

STT 481 HW 4

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Libraries

```
library(MASS)
library(ISLR)
library(gam)

## Loading required package: splines

## Loading required package: foreach

## Loaded gam 1.22-2

library(glmnet)

## Loading required package: Matrix

## Loaded glmnet 4.1-7

library(boot)
library(leaps)
library(splines)
```

Question 1

This question uses the variables dis (the weighted mean of distances to five Boston employment centers) and nox (nitrogen oxides concentration in parts per 10 million) from the Boston data (load the data from MASS package). We will treat dis as the predictor and nox as the response.

- Use the poly() function to fit a cubic polynomial regression to predict nox using dis. Plot the resulting data and polynomial fits

```
bos.poly <- lm(nox ~ poly(dis,3), data = Boston)
summary(bos.poly)
```

```

## 
## Call:
## lm(formula = nox ~ poly(dis, 3), data = Boston)
## 
## Residuals:
##       Min     1Q Median     3Q    Max 
## -0.121130 -0.040619 -0.009738  0.023385  0.194904 
## 
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)    
## (Intercept) 0.554695  0.002759 201.021 < 2e-16 ***
## poly(dis, 3)1 -2.003096  0.062071 -32.271 < 2e-16 ***
## poly(dis, 3)2  0.856330  0.062071  13.796 < 2e-16 ***
## poly(dis, 3)3 -0.318049  0.062071  -5.124 4.27e-07 ***
## --- 
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## 
## Residual standard error: 0.06207 on 502 degrees of freedom
## Multiple R-squared:  0.7148, Adjusted R-squared:  0.7131 
## F-statistic: 419.3 on 3 and 502 DF,  p-value: < 2.2e-16

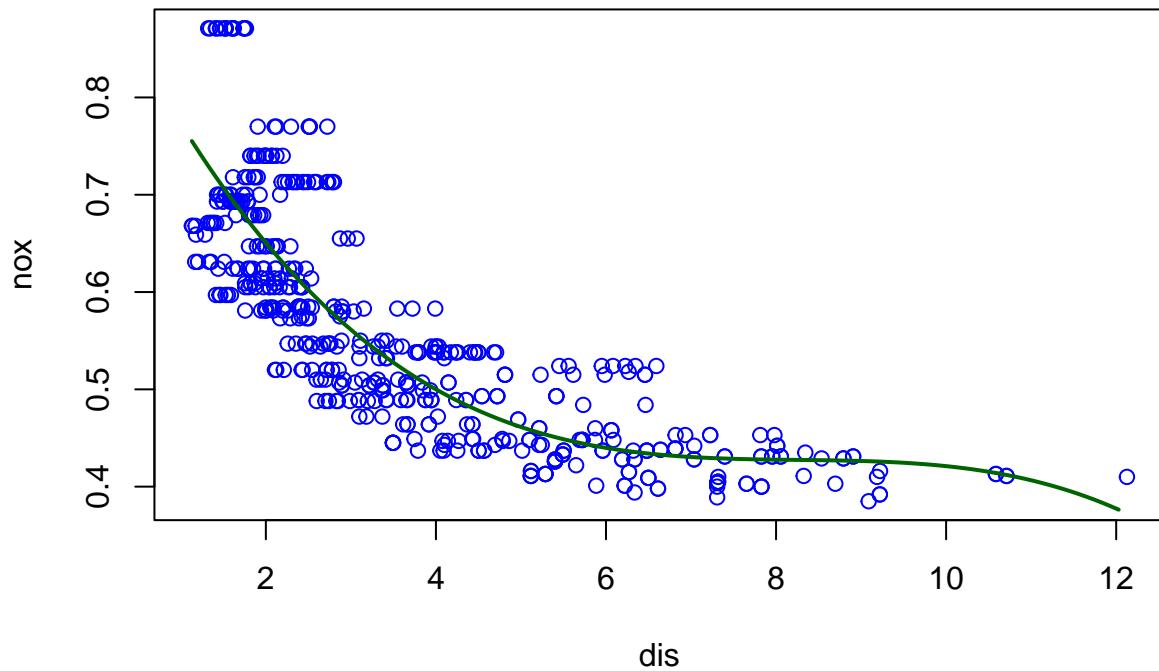
```

```

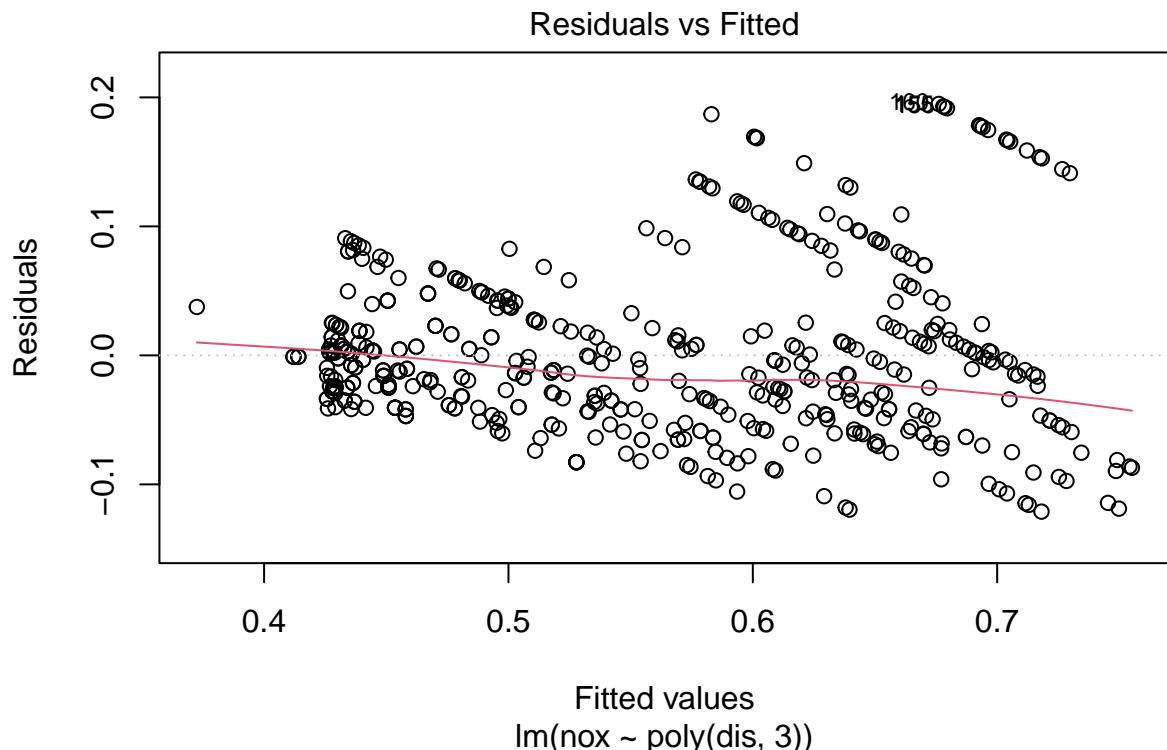
# using outline from Rlab

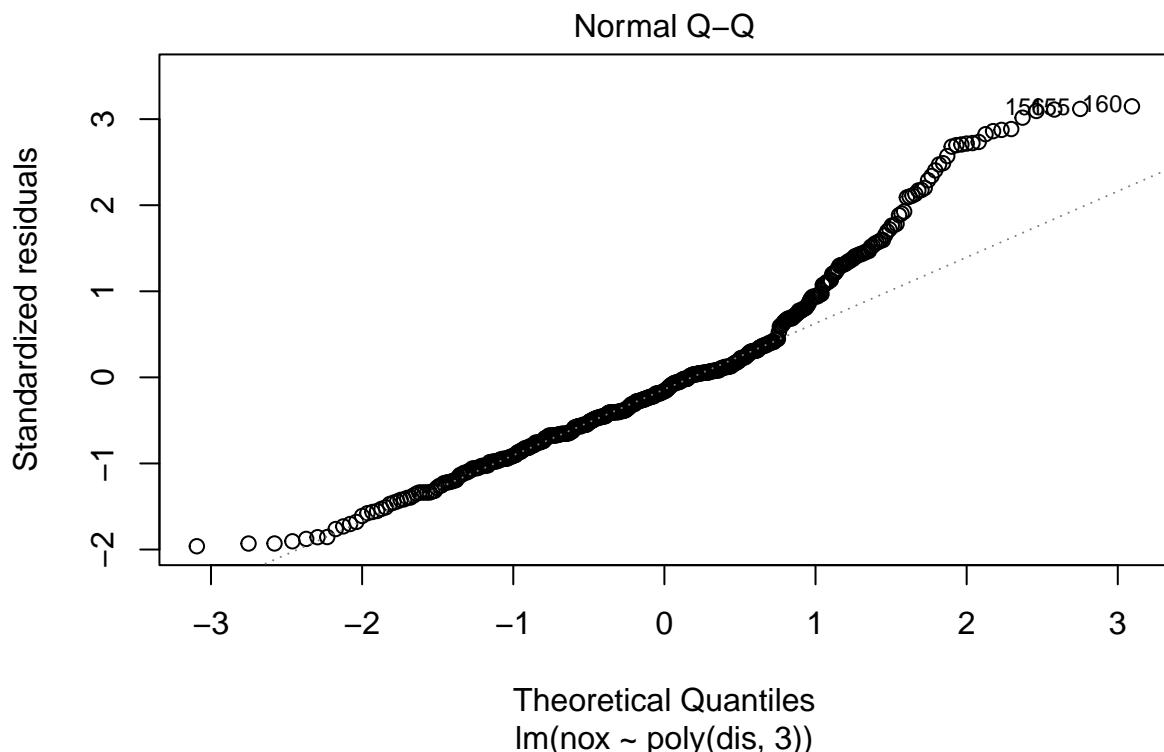
dislims <- range(Boston$dis)
dis.grid <- seq(from = dislims[1], to = dislims[2], by = 0.1)
preds <- predict(bos.poly, list(dis = dis.grid))
plot(nox ~ dis, data = Boston, col = "blue")
lines(dis.grid, preds, col = "darkgreen", lwd = 2)

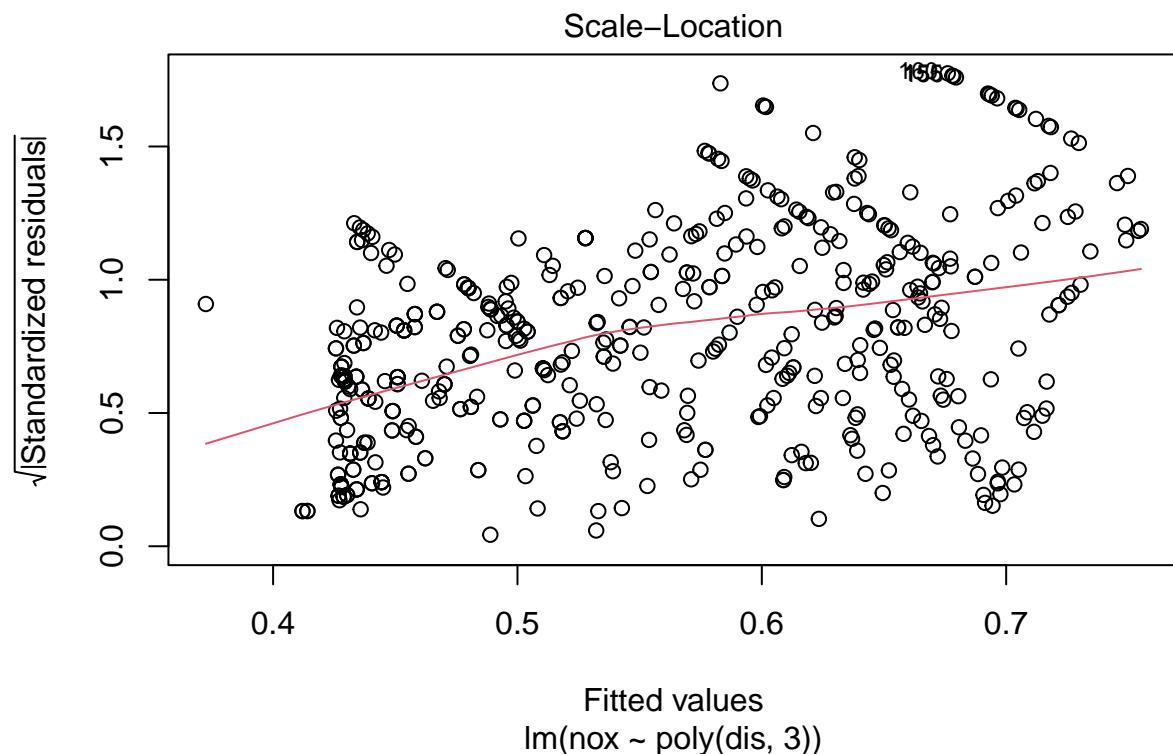
```

