e^{-j\omega})$

$\to 2X(e^{-j\omega}) \left[\frac{e^{-j\omega}+e^{j\omega}}{2}\right]$

$\to 2X(e^{-j\omega}) \cos\omega = \boxed{2\cos(\omega) X(e^{-j\omega})}$

b) $x_2[n] = \frac{x^*[-n] + x[n]}{2}$, $x^*[-n] \xleftrightarrow{FT} X^*(e^{j\omega})$

$\Rightarrow \frac{x^*[-n]+x[n]}{2} \xleftrightarrow{FT} \frac{1}{2}X^*(e^{j\omega}) + \frac{1}{2}X(e^{j\omega})$

$\Rightarrow x_2[n] \xleftrightarrow{FT} \frac{1}{2}\left[X^*(e^{j\omega}) + X(e^{j\omega})\right]$

$\to \boxed{\mathrm{Re}\{X(e^{j\omega})\}}$

5.13

impulse response: $h_1[n] = \left(\frac{1}{3}\right)^n u[n]$, $h_2[n] = ?$

result: $H(e^{jw}) = \dfrac{-12 + 5e^{-jw}}{12 - 7e^{-jw} + e^{-j2w}}$

, $h[n] = h_1[n] + h_2[n] \Rightarrow H(e^{jw}) = H_1(e^{jw}) + H_2(e^{jw})$

$h_1[n] = \left(\frac{1}{3}\right)^n u[n] = \left(1 - \frac{1}{3}e^{-jw}\right)^{-1}$, $\Rightarrow H_2(e^{jw}) = H(e^{jw}) - H_1(e^{jw})$

$$\Rightarrow H_2(e^{jw}) = \frac{-12 + 5e^{-jw}}{12 - 7e^{-jw} + e^{-j2w}} - \left(1 - \frac{1}{3}e^{-jw}\right)^{-1}$$

$$= \frac{1}{12}\left[\frac{-12 + 5e^{-jw}}{\left(1 - \frac{1}{3}e^{-jw}\right)\left(1 - \frac{1}{4}e^{-jw}\right)}\right] - \left(1 - \frac{1}{3}e^{-jw}\right)^{-1}$$

$$= \frac{-1 + \frac{5}{12}e^{-jw}}{\left(1 - \frac{1}{3}e^{-jw}\right)\left(1 - \frac{1}{4}e^{-jw}\right)} - \frac{1}{\frac{1}{3}e^{-jw}} = \left(1 - \frac{1}{3}e^{-jw}\right)^{-1}\left(\frac{-1 + \frac{5}{12}e^{-jw}}{1 - \frac{1}{4}e^{-jw}} - 1\right)$$

$$= \left(1 - \frac{1}{3}e^{-jw}\right)^{-1}\left(\frac{-1 + \frac{5}{12}e^{-jw} - 1 + \frac{1}{4}e^{-jw}}{1 - \frac{1}{4}e^{-jw}}\right)$$

$$= \left(1 - \frac{1}{3}e^{-jw}\right)^{-1}\left(\frac{1 - \frac{1}{3}e^{-jw}}{1 - \frac{1}{4}e^{-jw}}\right)(-2)$$

$$\Rightarrow H_2(e^{jw}) = \frac{-2}{1 - \frac{1}{4}e^{-jw}} = \boxed{-2\left(\frac{1}{4}\right)^n u[n]}$$