CSE 5350

Assignment #3

Daniel Delgado Acosta
00689598
4-10-23

Problem 1

a.) $ax = b \Rightarrow \begin{bmatrix} 0.16 & 0.10 \\ 0.17 & 0.11 \\ 2.02 & 1.29 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \cong \begin{bmatrix} 0.26 \\ 0.28 \\ 3.31 \end{bmatrix}$

$$A^TA = \begin{bmatrix} 0.16 & 0.17 & 2.02 \\ 0.1 & 0.11 & 1.29 \end{bmatrix} \begin{bmatrix} 0.16 & 0.10 \\ 0.17 & 0.11 \\ 2.02 & 1.29 \end{bmatrix} = \begin{bmatrix} 4.1349 & 2.6405 \\ 2.6405 & 1.6862 \end{bmatrix}$$

$$A^Tb = \begin{bmatrix} 0.16 & 0.17 & 2.02 \\ 0.1 & 0.11 & 1.29 \end{bmatrix} \begin{bmatrix} 0.26 \\ 0.28 \\ 3.31 \end{bmatrix} = \begin{bmatrix} 6.7754 \\ 4.3267 \end{bmatrix}$$

Augmented matrix: $\begin{bmatrix} 4.1349 & 2.6405 & 6.7754 \\ 2.6405 & 1.6862 & 4.3267 \end{bmatrix} \xrightarrow{\text{RREF}} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \boxed{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}$$

b.) $b = \begin{bmatrix} 0.27 \\ 0.25 \\ 3.33 \end{bmatrix} \rightarrow ax = b \Rightarrow \begin{bmatrix} 0.16 & 0.10 \\ 0.17 & 0.11 \\ 2.02 & 1.29 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \cong \begin{bmatrix} 0.27 \\ 0.25 \\ 3.33 \end{bmatrix}$

$$A^Tb = \begin{bmatrix} 0.16 & 0.17 & 2.02 \\ 0.1 & 0.11 & 1.29 \end{bmatrix} \begin{bmatrix} 0.27 \\ 0.25 \\ 3.33 \end{bmatrix} = \begin{bmatrix} 6.8123 \\ 4.3502 \end{bmatrix}, \quad A^TA = \begin{bmatrix} 4.1340 & 2.6405 \\ 2.6405 & 1.6862 \end{bmatrix}$$

Augmented matrix: $\begin{bmatrix} 4.1349 & 2.6405 & 6.8123 \\ 2.6405 & 1.6862 & 9.3502 \end{bmatrix} \xrightarrow{\text{RREF}} \begin{bmatrix} 1 & 0 & 7.0089 \\ 0 & 1 & -8.3957 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \cong \boxed{\begin{bmatrix} 7.0089 \\ -8.3957 \end{bmatrix}}$$

c.) Acknowledging the small difference in vector b, Part a and Part b actually have very different solutions. It appears that vector b highly affects $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$.

$$\begin{bmatrix} 0.26 \\ 0.28 \\ 3.31 \end{bmatrix} - \begin{bmatrix} -0.01 \\ 0.03 \\ -0.02 \end{bmatrix} = \begin{bmatrix} 0.27 \\ 0.25 \\ 3.33 \end{bmatrix}$$