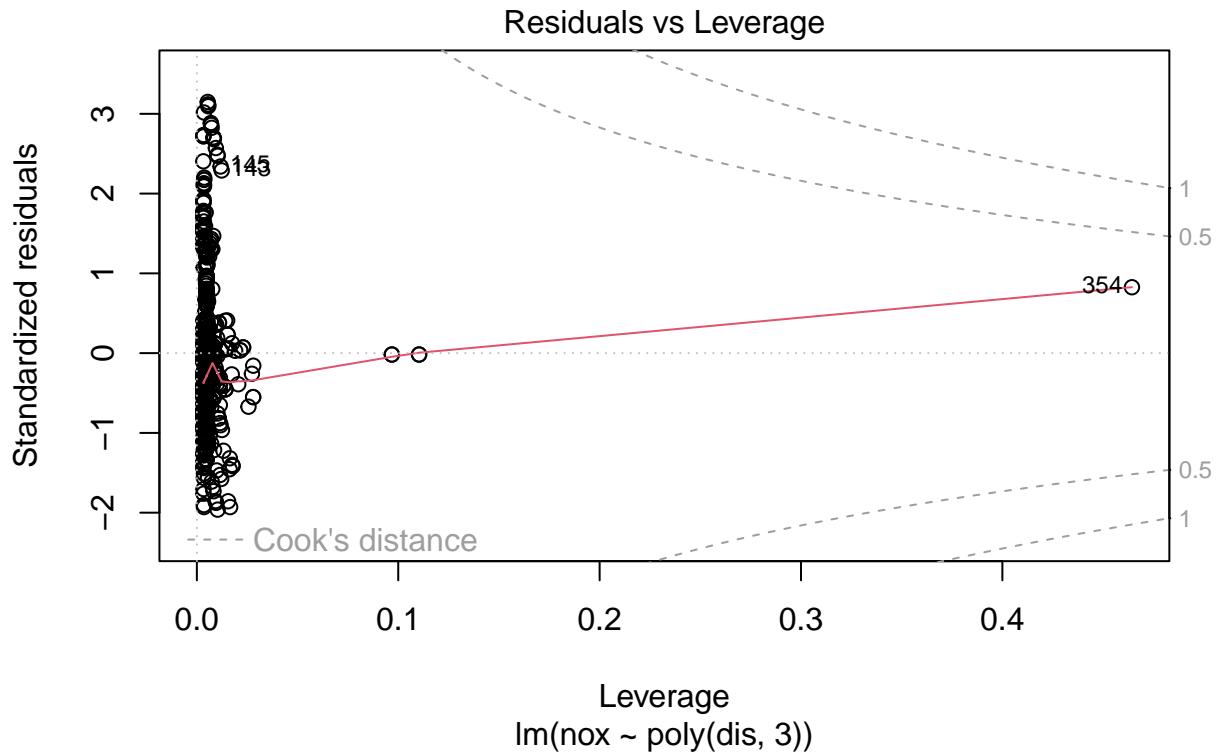


```
plot(bos.poly) # residuals
```







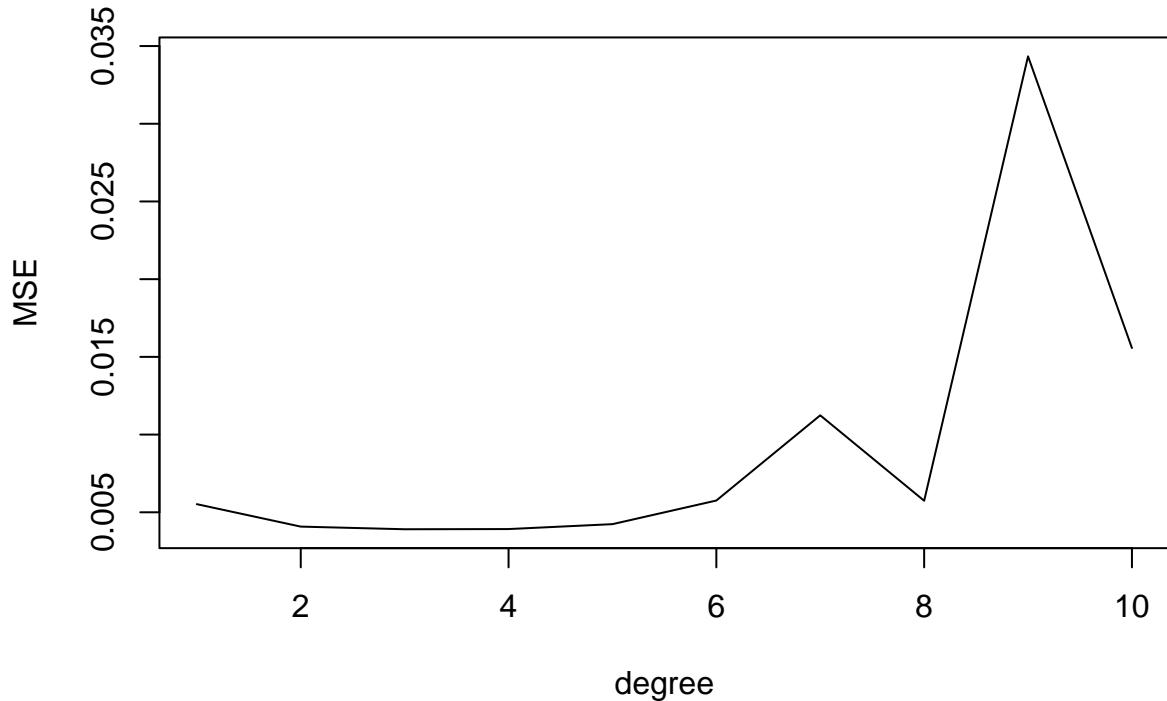


- b) Perform cross-validation to select the optimal degree for the polynomial. Plot the resulting polynomial fits with the optimal degree.

```

errors <- rep(0, 10)
for (i in 1:10) {
  bos.cv <- glm(nox ~ poly(dis, i), data = Boston)
  errors[i] <- cv.glm(Boston, bos.cv, K = 10)$delta[1]
}
plot(1:10, errors, xlab = "degree", ylab = "MSE", type = "l")

```



```
which.min(errors)
```

```
## [1] 3
```

Using cross-validation, it seems like 3 is the best degree for the model.

- c) Use the `bs()` function to fit a cubic spline with six degrees of freedom to predict nox using dis. Report the knot locations. Plot the resulting fit.

```
dis <- Boston$dis
nox <- Boston$nox
bs.fit <- lm(data = Boston, nox ~ bs(dis, 6))
summary(bs.fit)
```

```
##
## Call:
## lm(formula = nox ~ bs(dis, 6), data = Boston)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.128538 -0.037813 -0.009987  0.022644  0.195494
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
```

```

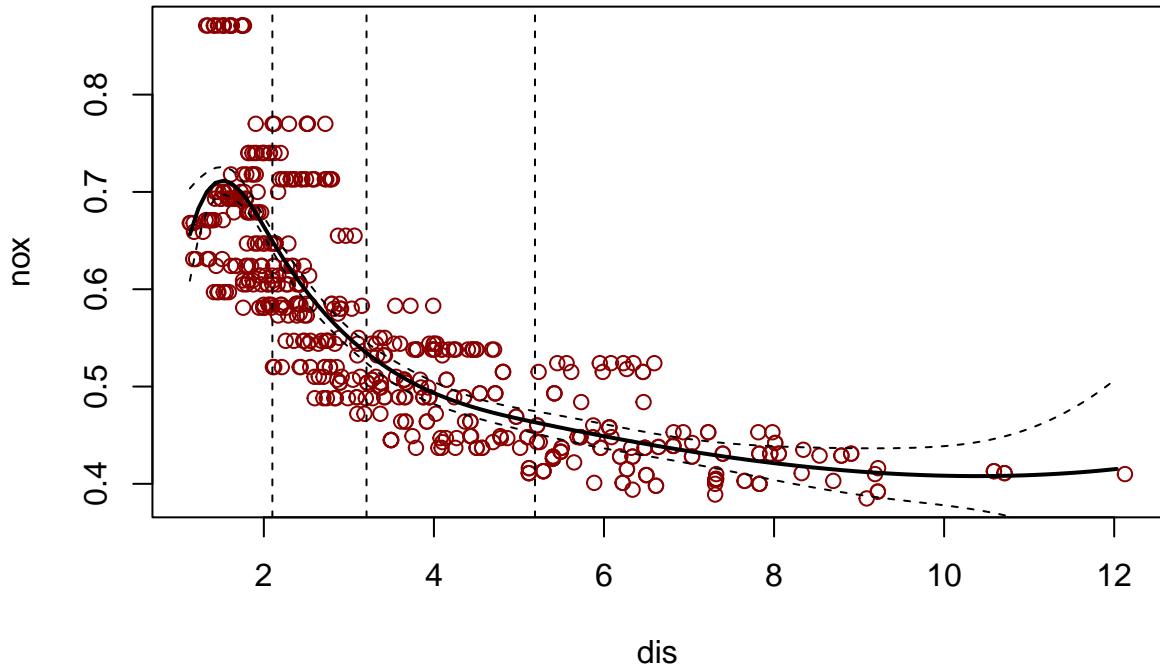
