CSE 3350

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Assignment 8

(3.5) $x_1(t)$ w/ ω_1 & a_k, $x_2(t) = x_1(1-t) + x_1(t-1)$

Since $x_1(1-t)$ & $x_1(t-1)$ is periodic w/ ω_1 and the signal $x_2(t)$ is $x_1(1-t) + x_1(t-1)$ has the same ω as $x_1(t)$.

$\Rightarrow \boxed{\omega_1 = \omega_2}$

Using table: $x_1(1-t) = x_1(-t-(-1))$

$\rightarrow a'_k = a_{-k} e^{-jk\omega_1}$ coef. $x_1(1-t)$

$\rightarrow a''_k = a_k e^{-jk\omega_1}$ coef. $x_1(t-1)$

coef. $x_2(t)$: $b_k = a'_k + a''_k = \boxed{e^{-jk\omega_1}(a_{-k} + a_k)}$

(3.6) a.) $x_1(t) = \sum_{k=0}^{100} \left(\frac{1}{2}\right)^k e^{jk\frac{2\pi}{50}t}$, $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$

Coef.: $a_k = \begin{cases} \left(\frac{1}{2}\right)^k & 0 < k < 100 \\ 0 & \text{otherwise} \end{cases}$, $a_k = a_{-k}$

check: $a_k = \left(\frac{1}{2}\right)^k$ & $a_{-k} = \left(\frac{1}{2}\right)^{-k}$ therefore $a_k \neq a_{-k}$ and $\boxed{x_1(t) \text{ is not a real value}}$

$x_2(t) = \sum_{k=-100}^{100} \cos(k\pi) e^{jk\frac{2\pi}{50}t} \rightarrow a_k = \begin{cases} \cos(k\pi) & -100 \le k \le 100 \\ 0 & \text{otherwise} \end{cases}$

check: $a_k = \cos(k\pi)$ & $a_{-k} = \cos(-k\pi) = \cos(k\pi)$

therefore $a_k = a_{-k}$ and $\boxed{x_2(t) \text{ is a real value}}$

$x_3(t) = \sum_{k=-100}^{100} j\sin\left(\frac{k\pi}{2}\right) e^{jk\frac{2\pi}{50}t} \rightarrow a_k = \begin{cases} j\sin\left(\frac{k\pi}{2}\right) & -100 \le k \le 100 \\ 0 & \text{otherwise} \end{cases}$

check: $a_k = j\sin\left(\frac{k\pi}{2}\right)$ & $a_{-k} = -j\sin\left(\frac{-k\pi}{2}\right) = j\sin\left(\frac{k\pi}{2}\right)$

therefore $a_k = a_{-k}$ and $\boxed{x_3(t) \text{ is a real value}}$

b.) If $x_1(t)$ is even then a_k is even. Since $a_k \neq a_{-k}$

$\rightarrow \boxed{x_1(t) \text{ is not even}}$

Since coef. for $x_2(t)$ are even $\rightarrow \boxed{x_2(t) \text{ is even}}$

since coef. for $x_3(t)$ are not even $\rightarrow \boxed{x_3(t) \text{ is not even}}$

$$g(t) = \frac{dx(t)}{dt} \quad \& \quad x(t) \xleftrightarrow{FS} a_k$$

3.7 Given $\int_T^{2T} x(t)\,dt = 2$, find a_k in terms of b_k & T.

$\rightarrow g(t) \xleftrightarrow{FS} b_k = jk\frac{2\pi}{T} a_k$

$\rightarrow a_k = b_k / j\left(\frac{2\pi}{T}\right)k, \quad k \neq 0$

when $k=0 \rightarrow a_k = \frac{1}{T}\int_T x(t)\,dt = \frac{2}{T}$

$$\Rightarrow \boxed{a_k = \begin{cases} \frac{2}{T} & k=0 \\ b_k / j\left(\frac{2\pi}{T}\right)k & k \neq 0 \end{cases}}$$

3.8 Parsevals relation: $\frac{1}{T}\int_T |x(t)|^2 dt = \sum_{k=-\infty}^{\infty} |a_k|^2, \quad T=2$

$\Rightarrow \frac{1}{2}\int_0^2 |x(t)|^2 dt = 1 \Rightarrow \sum_{k=-\infty}^{\infty} |a_k|^2 = 1$

where $a_k = 0$ for $|k|>1 \rightarrow |a_k| = |a_{-k}|$

$\rightarrow |a_{-1}|^2 + |a_1|^2 = 1 \Rightarrow |a_1| = \frac{1}{\sqrt{2}}$

$\rightarrow a_1 = -a_{-1} = \pm\frac{1}{j\sqrt{2}}, \quad x(t) = \sum_{k=-1}^{k=1} a_k e^{jk\left(\frac{2\pi}{T}\right)t}$

$\Rightarrow x(t) = a_{-1}e^{-j\left(\frac{2\pi}{2}\right)t} + \cancel{a_0} + a_1 e^{j\left(\frac{2\pi}{2}\right)t}$

$\Rightarrow x_1(t) = -\frac{1}{j\sqrt{2}}e^{-j\left(\frac{2\pi}{2}\right)t} + \frac{1}{j\sqrt{2}}e^{j\left(\frac{2\pi}{2}\right)t} = \frac{2}{\sqrt{2}}\left(\frac{e^{j\pi t} - e^{-j\pi t}}{2j}\right)$

$\Rightarrow x_1(t) = \sqrt{2}\sin(\pi t), \quad \sin(\pi t) = \frac{e^{j\pi t} - e^{-j\pi t}}{2j}$

$\rightarrow x_2(t) = \frac{1}{j\sqrt{2}}e^{-j\left(\frac{2\pi}{2}\right)t} - \frac{1}{j\sqrt{2}}e^{j\left(\frac{2\pi}{2}\right)t} = \frac{-1}{j\sqrt{2}}\left(e^{j\pi t} - e^{-j\pi t}\right)$

$\Rightarrow x_2(t) = -\sqrt{2}\sin(\pi t), \quad \sin(\pi t) = \frac{e^{j\pi t} - e^{-j\pi t}}{2j}$

$\rightarrow \boxed{x_1(t) = \sqrt{2}\sin(\pi t)} \quad \& \quad \boxed{x_2(t) = -\sqrt{2}\sin(\pi t)}$