

ICA 7

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Question 1

Define these matrices in R (using whichever technique you prefer)

```
# define matrices
A <- matrix(c(5,-1,5,-1,10,1,5,1,-3), nrow = 3, ncol = 3)
B <- matrix(c(5,1,3,1,-2,0), nrow = 2, ncol = 3)
C <- matrix(c(2,1,-3,4), nrow = 2, ncol = 2)
x <- matrix(c(5,1,4), nrow = 3, ncol = 1)
```

Question 2

Compute CB

```
CB <- C %*% B
CB
```

```
##      [,1] [,2] [,3]
## [1,]    7    3   -4
## [2,]    9    7   -2
```

Question 3

Compute Ax

```
Ax <- A %*% x
Ax
```

```
##      [,1]
## [1,]   44
## [2,]    9
## [3,]   14
```

Question 4

Find $\det(C)$

```
det(C)
```

```
## [1] 11
```

Question 5

Calculate BtB and BBt . What kind of matrices are these results?

```
Bt <- t(B)
BtB <- Bt %*% B
BtB
```

```
##      [,1] [,2] [,3]
## [1,]   26  16 -10
## [2,]   16  10  -6
## [3,]  -10  -6   4
```

```
BBt <- B %*% Bt
BBt
```

```
##      [,1] [,2]
## [1,]   38   8
## [2,]    8   2
```

Question 6

Compute xtx . Note that this is the square of the norm of x .

```
xt <- t(x)
xtx <- xt %*% x
xtx
```

```
##      [,1]
## [1,]   42
```

Question 7

Solve the following for the unknown y : $Ay = x$.

```
y <- solve(A,x)
y
```

```
##      [,1]
## [1,] 0.8665049
## [2,] 0.1699029
## [3,] 0.1674757
```

Question 8

Find the eigenvalues and eigenvectors of A. Note that the eigenvectors are perpendicular (this can be checked by seeing that the inner product is zero).

```
eigen(A)$values
```

```
## [1] 10.192741 7.325270 -5.518011
```

```
eigen(A)$vectors
```

```
##           [,1]      [,2]      [,3]
## [1,] -0.185056453 0.8815093 -0.43439095
## [2,] 0.982718248 0.1640362 -0.08577273
## [3,] 0.004353605 0.4427567 0.89663122
```

Question 9

Find the eigenvalues and eigenvectors of A2. How do the eigenvalues compare to that of A?

```
A2 <- A %*% A
eigen(A2)$values
```

```
## [1] 103.89197 53.65958 30.44845
```

```
eigen(A2)$vectors
```

```
##           [,1]      [,2]      [,3]
## [1,] 0.185056453 0.8815093 -0.43439095
## [2,] -0.982718248 0.1640362 -0.08577273
## [3,] -0.004353605 0.4427567 0.89663122
```

Question 10

Take a vector $z = (1, -3, 4)$. Re-write z using the eigenvectors of A as the new basis.

```
z <- c(1,-3,4)
```

```
solve(eigen(A)$vectors, z) * z
```

```
## [1] -3.115797 -6.481282 13.637808
```