## (Intercept) 0.65622    0.02370 27.689 < 2e-16 ***
## bs(dis, 6)1 0.10222    0.03516  2.907  0.00381 **
## bs(dis, 6)2 -0.02963    0.02338 -1.267  0.20571
## bs(dis, 6)3 -0.15959    0.02791 -5.718 1.86e-08 ***
## bs(dis, 6)4 -0.22815    0.03324 -6.864 1.99e-11 ***
## bs(dis, 6)5 -0.26272    0.04930 -5.329 1.50e-07 ***
## bs(dis, 6)6 -0.24002    0.05434 -4.417 1.23e-05 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.06062 on 499 degrees of freedom
## Multiple R-squared: 0.7295, Adjusted R-squared: 0.7263
## F-statistic: 224.3 on 6 and 499 DF, p-value: < 2.2e-16

```

```

pred <- predict(bs.fit, newdata = list(dis = dis.grid), se = T)
plot(dis, nox, col = "darkred")
lines(dis.grid, pred$fit, lwd = 2)
lines(dis.grid, pred$fit + 2*pred$se, lty = "dashed")
lines(dis.grid, pred$fit - 2*pred$se, lty = "dashed")
abline(v = attr(bs(dis, df = 6), "knots"), lty = 2) # knots placement

```

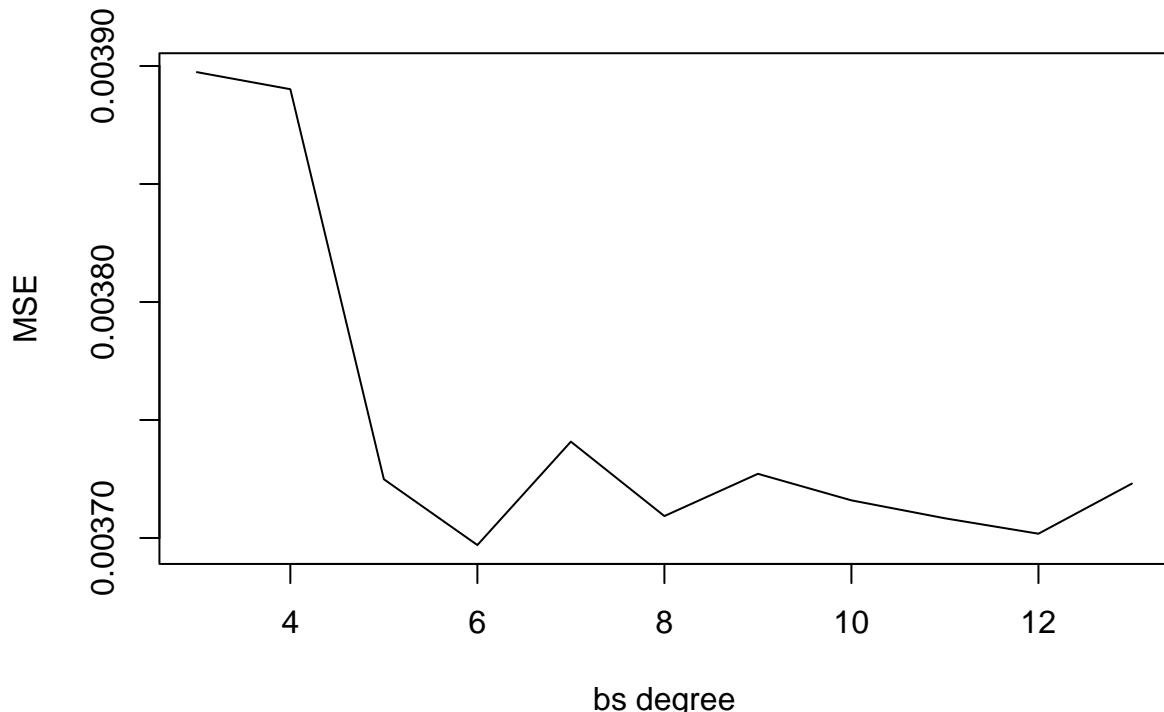


- d) Perform cross-validation in order to select the best degrees of freedom for a cubic spline on this data.
Plot the resulting cubic spline fits with the best degrees of freedom.

```

set.seed(1)
errors_2 <- rep(3, 13)
for (i in 3:13) {
  bs.cv <- glm(nox ~ bs(dis, df = i), data = Boston)
  errors_2[i] <- cv.glm(Boston, bs.cv, K = 10)$delta[1]
}
plot(3:13, errors_2[-c(1,2)], xlab = "bs degree", ylab = "MSE", type = "l")

```



```
which.min(errors_2)
```

```
## [1] 6
```

Using cross-validation, it looks like 6 is the best degree of freedom for the cubic spline model.

