CSE 3350

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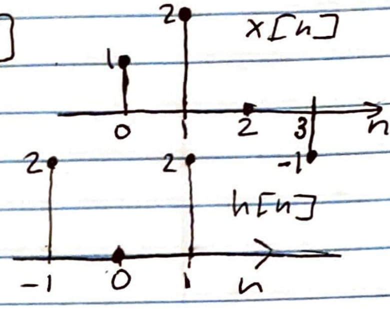
Assignment 4

2.1

$x[n] = \delta[n] + 2\delta[n-1] - \delta[n-3]$

$h[n] = 2\delta[n+1] + 2\delta[n-1]$

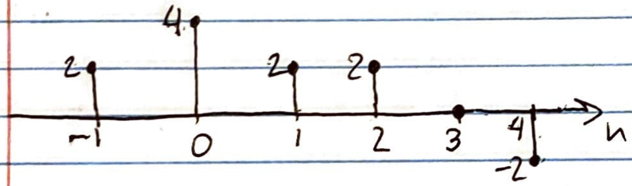


a.) $y_1[n] = x[n] * h[n]$

$= (\delta[n] + 2\delta[n-1] - \delta[n-3]) * (2\delta[n+1] + 2\delta[n-1])$

$= 2\delta[n+1] + 2\delta[n-1] + 4\delta[n] + 4\delta[n-2] - 2\delta[n-2] - 2\delta[n-4]$

$\boxed{y_1[n] = 2\delta[n+1] + 4\delta[n] + 2\delta[n-1] + 2\delta[n-2] - 2\delta[n-4]}$

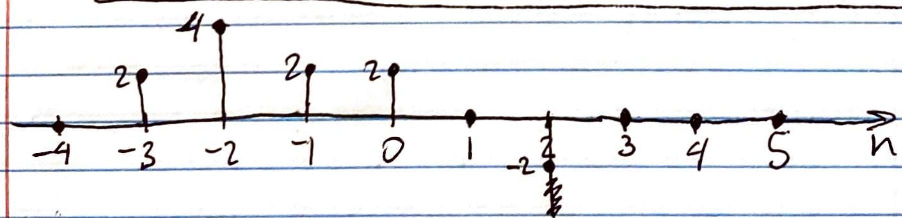


b.) $y_2[n] = x[n+2] * h[n]$

$= (2\delta[n+1] + \delta[n+2] - \delta[n-1]) * (2\delta[n+1] + 2\delta[n-1])$

$= (4\delta[n+2] + 4\delta[n] + 2\delta[n+3] + 2\delta[n+1] - 2\delta[n] - 2\delta[n-2])$

$= \boxed{2\delta[n+3] + 4\delta[n+2] + 2\delta[n+1] + 2\delta[n] - 2\delta[n-2]}$

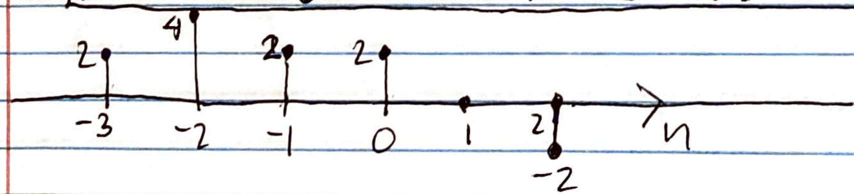


c.) $y_3[n] = x[n] * h[n+2]$

$= (\delta[n] + 2\delta[n-1] - \delta[n-3]) * (2\delta[n+3] + 2\delta[n+1])$

$= 2\delta[n+3] + 2\delta[n+1] + 4\delta[n+2] + 4\delta[n] - 2\delta[n] - 2\delta[n-2]$

$= \boxed{2\delta[n+3] + 4\delta[n+2] + 2\delta[n+1] + 2\delta[n] - 2\delta[n-2]}$



2.2 $h[n] = \left(\frac{1}{2}\right)^{n-1} \{u[n+3] - u[n-10]\}$

$$u[n+3] - u[n-10] \begin{cases} 1, & -3 \le n < 10 \\ 0, & n \ge 10,\ n < -3 \end{cases}$$

$$\Rightarrow h(n-k) = \left(\tfrac{1}{2}\right)^{n-k-1} \{u[n-k+3] - u[n-k-10]\}$$

$$\left(\tfrac{1}{2}\right)^{n-k-1} \begin{cases} k \le (n+3) \\ k > (n-10) \\ k \ge (n-9) \end{cases} \rightarrow \underset{A}{n-9} \le k \le \underset{B}{n+3}$$

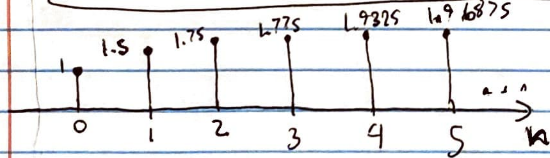
$$\boxed{A = n-9 \ \&\ B = n+3}$$

2.3 $x[n] = \left(\frac{1}{2}\right)^{n-2} u[n-2]$, $h[n] = u[n+2]$

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} \left(\tfrac{1}{2}\right)^{k-2} u[k-2]\, u[n-k+2] \qquad \begin{cases} u[k-2] = 0, & k < 2 \\ u[n-k+2] = 0, & k > n+2 \end{cases}$$

