CSE 3350

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# Assignment 3

**1.16** $y[n] = x[n]\,x[n-2]$; input $x[n]$ & output $y[n]$

a.) This system is <u>not</u> memoryless because the system is not static. Therefore, dynamic.

b.) $x[n] = A\delta[n] \Rightarrow y[n] = A\delta[n]A\delta[n-2]$

$$\Rightarrow y[n] = A^2\delta[n]\delta[n-2]; \quad n=0$$

$$\Rightarrow \boxed{y[n] = 0}$$

c.) The system is <u>not</u> invertible because the output is always zero.

**1.17** $y(t) = x(\sin(t))$.

a.) This system is <u>not</u> causal because it ranges from $-\infty$ to $\infty$.

b.) Check for linearity:

$y_1(t) = x_1(\sin(t))$, $y_2(t) = x_2(\sin(t))$

$$\Rightarrow \underline{y_1(t) + y_2(t) = x_1(\sin(t)) + x_2(\sin(t))}$$

The system <u>is Linear</u>

**1.18** $y[n] = \sum_{k=n-n_0}^{n+n_0} x[k]$

a.) $y_1[n] = \sum_{k=n-n_0}^{n+n_0} x_1[k]$ & $y_2[n] = \sum_{k=n-n_0}^{n+n_0} x_2[k]$

$$\Rightarrow y_1[k] + y_2[k] = \sum_{k=n-n_0} x_1[k] + x_2[k]$$

<u>The system is linear</u>

If $y[n-n_1] = y_1[n]$ then the system is time-invariant

b.) $x[n-n_1] \rightarrow \sum_{k=n-n_0}^{n+n_0} x[k-n_1] = y_1[n]$

$\Rightarrow y[n-n_1] = \sum_{k=n-n_1-n_0}^{n-n_1+n_0} x[k]$

$\Rightarrow y_1[n] = \sum_{k-n_1=n-n_1-n_0}^{n-n_1+n_0} x[k-n_1]$

The system is time-invariant

c.) If $|x[n]| < B$ for all $n$ ; $y[n] < C$

$\rightarrow y[n] = \sum_{k=n-n_0}^{n+n_0} x[k] < (B)(2n_0+1)$

$\rightarrow \boxed{C = B(2n_0+1)}$

1.19 a.) $y(t) = t^2 x(t-1)$ : check linearity

$y_1(t) = t^2 x_1(t-1)$ & $y_2(t) = t^2 x_2(t-1)$

$\Rightarrow y_1(t) + y_2(t) = t^2 x_1(t-1) + t^2 x_2(t-1)$ linear

Check Time-invariance : $y_1(t-t_0) = (t-t_0)^2 x_1(t-t_0-1)$...

$y_1(t-t_0) \neq y(t)$ not time-invariant This system is only linear

b.) $y[n] = x^2[n-2]$ : check linearity

$\Rightarrow y_1[n] + y_2[n] = x_1^2[n-2] + x_2^2[n-2]$ not linear

Check Time-invariance :

$y_1[n] = x_1^2[n-2]$ ; $x_2[n = x_1[n-n_0] \Rightarrow$

$\Rightarrow y_1[n-n_0] = x_1^2[n-n_0-2]$ ; $y_2[n] = y_1[n-n_0]$ yes

This system is only Time-Invariant

c.) $y[n] = x[n+1] - x[n-1]$ : check linearity

$\Rightarrow y_1[n] + y_2[n] = x_1[n+1] - x_1[n-1] + x_2[n+1] - x_2[n-1]$

yes

check Time-invariance

If $x_2[n] = x_1[n-n_0] \Rightarrow$ ~~$y_1[n-n_0] = x_1[n-n_0+1]$~~

$\Rightarrow y_1[n-n_0] = x_1[n-n_0+1] + x_1[n-n_0-1]$,

$y_2[n] = y_1[n-n_0]$ yes

This system is both linear and time-invariant

d.) $y(t) = \mathcal{O}d\{x(t)\} = \frac{1}{2}[x(t) - x(-t)]$ : check linearity

$\Rightarrow y_1(t) + y_2(t) = \frac{1}{2}[x_1(t) - x_1(-t)] + \frac{1}{2}[x_2(t) - x_2(-t)]$

Yes

check time-invariance:

If $x_2(t) = x_1(t-t_0) \Rightarrow y(t-t_0) = \frac{1}{2}[x_1(t-t_0) - x_1(-t+t_0)]$

$\Rightarrow y_2(t) \neq y_1(t-t_0)$ no

This system is only linear

1.20 $x_1(t) = e^{j2t} \xrightarrow{S} y(t) = e^{j3t}$

input $\rightarrow$ output

$x(t) = e^{-j2t} \xrightarrow{S} y(t) = e^{-j3t}$

$y(t) = x(\frac{3}{2}t)$ : $y(t) = e^{j2\frac{3}{2}t} = e^{j3t}$

$y(t) = e^{-j2\frac{3}{2}t} = e^{-j3t}$

a.) $x_1(t) = \cos(2t) \Rightarrow y_1(t) = \cos(2 \cdot \frac{3}{2}t)$

$\Rightarrow \boxed{y_1(t) = \cos(3t)}$

b.) $x_2(t) = \cos(2(t - \frac{1}{2})) \Rightarrow y_2(t) = \cos(2(\frac{3}{2}t - \frac{1}{2}))$

$\Rightarrow \boxed{y_2(t) = \cos(3t - 1)}$

1.27 a) $y(t) = x(t-2) + x(2-t)$ : ~~Check for linearity~~

check for linearity:

$\approx y_1(t) + y_2(t) = x_1(t-2) + x_1(2-t) + x_2(t-2) + x_2(2-t)$ $\boxed{\text{yes}}$

check for time-variant:

If $x_1(t-t_0) = x_2(t) \Rightarrow y_2(t) = x_1(t-t_0-2) + x_2(2-t-t_0)$

$y_1(t-t_0) = x_1(t-t_0-2) + x_1(2-t+t_0)$ $\boxed{\text{yes}}$

check stability: If $|x(t)| < B$ then $y(t) < 2B$ $\boxed{\text{yes}}$

check memoryless: $\boxed{\text{No}}$ because $t-2$ is dependent

check Causal: $\boxed{\text{No}}$ because $y(-1)$ is dependent

c) $y(t) = \int_{-\infty}^{2t} x(\tau)\,d\tau$ :

Check linearity:

$y_1(t) + y_2(t) = \int_{-\infty}^{2t} (x_1(\tau) + x_2(\tau))\,d\tau$ $\boxed{\text{yes}}$

check time invariant:

If $x_1(t-t_0) = x_2(t) \Rightarrow y_2(t) = \int_{-\infty}^{2t-t_0} x_1(\tau)\,d\tau$

$\rightarrow y_1(t-t_0) = \int_{-\infty}^{2(t-t_0)} x_1(\tau)\,d\tau$ $\boxed{\text{yes}}$

check stability: If $x(t) = 1$ the $y(1) = \infty$, $\boxed{\text{No}}$

check memoryless: ~~because of~~ $\boxed{\text{No}}$ b/c of integral

check causal: $\boxed{\text{No}}$, b/c of $x(t)$ @ $t=2$