- e) Use the ns() function to fit a natural cubic spline with six degrees of freedom to predict nox using dis. Report the knot locations. Plot the resulting fit.

```

ns.fit <- lm(data = Boston, nox ~ ns(dis, 6))
summary(ns.fit)

```

```

##
## Call:
## lm(formula = nox ~ ns(dis, 6), data = Boston)

```

```

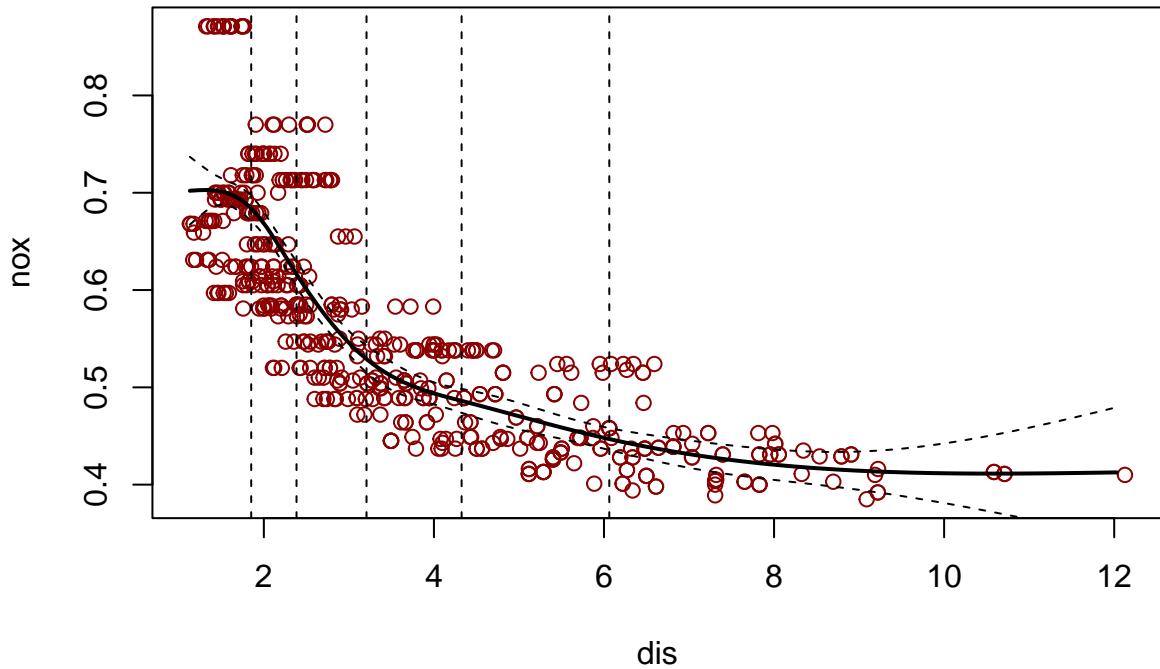
## 
## Residuals:
##       Min      1Q   Median      3Q     Max
## -0.135007 -0.039356 -0.006573  0.024535  0.196108
## 
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)    
## (Intercept)  0.70205   0.01744  40.263 < 2e-16 ***
## ns(dis, 6)1 -0.10413   0.01807 -5.764 1.44e-08 ***
## ns(dis, 6)2 -0.19401   0.02354 -8.242 1.50e-15 ***
## ns(dis, 6)3 -0.21955   0.02053 -10.694 < 2e-16 ***
## ns(dis, 6)4 -0.29707   0.02070 -14.352 < 2e-16 ***
## ns(dis, 6)5 -0.28848   0.04286 -6.731 4.63e-11 ***
## ns(dis, 6)6 -0.29134   0.03528 -8.258 1.33e-15 ***
## ---    
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## 
## Residual standard error: 0.06096 on 499 degrees of freedom
## Multiple R-squared:  0.7266, Adjusted R-squared:  0.7233 
## F-statistic:  221 on 6 and 499 DF,  p-value: < 2.2e-16

```

```

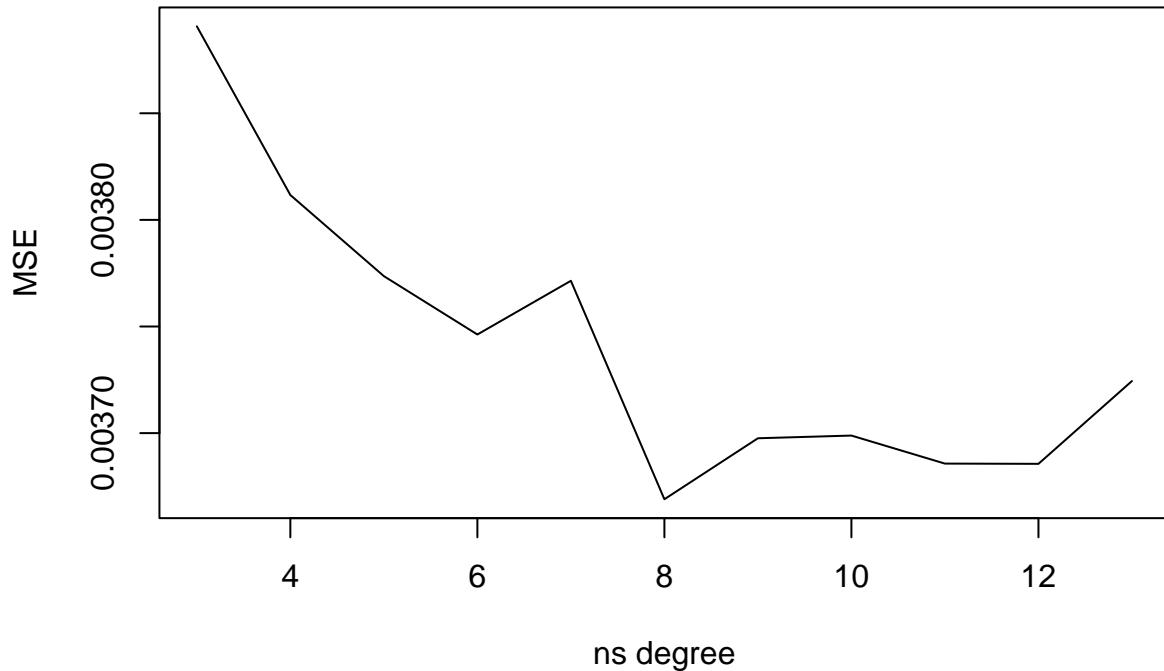
pred <- predict(ns.fit, newdata = list(dis = dis.grid), se = T)
plot(dis, nox, col = "darkred")
lines(dis.grid, pred$fit, lwd = 2)
lines(dis.grid, pred$fit + 2*pred$se, lty = "dashed")
lines(dis.grid, pred$fit - 2*pred$se, lty = "dashed")
abline(v = attr(ns(dis, df = 6), "knots"), lty = 2) # knots placement

```



- f) Perform cross-validation in order to select the best degrees of freedom for a natural cubic spline on this data. Plot the resulting natural cubic spline fits with the best degrees of freedom.

```
set.seed(1)
errors_3 <- rep(3, 13)
for (i in 3:13) {
  ns.cv <- glm(nox ~ ns(dis, df = i), data = Boston)
  errors_3[i] <- cv.glm(Boston, ns.cv, K = 10)$delta[1]
}
plot(3:13, errors_3[-c(1,2)], xlab = "ns degree", ylab = "MSE", type = "l")
```



```
which.min(errors_3)
```

```
## [1] 8
```

Using cross-validation, it looks like the best degree of freedom is 8 for this model.

Question 2

```
data("College")
set.seed(123)
train <- sample(nrow(College), 600)
College.train <- College[train,]
College.test <- College[-train,]
```

This question relates to the College data set.