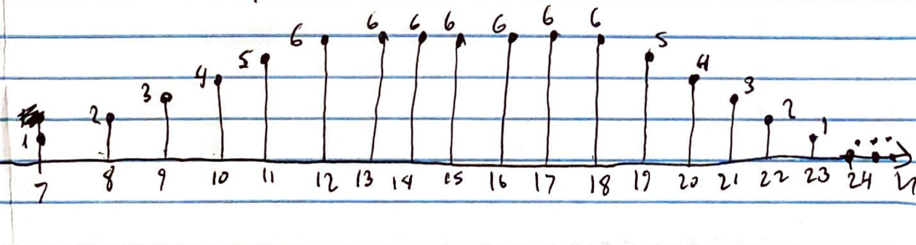
$$\Rightarrow y[n] = \sum_{k=2}^{n+2} \left(\tfrac{1}{2}\right)^{k-2} = 1 - \left(\tfrac{1}{2}\right)^{n+1}$$

$$\boxed{y[n] = 2\left[1 - \left(\tfrac{1}{2}\right)^{n+1}\right] u[n]}$$



2.4. $y[n] = x[n] * h[n]$, where $\left\{ x[n] = \begin{cases} 1, & 3 \le n \le 8 \\ 0, & \text{otherwise} \end{cases} \quad h[n] = \begin{cases} 1, & 4 \le n \le 15 \\ 0, & \text{otherwise} \end{cases} \right.$

$$\Rightarrow y[n] = \begin{cases} n-6 & \text{for } 7 \le n \le 11 \\ 6 & \text{for } 12 \le n \le 18 \\ 24-n & \text{for } 19 \le n \le 23 \\ 0, & \text{otherwise} \end{cases}$$



$x[n] = \begin{cases} 1, & 0 \le n \le 9 \\ 0, & \text{elsewhere} \end{cases}$ & $h[n] = \begin{cases} 1, & 0 \le n \le N \\ 0, & \text{elsewhere} \end{cases}$

where $N \le 9$, given $y[n] = x[n] * h[n]$ & $y[4]=5$, $y[14]=0$.

2.5 ~~$y[n] = x[n] * h[n]$~~

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] \Rightarrow y[n = \sum_{k=0}^{9} x[k]h[n-k]$$

$$\Rightarrow y[n] = \sum_{k=0}^{9} h[n-k] \xrightarrow{n=14} y[14] = \left\{ \begin{array}{l} h[14] + h[13] \\ + h[12] + h[11] \\ + \dots + h[5] \end{array} \right\}$$

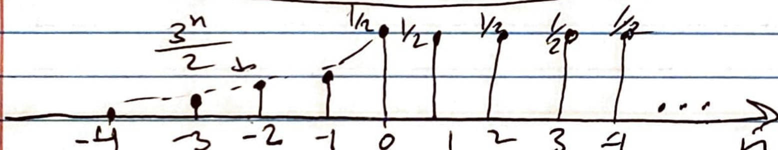
$$h[n] = \left\{ \begin{array}{ll} 1, & 0 \le n \le N \\ 0, & \text{elsewhere} \end{array} \right\}, N \le 9$$

$$\Rightarrow y[4] = 5, \rightarrow \boxed{N = 4}$$

2.6 $y[n] = x[n] * h[n]$, where $x[n] = \left(\frac{1}{3}\right)^{-n} u[-n-1]$ & $h[n] = u[n-1]$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{l=0}^{\infty} \left(\frac{1}{3}\right)^{l+3} u[n+l] \text{ where } l = k-1$$

$$\Rightarrow \boxed{y[n] = \frac{1}{2} \text{ for } n \ge 0 \ \& \ y[n] = \frac{3^n}{2} \text{ for } n < 0}$$



2.7 $y[n] = \sum_{k=-\infty}^{\infty} x[k]g[n-2k]$, where $g[n] = u[n] - u[n-4]$

$$\Rightarrow g[n-2k] = u[n-2k] - u[n-2k-4]$$

a.) $x[n] = \delta[n-1]$; $y[n] = \sum_{k=-\infty}^{\infty} x[k]g[n-2k]$

$$= \sum_{k=-\infty}^{\infty} \delta[k-1]\{u[n-2k] - u[n-2k-4]\}; \ \delta[k-1] \rightarrow k=1$$

$$\Rightarrow y[n] = u[n-2] - u[n-2-4] \Rightarrow \boxed{y[n] = u[n-2] - u[n-6]}$$

b.) $x[n] = \delta[n-2]$: $y[n] = \sum_{k=-\infty}^{\infty} x[k]g[n-2k]$

$$= \sum_{k=-\infty}^{\infty} \delta[k-2]\{u[n-2k] - u[n-2k-4]\}; \ k=2$$

$$\Rightarrow y[n] = u[n-2(2)] - u[n-2(2)-4] = \boxed{u[n-4] - u[n-8]}$$

c.) system S is not LTI because the input shifts & ouput shifts are not the same.

d.) $x[n] = u[n]$: $y[n] = \sum_{k=-\infty}^{\infty} x[k]\, g[n-2k]$

$= \sum_{k-\infty}^{\infty} u[k] \{ u[n-2k] - u[n-2k-4] \}$: $k = 0,1,2$

$$\Rightarrow \boxed{y[n] = \begin{cases} 1 & n = 0,1 \\ 2 & n > 1 \\ 0 & \text{elswhere} \end{cases}}$$