CSE3350

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Assignment 9

(3.2) $N=5$; $x[n]$: $a_0=1$, $a_2=a_{-2}=e^{j\pi/4}$, $a_4=a_{-4}=2e^{j\pi/3}$

$\omega=\frac{2\pi}{N}=\frac{2\pi}{5}$, $a_2=a_{-2}=e^{j\pi/4}=\frac{1+j}{\sqrt{2}}$,

$a_4=a_{-4}=2^{e^{j\pi/3}}=1+j\sqrt{3}$

$\rightarrow x[n]=\sum_{k=N} a_k e^{jk\omega n}=\sum_{k=-4}^{4} a_n e^{jk\omega n}$

$=a_{-4}e^{-j4\omega n}+a_{-2}e^{-j2\omega n}+a_0+a_2e^{j2\omega n}+a_4e^{j4\omega n}$

$=1+\left(e^{j\frac{8\pi}{5}n}+e^{-j\frac{8\pi}{5}n}\right)+\sqrt{3}j\left(e^{j\frac{8\pi}{5}n}-e^{-j\frac{8\pi}{5}n}\right)+\frac{1}{\sqrt{2}}\left(e^{j\frac{4\pi}{5}n}+e^{-j\frac{4\pi}{5}n}\right)+\frac{j}{\sqrt{2}}\left(e^{j\frac{4\pi}{5}n}-e^{-j4\pi/5n}\right)$

$\Rightarrow \boxed{x[n]=1+2\sin\left(\frac{4\pi}{5}n+\frac{3\pi}{4}\right)+4\sin\left(\frac{8\pi}{5}n+\frac{5\pi}{6}\right)}$

(3.9) $x[n]=\sum_{m=-\infty}^{\infty}\{4\delta[n-4m]+8\delta[n-1-4m]\}$

$a_k=\frac{1}{N}\sum_{n=0}^{N-1} x[n]e^{-jk\omega n}$, $N=4$, $\omega=\frac{2\pi}{N}$

$\rightarrow a_k=\frac{1}{4}\sum_{n=0}^{3} x[n]e^{-jk\frac{2\pi}{4}n}=1+2e^{-jk\frac{\pi}{2}}$

$a_0=1+2=3$, $a_1=1+2e^{-j\pi/2}=1-2j$, $a_2=1+2e^{-j\pi}=1-2=-1$,

$a_3=1+2e^{-j3\pi/2}=1+2j$

$\Rightarrow \boxed{a_0=3,\ a_1=1-2j,\ a_2=-1,\ a_3=1+2j}$

(3.10) $N=7$; $x[n]$: $a_{15}=j$, $a_{16}=2j$, $a_{17}=3j$

$$x[n]=\sum_{k=-N}^{N} a_k e^{jk\left(\frac{2\pi}{N}\right)n}, \quad a_k=\frac{1}{N}\sum_{n=-N}^{N} x[n]e^{-jk\left(\frac{2\pi}{N}\right)n}$$

$a_{15}, a_{16}, a_{17} = a_1; a_2, a_3$

$a_0=0$, $\rightarrow a_{-1}=-a_1=-j$; $a_{-2}=-a_2=-2j$, $a_{-3}=-a_3=-3j$

$\rightarrow \boxed{a_{-1}=-j,\ a_{-2}=-2j,\ a_{-3}=-3j}$

(3.11) $N=10$; $x[n]$: real & even, $a_{11}=5$, $\frac{1}{10}\sum_{n=0}^{9}|x[n]|^2=50$

$a_{11}=a_1=5$; $a_k=a_{-k} \rightarrow a_{-11}=a_{-1}=5$

$$\rightarrow \frac{1}{N}\sum_{n=<N>}|x[n]|^2=\sum_{k=<N>}|a_k|^2 \Rightarrow \frac{1}{10}\sum_{n=0}^{9}|x[n]|^2=50$$

$$\Rightarrow \sum_{k=-1}^{8}|a_k|^2=50 \Rightarrow 50+|a_0|^2+\sum_{k=2}^{8}|a_k|^2=50$$

$$\Rightarrow |a_0|^2+\sum_{k=8}^{8}|a_k|^2=0 \rightarrow a_k=0,\ k=2,\dots,8$$

$$x[n]=\sum_{k=<10>}a_k e^{jk\frac{\pi}{5}n}=a_{-1}e^{-j\frac{\pi}{5}n}+a_1 e^{j\frac{\pi}{5}n}=10\cos\left(\frac{\pi}{5}n\right)$$

$x[n]=A\cos(Bn+C) \rightarrow \boxed{A=10,\ B=\frac{\pi}{5},\ C=0}$

(3.12) $N=4$; $x_1[n] \leftrightarrow a_k$, $x_2[n] \leftrightarrow b_k$; $a_0=a_3=\frac{1}{2}a_1=1$ & $b_0=b_1=b_2=b_3=1$

$$g[n]=x_1[n]x_2[n] \xleftrightarrow{FS} \sum_{l=<N>}^{N-1} a_l b_{k-l}=\sum_{l=0}^{3} a_l b_{k-l}$$

$$\rightarrow C_k=\sum_{l=0}^{3} a_l b_{k-l};\ k=0 \Rightarrow C_k=6;\ k=1 \Rightarrow C_k=6;\ k=2 \Rightarrow C_k=6;$$
$k=3 \Rightarrow C_k=6$; $k=4 \Rightarrow C_k=6$; $k=5 \Rightarrow C_k=6$; $k=6 \Rightarrow C_k=6$;
$k=7 \Rightarrow C_k=6$; $k=8 \Rightarrow C_k=6$.

$\rightarrow \boxed{C_k=\{6, \text{ for all } k}$