- Using out-of-state tuition as the response and the other variables as the predictors, perform forward stepwise selection on the training set in order to identify a satisfactory model that uses just a subset of the predictors.

```

fwd.college <- regsubsets(Outstate ~ ., data = College.train, method = 'forward', nvmax = ncol(College))
fwd.summary <- summary(fwd.college)

which.min(fwd.summary$bic) # lowest point on bic

## [1] 10

which.min(fwd.summary$cp) # lowest point on cp

## [1] 12

which.min(fwd.summary$adjr2) # lowest point on adj r^2

## [1] 1

par(mfrow=c(2, 2))

plot(fwd.summary$bic, type='b', xlab='# of Variables', ylab='BIC') # Plot BIC
axis(1, at = seq(1,17,by=1))
points(10, fwd.summary$bic[10], col = "darkgreen", cex = 2, pch = 20)

plot(fwd.summary$cp, type='b', xlab='# of Variables', ylab='CP') # Plot CP
axis(1, at = seq(1,17,by=1))
points(12, fwd.summary$cp[12], col = "blue", cex = 2, pch = 20)

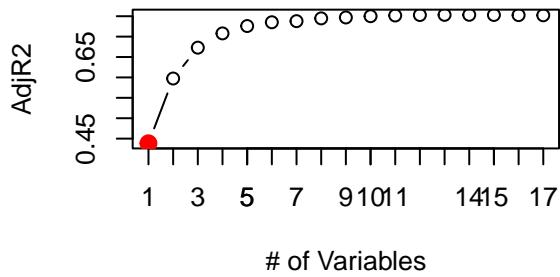
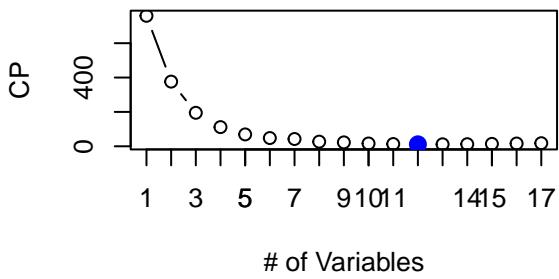
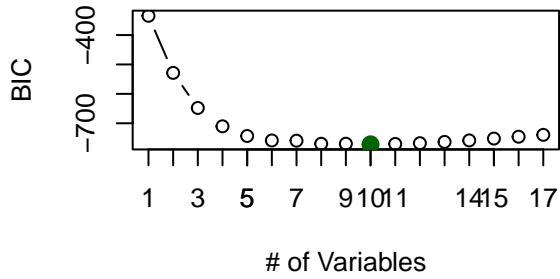
plot(fwd.summary$adjr2, type='b', xlab='# of Variables', ylab='AdjR^2') # Plot AdjR^2
axis(1, at = seq(1,17,by=1))
points(1, fwd.summary$adjr2[1], col = "red", cex = 2, pch = 20)

# Find the 10 variables used for the GAM model

coefs <- coef(fwd.college, id = 10)
names(coefs)

## [1] "(Intercept)" "PrivateYes"   "Apps"          "Accept"        "Enroll"
## [6] "Top10perc"    "Room.Board"   "PhD"           "perc.alumni"   "Expend"
## [11] "Grad.Rate"

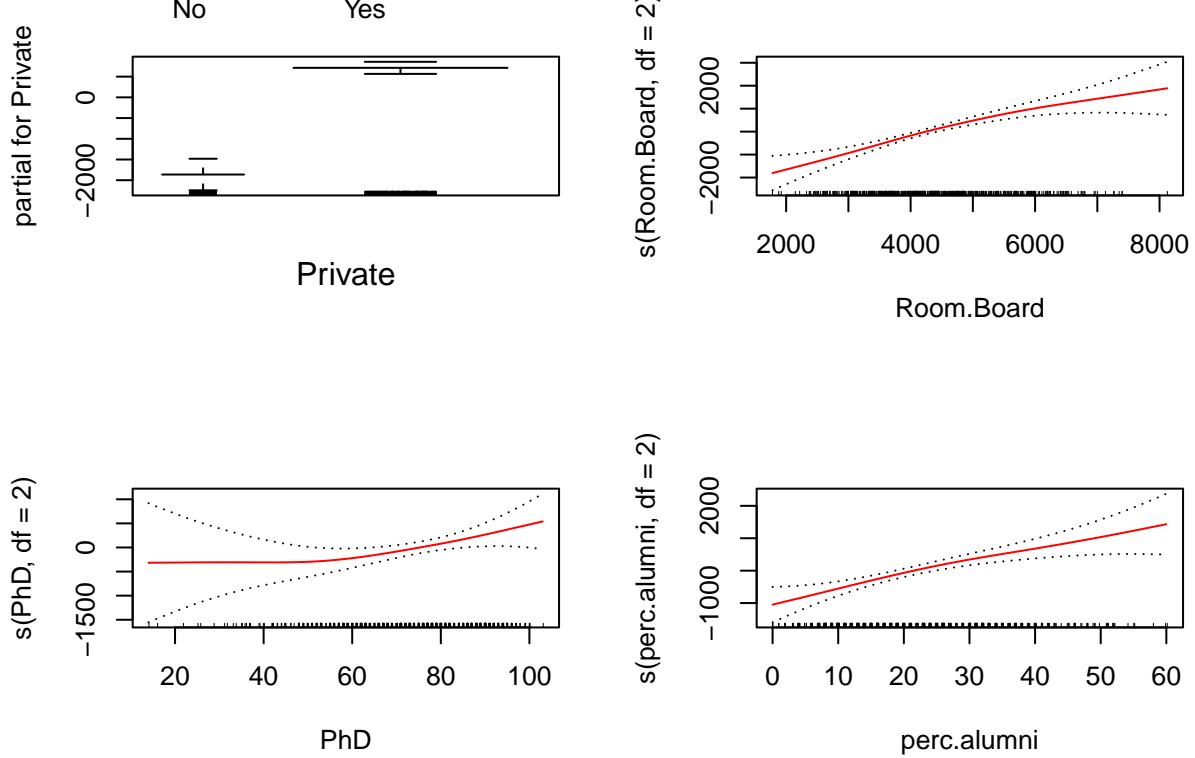
```

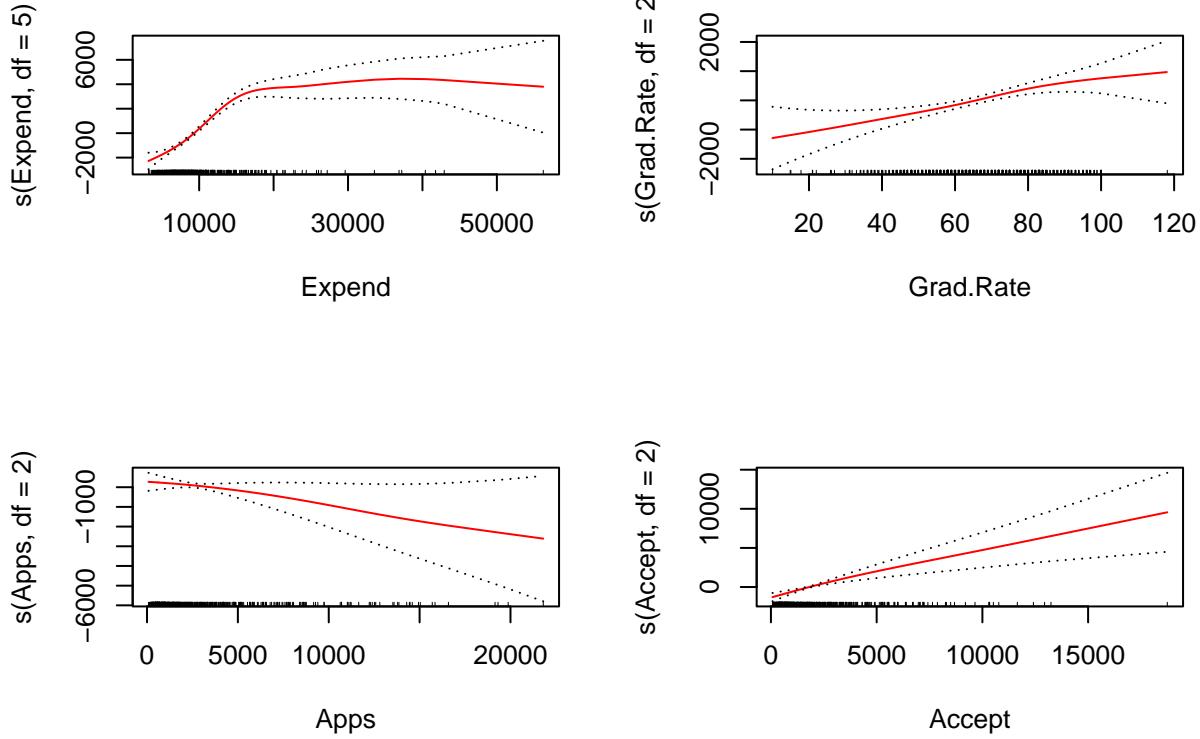


According to BIC, the number of variables best fit for the model is 10, while for cp the number of variables is 12, and for Adjusted R² the number of variables is only 1. In this case, I believe I am going with BIC is the best. It is not too many variables, while at the same time not excluding too many variables. The selected predictors are: "Private", "Apps", "Accept", "Enroll", "Top10perc", "Room.Board", "PhD", "perc.alumni", "Expend", "Grad.Rate"

- b) Fit a GAM on the training data, using out-of-state tuition as the response and the features selected in the previous step as the predictors.

```
fit.gam <- gam(Outstate ~ Private + s(Room.Board, df = 2) + s(PhD, df = 2) + s(perc.alumni, df = 2) + s(Top10perc, df = 2)
# I use s() to indicate I want a smoothing spline
par(mfrow = c(2, 2))
plot(fit.gam, se = T, col = "red")
```





```
summary(fit.gam)
```

```
##
## Call: gam(formula = Outstate ~ Private + s(Room.Board, df = 2) + s(PhD,
##          df = 2) + s(perc.alumni, df = 2) + s(Expend, df = 5) + s(Grad.Rate,
##          df = 2) + s(Apps, df = 2) + s(Accept, df = 2) + s(Enroll,
##          df = 2) + s(Top10perc, df = 2), data = College.train)
## Deviance Residuals:
##      Min       1Q     Median       3Q      Max
## -6804.14 -1072.08    83.49   1236.38  7831.31
##
## (Dispersion Parameter for gaussian family taken to be 3455825)
##
## Null Deviance: 9694792870 on 599 degrees of freedom
## Residual Deviance: 1994010747 on 576.9999 degrees of freedom
## AIC: 10760.62
##
## Number of Local Scoring Iterations: NA
##
## Anova for Parametric Effects
##             Df     Sum Sq   Mean Sq F value    Pr(>F)
## Private           1 2732131662 2732131662 790.5874 < 2.2e-16 ***
## s(Room.Board, df = 2) 1 1796653320 1796653320 519.8913 < 2.2e-16 ***
## s(PhD, df = 2)     1 624690191 624690191 180.7644 < 2.2e-16 ***
## s(perc.alumni, df = 2) 1 390893264 390893264 113.1114 < 2.2e-16 ***
```

```

## s(Expend, df = 5)      1  735768872  735768872 212.9069 < 2.2e-16 ***
## s(Grad.Rate, df = 2)   1  95393137   95393137 27.6036 2.098e-07 ***
## s(Apps, df = 2)        1   8586354   8586354  2.4846 0.1155134
## s(Accept, df = 2)      1  21644616  21644616  6.2632 0.0126024 *
## s(Enroll, df = 2)      1  45696898  45696898 13.2232 0.0003012 ***
## s(Top10perc, df = 2)   1   9236598   9236598  2.6728 0.1026241
## Residuals             577 1994010747   3455825
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Anova for Nonparametric Effects
##                               Npar Df  Npar F  Pr(F)
## (Intercept)
## Private
## s(Room.Board, df = 2)      1   2.8824 0.0901 .
## s(PhD, df = 2)              1   1.8082 0.1792
## s(perc.alumni, df = 2)     1   1.0829 0.2985
## s(Expend, df = 5)           4  22.8533 <2e-16 ***
## s(Grad.Rate, df = 2)       1   2.2233 0.1365
## s(Apps, df = 2)             1   2.5250 0.1126
## s(Accept, df = 2)           1   5.5050 0.0193 *
## s(Enroll, df = 2)           1   0.6237 0.4300
## s(Top10perc, df = 2)       1   0.6516 0.4199
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

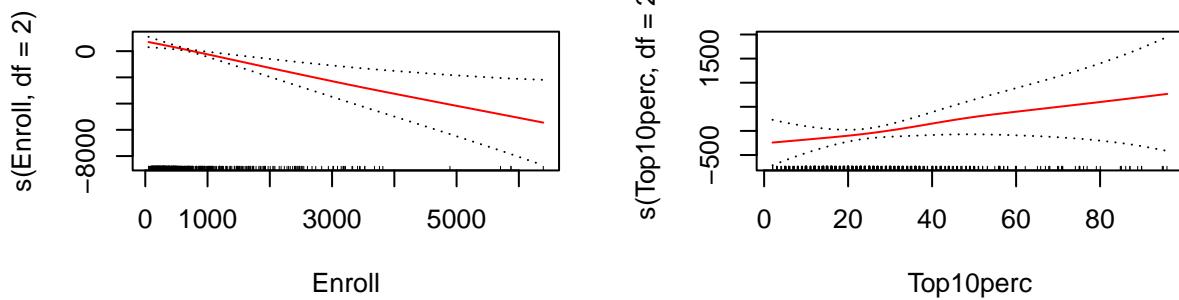
fit.gam$aic

```

```

## [1] 10760.62

```



With this model, the produced aic is 10760.62. In order to lower the AIC score, I could attempt to change around the degrees of freedom for some variables. Either decreasing some, increasing some, or removing some all together. I can do this by looking at what variables are and aren't significant.

- c) Evaluate the model obtain on the test set, and explain the results obtained.

```
test.pred <- predict(fit.gam, College.test)
errors_4 <- mean((College.test$Outstate - test.pred)^2)
errors_4

## [1] 2997372

rsq <- 1 - errors_4/mean(((College.test$Outstate - mean(College.test$Outstate)))^2)
rsq

## [1] 0.8143664
```

With 10 predictors, the rsquared for the test set is 0.8033426 using GAM. This means that roughly 80% of the variance within the data can be explained within the model of 10 predictors.

Question 3

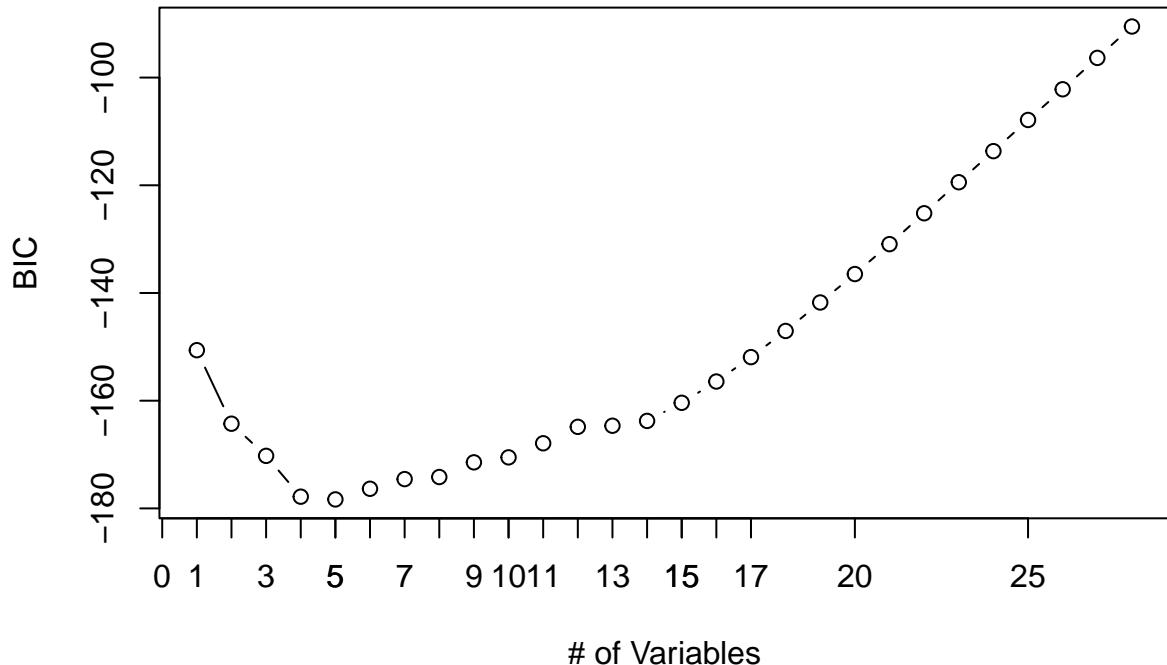
In this question, we wish to predict the 2019-20 salaries of five NBA players, who were free agents in 2019 summer, on the basis of various statistics associated with performance in the 2018-19 season.

- a) Run the following command lines to load the two data sets. Notice that here **row.names = 1** because by doing so, we can see those players' names.

```
train.dat <- read.csv("nbadata17.csv", row.names = 1)
test.dat <- read.csv("nbadata18.csv", row.names = 1)
```

- b) Using salary as the response and the other variables as the predicts, perform forward stepwise selection on the training set, and use BIC as the selection criterion. Report the non-zero coefficient estimates.

```
fwd.nba <- regsubsets(salary ~ ., data = train.dat, method = 'forward', nvmax = ncol(train.dat))
fwd.nbasum <- summary(fwd.nba)
plot(fwd.nbasum$bic, type='b', xlab='# of Variables', ylab='BIC') # Plot BIC
axis(1, at = seq(1,17,by=1))
```



```
which.min(fwd.nbasum$bic)
```

```
## [1] 5
```

```
coef(fwd.nba, id = 5)
```

	(Intercept)	age	g	gs	x2ppercent	pts
##	-10003.77541	360.81836	-69.17732	79.48794	8599.43163	644.44076

I use 5 predictors in the models moving forward, according to BIC. The coefficients are: “age”, “g”, “gs”, “x2ppercen”, and “pts”.

c) Predict the salaries of the five players in the test.dat using the fitted model in (b)

```

predict.regsubsets <- function (object, newdata , id, ...){
  form <- as.formula(object$call[[2]]) # formula of null model
  mat <- model.matrix(form, newdata) # building an "X" matrix from newdata
  coefi <- coef(object, id = id)      # coefficient estimates associated with the object model contain
  xvars <- names(coefi)              # names of the non-zero coefficient estimates
  return(mat[,xvars] %*% coefi)     # X[,non-zero variables] %*% Coefficients[non-zero variables]
}
fwd.pred <- predict.regsubsets(fwd.nba, newdata = test.dat, id = 5)
fwd.pred

##          [,1]
## Khris Middleton 16496.228
## Terrence Ross    8028.138
## Ish Smith       6595.110
## Myles Turner     11748.581
## Thaddeus Young   14625.969

```

d) Fit a ridge regression model on the training set, and predict the salaries in the test set.

```

x.fit <- model.matrix(salary ~., data = train.dat)[,-1]
y.fit <- model.matrix(salary ~., data = test.dat)[,-1] # the model matrix for test to make predictions
trs <- train.dat$salary

set.seed(1)
ridge.fit <- glmnet(x.fit, trs, alpha = 0)
names(ridge.fit)

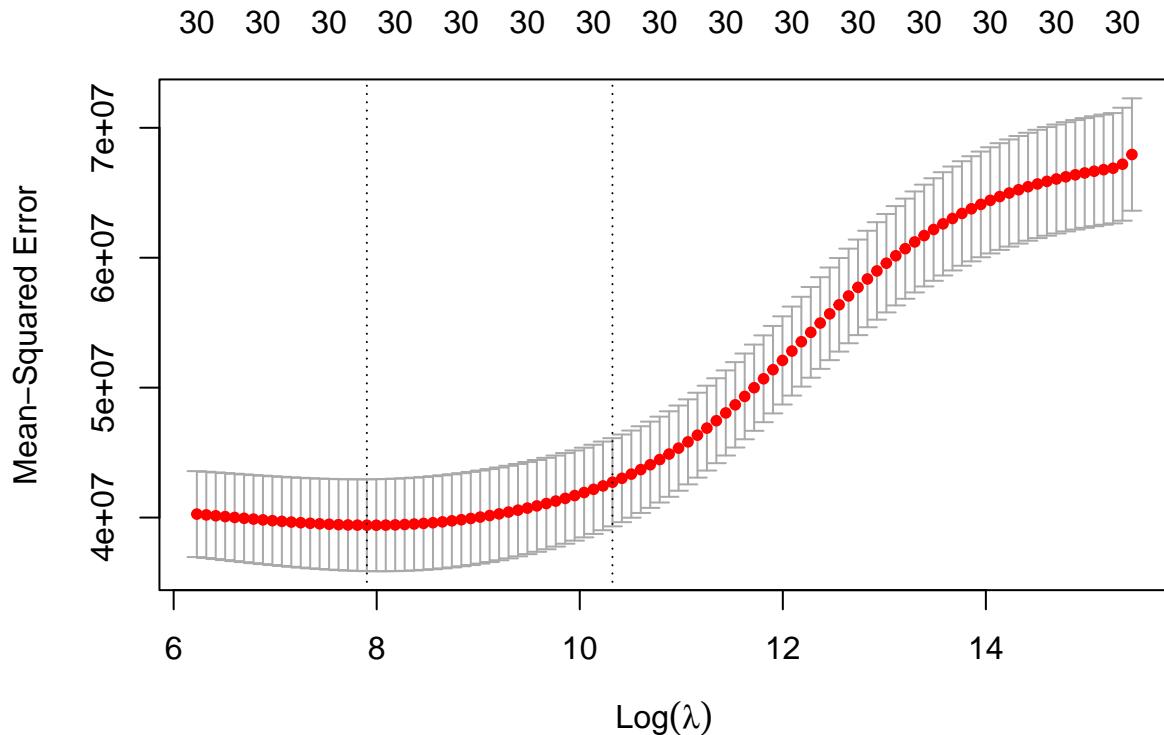
## [1] "a0"        "beta"       "df"         "dim"        "lambda"     "dev.ratio"
## [7] "nulldev"   "npasses"    "jerr"       "offset"     "call"       "nobs"

cv.ridge <- cv.glmnet(x.fit, trs, alpha = 0, nfolds = 10)
bestridge_lambda <- cv.ridge$lambda.min # best tuning parameter
bestridge_lambda

## [1] 2705.659

plot(cv.ridge)

```



```
# Predict
```

```
ridge.pred <- predict(ridge.fit, s = bestridge_lambda, newx = y.fit)
head(ridge.pred)
```

```
##                               s1
## Khris Middleton 16234.727
## Terrence Ross     9405.972
## Ish Smith        6576.191
## Myles Turner     12234.336
## Thaddeus Young   12683.456
```

```
# Coefficients
```

```
coef(ridge.fit, s = bestridge_lambda)
```

```
## 31 x 1 sparse Matrix of class "dgCMatrix"
##                               s1
## (Intercept) -5357.349462
## posPF       -35.771352
## posPG      -525.723930
## posSF       331.935355
## posSG      -433.625881
## age         265.637195
## g           -29.479467
```

```

## gs           39.150786
## mp          38.846573
## fg          192.520019
## fga         74.079614
## fgpercent   -1820.651544
## x3p          516.270084
## x3pa         312.439160
## x3ppercen   -1227.033143
## x2p          136.134341
## x2pa         3.452693
## x2ppercen   6498.212628
## efgpercent  390.182411
## ft           599.639512
## fta          414.380551
## ftpercent   -2973.031826
## orb          262.456020
## drb          186.074215
## trb          128.808577
## ast          446.806723
## stl          285.781085
## blk          696.780007
## tov          -327.040226
## pf           -786.604472
## pts          89.432425

```

Keep note that there are no zero coefficients in this model, this is important when comparing the predictions made by lasso.

- e) Fit a lasso regression model on the training set, and predict the salaries in the test set. Report the non-zero coefficient estimates.

```

set.seed(1)
lasso.fit <- glmnet(x.fit, trs, alpha = 1)
names(lasso.fit)

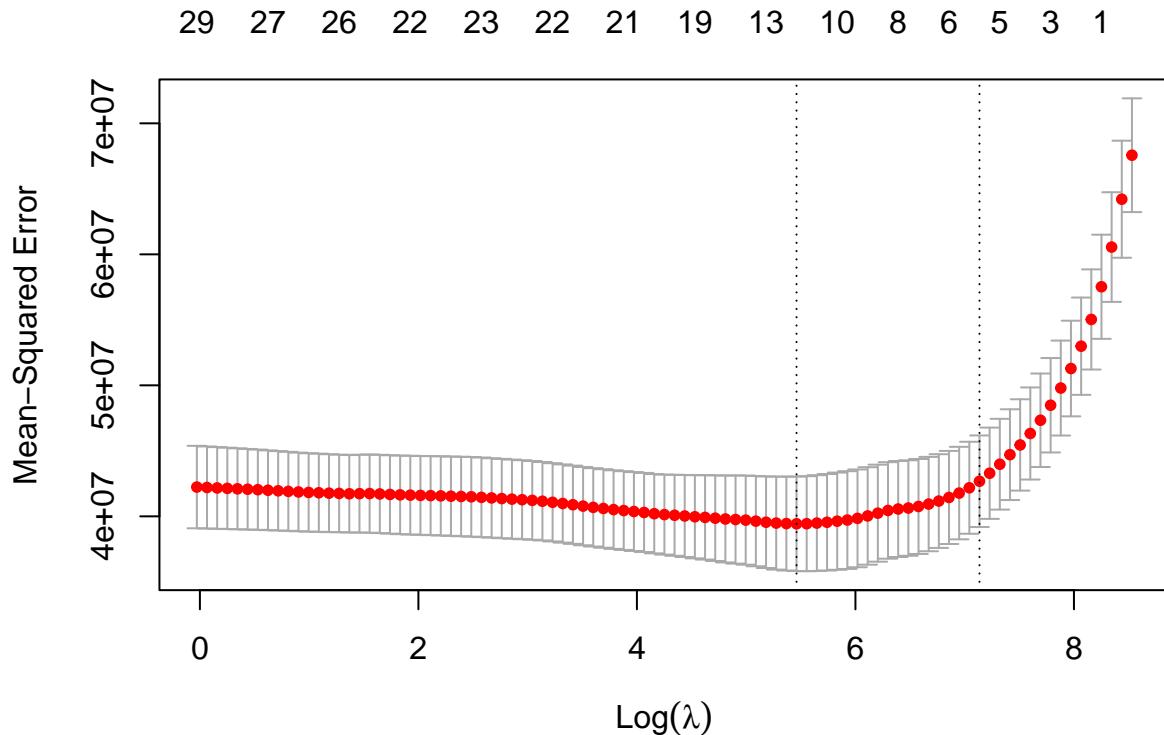
## [1] "a0"        "beta"       "df"         "dim"        "lambda"     "dev.ratio"
## [7] "nulldev"   "npasses"    "jerr"       "offset"     "call"       "nobs"

cv.lasso <- cv.glmnet(x.fit, trs, alpha = 1, nfolds = 10)
bestlasso_lambda <- cv.lasso$lambda.min # best tuning parameter
bestlasso_lambda

## [1] 235.3244

plot(cv.lasso)

```



```
# Predict
```

```
lasso.pred <- predict(lasso.fit, s = bestridge_lambda, newx = y.fit)
head(lasso.pred)
```

```
##                               s1
## Khris Middleton 11988.298
## Terrence Ross   10353.186
## Ish Smith      8051.272
## Myles Turner    10114.499
## Thaddeus Young  9895.244
```

```
# Lasso coefficients
```

```
coef(lasso.fit, s = bestlasso_lambda)
```

```
## 31 x 1 sparse Matrix of class "dgCMatrix"
##                               s1
## (Intercept) -6436.81525
## posPF       .
## posPG       .
## posSF       .
## posSG       .
## age         293.38081
## g           -31.12952
## gs          65.49790
```

```

## mp
## fg
## fga
## fgpercent
## x3p
## x3pa      398.96029
## x3ppercents
## x2p
## x2pa
## x2ppercents 7037.20394
## efgpercent
## ft      1356.26682
## fta
## ftpercent -3272.60106
## orb
## drb      146.58135
## trb      114.87792
## ast      321.99443
## stl
## blk      322.10480
## tov
## pf      -295.41155
## pts      156.00788

```

Lasso predictions are a lot lower than the ridge predictions. This could be because the way lasso works as a whole. Looking at the coefficients, it is obvious to tell that this is due to all the coefficients not lasso'd in. There are a lot less non-zero coefficients than in ridge.

- f) Fit a GAM model on the training set using the features selected in model (b) as the predictors, and plot the results and explain your findings, and then predict the salaries in the test set.

```

nba.gam <- gam(salary ~ age + s(g, df = 2) + s(gs, df = 2) + x2ppercents + s(pts, df = 5), data = train.dat)
# I use s() to indicate I want a smoothing spline
par(mfrow = c(2, 3))
plot(nba.gam, se = T, col = "red")
summary(nba.gam)

```

```

##
## Call: gam(formula = salary ~ age + s(g, df = 2) + s(gs, df = 2) + x2ppercents +
##           s(pts, df = 5), data = train.dat)
## Deviance Residuals:
##       Min      1Q      Median      3Q      Max
## -17235.1 -3237.3   -658.2   2911.9  21290.7
##
## (Dispersion Parameter for gaussian family taken to be 35810703)
##
## Null Deviance: 23146145548 on 340 degrees of freedom
## Residual Deviance: 11781715666 on 328.9998 degrees of freedom
## AIC: 6912.771
##
## Number of Local Scoring Iterations: NA
##
## Anova for Parametric Effects

```

```

##          Df     Sum Sq   Mean Sq   F value   Pr(>F)
## age           1 1.4671e+09 1467117652 40.9687 5.327e-10 ***
## s(g, df = 2)  1 1.0354e+09 1035385408 28.9127 1.439e-07 ***
## s(gs, df = 2) 1 6.0612e+09 6061185258 169.2562 < 2.2e-16 ***
## x2ppercents  1 2.4646e+08 246461171  6.8823  0.009111 **
## s(pts, df = 5) 1 2.1746e+09 2174625501 60.7256 8.685e-14 ***
## Residuals    329 1.1782e+10 35810703
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Anova for Nonparametric Effects
##          Npar Df Npar F   Pr(F)
## (Intercept)
## age
## s(g, df = 2)      1 0.2879 0.591950
## s(gs, df = 2)      1 0.6359 0.425770
## x2ppercents
## s(pts, df = 5)      4 3.8712 0.004347 **
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

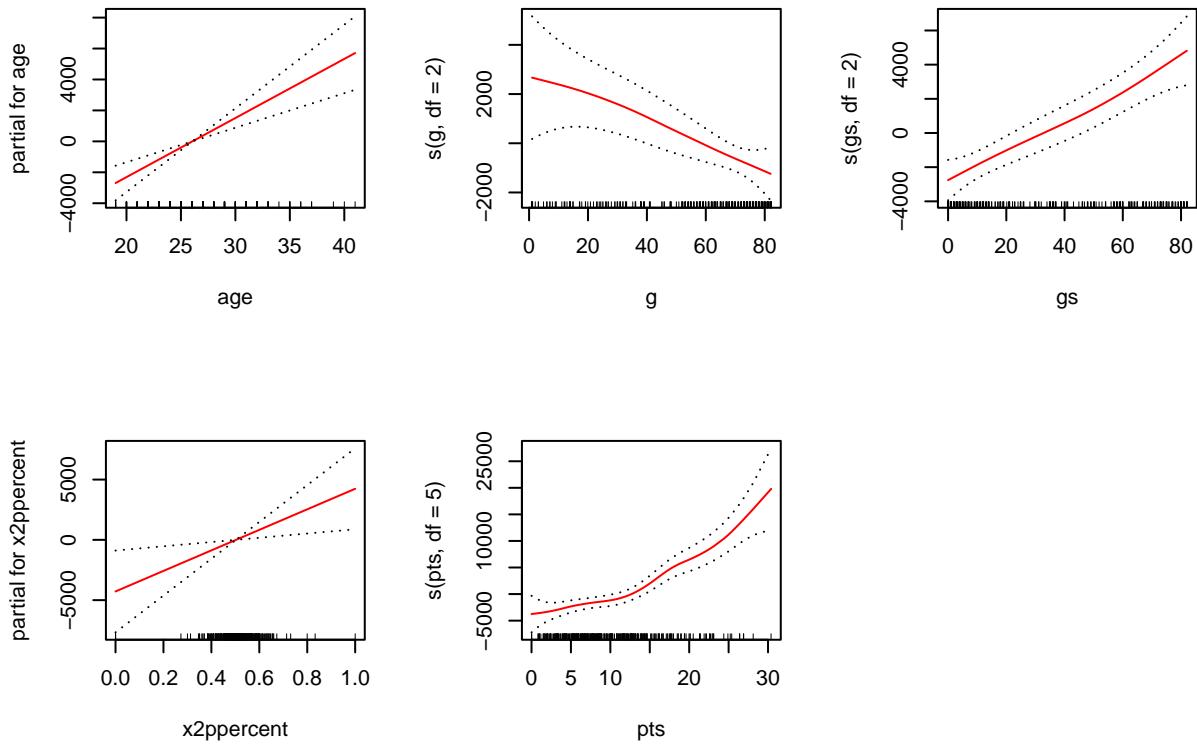
nba.gam$aic

## [1] 6912.771

test.predict <- predict(nba.gam, test.dat)
test.predict

## Khris Middleton   Terrence Ross       Ish Smith      Myles Turner Thaddeus Young
##        17487.701      7018.941      5746.448      10796.790     14151.335

```



Using the 5 predictors selected in the earlier fitted BIC model, I have produced a model with an AIC of 6912.771, which is a pretty good number for this. The predictions look fairly accurate too, this model seems to have produced the best predictions.